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# Dispersion Relations for Pion-Hyperon Production and a Possible Pion-Hyperon Resonance*

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Approximate $P$-wave dispersion relations are derived, but not proved, for the processes $\bar{K}+N \rightarrow \pi+\Sigma$ (or $\Lambda$) under the assumption that all pion-baryon and $K$-baryon interactions are of the odd intrinsic-parity, Yukawa type. By comparing these equations with the analogous equations for pion-hyperon scattering, it is shown that a low-energy resonance is likely to occur in a particular combination of the isotopic spin one, $P_{\frac{3}{2}}$, $\pi+\Lambda$ and $\pi+\Sigma$ scattering states, and this resonance should be recognizable in the $\pi+Y$ production processes. Despite complications associated with the unphysical region in the $\pi+Y$ production dispersion relations, and with the fact that the $K$ interactions are moderately strong, the resonance should occur in the production process in a direct and simple way. Measurements of the cross section for production of this resonance state can give information about the strengths of the strong interactions, particularly the pion-baryon interactions. The present experimental evidence concerning the existence of such a resonance effect in $\pi+Y$ production is favorable, but inconclusive. A short discussion is given of the additional experimental information needed to test the resonance hypothesis.

## I. INTRODUCTION

IF, as is generally believed, the primary mechanism for the binding of $\Lambda$ particles in light nuclei is the exchange of virtual $\pi$ mesons between the $\Lambda$ and the nucleons, the interactions of pions with $\Lambda$ and $\Sigma$ particles must be among the strongest of all particle interactions. Hence, a knowledge of the $\pi\Lambda\Sigma$ and the $\pi\Sigma\Sigma$ interactions is essential to understanding the strange particles. One of the best ways of investigating these interactions is to study the processes $\bar{K}+N \rightarrow \pi+Y$ (where $Y$ denotes either a $\Sigma$ or $\Lambda$ hyperon), since these are the only reactions involving a two-particle $\pi+Y$ state that can be produced copiously with present experimental techniques. The interpretation of these $\pi+Y$ production processes depends on the general nature of the pion and $K$-meson strong interactions; in this paper it is assumed that all meson-baryon interactions are of the odd intrinsic-parity, Yukawa type. It is further assumed that the spins of $K$ particles and hyperons are zero and one-half, respectively, so that the initial and final orbital angular momenta are the same in the $\pi+Y$ production reactions.

The $\pi+Y$ production amplitudes have poles at unphysical values of the center-of-mass system energy below the reaction threshold; the residues of these poles involve the product of one of the pion-baryon coupling constants ($F_{NN}$, $F_{\Sigma\Sigma}$, or $F_{\Lambda\Sigma}$) with one of the $K$-coupling constants ($G_{\Lambda N}$ or $G_{\Sigma N}$). Approximate values of the $K$-coupling constants may be obtained from the analysis of other processes, such as $K$-nucleon scattering. Hence, if the $\pi+Y$ production amplitudes satisfy dispersion relations, it may be possible to determine $F_{\Sigma\Sigma}$ and $F_{\Lambda\Sigma}$ by applying these relations to experimental hyperon-production data. Many authors have discussed the implications of the $S$-wave $K^{-}-p$ data with respect to the pion-hyperon interactions.[1] Unfortunately, it is difficult to relate the size of $S$-wave amplitudes to the strengths of coupling constants, because of the subtraction that must be made in deriving $S$-wave dispersion relations. In this paper we are concerned with the $P$-wave amplitudes for $\pi+Y$ production. The strong pion-hyperon interactions are likely to lead to a low-energy resonance in some $P$-wave state of pion-hyperon scattering. Such a resonance should appear in the corresponding pion-hyperon production amplitude, in much the same way that the pion-nucleon $P_{\frac{3}{2}}$ resonance appears in photopion production. The measured characteristics of pion-hyperon production in the resonance region would yield information concerning the pion-hyperon coupling constants. This procedure for measuring $F_{\Lambda\Sigma}$ and $F_{\Sigma\Sigma}$ has been suggested previously by Capps and Nauenberg.[2]

In Secs. II and III of this paper approximate $P$-wave dispersion relations are derived (but not proved) for pion-hyperon production by generalizing the procedure used for pion-nucleon scattering by Chew, Goldberger, Low, and Nambu.[3] The starting point of the derivation is the fixed momentum-transfer hyperon-production dispersion relations of Jin.[4] The $P$-wave relations are static in the sense that the particle center-of-mass momentum is considered small compared to the average baryon mass; however, it is not assumed that the $K$-meson total energy, $\pi$-meson total energy, or the baryon mass-difference is small.

Methods of solving the $P$-wave equations are discussed in Secs. IV through VII. Since crossing symmetry relates the processes $\bar{K}+N \rightarrow \pi+Y$ and $\pi+N \rightarrow$

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[1] See, for example, D. Amati and B. Vitale, Nuovo cimento **9**, 895 (1958); Ken Kawarabayashi, Progr. Theoret. Phys. (Kyoto) **22**, 451 (1959); R. H. Capps, Phys. Rev. **118**, 1097 (1960).
[2] R. H. Capps and M. Nauenberg, Phys. Rev. **118**, 593 (1960). This paper will be referred to by the symbol CN.
[3] G. F. Chew, M. L. Goldberger, F. E. Low, and Y. Nambu, Phys. Rev. **106**, 1337 (1957).
[4] Y. S. Jin, Nuovo cimento **12**, 455 (1959).

