Velocity Dependence of the Bubble Density for Charged Particle Tracks in Liquid Hydrogen*

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(Received December 21, 1959; revised manuscript received March 31, 1960)

Bubble densities of tracks of 635-Mev/c protons and pions in a liquid hydrogen bubble chamber operated at 26.5°K, 62 psig have been determined from measurements of the distribution in spacing of the individual bubbles. The velocity dependence of the bubble density has been obtained by fitting the bubble densities observed to the expression $m=A/\beta^b$ by the least-squares method, yielding the values $A=8.64$ bubbles/cm, and exponent $b=1.935\pm0.077$. The constant A is a function of the temperature of the liquid hydrogen, varying ~30% per 0.1°K. If the number of bubbles per unit track length observed is correlated with the rate of delta-ray formation, it would appear that an energy of the order of 400 ev is necessary for bubble nucleation in liquid hydrogen.

I. INTRODUCTION

A KNOWLEDGE of the form of velocity dependence of the track density, the number of bubbles produced per unit length along the paths of charged particles, in liquid hydrogen is of interest not only from the point of view of providing information from which the velocity of charged particles can be determined, but also because of the light that can be shed on the fundamental mechanism of bubble nucleation in superheated liquids.

The earliest measurements[1] of the bubble densities of particle tracks in a propane bubble chamber by direct bubble counting resulted in overestimates of track density due to the effects of limited optical resolution and possible bubble coalescence. Subsequent measurements[2] with improved technique have shown the velocity dependence of the track density m in propane to be of the form $m=A/\beta^2$, where β is the relativistic velocity of the particle responsible for the track. Bugg[3] has used the track densities reported by these authors to show that if the bubble density is correlated with the rate of delta-ray formation, an estimate of the energy required for bubble nucleation in superheated propane is obtained which is in agreement with the value predicted by a "hole" theory of liquids.

In view of the widespread use of liquid hydrogen in bubble chamber experiments it is of interest to extend the determination of the velocity dependence of charged particle tracks to this material, and to compare these results with the theory of bubble nucleation presented by Bugg.

* Work performed under the auspices of the U. S. Atomic Energy Commission; supported in part by the National Science Foundation.

[1] D. A. Glaser, D. C. Rahm, and C. Dodd, Phys. Rev. **102**, 1653 (1956).

[2] W. J. Willis, E. C. Fowler, and D. C. Rahm, Phys. Rev. **108**, 1046 (1957); M. F. Blinov, Iu. S. Krestnikov, and G. A. Lomanov, Soviet Phys.—JETP 4, 661 (1957).

[3] D. V. Bugg, *Progress in Nuclear Physics* (Butterworths-Springer, London, 1959), Vol. 7, p. 1.

II. MEASUREMENT PROCEDURE

The tracks chosen for track-density measurement were produced by primary and secondary particles of a 635 Mev/c momentum-analyzed positive particle beam from the Brookhaven Cosmotron passing through a 6-in. hydrogen bubble chamber operated by the Brookhaven Cloud Chamber Group. The chamber was well controlled at a temperature of 26.5°K and a pressure of 62 psig; a 650 μsec time delay was introduced between the arrival of the beam, whose time spread was $<50\,\mu$sec, and the illumination of the chamber. No magnetic field was provided. The experiment in connection with which the pictures used were originally taken,[4] and the performance characteristics of the bubble chamber itself,[5] have been described elsewhere.

Tracks from these pictures were selected for measurement on the basis of the following criteria: (1) in every case the particle observed either initiated, or was produced in an elastic scattering interaction as demonstrated by agreement of track angles and ranges with the kinematic relationships for 635 Mev/c π-p or p-p elastic scattering; (2) the scattering event was induced by a particle consistent in direction and apparent density with the appropriate beam particles, travelling at least 20 mm but no more than 115 mm into the visible region of the chamber, and producing secondaries no less than 10 mm long unless such secondaries stopped in the chamber; and (3) the outgoing particles made an angle ϕ with respect to the plane of the beam no greater than ±30°. Some 91 π-p scatterings and 138 p-p scatterings satisfied these requirements and for each such interaction, density measurements were made on the incoming primary and both secondary particle tracks in each of two stereo views; in the case of the p-p scatterings a measurement was made as well of the density of the nearest minimum-density track for normalization purposes, as discussed below. The momenta of the secondary particles, determined by the measured scattering angles, were recorded.

