Theoretical Predictions for the Spectra of the Odd-Mass Xenon and Tellurium Isotopes

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The spectra of the isotopes $Te^{123,125}$, $Xe^{127,129}$ are calculated by assuming that the odd neutron, having available the $3s_{\frac{1}{2}}$ and $2d_{\frac{3}{2}}$ states, is coupled to collective surface vibrations of the core. Good agreement is obtained with the known levels in these nuclei using a reasonable value for the coupling parameter. To obtain the agreement, the $d_{\frac{3}{2}}-s_{\frac{1}{2}}$ splitting, ϵ, must be regarded as a function of neutron number. The manner in which ϵ varies, as found in the intermediate coupling calculation is compared with the predictions of the pairing correlation theory originally introduced in connection with superconductivity. Agreement as to the general trend is found. This may be regarded to some extent as an indication of the applicability of the pairing correlation theory to nuclear structure calculations.

I. INTRODUCTION

MOST of the even-even nuclei in the mass region $108 \lesssim A \lesssim 136$ exhibit a vibrational spectrum up to the second excited state. Also, neighboring odd mass isotopes of a given nucleus in this region have similar spectra aside from systematic variations. It seems plausible therefore to treat the odd-N isotopes as a coupled system comprising an even-even core, capable of collective surface vibrations with frequencies of the corresponding even-even nucleus, plus an extra-core neutron which has available several single-particle levels. The energy of the resulting states depends on the particle state occupied by the odd neutron, on the state of the core excitation characterized by the number of phonons, on the strength of the particle-collective coupling, and on the total spin to which the particle and core angular momenta are coupled. If the coupling between the odd nucleon and the core is weak, we may treat it in the perturbation approximation. Such a model is called an intermediate-coupling model, and was briefly outlined by Bohr and Mottelson,[1] and later elaborated by Choudhury,[2] and Ford and Levinson.[3] Recently, the theory has been applied in the region of gold by Alaga.[4]

Three kinds of parameters appear in such a model: the strength of the coupling, ξ, between the odd nucleon and the core vibrations, the quantum of energy, $\hbar\omega$, associated with the core vibrations, and the single-particle energies, E_j, available to the odd nucleon. The first of these parameters may be regarded as adjustable, and should be roughly constant for a given mass region. The phonon energy $\hbar\omega$ may be deduced from the neighboring even-even nucleus, which forms the core of the odd-A nucleus. The single-particle energies E_j, however, cannot be deduced from experimental information if the odd nucleon does in fact interact with the core vibrations. Therefore, this set of parameters must be regarded as adjustable and peculiar to each isotope, or else they must be calculated from a model which predicts their effective spacing as a function of the number of particles occupying them.

We speak of an "effective" single-particle spacing in the following sense. If we begin to fill a set of levels in a potential well with noninteracting particles, the levels will be filled in order, starting with the level of lowest energy. But if the particles interact with each other, their motion can no longer be described in terms of the uncorrelated wave functions of the potential well. Additional states are required to describe the motion, namely the unoccupied higher states. Therefore, the ground state for the interacting particles cannot be found by simply filling the levels in order; instead, higher levels will begin to be filled before some of the lower ones are completely occupied. Moreover, the difference in the energy when one adds a particle in turn to two different single-particle levels will not be the difference in the energies of the levels in the well, but will depend in a complicated way on the particular levels (i.e., their maximum occupation numbers), their spacings, the nature of the interparticle interaction, and the number of interacting particles. Essentially, then, the problem of finding the effective single-particle spacings is a many-body problem.

Progress has been made recently in the treatment of the many-body problem by Bardeen, Cooper, and Schrieffer[5] and with specific reference to nuclei by Belyaev[6] and Mottelson.[7] Very briefly we may describe the treatment as follows. The interaction between the real nucleons may be divided into two parts. The one defines the self-consistent field in which the particles move (i.e., the potential well with its unperturbed levels). The remaining part is essentially a coherent pairing interaction (e.g., a very short-range interaction),

[1] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Met.-fys. Medd. **27**, No. 16 (1953), Chap. II.
[2] D. C. Choudhury, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **28**, No. 4 (1954).
[3] K. W. Ford and C. Levinson, Phys. Rev. **100**, 1 (1955).
[4] G. Alaga, Institute Ruder Boskovic, Zagreb, Yugoslavia (private communication).
[5] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).
[6] S. T. Belyaev, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **31**, No. 11 (1959).
[7] B. R. Mottelson, lectures delivered at University of California, Berkeley, Spring, 1959 (unpublished).

and gives rise to the "effective" spacings spoken of above. A transformation is made to new noninteracting particles, the quasi-particles, whose intrinsic structure is determined by the pairing interaction. The spectrum of the quasi-particles yields directly the effective single-particle spacing between the real nucleon levels as is described in more detail below.