$K+Y$, and since unitarity relates $\pi+Y$ production, $\pi-Y$ scattering, and $\bar{K}-N$ scattering on the one hand, and $K+Y$ production, $\pi-N$ scattering, and $K-Y$ scattering on the other, the dispersion relations are coupled integral equations for all the above processes. Fortunately, approximate solutions for the $\pi+Y$ production amplitudes may be obtained by considering only these amplitudes and the pion-hyperon scattering amplitudes. These $P$-wave $\pi+Y$ production dispersion relations will be useful only if a resonance exists in some $\pi-Y$ $P$-wave state. The theoretical possibility of such a resonance is discussed in Sec. VI and the experimental evidence concerning the possible presence of a resonance effect in $\pi+Y$ production is discussed in Sec. VIII.

## II. THE COVARIANT DISPERSION RELATIONS

We first express the reaction amplitudes and dispersion relations in terms of Lorentz-invariant combinations of the momentum variables; later the relations will be specialized to the center-of-mass Lorentz system. We consider any strong reaction in which both the initial and final states consist of one spin-zero meson and one spin-$\frac{1}{2}$ baryon. The four-momenta of the initial and final mesons are denoted by $k_i$ and $k_f$, those of the baryons by $p_i$ and $p_f$. The four-momentum-transfer squared $q^2$ and the covariant energy parameter $\nu$ are defined by the relations,

$$q^2=(p_i-p_f)^2,$$
$$\nu=-(k_i+k_f)\cdot(p_i+p_f),$$

where the scalar product of two four-vectors is defined by the relation $\alpha\cdot\beta=\boldsymbol{\alpha}\cdot\boldsymbol{\beta}-\alpha_0\beta_0$. The constants $\hbar$ and $c$ are set equal to unity. If the relative parity of the reaction is even, the covariant reaction amplitude $T$ may be written in the form,

$$T=A+\tfrac{1}{2}\gamma\cdot(k_i+k_f)B,$$

where $\gamma$ is the Dirac gamma-matrix four-vector, and $T$ is considered as operating between initial and final baryon spinors. The gamma-matrices and spinors are defined in the manner of Schweber, Bethe, and de Hoffmann.[5]

Crossing symmetry relates the process $M_1+B_1\rightarrow M_2+B_2$ to the process $\bar{M}_2+B_1\rightarrow\bar{M}_1+B_2$, where $M_i$ and $B_i$ represent definite charge states of the mesons and baryons, respectively, and $\bar{M}_i$ is the antiparticle of $M_i$ (i.e., $\bar{\pi}^+=\pi^-$, $\bar{\pi}^0=\pi^0$, $\bar{K}^+=K^-$, $\bar{K}^0=\bar{K}^0$). Hence we define the crossed amplitude $T^{\mathrm{cr}}$ corresponding to the amplitude $T$ by the relation,

$$T^{\mathrm{cr}}(M_1+B_1\rightarrow M_2+B_2)=T(\bar{M}_2+B_1\rightarrow\bar{M}_1+B_2).$$

If the amplitudes $T$ are defined for complex values of $\nu$ in such a way that $T$ is analytic in the entire upper half $\nu$-plane, it may be shown by well-known methods that the crossing symmetry for the covariant amplitudes $A$ and $B$ is[4]

$$A^{\mathrm{cr}}(\nu,q^2)=A^*(-\nu,q^2),\qquad B^{\mathrm{cr}}(\nu,q^2)=-B^*(-\nu,q^2).\tag{1}$$

In order to express the crossing symmetry in terms of hyperon-production amplitudes corresponding to definite isotopic spin states, it is useful to consider the following three processes: (a) $K^-+p\rightarrow\pi^0+\Sigma^0$, (b) $K^-+p\rightarrow\pi^0+\Lambda$, and (c) $K^-+n\rightarrow\pi^0+\Sigma^-$. The amplitudes for these three processes are given, respectively, by $6^{-\frac{1}{2}}T_{0\Sigma}$, $2^{-\frac{1}{2}}T_{1\Lambda}$, and $2^{-\frac{1}{2}}T_{1\Sigma}$, where the first subscript denotes the isotopic spin, and the second denotes the nature of the hyperon involved.[6] By considering these processes, one can show easily that the crossing relations for the different hyperon-production amplitudes are:

$$\begin{aligned}T_{0\Sigma}{}^{\mathrm{cr}}&=(\tfrac{2}{3})^{\frac{1}{2}}(2T_{\frac{3}{2}\Sigma}+T_{\frac{1}{2}\Sigma}),\\ T_{1\Sigma}{}^{\mathrm{cr}}&=\tfrac{2}{3}T_{\frac{3}{2}\Sigma}-\tfrac{2}{3}T_{\frac{1}{2}\Sigma},\\ T_{1\Lambda}{}^{\mathrm{cr}}&=-(\tfrac{2}{3})^{\frac{1}{2}}T_{\frac{1}{2}\Lambda},\end{aligned}\tag{2}$$

where $T_{\frac{3}{2}\Sigma}$, $T_{\frac{1}{2}\Sigma}$, and $T_{\frac{1}{2}\Lambda}$ refer to the processes $\pi+N\rightarrow K+Y$.

