Spin-Orbit Force and a Neutral Vector Meson

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The coefficient of $(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\hat{n}$ in the pp scattering amplitude at 310 Mev can be completely accounted for by a strongly interacting neutral vector meson (or a sharp three-pion resonance in the $T=0$, $J=1$ state) of mass $\approx 3\mu_\pi - 4\mu_\pi$.

RECENTLY several authors have speculated on the possible existence of a neutral vector meson or a sharp three-pion resonance that has the same symmetry properties as a neutral vector meson (i.e., $T=0$, $J=1$, odd G conjugation parity).[1–5] As is well known, an exchange of such a meson between two nucleons would produce a short-range repulsion in all spin and parity states in agreement with observation. It has been shown by the present author that such a vector meson could also account for the observed spin-orbit force, at least qualitatively.[4] The purpose of the present paper is to explore the connection between the spin-orbit force and the vector meson in a more quantitative manner. In particular, we shall show that Wolfenstein's C amplitude [coefficient of $(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot\mathbf{n}$] in pp scattering at $E_{\mathrm{lab}}{}^{(\mathrm{kin})}=310$ Mev can be completely accounted for if there exists a vector meson (or a sharp three-pion resonance) of mass $\approx 3\mu_\pi$-$4\mu_\pi$.

Throughout this paper we assume that the spin-orbit force arising from an exchange of uncorrelated pions between two nucleons is much too weak to account for the observed spin-orbit force.[6] Then the most likely candidate for the longest-range contribution to the spin-orbit force is precisely the single-vector-meson state. Following Chew's extremely plausible conjecture on one-particle singularities,[7] we argue that the nucleon-nucleon scattering matrix regarded as a function of the square of the momentum transfer must have a pole at $q^2=-\mu_V{}^2$ (where μ_V is the mass of the vector meson), the residue of which is given by the usual Born prescription. We should like to emphasize that our approach based on Chew's conjecture is independent of the validity of perturbation theory (even though the contribution to the scattering matrix we obtain turns out to be formally identical to that expected from the lowest-order Born approximation) and that the method employed is completely covariant and therefore free from the usual difficulties and ambiguities associated with the potential approach.

Even if the postulated vector meson is too short-lived to be regarded as a "particle," a system of three well-correlated pions in resonance is expected to behave like a particle as far as virtual effects are concerned. In the dispersion-theoretic language this means that we can justifiably replace the continuum contribution corresponding to the resonance by a single pole, provided that the resonance is sufficiently sharp.

The one-vector-meson exchange contribution to the pp scattering amplitude in the center-of-mass system can be written down as follows:

$$T_{fi}=-\left(\frac{f_V{}^2}{4\pi}\right)\frac{m_N{}^2}{E}\left[\frac{\bar{u}(\mathbf{p}_f)\gamma_\mu u(\mathbf{p}_i)\bar{u}(-\mathbf{p}_f)\gamma_\mu u(-\mathbf{p}_i)}{(\mathbf{p}_f-\mathbf{p}_i)^2+\mu_V{}^2} -\frac{\bar{u}(\mathbf{p}_f)\gamma_\mu u(-\mathbf{p}_i)\bar{u}(-\mathbf{p}_f)\gamma_\mu u(\mathbf{p}_i)}{(\mathbf{p}_f+\mathbf{p}_i)^2+\mu_V{}^2}\right], \quad (1)$$

where $f_V{}^2/4\pi$ is the coupling constant (analogous to $e^2/4\pi=1/137$) that characterizes the vector coupling of the postulated meson to the nucleon. Now consider the scattering matrix of the form written by Wolfenstein[8]:

$$\begin{aligned}M=BS+C(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot\mathbf{n}+\tfrac{1}{2}G(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{K}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{K}\\+\boldsymbol{\sigma}^{(1)}\cdot\mathbf{P}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{P})T+\tfrac{1}{2}H(\boldsymbol{\sigma}^{(1)}\cdot\mathbf{K}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{K}\\-\boldsymbol{\sigma}^{(1)}\cdot\mathbf{P}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{P})T+N\boldsymbol{\sigma}^{(1)}\cdot\mathbf{n}\boldsymbol{\sigma}^{(2)}\cdot\mathbf{n}T, \quad (2)\end{aligned}$$

where S and T are singlet and triplet projection operators and $\mathbf{n}$, $\mathbf{K}$ and $\mathbf{P}$ stand for unit vectors along $\mathbf{p}_i\times\mathbf{p}_f$, $\mathbf{p}_f-\mathbf{p}_i$, and $\mathbf{p}_f+\mathbf{p}_i$, respectively. Our basic assumption is that the amplitude C which is the direct and major manifestation of the spin-orbit force has its sole origin in (1). Then

$$C=-i\left(\frac{f_V{}^2}{4\pi}\right)\frac{3\sin\theta}{4E}\left[x_0+\frac{k^2x^2}{3(E+m_N)^2}\right]\Big/(x_0{}^2-x^2), \quad (3)$$

where

$$x_0=1+\mu_V{}^2/2k^2, \quad x=\cos\theta,$$
$$k=|\mathbf{p}_i|=|\mathbf{p}_f|, \quad E=(k^2+m_N{}^2).$$

We are now in a position to compare our theoretical prediction (3) with the results of the phase-shift

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[1] Y. Nambu, Phys. Rev. **106**, 1366 (1957).
[2] Y. Fujii, Progr. Theoret. Phys. (Kyoto) **21**, 232 (1959).
[3] G. F. Chew, Phys. Rev. Letters **4**, 142 (1960).
[4] J. J. Sakurai, Ann. Phys. (to be published).
[5] J. J. Sakurai, Nuovo cimento **16**, 388 (1960).
[6] While the present work was in progress the author has been informed by Dr. H. P. Noyes that a rather extensive treatment of two-pion exchange contributions is being carried out at Livermore and Berkeley. Our work will be of particular interest if conventional two-pion exchange contributions fail to give a sufficiently large spin-orbit force.
[7] G. F. Chew, Phys. Rev. **112**, 1380 (1958).
[8] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954), see Eq. (3.4).

analysis at 310 Mev carried out by Stapp, Ypsilantis, and Metropolis[9] (hereafter referred to as SYM). We consider only the SYM solutions 1 and 2 modified by Czifra et al.[10] and by MacGregor et al.[11] since they seem to be the only acceptable sets of phase shifts. The real and imaginary parts of the Wolfenstein amplitudes are tabulated in Table V of reference 11.