II. INTERMEDIATE COUPLING IN THE UNIFIED MODEL

The Hamiltonian for the system of core plus extra nucleon is written

$$H=H_s+H_p+H_{\mathrm{int}}. \quad (1)$$

Here H_s, the energy of surface vibrations of the core, is

$$H_s=(1/2)\sum_\mu (B|\dot{\alpha}_{2\mu}|^2+C|\alpha_{2\mu}|^2), \quad (2)$$

H_p is the energy of the odd nucleon with eigenvalues corresponding to the single-particle levels in the presence of the spherical core, and H_{int} is the interaction energy which couples the particle motion to the core oscillations:

$$H_{\mathrm{int}}=-k\sum_\mu \alpha_{2\mu}Y_{2\mu}^*(\theta,\phi). \quad (3)$$

We adopt as basic wave functions the eigenvectors of the Hamiltonian for the uncoupled system

$$H_0=H_s+H_p, \quad (4)$$

so that

$$H_0|j,NR;IM\rangle=(E_j+N\hbar\omega)|j,NR;IM\rangle. \quad (5)$$

Here j denotes the nucleon quantum numbers, N is the number of phonons of vibrational motion, R is the total angular momentum associated with the core vibrations, and $\mathbf{I}=\mathbf{j}+\mathbf{R}$ is the total angular momentum of the nucleus. The single-particle energies are denoted by E_j, and $\hbar\omega$ is the energy of one phonon of surface vibration.

The energy eigenvalues are found by diagonalizing the total Hamiltonian in the space of the basic states. In the present calculation this was done for states having up to three phonons. The diagonal matrix elements are given by Eq. (5). The off-diagonal elements are

$$\langle j,NR;IM|H|j',N'R';IM\rangle = (-)^{j+R'-I+1}\xi\hbar\omega(\pi/5)^{\frac{1}{2}}\langle j\|Y_2\|j'\rangle \times\langle NR\|b\|N'R'\rangle W(jRj'R';I2), \quad \text{for } N'>N, \quad (6)$$

where

$$\xi=k(5/2\pi\hbar\omega C)^{\frac{1}{2}}, \quad (7)$$

is the dimensionless coupling parameter that measures the strength of the particle-surface interaction, b is the annihilation operator for phonons,[8] and W is a Racah coefficient. Racah's definition of reduced matrix elements has been used.[9] The reduced matrix elements of Y_2 vanish unless $\mathbf{l}+\mathbf{l}'+\mathbf{2}=0$ and may be written

$$\langle j\|Y_2\|j'\rangle=(-)^{j-\frac{1}{2}}[(2j+1)(2j'+1)/4\pi]^{\frac{1}{2}} \times C(jj'2;\tfrac{1}{2}-\tfrac{1}{2}0), \quad (\text{for } l+l'=\text{even}) \\ =0, \quad (\text{for } l+l'=\text{odd}). \quad (8)$$

The reduced matrix elements of b are listed in Choudhury's paper[2] and differ by a factor from those given

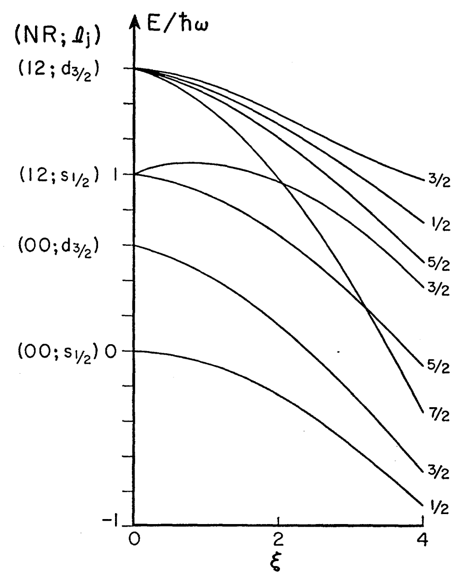

FIG. 1. The energy levels of the coupled system of one neutron and oscillating core are plotted as a function of the coupling parameter ξ. The neutron has available the $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$ states separated by ϵ, which for this figure is $\epsilon=0.6\hbar\omega$.