The residues of the poles of the hyperon-production amplitudes explicitly depend on the nature of the interactions between the particles. We assume the usual five local, pseudoscalar meson-baryon interactions; the dependence of these interactions on the charge states of the particles is represented by the equation,

$$\begin{aligned}H=&F_N\boldsymbol{\pi}\cdot(\bar{N}\boldsymbol{\tau}N)-iF_\Sigma\boldsymbol{\pi}\cdot(\bar{\boldsymbol{\Sigma}}\times\boldsymbol{\Sigma})\\ &+[F_\Lambda\boldsymbol{\pi}\cdot\boldsymbol{\Sigma}\bar{\Lambda}+G_\Lambda(K\mathbf{1}\bar{N})\Lambda+G_\Sigma(K\boldsymbol{\tau}\bar{N})\cdot\boldsymbol{\Sigma}+\mathrm{H.c.}.]\end{aligned}\tag{3}$$

The quantities $F$ and $G$ are the $\pi$ and $K$ coupling constants, $\boldsymbol{\pi}$ and $\boldsymbol{\Sigma}$ represent the isotopic vector $\pi$ and $\Sigma$ field operators, and $\boldsymbol{\tau}$ and $\mathbf{1}$ are the isotopic spin and unit operator that operate between two isotopic spinors. Jin[4] has derived the fixed momentum-transfer dispersion relations for $\Sigma$ and $\Lambda$ production, though these relations have not been proved. Expressed in terms of the quantities defined in this section, the $\pi+Y$ production dispersion relations are:

$$\begin{aligned}\mathrm{Re}A_j(\nu,q^2)&=\frac{2(m_\Lambda-m)\mathcal{G}_j(\Lambda)}{\nu-\nu_\Lambda}+\frac{2(m_\Sigma-m)\mathcal{G}_j(\Sigma)}{\nu-\nu_\Sigma}\\ &+\frac{2(m-m_N)\mathcal{G}_j(N)}{\nu+\nu_N}+\frac{P}{\pi}\int_{\nu_{\Lambda\pi}}^{\infty}\frac{\mathrm{Im}A_j(\nu',q^2)d\nu'}{\nu'-\nu}\\ &+\frac{P}{\pi}\int_{\nu_{N\pi}}^{\infty}\frac{\mathrm{Im}A_j{}^{\mathrm{cr}}(\nu',q^2)d\nu'}{\nu'+\nu},\end{aligned}\tag{4a}$$

[5] S. S. Schweber, H. A. Bethe, and F. de Hoffmann, *Mesons and Fields* (Row, Peterson, and Company, Evanston, Illinois, 1956), Vol. 1, pp. 17–20, 46–59.

[6] Throughout this paper the relative phases of the different charge states differ from those that correspond to the Clebsch-Gordan coefficients of Condon and Shortley only in that the $\Sigma^+$ and $\pi^+$ phases are chosen opposite to those of this reference. See E. U. Condon and G. E. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, New York, 1935).

$$\mathrm{Re}B_j(\nu,q^2)=\frac{2\mathcal{G}_j(\Lambda)}{\nu-\nu_\Lambda}+\frac{2\mathcal{G}_j(\Sigma)}{\nu-\nu_\Sigma}+\frac{2\mathcal{G}_j(N)}{\nu+\nu_N}+\frac{\mathrm{P}}{\pi}\int_{\nu_{\Lambda\pi}}^{\infty}\frac{\mathrm{Im}B_j(\nu',q^2)d\nu'}{\nu'-\nu}-\frac{\mathrm{P}}{\pi}\int_{\nu_{N\pi}}^{\infty}\frac{\mathrm{Im}B_j^{\mathrm{cr}}(\nu',q^2)d\nu'}{\nu'+\nu}, \quad (4b)$$

where P denotes the principal part of the integral, and $j$ denotes the nature of the hyperon and the total isotopic spin. The quantities $m_\Lambda$, $m_\Sigma$ and $m_N$ are the masses of the $\Lambda$, $\Sigma$, and nucleon, while $m$ is the average of the initial and final baryon masses. The values of the energy parameter at the poles ($\nu_\Lambda$, $\nu_\Sigma$, and $\nu_N$) are functions of momentum-transfer and are given by the formula, $\nu_\alpha=2m_\alpha^2-m_i^2-m_f^2-\mu_i^2-\mu_f^2-q^2$ where $m_\alpha$ is the mass of the baryon denoted by $\alpha$, and $m_i$, $m_f$, $\mu_i$, and $\mu_f$ are the masses of the initial baryon, final baryon, initial meson, and final meson. The coefficients $\mathcal{G}_j$ in the Born approximation terms (pole terms) of the dispersion relations are given in terms of the renormalized coupling constants of Eq. (3) by the equations,

$$\begin{aligned}&\mathcal{G}_{0\Sigma}(\Lambda)=6^{\frac{1}{2}}F_\Lambda G_\Lambda,\quad \mathcal{G}_{0\Sigma}(\Sigma)=0,\quad \mathcal{G}_{0\Sigma}(N)=6^{\frac{1}{2}}F_N G_\Sigma,\\&\mathcal{G}_{1\Sigma}(\Lambda)=0,\ \mathcal{G}_{1\Sigma}(\Sigma)=-2F_\Sigma G_\Sigma,\ \mathcal{G}_{1\Sigma}(N)=-2F_N G_\Sigma,\\&\mathcal{G}_{1\Lambda}(\Lambda)=0,\quad \mathcal{G}_{1\Lambda}(\Sigma)=2^{\frac{1}{2}}F_\Lambda G_\Sigma,\quad \mathcal{G}_{1\Lambda}(N)=2^{\frac{1}{2}}F_N G_\Lambda.\end{aligned} \quad (5)$$

The limits of integration $\nu_{\Lambda\pi}$ and $\nu_{N\pi}$ in Eqs. (4) are given by the formula $\nu_{\alpha\pi}=2(m_\alpha+\mu_\pi)^2-m_i^2-m_f^2-\mu_i^2-\mu_f^2-q^2$. It should be noted that the lower portions of the dispersion integrals refer to unphysical energy regions below the thresholds of the $\bar{K}+N\rightarrow\pi+Y$ and $\pi+N\rightarrow K+Y$ processes.