First of all, in order that our approach has any merit and validity at all, it is crucial that the Wolfenstein amplitude C deduced from the SYM phase-shift analysis be purely imaginary to a good approximation. This requirement is indeed satisfied for the SYM solutions 1 and 2; in each case the real part of C is smaller than the imaginary part of C by a factor of 10 to 80. Note that for each of the four other amplitudes the real and imaginary parts are comparable, which means that the C amplitude is the *only* Wolfenstein amplitude that *can* be interpreted *in its entirety* in terms of a simple pole contribution.[12]

As far as $\mathrm{Im}(C)$ is concerned, the SYM solutions 1 and 2 differ at most by 5% between $\theta=0°$ and $\theta=60°$, so we consider the solution 1 only. In Fig. 1, $-\mathrm{Im}(C)/\sin\theta$ is plotted for the SYM solution 1 as a dashed line, and our theoretical predictions based on (3) are given for various values of the mass of the postulated vector meson in unbroken lines. The quantity plotted would be constant if the range of the spin-orbit force were much shorter than the center-of-mass de Broglie wavelength of the nucleon (which is 0.52×10^{-13} cm at 310 Mev lab) in which case only the triplet P states would be affected. $\mathrm{Im}(C)/\sin\theta$ corresponding to the SYM solution 1 deviates appreciably from a constant value, and we can pin down the range of the spin-orbit force, hence the mass of the vector meson. The mass value of $\approx 3\mu_\pi$ to $4\mu_\pi$ seems to be most reasonable from Fig. 1. Needless to say, this value should not be taken too seriously since the amount of anisotropy in $\mathrm{Im}(C)/\sin\theta$ is very sensitive to the ^3F phase shifts. Our analysis does point out, however, that the range of the spin-orbit force is neither as short as $1/m_N$ nor as long as $1/\mu_\pi$.

Once μ_V is given, we can determine the coupling constant, which turns out to be

$$f_V^2/4\pi\approx 4 \quad \text{for} \quad \mu_V=3\mu_\pi, \quad f_V^2/4\pi\approx 7 \quad \text{for} \quad \mu_V=4\mu_\pi.$$

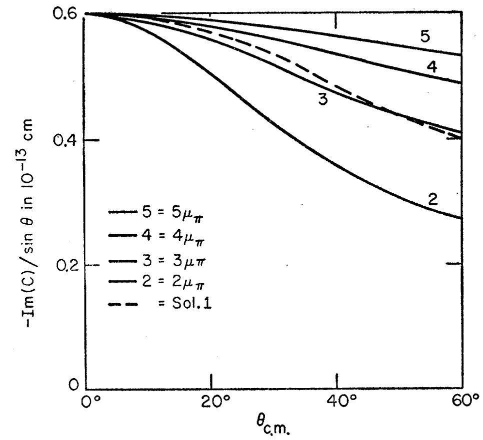

FIG. 1. $-\mathrm{Im}(C)/\sin\theta$ is plotted as a function of θ. The broken line represents the result obtained from the phase-shift analysis for the SYM solution 1 as given in MacGregor et al. (see reference 11). The solid lines are based on our theoretical predictions normalized at $\theta=0°$.

These values are smaller than a crude estimate made earlier (which gave $f_V^2/4\pi\approx 20$) from "potential" considerations.[4,13] The disagreement can be easily traced back to the fact that previously only the Thomas-type spin-orbit force arising from the repulsive *static* potential (responsible for the hard core) was considered. The spin-orbit force arising from the "radiation" field (the so-called Breit term) is twice as large (but fortunately of the same sign) as the Thomas term, as is well known from relativistic two-electron problems.[14]

To sum up, the existence of the spin-orbit force between two nucleons is no more mysterious than that of the spin-orbit force in atomic physics, provided that there exists a strongly interacting neutral vector meson (or a sharp resonance in the $T=0$, $J=1$ state of the three-pion system) with mass $\approx 3\mu_\pi$-$4\mu_\pi$. In the future we plan to carry out similar analyses at different energies and also for the np (triplet even) case.

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Note added in proof.—Recently G. Breit has also proposed that the repulsive core and the spin-orbit force may be understood in terms of a neutral vector meson field [Proc. Nat. Acad. Sci. **46**, 746 (1960)].

[9] H. P. Stapp, T. J. Ypsilantis, and N. Metropolis, Phys. Rev. **105**, 302 (1957).

[10] P. Cziffra, M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **114**, 880 (1959).

[11] M. H. MacGregor, M. J. Moravcsik, and H. P. Stapp, Phys. Rev. **116**, 1248 (1959).

[12] As is well known, the lowest-order Born-type calculations give purely real amplitudes for B, iC, G, H, and N.

[13] P. S. Signell, R. Zinn, and R. E. Marshak, Phys. Rev. Letters **1**, 416 (1958).

[14] See G. Breit, Phys. Rev. **34**, 55 (1929), and Eqs. (39.14) and (40.3) of H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-electron Atoms* (Academic Press, Inc., New York, 1957), p. 170–185.