[4] W. J. Willis, Phys. Rev. **116**, 753 (1959).

[5] H. Courant, J. E. Jensen, R. I. Louttit, and J. R. Sanford, Rev. Sci. Instr. **30**, 280 (1959).

Density measurements were made by examining each track under a microscope fitted with a ruled reticle and counting the distances x_i between the centers of adjacent bubbles. The least count of this microscope-reticle arrangement was $\sim 50\,\mu$, which was of the order of the diameter of the image of a single bubble on the film. The film-to-chamber magnification factor was 5; in the discussion which follows, all distances are referred to dimensions in the chamber.

Since very short gaps are apt to be missed due to overlap of bubble images, a cutoff length $x_0 = 625\,\mu$ was introduced such that gaps of length $> x_0$ would surely have been observed. If N is the number of such gaps, of total length $\sum x_i$, then the true track density as given by the maximum likelihood method has been shown[4] to be given by the expression

$$m = N/(\sum x_i - m x_0). \tag{1}$$

The mean value of m for each stereo pair was corrected for apparent foreshortening due to track dip and normalized in the ratio $(m_0)/\bar{m}_0$, where m_0 is the density of the minimum-density track nearest the event, in the case of p-p scatterings, or of the incident pion in π-p scatterings, and $\bar{m}_0$ is the mean of such densities.

The percentage average deviation of the measured values m_0 from the mean $\bar{m}_0$ was 13.9%. Since the total number of bubbles in the tracks of particles initiating π-p scatterings averaged 58, corresponding to a relative error $n^{\frac{1}{2}}/n = 13.1\%$, it appears that variation in the density of minimum-density tracks was largely statistical. The deviation of the mean of measurements in the two stereo views of each such individual track, which is an indication of the measurement accuracy and judgement of the observer, averaged 4.7%.

III. RESULTS

Density measurements were made on the tracks of 91 beam pions, 138 beam protons, 91 pions scattered from π-p collisions, and 367 protons from both π-p and p-p collisions. The mean value of the track density of the beam pions, for which $\beta = 0.976$, was $m_1 = 9.06 \pm 0.116$ bubbles/cm; for the beam protons, $\beta = 0.558$, the mean track density $m_2 = 26.72 \pm 0.290$ bubbles/cm. A least-squares fit of these values to an expression of the form $m = A/\beta^b$, where β is the relativistic velocity of the particle which produces the track, gives $A = 8.64$ bubbles/cm, and $b = 1.935 \pm 0.077$. The error given for the exponent b is the probable error. This velocity dependence is consistent with the A/β^2 form reported for propane by Willis *et al.* and by Blinov *et al.*[2]; in the latter study the experimental value of b was actually 2.07 ± 0.17.

The 91 pions scattered from π-p collisions were sufficiently relativistic that they showed essentially the same track density as did the beam pions; for these particles the mean relativistic velocity $\beta = 0.965$.

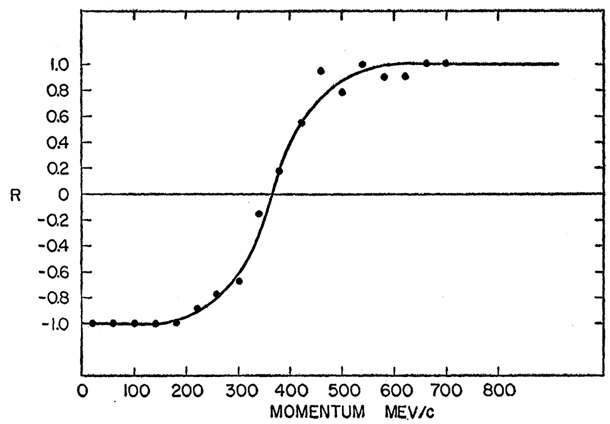

FIG. 1. Track measurability factor R = (measurable) − (unmeasurable) tracks/(measurable) + (unmeasurable) tracks as a function of particle momentum.