If one takes the energy parameter $\nu$ in Eqs. (4) to be negative, and makes use of the crossing relations [Eqs. (1) and (2)], then Eqs. (4) become the dispersion relations for the processes $\pi+N\rightarrow K+Y$.

### III. THE *P*-WAVE EQUATIONS

In order to write approximate dispersion relations for $P$ waves we must express the various quantities in terms of center-of-mass system parameters. We use the following notation for the particle center-of-mass momenta and energies in any of the various meson-baryon production and scattering reactions: $\mathbf{k}_i$ and $\mathbf{k}_f$, three momenta of initial and final mesons; $k_{0i}$, $k_{0f}$, and $k_0$, initial, final, and average meson energies; $E_i$, $E_f$, and $E$, initial, final, and average baryon energies; $W=E+k_0$, total energy. Henceforth $k_i$ and $k_f$ denote the magnitudes of $\mathbf{k}_i$ and $\mathbf{k}_f$, rather than four-momenta. The most important energy parameter to be used (denoted by $\omega$) is defined as the average meson total energy plus the average baryon kinetic energy, i.e.,

$$\omega=W-m. \quad (6)$$

The center-of-mass pion-hyperon production amplitudes may be expressed in terms of the baryon spin matrices by the relation,

$$T=T_a(W,\cos\theta)+T_b(W,\cos\theta)\boldsymbol{\sigma}\cdot\mathbf{k}_f\boldsymbol{\sigma}\cdot\mathbf{k}_i,$$

where $\theta$ is the angle between the directions of $\mathbf{k}_i$ and $\mathbf{k}_f$. The expansions of $T_a$ and $T_b$ in terms of partial wave amplitudes are[7]:

$$T_a=k_ik_f\sum_{l=0}^{\infty}t_{l+}P_{l+1}'(\cos\theta)-\sum_{l=2}^{\infty}t_{l-}P_{l-1}'(\cos\theta), \quad (7a)$$

$$T_b=\sum_{l=1}^{\infty}(t_{l-}-t_{l+})P_l'(\cos\theta), \quad (7b)$$

where the $P_l$ are Legendre polynomials and the prime denotes differentiation with respect to $\cos\theta$. The $t_{l\pm}$ are the $\pi+Y$ production amplitudes for orbital angular momentum $l$ and total angular momentum $l\pm\frac{1}{2}$. The partial wave amplitudes $t$ for all processes discussed in this paper ($\pi+Y$ production, $\pi-Y$ scattering, etc.) are normalized in terms of the corresponding matrix elements of the unitary $S$ matrix by the equation,

$$2it=(S-\mathbf{1})(k_ik_f)^{-\frac{3}{2}}, \quad (8)$$

where $\mathbf{1}$ is the unit, diagonal matrix. This normalization is not the most common one; it is convenient for $P$-wave amplitudes, however.

We now give the relations between the covariant quantities of Sec. II and the center-of-mass quantities defined above. These relations, for the covariant momentum-transfer and energy parameter, are[4]:

$$q^2=-\mu_i^2-\mu_f^2+2k_{0i}k_{0f}-2\mathbf{k}_i\cdot\mathbf{k}_f, \quad (9a)$$

$$\nu=2W^2-m_i^2-m_f^2-\mu_i^2-\mu_f^2-q^2. \quad (9b)$$

The relations between the covariant and center-of-mass amplitudes may be found by well-known methods,[8] and are given by the equations,

$$T_a=\frac{(E_i+m_i)^{\frac{1}{2}}(E_f+m_f)^{\frac{1}{2}}}{2m}\tau_a;\quad \tau_a=\frac{m}{W}[A-B(W-m)], \quad (10a)$$

$$T_b=\frac{2m}{(E_i+m_i)^{\frac{1}{2}}(E_f+m_f)^{\frac{1}{2}}}\tau_b;\quad \tau_b=\frac{1}{4mW}[-A-B(W+m)]. \quad (10b)$$

One may write fixed momentum-transfer dispersion relations for the center-of-mass amplitudes $T_a$ and $T_b$ by taking the real parts of Eqs. (10), expressing ReA

[7] These equations are equivalent to Eqs. (2.18) and (2.19) of reference 3.

