sample of Powell *et al.*[1] This difference is also reflected in the maximum value of heat conductivity (see inset figure), which is about $190W\ \mathrm{cm}^{-1}\ \mathrm{deg}^{-1}$ for our sample and about $135W\ \mathrm{cm}^{-1}\ \mathrm{deg}^{-1}$ for their annealed sample.

The important factor is, however, the comparison of the thermal and electrical resistivities. The values of $L=\rho/WT$ are shown in Fig. 1, together with curves obtained by Berman and MacDonald[3] and by Powell *et al.*[1] The third curve shown is that calculated from existing data on the "ideal" resistivities for copper,[11] and from the known residual resistivity of our specimen, i.e., from the equation $L=(\rho_0+\rho_i)/(W_0+W_i)T$, assuming $\rho_0=0.87\times10^{-9}$ ohm cm, $W_0=\rho_0/2.45\times10^{-8}T$. The experimental points below about 4°K indicate that at the lowest temperatures L lies between 2.4 and $2.5\times10^{-8}W$ ohm deg^{-2}, not differing more than 2% from the theoretical Sommerfeld value of 2.45×10^{-8}.

Note that only at very low and at high temperatures should L approach the Sommerfeld value. At intermediate temperatures, where scattering is not elastic, and hence where relaxation times are not the same for thermal and electrical resistivity, L will fall below this value. The temperature at which L is a minimum, and hence the minimum value of L, decrease with increase in purity. Allowing for the differing purity of our sample and that of Berman and MacDonald, the temperature variation of L is consistent.

5. CONCLUSION

In agreement with the theory, it appears that this specimen of high purity copper exhibits the same value of the Wiedemann-Franz-Lorenz ratio at liquid helium temperatures as do less pure copper samples, and as do many specimens of other metallic elements, namely $L\simeq2.45\pm0.05\times10^{-8}W$ ohm deg^{-2}. The reason for a departure from this pattern in the experiments of Powell *et al.*[1] seems obscure; perhaps it arises from spurious heating effects.

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Possibility of the Existence of Attractive Forces Between Dislocations of Like Sign

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It is concluded that two moving dislocations of the same sign and on the same slip plane can sometimes attract rather than repel one another (and two of unlike sign repel rather than attract each other). This reversal over the usual behavior will occur at velocities where the kinetic energy in the displacement field of an isolated moving dislocation is larger than the strain energy in the same field.

It is usually considered that dislocations of like sign repel each other and those of unlike sign attract each other. We wish to point out that, at high dislocation velocities, the reverse situation may occur. A consideration of the energy of a moving dislocation shows under what circumstances like dislocations can attract and unlike repel. The energy of a uniformly moving dislocation can be divided into two parts, a potential energy associated with the strains existing in the elastic displacement field and a kinetic energy associated with the velocity of these elastic displacements. Let E_1 and E_2 represent these two energies. In general the values of E_1 and E_2 will increase monotonically as the velocity of the dislocation increases. Now consider the following thought experiment. Let two dislocations which originally are widely separated each be given a velocity V_0 with respect to the crystal lattice and set into motion towards each other so that ultimately they will meet. If it is assumed that energy is conserved, the dislocations will pass through each other and eventually regain the original velocity V_0 when they again are widely separated. The criterion as to whether the forces between these dislocations are attractive or repulsive simply is the following: If at the moment of meeting the velocity V_1 of the dislocations is greater than the original velocity V_0, the forces must be attractive. If the velocity V_1 is smaller, the forces are repulsive. (We assume, of course, that the initial energy is sufficiently great that the two dislocations can meet in the repulsive case.)

If linear elasticity theory is applicable and energy loss by the radiation of sound waves neglected, the solution of the displacement field of two moving dislocations is simply the sum of the displacements of each dislocation considered alone. Hence when two unlike dislocations collide, the displacements and stresses go to zero and the potential energy at the moment of impact is zero. The velocity of the displacements does not go to zero for unlike dislocations, but is double the velocity of the displacements of an isolated, uniformly moving dislocation with the velocity V_1. Since kinetic energy goes as the square of the displacement velocity one has

$$4E_2(V_1)=2[E_1(V_0)+E_2(V_0)], \tag{1}$$

if it is assumed that energy is conserved. Under ordinary circumstances the potential energy E_1 of an isolated dislocation moving at an arbitrary velocity is larger than the kinetic energy E_2. Hence, if Eq. (1) is to be valid, V_1 has to be larger than V_0 provided that the energies are increasing functions of the velocity. However, should the kinetic energy E_2 be larger than E_1 Eq. (1) would apply only if V_1 is less than V_0. If V_1 is less than V_0, there is a repulsive force between two unlike dislocations.