The 367 protons from both π-p and p-p collisions ranged in relativistic velocity from $\beta = 0.681$ to $\beta = 0.183$, with a broad peak in the vicinity of $\beta = 0.490$. Not all these yielded useful track density data, since slow particles produce tracks so dense as to be essentially continuous. In order for a track to be considered "measurable," it was required that more than one gap of length greater than the cutoff value $x_0 = 0.625$ mm be observed over the length of the track. Equation (1) converges to a value $m_\infty = 80$ bubbles/cm when a track is sufficiently dense that just one gap, of minimum length 0.75 mm is observed in its entire length. Consequently, tracks of density equal to or greater than 80 bubbles/cm were considered essentially continuous and unmeasurable by this procedure. The measurability of the tracks of outgoing protons from the π-p and p-p collisions is shown in Fig. 1, where the ratio

$$R = \frac{\text{(measurable)} - \text{(unmeasurable) tracks}}{\text{(measurable)} + \text{(unmeasurable) tracks}}$$

is shown as a function of particle momentum.

Above 590 Mev/c, or $\beta > 0.535$, none of the 44 proton tracks were so dense as to be unmeasurable. All of the 257 proton tracks below 140 Mev/c momentum, or $\beta < 0.148$ were too dense for measurement. Between these limits lie 66 tracks, which due to foreshortening corrections, $m_0/\bar{m}_0$ normalization, and simple statistics include unmeasurable tracks of apparent density ≥ 80 bubbles/cm; the proportion of these unmeasurable tracks increases sharply with decreasing momentum.

Failure to account for these unmeasurable events would bias a distribution of track densities toward low track density values in the region of $\beta < 0.535$. Exact procedures for correcting such a distribution are limited by statistical considerations. An approximate correction, which should be valid near the high momentum limit, would consider that each track observed to be unmeasurable had just the limiting track density

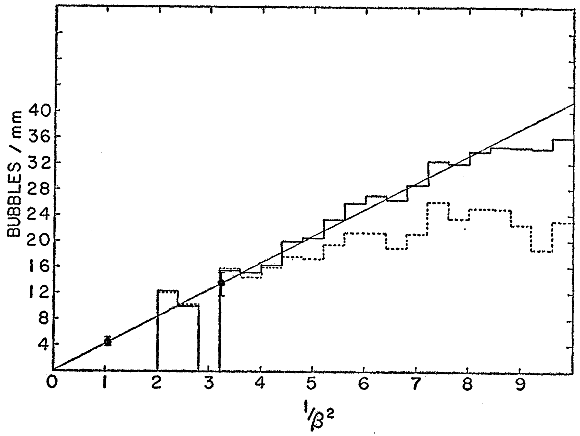

FIG. 2. Track density as a function of the velocity parameter $1/\beta^2$. The uncorrected track density distribution for secondary protons is shown by the dashed histogram, while the solid histogram shows the distribution corrected for unmeasurable tracks. The solid line is the least-squares fit to the beam pion and proton track densities at $1/\beta^2=1.049$ and 3.212 respectively. Errors shown are the probable errors; distances are referred to the film image.