[8] See R. H. Capps and G. Takeda, Phys. Rev. **103**, 1877 (1956), Appendix A.

and ReB in terms of the covariant dispersion relations, [Eqs. (4)], and then using the inverses of Eqs. (10) to express ImA and ImB back in terms of Im$T_a$ and Im$T_b$. A variety of dispersion relations for different combinations of the partial wave amplitudes may then be derived by multiplying by different Legendre polynomials in $\cos\theta$ and integrating over $\cos\theta$. However, in order to obtain simple equations involving only $P$-wave amplitudes, some sort of static approximation must be made. Since the $K$-meson mass is not small compared to the baryon masses, the usual procedure of neglecting all terms of order $\omega/m$ is not satisfactory here. We follow an alternate procedure, choosing as expansion parameters the two quantities,

$$\epsilon_1=(k_i^2+k_f^2)/(2\omega m) \quad \text{and} \quad \epsilon_2=(k_ik_f)/(\omega m),$$

which are considered to be of the same order. (The symbol $\epsilon$ will often be used to denote either of these quantities.) These two parameters remain fairly small in the low-energy region for the reactions $\bar{K}+N\rightarrow\pi+Y$, e.g., at 420 Mev/$c$ lab $K$-meson momentum, each of the two parameters is about 0.14 for $\pi+\Sigma$ production, and about 0.17 for $\pi+\Lambda$ production. The approximation of neglecting $\epsilon_1$ and $\epsilon_2$ compared to unity will be termed the "small-momentum approximation."

The parameters $\nu'$ and $\nu'\pm\nu$ occurring in the dispersion integrals of Eqs. (4) can be expressed in terms of the center-of-mass variables $\omega'$, $\omega$, and $\mathbf{k}_i\cdot\mathbf{k}_f$ by the relations:

$$d\nu'=4(m+\omega')d\omega', \tag{11a}$$

$$\nu'-\nu=2(\omega'-\omega)(2m+\omega'+\omega), \tag{11b}$$

$$\nu'+\nu=4m(\omega'+\omega)[1+O(\epsilon)]+4\mathbf{k}_i\cdot\mathbf{k}_f, \tag{11c}$$

where $O(\epsilon)$ denotes a quantity of order $\epsilon_1$ or $\epsilon_2$ that depends only on $\omega$ (i.e., is independent of $\cos\theta$). In deriving Eqs. (11), explicit use is made of the fact that the momentum-transfer is fixed. We now write the fixed momentum-transfer dispersion relations for the $\pi+Y$ production amplitudes by following the procedure described above, i.e., by making use of Eqs. (4), (10), and (11). For simplicity the relations are given for the amplitudes $\tau_a$ and $\tau_b$ of Eqs. (10) rather than for $T_a$ and $T_b$, and the isotopic spin index $j$ is suppressed.

$$\mathrm{Re}\tau_b(\omega)=\mathrm{B.A.}-\frac{P}{\pi}\int_{\mu_\pi+m_\Lambda-m}^{\infty}\frac{d\omega'(m+\omega')\,\mathrm{Im}[(2m+\omega+\omega')\tau_b(\omega')+(4m^2)^{-1}(\omega-\omega')\tau_a(\omega')]}{(\omega'-\omega)(2m+\omega+\omega')(m+\omega)}$$
$$-\frac{P}{\pi}\int_{\mu_\pi+m_N-m}^{\infty}\frac{d\omega'(m+\omega')\,\mathrm{Im}[(2m+\omega-\omega')\tau_b^{\mathrm{cr}}(\omega')+(4m^2)^{-1}(4m+\omega+\omega')\tau_a^{\mathrm{cr}}(\omega')]}{\{(\omega'+\omega)[1+O(\epsilon)]m+\mathbf{k}_i\cdot\mathbf{k}_f\}2(m+\omega)}, \tag{12}$$

$$\mathrm{Re}\tau_a(\omega)=\mathrm{B.A.}+\frac{P}{\pi}\int_{\mu_\pi+m_\Lambda-m}^{\infty}\frac{d\omega'(m+\omega')\,\mathrm{Im}[(2m+\omega+\omega')\tau_a(\omega')+4m^2(\omega-\omega')\tau_b(\omega')]}{(\omega'-\omega)(2m+\omega+\omega')(m+\omega)}$$
$$+\frac{P}{\pi}\int_{\mu_\pi+m_N-m}^{\infty}\frac{d\omega'(m+\omega')\,\mathrm{Im}[(2m+\omega'-\omega)\tau_a^{\mathrm{cr}}(\omega')-4m^2(\omega'+\omega)\tau_b^{\mathrm{cr}}(\omega')]}{\{(\omega'+\omega)[1+O(\epsilon)]m+\mathbf{k}_i\cdot\mathbf{k}_f\}2(m+\omega)}, \tag{13}$$

where the amplitudes are all evaluated at a fixed momentum-transfer $q^2$. The Born approximation terms in these equations (denoted by B.A.) have not been written out; they may be computed easily from the Born terms of Eq. (4), by making use of the following relations,

$$\nu-\nu_Y=2[\omega-(m_Y-m)][m+m_Y+\omega),$$
$$\nu+\nu_N=2[\omega+(m_N-m)][m+m_N-\omega][1+O(\epsilon)]+4\mathbf{k}_i\cdot\mathbf{k}_f, \tag{14}$$

where $Y$ denotes either the $\Sigma$ or $\Lambda$ particle. Nothing has been neglected in writing down Eqs. (12), (13), and (14); the only small terms not written out explicitly are represented by the symbol $O(\epsilon)$.

In the small-momentum approximation the amplitudes $\tau_a$ and $\tau_b$ in the dispersion relations may be replaced by $T_a$ and $T_b$. In order to obtain simple equations some assumption must be made concerning the relative magnitudes of the $t_{l\pm}$. We shall use the Born approximation as a guide in making such an assumption. It is easily shown from Eqs. (4) and (14) that in Born approximation, $t_{l\pm}$ is proportional to the $l$th power of the expansion parameters $\epsilon$, i.e., $k_ik_ft_{l\pm}\approx FGm^{-1}(\epsilon)^l$. Hence, we assume that the actual amplitudes for all angular momenta greater than one are smaller than the $P$-wave amplitudes by at least one power of $\epsilon$. We further assume that the imaginary parts of the $D$ amplitudes are smaller than the imaginary parts of the $P$ amplitudes by a factor of $\epsilon^2$, since this condition is true in perturbation theory.