One can use similar reasoning to consider two dislocations of like sign which are set into a collision motion. At the moment of meeting, the displacement velocities now cancel one another and the kinetic energy will go to zero. The stresses in the displacement field are double those of an isolated dislocation moving at the velocity V_1. Since potential energy goes as the square of the strain, one has

$$4E_1(V_1)=2[E_1(V_0)+E_2(V_0)]. \qquad (2)$$

If at any given velocity E_2 is larger than E_1, V_1 must be larger than V_0. Under this circumstance an attractive force exists between the two like dislocations, whereas if E_1 is greater than E_2, the force is repulsive.

In our arguments we have considered that the energy is constant and have allowed the dislocation velocity to vary. Our model can also be used for the situation where the dislocation velocity is kept constant and the total dislocation energy allowed to vary. If a force is applied to the two colliding dislocations which is always equal and opposite to the force of interaction between them, their velocity would remain unchanged but their total energy would change. This force could be applied by, say, allowing the dislocations to run in an alloy in which the short-range order on the slip plane varies as an appropriate function of distance to give the desired magnitude and direction of the force. If V is the dislocation velocity, the initial energy will be $2[E_1(V)+E_2(V)]$ and the energy upon meeting will be either $4E_1(V)$ or $4E_2(V)$, depending on whether the dislocations are of the same or opposite sign. If the energy at collision is greater than the initial energy, the force between dislocations must have been repulsive, and if the energy is less the force must have been attractive. If we repeat our argument when either E_1 or E_2 is the larger energy of an isolated moving dislocation, one sees that we come up with the same conclusions as before.

The potential energy of a moving screw dislocation always is greater than its kinetic energy except when the velocity of the dislocation is that of the slowest sound velocity, in which case the two energies are equal and infinite. Hence screw dislocations are always well behaved. Like screw dislocations repel each other and unlike attract.

In the case of edge dislocations a calculation[1] shows that above the Rayleigh wave velocity (approximately 0.9 times the slow sound velocity) the kinetic energy becomes greater than the potential energy. Thus at high velocities edge dislocations of like sign can attract each other and unlike repel, in contrast to the usual situation. This result also can be shown directly[1] by calculating the shear stress on the slip plane around a moving edge dislocation from the exact solution of the displacement field of an edge dislocation given by Eshelby.[2] The shear stress goes to zero as the Rayleigh wave velocity is approached and reverses its sign and increases monotonically its magnitude above this velocity.

It has been claimed by Eshelby that the Rayleigh wave velocity actually is a limiting velocity for edge dislocations because the "width" of a dislocation goes to zero at this speed. However since the energy of a moving edge dislocation does not go to infinity at this speed there really is no physical reason for this velocity to be an upper limit. The reason the "width" of the dislocation goes to zero arises from the fact that the shear stress on the slip plane of the dislocation approaches zero as the Rayleigh wave velocity is approached. The width, which simply is a measure of this shear stress, thus will decrease, reach zero, and then increase as the velocity is increased beyond the Rayleigh wave velocity. The sign of the width becomes negative rather than positive above the Rayleigh wave velocity because the shear stress reverses itself. (The velocity at which the width of a screw dislocation goes to zero is truly a limiting velocity because the width becomes a complex number at higher speeds. This velocity is identical to the one at which the energy of the screw dislocation becomes infinite.)

In conclusion, dislocations of like sign can be attracted to each other, and unlike repel if the kinetic energy in the displacement field of an isolated dislocation is greater than the potential energy. If the potential energy is the greater the dislocations behave normally.

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[1] J. Weertman, American Institute of Mining, Metallurgical, and Petroleum Engineers Conference on High Velocity Deformation, Estes Park, Colorado, July, 1960 (to be published).

[2] J. D. Eshelby, Proc. Phys. Soc. (London) **A62**, 307 (1949).