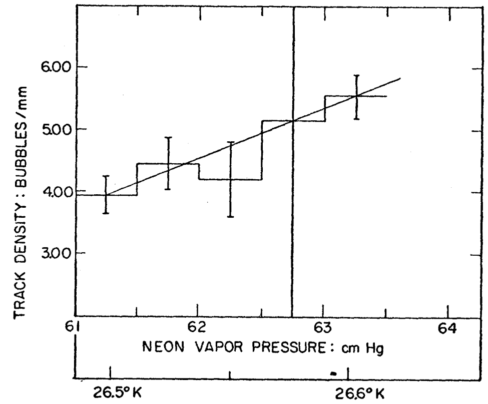

FIG. 3. Charged particle track density as a function of temperature in superheated liquid hydrogen; distances are referred to the film image.

value $m_\infty=80$ bubbles/cm, and would weight each interval by a number of cases of this density m_∞ equal to the proportion of unmeasurable tracks at the corresponding momentum, as determined from Fig. 1.

The track density of charged particles in liquid hydrogen is shown as a function of the velocity parameter $1/\beta^2$ in Fig. 2. The beam pion and beam proton track densities are shown at $\beta^{-2}=1.049$ and 3.212, respectively. A least squares fit of these densities to a velocity dependence $m=A/\beta^2$ gives the slope A, shown by the solid line, as 8.44 ± 0.044 bubbles/cm measured at the chamber. The track density distribution for the protons from π-p and p-p collisions is shown uncorrected by the dashed histogram, and corrected for unmeasurable events by the solid histogram. Agreement of the corrected histogram with the A/β^2 slope is good up to the value $\beta^{-2}=8.2$, beyond which point the correction presumably fails due to the increasing significance of bubble densities $>m_\infty=80$ bubbles/cm at momenta below 350 Mev/c.

Although the temperature variation in the chamber permitted during the experiment was small, some idea of the temperature dependence of the slope A can be gained from Fig. 3, where the track density of minimum-density pions is shown as a function of the mean temperature of the chamber during the period when the tracks were photographed. From the slope of this curve a very approximate value of $\Delta m/\Delta T=3.0$ bubbles/cm-°K, amounting to a 30% variation in track density per 0.1°K change in temperature, is obtained.

IV. DISCUSSION

As Bugg[3] has pointed out, it is of interest to investigate the possibility of correlation between an A/β^2 dependence of the track density produced by charged particles in superheated liquids, and the number of secondary electrons produced per unit track length of a primary ionizing particle. The rate of this "delta-ray" formation is also velocity-dependent as A/β^2, if one excludes secondaries energetic enough to leave a recognizable track. Inasmuch as the slow rate of bubble growth apparently rules out the surface repulsion of ionic charge[6] as the primary mechanism of bubble nucleation, unless some unknown process acts to inhibit the recombination of ions in the superheated liquid, delta rays may be considered to act as highly localized thermal spikes, providing energy sufficient to permit vapor bubbles to grow past the critical radius $r_c=2\sigma/(p_v-p_l)$, where σ is the surface tension of the liquid and (p_v-p_l) represents the pressure difference inside and outside the bubble. Since charged secondaries lose much of their energy in collisions of the second kind, exciting the rotational and vibrational degrees of freedom of the molecules with which they interact, the heat required for bubble nucleation is readily available from this source.

Bugg has calculated an approximate lower limit to the energy required for bubble nucleation by considering the work necessary to open a spherical hole against the pressure of the liquid, less the work returned by vapor filling it in, and adding to this the heat required to raise the temperature and evaporate liquid into the bubble, to overcome surface tension, and to provide for kinetic energy of radial expansion. At the lower limit of sensitivity this energy ranges from ~310–350 electron volts for liquid hydrogen, depending on the assumptions made for the kinetic energy term.