Similar assumptions are made concerning the partial wave amplitudes $t_{l\pm}^{\mathrm{cr}}$ that refer to the "crossed" processes $\pi+N\rightarrow K+Y$. The threshold for $K+Y$ production is sufficiently high that the small-momentum approximation is not accurate in the physical region for these processes. However, in the low-energy dispersion relations for $\pi+Y$ production, the only large contributions to the crossed dispersion integrals are expected to come from the $K+Y$ production amplitudes in the low-energy part of the unphysical region, where the small-momentum approximation is valid. We further assume that the $S$-wave quantity $\frac{1}{2}k_ik_ft_0^{\text{cr}}/m^2$ is small compared with $t_{1+}^{\text{cr}}$, since this quantity must be small compared to a large $P$-wave amplitude, and only large $P$ amplitudes are expected to contribute appreciably to the "crossed" dispersion integral.

In the small-momentum approximation the dispersion relation for the $P$-wave spin-flip amplitude $t_{1S}=(t_{1-}-t_{1+})$ may be derived simply by neglecting all terms of order $\epsilon$ in Eq. (12), and making the above assumptions concerning the relative sizes of the $t_{l\pm}$. The result is

$$\begin{aligned}\text{Re}t_{j,1s}(\omega)&\\=&-\frac{\mathcal{G}_j(\Lambda)}{4(m_im_f)^{\frac{1}{2}}(m+\omega)}\frac{1}{[\omega-(m_\Lambda-m)]}\\&-\frac{\mathcal{G}_j(\Sigma)}{4(m_im_f)^{\frac{1}{2}}(m+\omega)}\frac{1}{[\omega-(m_\Sigma-m)]}\\&-\frac{\mathcal{G}_j(N)(3m-m_N+\omega)}{4(m_im_f)^{\frac{1}{2}}(m+\omega)(m+m_N-\omega)}\frac{1}{[\omega+(m_N-m)]}\\&+\frac{P}{\pi}\int_{\mu_\pi+m_\Lambda-m}^{\infty}\frac{d\omega'\,\text{Im}t_{j,1s}(\omega')}{\omega'-\omega}\\&-\frac{P}{\pi}\int_{\mu_\pi+m_N-m}^{\infty}\frac{d\omega'\,\text{Im}t_{j,1s}^{\text{cr}}(\omega')}{\omega'+\omega},\end{aligned}\quad(15)$$

where $j$ again denotes the nature of the hyperon and the total isotopic spin, and the $\mathcal{G}_j$ are given in Eq. (5).

The dispersion relations for the amplitudes $t_{1+}$ may be derived by multiplying Eq. (13) by $\cos\theta$, integrating over $\cos\theta$, and dividing by $3k_ik_f$. Terms of order $\epsilon$ may then be neglected. In carrying out the integral it must be kept in mind that $\theta'$ (the angle corresponding to $\omega'$) and $\theta$ refer to the same momentum-transfer $q^2$ and hence are related to each other by the equation

$$k_i'k_f'\cos\theta'-k_{0i}'k_{0f}'=k_ik_f\cos\theta-k_{0i}k_{0f}.$$

The resulting small-momentum approximation dispersion relations for $t_{1+}$ are,

$$\begin{aligned}\text{Re}t_{j,1+}(\omega)&\\=&\frac{2(m_im_f)^{\frac{1}{2}}\mathcal{G}_j(N)}{3(m+\omega)(m+m_N-\omega)^2}\frac{1}{[\omega+(m_N-m)]}\\&+\frac{P}{\pi}\int_{\mu_\pi+m_\Lambda-m}^{\infty}\frac{d\omega'\,\text{Im}t_{j,1+}(\omega')}{\omega'-\omega}\\&+\frac{P}{\pi}\int_{\mu_\pi+m_N-m}^{\infty}\frac{d\omega'\,\text{Im}[t_{j,1+}^{\text{cr}}(\omega')+2t_{j,1-}^{\text{cr}}(\omega')]}{3(\omega'+\omega)}.\end{aligned}\quad(16)$$

If the energy $\omega$ lies in the unphysical region $(\omega<\mu_K+m_N-m)$ for the process $t_j$, a slight modification of the above procedure is required. In this region the momentum in the $\bar{K}+N$ state is positive imaginary $k_K=i|k_K|$, so that the $P$-wave term of $T_a$ is equal to $i|k_K|k_\pi(3\cos\theta)t_{j,1+}$. Hence, one writes the dispersion relation for $-iT_a$, rather than for $T_a$, multiplies by $\cos\theta$ and integrates over $\cos\theta$ and then divides by $3|k_K|k_\pi$. The result is identical in form to Eq. (16).