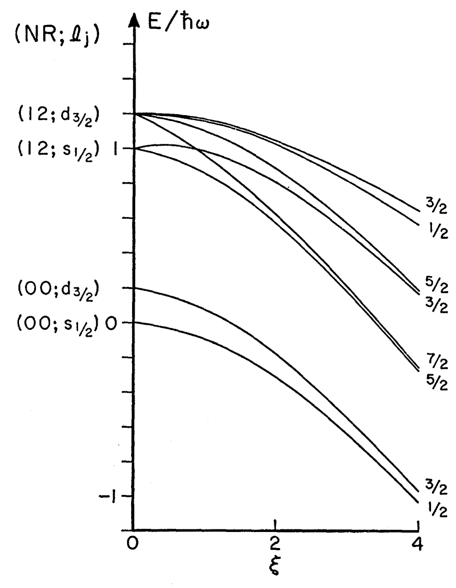

FIG. 2. The energy levels of the coupled system of one neutron and oscillating core are plotted as a function of the coupling parameter ξ. The neutron has available the $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$ states separated by ϵ, which for this figure is $\epsilon=0.2\hbar\omega$.

[8] The usual definition of b is used, *viz.*

$$\alpha_\mu=(\hbar\omega/2C)^{\frac{1}{2}}[b_\mu+(-)^\mu b_{-\mu}^*].$$

[9] G. Racah, Phys. Rev. **62**, 438 (1942); **63**, 367 (1943).

here:

$$\langle NR\|b\|N'R'\rangle=(2R'+1)^{\frac{1}{2}}\langle NR\|b\|N'R'\rangle_{\text{Choudhury}}. \quad (9)$$

The energy eigenvalues of the system are a function of the coupling parameter ξ. For one nucleon having available two single-particle levels, $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$ with separation ϵ, the lower energy eigenvalues of the system of oscillating core plus nucleon are shown in Figs. 1 and 2 corresponding to $\epsilon=0.6\ \hbar\omega$ and $0.2\ \hbar\omega$, respectively. Notice the doublet structure that the spectrum develops at the larger coupling strengths, when ϵ is small compared to $\hbar\omega$.

The eigenvectors of the coupled system have the form

$$|E,IM>=\sum_{j,N,R} a_{j,N,R}(E)\,|\,j,NR;IM>, \quad (10)$$

where the expansion coefficients $a_{j,N,R}$ are given by the diagonalization of H. In the present application of the theory, the sum extends on j over the $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$ single-particle states and over the core-oscillator states $N=0,\cdots 3$. For intermediate and strong coupling strengths, a given level (E,I) will contain large admixtures of several oscillator and particle states. This configuration mixing has, of course, a marked effect on gamma-transition rates. However, since so little is known about the reduced transition rates of the tellurium and xenon isotopes, they have not been calculated in this work.

We may assess the validity of the perturbation approach by examining the expansion coefficients in Eq. (10) corresponding to the highest oscillator state considered. Their squares should be small compared to unity. This condition was satisfied in the calculations reported in the following section.

III. THE ODD-MASS TELLURIUM AND XENON ISOTOPES

The experimental information on the energy levels of the odd-mass tellurium and xenon isotopes is summarized in Figs. 3 and 4. The 11/2− level is omitted when known, since this corresponds to exciting a particle from the core and does not come within the framework of the present model. It is interesting to note that the first $\frac{3}{2}+$ level migrates to lower energies with increasing N, until it finally becomes the ground state. This migration is not caused by the coupling to the surface but can be explained only in terms of many nucleon interactions as mentioned in the introduction. A more quantitative discussion of this effect is given in Sec. IV. For the present, the single-particle spacing will be regarded as an adjustable parameter.

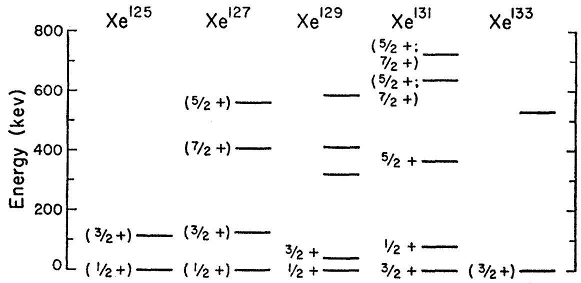

FIG. 4. The experimental energy spectra of several of the odd-N xenon isotopes are shown. The 11/2− state has been omitted as explained in the text. The spectra are taken from reference 10, where references to individual experimenters can be found.