If one assumes that the track densities observed are correlated with the number of delta rays per unit track length n having energies between the minimum energy E_a required for bubble nucleation and the maximum energy E_b which would permit a secondary to stop in

[6] D. Glaser, Phys. Rev. **91**, 762 (1953); Suppl. Nuovo cimento **11**, 361 (1954).

a range $2r_c$, where[3]

$$n=\frac{153}{\beta^2}\frac{\rho z}{A}\left[\frac{1}{E_a}-\frac{1}{E_b}\right] \tag{2}$$

one obtains $E_a=400$ electron volts. In view of the somewhat arbitrary criterion used in defining E_b in Eq. (2), the agreement with the calculated value for the lower value limit of sensitivity is quite reasonable. By comparison, Bugg has shown that the measurements of Willis *et al.*[2] in propane give a value $E_a \simeq 680$ electron volts, while the calculated value for propane is $\simeq 530$ electron volts, including kinetic energy terms.

It may be concluded that both the velocity dependence and the actual track density values for charged particles in superheated liquid hydrogen are consistent with the concept of bubble nucleation by delta rays having energies of the order of 400 electron volts. The use of track density measurement as an effective means of distinguishing particles whose velocities are appreciably different is limited only by the statistics associated with the number of bubble gaps to be counted in the tracks observed.

V. ACKNOWLEDGMENTS

The author wishes to express his appreciation to R. P. Shutt and the members of the Bubble Chamber Group of the Brookhaven National Laboratory for their advice, assistance, and many helpful services. He is particularly indebted to W. L. Willis for providing the original data film and for innumerable helpful suggestions, and to M. R. Burns and M. Vitols, who made the microscope measurements. The assistance of R. Hanau of the University of Kentucky in the statistical analysis is gratefully acknowledged.

PHYSICAL REVIEW VOLUME 119, NUMBER 1 JULY 1, 1960

Theory of Allowed and Forbidden Transitions in Muon Capture Reactions. II*

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The general formalism of the first paper in this series is applied to the calculation of the angular distribution of the recoils in muon capture. Only the unique nth forbidden transitions [spin change $0\rightarrow J$, parity change $(-)^{J+1}$] are considered. As an example the special case of C^{12} is discussed. The angular distribution of the recoils depends strongly on the strength of the induced pseudoscalar interaction, but is rather insensitive to the assumption of conserved vector current.

I. INTRODUCTION

IN the first paper of this series,[1] a general formalism was developed for the treatment of muon capture reactions. The application of this formalism to capture by C^{12} to the ground state of B^{12} yielded results which differed by 9–13% in the capture rate from those previously calculated.[2] The difference arose from a detailed consideration of nuclear matrix elements involving the differential operator acting on the nuclear wave function, and of interference terms among the nuclear matrix elements. Both factors had been neglected in earlier calculations by Fujii and Primakoff.[3] The situation in C^{12} was also considered by Wolfenstein.[4]

Recently, Rose and Good[5] have calculated the angular distribution of recoils from muon capture, for the unique nth forbidden transitions [spin change $0\rightarrow J$, parity change $(-)^{J+1}$]. Since their aim was to find parity nonconservation and lepton conservation in muon capture reaction, they considered the sign of the asymmetry and its magnitude, neglecting matrix elements of the type mentioned above.

We apply the formalism of I, taking into account the matrix elements involving the differential operator acting on the nuclear wave functions, to the calculation of the angular distribution of recoils for the same case treated by Rose and Good. The numerical results again bring about 5–20% correction in the asymmetry coefficient for C^{12}. However, the results given both by Rose and Good and by us suffer from a theoretical uncertainty due to the nuclear wave function, which is

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* This work was partially supported by the U. S. Atomic Energy Commission.
[1] M. Morita and A. Fujii, Phys. Rev. **118**, 606 (1960). We refer to it as I, hereafter.
[2] See Sec. 10 of reference 1.
[3] A. Fujii and H. Primakoff, Nuovo cimento **12**, 237 (1959); H. Primakoff, Revs. Modern Phys. **31**, 802 (1959).
[4] L. Wolfenstein, Nuovo cimento **13**, 319 (1959).
[5] M. E. Rose and R. H. Good, Jr., Ann. Phys. **9**, 211 (1960). The authors would like to express their sincere thanks to them for sending a preprint.