Equations (15) and (16) also apply to the $\pi-Y$, $\pi-N$, $K-Y$, and $K-N$ elastic scattering processes if the constants $\mathcal{G}_j$, intermediate baryon masses ($m_N$, $m_\Lambda$, and $m_\Sigma$), and the limits of integration are suitably modified. If the equations are applied to $\pi-N$ scattering and $\omega/m$ is neglected, the equations become identical with the usual static equations, except that the energy variable is $\omega=W-m$ rather than the meson energy.[9]

## IV. THE UNITARITY CONDITIONS

The approximate $P$-wave dispersion relations of Sec. III can be used to study the possible behavior of the hyperon-production amplitudes only after the real and imaginary parts of the various amplitudes are related by means of the unitarity condition. In applying unitarity to inelastic processes, we assume that the phases of the various states are so chosen that the scattering matrix is symmetric in the angular momentum representation; the time reversal invariance of the strong interactions assures that this can be done.[10] This phase choice already has been made implicitly in the original derivation of the covariant dispersion relations in reference 4.

We denote by $\mathbf{t}_{\alpha\beta}$ the amplitude for the process $\alpha\rightarrow\beta$, normalized as in Eq. (8), ($\alpha$ and $\beta$ may refer to the same state). Multiple meson processes and all weak processes are neglected, so that $\alpha$ and $\beta$ and all states coupled to them are two-particle $P$-wave states. It may be shown directly from Eq. (8) that the unitarity condition in the physical region for the process $\alpha\rightarrow\beta$ is,

$$\text{Im}\mathbf{t}_{\alpha\beta}=\textstyle\sum_\gamma k_\gamma^3\mathbf{t}_{\alpha\gamma}^*\mathbf{t}_{\beta\gamma},\quad(17)$$

where the sum is over all open channels, and $k_\gamma$ is the momentum in the state $\gamma$. All amplitudes refer to the same total energy.

We will assume that this unitarity condition remains valid in the unphysical energy region below the threshold for the process $\alpha\rightarrow\beta$. This assumption has not been proved. However, a similar condition, used in the unphysical region of the process $N+\bar{N}\rightarrow2\pi$ by Fraser and Fulco,[11] has been justified by Mandelstam on the basis of a few plausible assumptions and the known analytic properties of Green's functions and reaction amplitudes.[12]

If $\alpha$ and $\beta$ differ, and if only these two states contribute to the sum over $\gamma$, Eq. (17) implies that the

[9] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956).
[10] F. Coester, Phys. Rev. **89**, 619 (1953).
[11] W. R. Frazer and J. R. Fulco, Phys. Rev. Letters **2**, 365 (1959).
[12] S. Mandelstam, Phys. Rev. Letters **4**, 84 (1960).

phase of the inelastic amplitude $\mathbf{t}_{\alpha\beta}$ is equal (within an additive factor of $\pi$) to the sum of the real parts of the phase shifts for elastic scattering in the channels $\alpha$ and $\beta$. Such a phase relation applies in the physical region of the $I=0$, $\pi+Y$ production amplitude, and is approximately valid for production in the $I=1$ state also, provided that the two $I=1$, $\pi+Y$ states are chosen in such a way that the amplitude for transitions between them is small. In the unphysical region below $\bar{K}+N$ threshold the $\bar{K}+N$ state does not occur in the sum over $\gamma$, in which case no $\bar{K}-N$ phase shift appears in the phase condition.

Pion-hyperon production, $\pi-Y$ scattering, and $\bar{K}-N$ scattering are coupled by the unitarity conditions, as are $K$-hyperon production, $\pi-N$ scattering, and $K-Y$ scattering. Hence one must write dispersion relations for all these processes in order to get a closed system of equations for the $P$-wave amplitudes. All these $P$-wave dispersion relations may be written in the form of Eqs. (15) and (16) if the Born approximation terms and the limits of the dispersion integrals are changed appropriately. The Born approximation terms are given for pion-nucleon scattering by Chew, Goldberger, Low, and Nambu[3] and for pion-hyperon scattering in CN.

## V. OUTLINE OF METHOD FOR SOLVING EQUATIONS

When the unitarity conditions are all specified, the dispersion relations for the various $P$-wave processes are a closed system of coupled equations for the amplitudes. If explicit values are assumed for the coupling constants of Eq. (3), some type of successive approximation procedure can be used to seek approximate solutions. For illustrative purposes we outline below an iteration method that is useful if the ratios of $K$-coupling constants to $\pi$-coupling constants are small.

The pion-hyperon and pion-nucleon scattering equations are first solved, with the contribution of $K$-particle processes neglected in the unitarity conditions. Appropriate linear combinations of the $I=1$, $\pi+\Lambda$ and $\pi+\Sigma$ states are chosen so that the $\pi-Y$ scattering is diagonalized, approximately. One then solves the equations for $\pi+Y$ and $K+Y$ production, neglecting the $\bar{K}-N$ and $K-Y$ scattering phase shifts in the unitarity conditions. Since the unitarity conditions specify the phases of the hyperon-production amplitudes, the method of Omnès[13] should be convenient for solving these equations. Because of the crossing terms, even this step involves an iteration procedure, i.e., one first solves for the $\pi+Y$ and $K+Y$ production amplitudes by neglecting crossing terms; these amplitudes are then inserted into the crossing terms and the equations are solved again, etc. The result of this sub-procedure gives the first approximation to the general iteration method for the $\pi+Y$ production amplitudes.