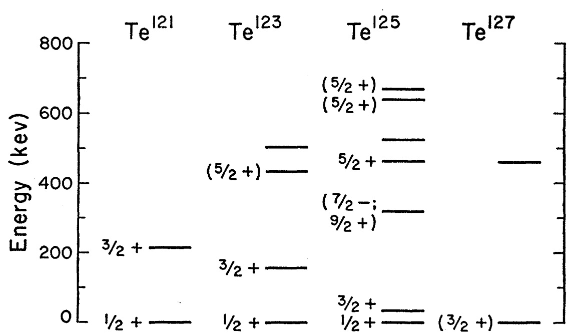

FIG. 3. The experimental energy spectra of several of the odd-N tellurium isotopes are shown. The 11/2− state has been omitted as explained in the text. The spectra are taken from reference 10, where the references to individual experimenters can be found.

The intermediate-coupling model described in Sec. II is applied here to $Te^{123,125}$ and $Xe^{127,129}$. We assume that the low-energy spectrum of each of these nuclei can be described in terms of having the odd neutron occupy the $s_{\frac{1}{2}}$ or $d_{\frac{3}{2}}$ level in the shell ending at 82. All lower levels are occupied (but not filled) by an even number of nucleons which collaborate in a collective surface vibration with frequencies given approximately by the corresponding even-even nucleus.

The Hamiltonian was diagonalized for various values of the spacing ϵ of the $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$ levels and the energy eigenvalues were plotted as a function of the coupling parameter ξ, as in Figs. 1 and 2.

It was found that the spectra of the four nuclei $Te^{123,125}$ and $Xe^{127,129}$ could all be fitted as shown in Figs. 5 and 6 with the same coupling parameter $\xi=2.5$

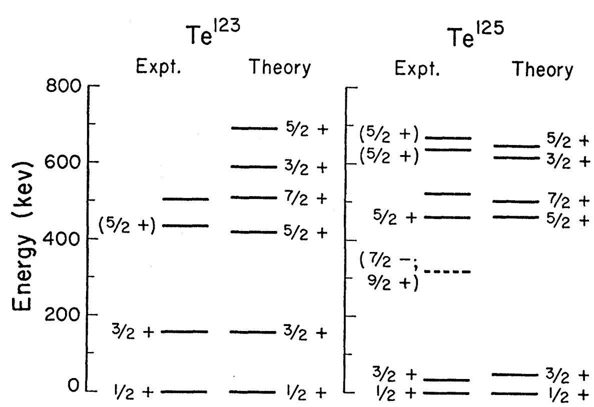

FIG. 5. Comparison of theoretical with experimental spectra of Te^{123} and Te^{125}. The parameters used in the calculation are listed in Table I. The 11/2− state has been omitted as explained in the text.

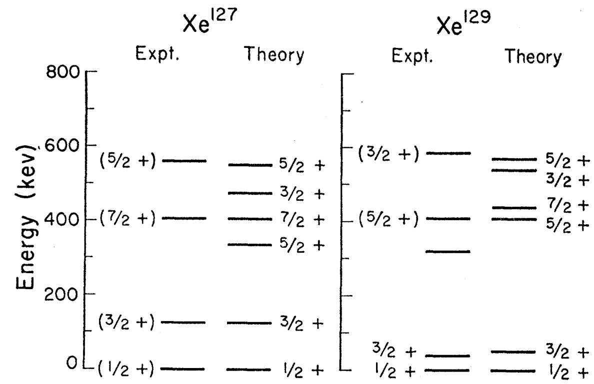

FIG. 6. Comparison of theoretical with experimental spectra of Xe^{127} and Xe^{129}. The parameters used in the calculation are listed in Table I. The 11/2− state has been omitted as explained in the text.

and the single-particle spacings and phonon energies listed in Table I. The quoted value of ξ implies a value for the nuclear deformability:

$$C \simeq 200/\hbar\omega \text{ Mev.}$$

This is consistent with our rough knowledge of this parameter. The phonon energies used are compared in Fig. 7 with the energies of the first excited state in the corresponding even-even nuclei.

The agreement of the theory with the experimental energy levels is quite good. The theory usually corroborates those spin assignments that are indicated as tentative in the literature,[10] and agrees with all the firm assignments. The level at 320 kev in Te^{125} is listed as either 7/2− or 9/2+. Since the theory, which should predict the low-lying, even-parity states, does not give a state near this energy, the present work

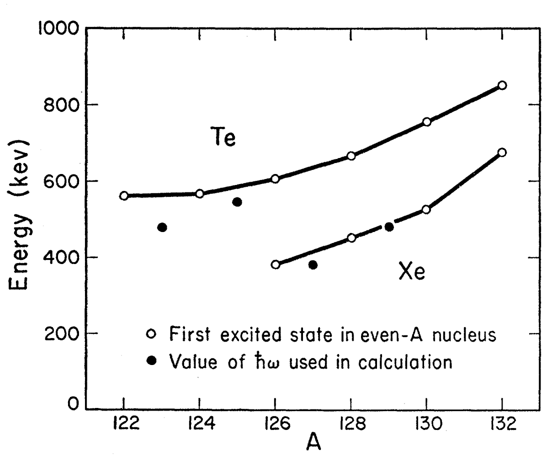

FIG. 7. The values of $\hbar\omega$ used in the calculation to obtain the fits to the experiments shown in Figs. 5 and 6 are compared here with the energy of the first excited states of the even-even nuclei.

[10] *Nuclear Data Cards* (National Research Council, Washington, D. C., 1958).

suggests an odd-parity assignment. Such a state, as with the 11/2−, would correspond to exciting an $h_{11/2}$ particle out of the core. The level at the same energy in Xe^{129} probably has the same character. Some new states are predicted in the low-energy spectra ($E \lesssim 700$ kev) of these nuclei. The theory also predicts many states at higher excitation, though it is not known to what extent the calculation can be trusted at higher energies. Some of the higher states together with those shown in the figures are listed in Table II.

IV. APPLICATION OF THE PAIRING CORRELATION THEORY

In the preceding section the spacing, ϵ, of the $s_{\frac{1}{2}}$ and $d_{\frac{3}{2}}$ levels was used as an adjustable parameter which was varied, as shown in Table I, to obtain the best fits to the experimental energy spectra. In this section we shall exploit some recent developments in the treatment of the many-body problem to calculate, from a given set of independent-particle levels, the variation of the single-particle spacing for the last neutron as a function of nucleon number. The theory is found to agree with the empirically determined dependence found in Sec. III.

The method of solving the many-particle system was briefly explained in the Introduction. The properties of the interacting system are completely determined in this theory by the "chemical potential," λ, and the "energy gap," Δ, which are simultaneous solutions of the two equations

$$\frac{G}{2}\sum_k \frac{\Omega_k}{[(E_k{}^0-\lambda)^2+\Delta^2]^{\frac{1}{2}}}=1, \qquad (11)$$

$$\sum_k \Omega_k\left(1-\frac{E_k{}^0-\lambda}{[(E_k{}^0-\lambda)^2+\Delta^2]^{\frac{1}{2}}}\right)=N. \qquad (12)$$

The sum on k extends over the independent-particle levels whose energies are $E_k{}^0$; $\Omega_k=(2j_k+1)/2$ is the number of pairs of nucleons that can occupy the k'th level, N is the number of nucleons that occupy the set of levels, and G is the interaction energy of paired nucleon states. The only interactions included are those between nucleons in the conjugate states $(m, -m)$. It is estimated[7] that $G \sim 25/A$ Mev.

Unfortunately the Hamiltonian for the system of interacting nucleons in the above theory does not conserve the particle number. This is the price that is paid for a solution to the many-body problem. However, Eq. (12) insures that the average number of particles

TABLE I. The $s_{\frac{1}{2}}-d_{\frac{3}{2}}$ splitting, ϵ($s_{\frac{1}{2}}$ lower), and phonon energy $\hbar\omega$ used in the fit to the experimental spectra shown in Figs. 3 and 4.

	Te^{123}	Te^{125}	Xe^{127}	Xe^{129}
$\epsilon/\hbar\omega$	0.6	0.2	0.6	0.2
$\hbar\omega$(Mev)	0.48	0.55	0.38	0.48

is equal to N, while Eq. (11) insures that the energy of the ground state is a minimum, subject to the condition on N. Because of the nonconservation of particle number, the nuclear properties that are calculated in this theory represent an average over neighboring nuclei differing from the one of interest by pairs of nucleons. However, since the amplitudes in the wave function corresponding to N, $N\pm2$, $N\pm4$, $\cdots$ particles are successively smaller, in general, the average is weighted at the correct nucleon number.

The "effective" single-particle spacing can be calculated from the two parameters λ and Δ [solution of Eqs. (11) and (12) for even-N] in the following way. When the odd nucleon is added to the existing system of nucleons, it does not interact with them in the pairing-force approximation used. It does however occupy a state which, because of the Pauli exclusion principle, prevents the pairs of nucleons from scattering into this and the conjugate state. Reducing the number of ways in which interacting nucleons can scatter reduces the (attractive) interaction energy. Therefore when an odd nucleon is added to a nucleus in the state j, the energy of the nucleus is increased by the amount

$$\delta E_j=[(E_j^0-\lambda)^2+\Delta^2]^{\frac{1}{2}}. \tag{13}$$

The "effective" single-particle spacing between the states j and j' is just the difference

$$\epsilon_{jj'}=\delta E_j-\delta E_{j'}. \tag{14}$$

Figure 8 shows the result of such a calculation for the tellurium isotopes of interest in this work. All the neutrons beyond the closed magic core of 50 are included in the calculation. The independent-particle spectrum shown in Fig. 8 refers to the levels, E_j^0, that these neutrons occupy if we assume that the interparticle interaction is completely exhausted by the self-consistent field. This situation in general is not realized, and inclusion of a residual interaction in the form of a

TABLE II. Calculated energy levels of the positive-parity states in some tellurium and xenon isotopes.

Spin	Te^{123} Energy (kev)	Te^{125} Energy (kev)	Xe^{127} Energy (kev)	Xe^{129} Energy (kev)
1/2	0	0	0	0
3/2	154	58	122	51
5/2	418	462	331	403
7/2	527	490	417	427
3/2	592	628	469	548
5/2	681	659	539	576
1/2	722	763	572	666
3/2	773	783	612	683
9/2	843	931	667	813
5/2	959	1060	759	928
7/2	1060	1110	838	967
1/2	1100	1210	871	1060
9/2	1260	1330	995	1160

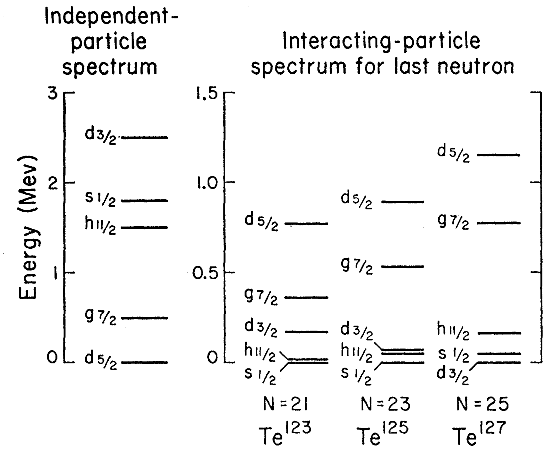

FIG. 8. The spectrum of the last odd neutron in the major shell 50–82 is shown for several odd-N systems. The spectra are calculated from the pairing correlation theory using the set of independent particle levels shown at the left of the figure. N refers to the number of neutrons that occupy the levels. The calculation corresponds to a value of $G=0.25$ Mev.

pairing correlation gives rise to the interacting-particle spectra for the last neutron shown on the right side of Fig. 8. Notice in particular that the $d_{\frac{3}{2}}$ state moves toward the $s_{\frac{1}{2}}$ state with increasing N until it becomes the ground state. This is the trend that was found in the previous section and is also indicated in the experimental spectra (Fig. 1).

Of course it is the spectrum of the coupled system of particle motion and core oscillations that one observes experimentally, not the spectra of Fig. 8. But the magnitude of the $s_{\frac{1}{2}}-d_{\frac{3}{2}}$ splitting compared to $\hbar\omega$ is reflected in the coupled system as may be seen by comparing Figs. 1 and 2.

V. SUMMARY

The intermediate coupling approach in the unified model provides a good account of the energy spectra of the several odd-N nuclei considered in this work. Undoubtedly many odd-A nuclei in the mass regions where the even-A nuclei display vibrational spectra can be similarly treated, though it may be necessary in some cases to consider explicitly the particle coordinates of more than one nucleon.[11]

The pairing correlation theory was used to calculate the spectrum of an odd nucleon in the presence of interactions among the other nucleons. The agreement with the information deduced in the intermediate coupling calculation suggests at least the semiquantitative value of the pairing correlation theory in nuclear structure calculations.

[11] G. Alaga found it necessary to couple the motion of the last three protons (holes) to the core oscillations in his calculations for Ag^{197} (see reference 4).