The second approximation is carried out in the following way. The results of the first approximation for $\pi+Y$ production are now included in the unitarity conditions for the $\pi-Y$ scattering equations, and these equations are solved again to give a second approximation to the $\pi-Y$ scattering amplitudes. A simultaneous step is the solving of the $\bar{K}-N$ scattering dispersion relations, including in the unitarity condition only the $\bar{K}+N\rightarrow\pi+Y$ amplitudes calculated in the first approximation. One then uses these $\bar{K}-N$ phase shifts, and the improved $\pi-Y$ phase shifts, in the hyperon-production dispersion relations to calculate the second approximation for $\pi+Y$ production. One could continue in this manner to higher approximations. The effective expansion parameters in this general procedure are the squares of the coupling constant ratios $G/F$ [See Eq. (3)] so that the procedure is expected to be convergent if the pion-hyperon interactions are of comparable strength to the pion-nucleon interaction. The fact that the $\bar{K}+N$ rest mass is larger than the $\pi+Y$ rest mass also helps the convergence.

The existing experimental data is so sparse that a lengthy calculation using the above-outlined method, or some similar method, is not justified at present. Furthermore, because of the neglected high-energy contributions, the solutions to the disperison relations are expected to be reliable only for a channel in which a low-energy resonance occurs. The considerations of Sec. IV and this section are useful primarily because they can be used to show that a $P$-wave $\pi+Y$ resonance should produce easily recognizable effects in the $\bar{K}+N\rightarrow\pi+Y$ amplitudes, despite the complications of unphysical regions, moderately strong $K$ coupling, and many coupled channels. In the next section we will discuss the possibility of such a resonance and the information obtainable if one exists.

## VI. A POSSIBLE RESONANT AMPLITUDE

We now consider the questions of whether or not a $\pi+Y$ $P$-wave scattering resonance exists, and in which angular momentum state it may occur, using as a guide the static $\pi-Y$ scattering dispersion relations of CN, and neglecting $K$-meson effects. Throughout the rest of this paper, the $\Sigma-\Lambda$ mass difference will be neglected, and the hyperon mass taken as the average of the experimental $\Sigma$ and $\Lambda$ masses.

In order for a low-energy resonance to exist, it is necessary that the Born approximation terms for the elastic scattering amplitude in question be positive, i.e., represent an attractive interaction. The signs of the Born contributions to many of the $\pi-Y$ amplitudes depend on the value of the coupling constant ratio $F_\Sigma/F_\Lambda$; several authors have listed the various states which may be resonant for different values of this ratio.[14] If the coupling constant ratio is in the range $\frac{1}{2}<(F_\Sigma/F_\Lambda)^2$

[13] R. Omnès, Nuovo cimento 8, 316 (1958).

[14] Michael Nauenberg, Phys. Rev. Letters 2, 351 (1959); A. Komatsuzawa, R. Sugano, and Y. Nogami, Progr. Theoret. Phys. (Kyoto) 21, 151 (1959); Yukihisi Nogami, Progr. Theoret. Phys. (Kyoto) 22, 25 (1959); D. Amati, A. Stanghellini, and B. Vitale, Nuovo cimento 13, 1143 (1959).

<2, it may be seen from Table I of CN that a $\pi-Y$ $P$-wave resonance is possible only for angular momentum $\frac{3}{2}$ and isotopic spin 1 or 2. The possibility of a $P_{\frac{3}{2}}$ $I=1$ resonance is most easily investigated if orthogonal combinations of the $\pi+\Lambda$ and $\pi+\Sigma$ states are chosen in such a way as to approximately diagonalize that part of the scattering matrix referring to these two channels. For simplicity we choose the states (denoted by $\psi_r$ and $\psi_s$) that lead to diagonalization in the Born approximation; the coefficients relating these states to the $\pi+\Lambda$ and $\pi+\Sigma$ states may be determined from Table I of CN, and are given by

$$(F_\Sigma^2+2F_\Lambda^2)^{\frac{1}{2}}\psi_r=\sqrt{2}F_\Lambda\psi_{1\Lambda,1+}+F_\Sigma\psi_{1\Sigma,1+}, \quad (18a)$$

$$(F_\Sigma^2+2F_\Lambda^2)^{\frac{1}{2}}\psi_s=F_\Sigma\psi_{1\Lambda,1+}-\sqrt{2}F_\Lambda\psi_{1\Sigma,1+}. \quad (18b)$$

Even though the amplitudes are expected to differ markedly from the predictions of the Born approximation, we believe this choice of $\psi_r$ and $\psi_s$ approximately diagonalizes the actual scattering, for the following two reasons: (i) In the first approximation for the solution of the $\pi-Y$ scattering dispersion relations, in which the crossing terms are neglected, diagonalization is achieved by the same choice of states that produces diagonalization in Born approximation. (ii) Even if the crossing terms are included, this choice of $\psi_r$ and $\psi_s$ leads to diagonalization in the three special cases $F_\Sigma=0$, $F_\Lambda=0$, and the global symmetry case $F_\Sigma=F_\Lambda$.[15]

We denote the $\pi-Y$ elastic scattering amplitudes by $\mathbf{t}_{j,l\pm}{}^\pi$, where $j$ indicates the isotopic spin and the nature of the initial and final hyperons. If the $P$-wave equations of CN are written in the form used in Sec. III of this paper (i.e., $\omega/m$ terms included) and $m_\Sigma-m_\Lambda$ neglected, the equations for the $P_{\frac{3}{2}}$, $\pi-Y$ scattering amplitudes become

$$\mathrm{Re}\mathbf{t}_{j,1+}{}^\pi(\bar\omega) =\frac{2m_Y\mathfrak{F}_j}{3(m_Y+\bar\omega)(2m_Y-\bar\omega)^2}\frac{1}{\bar\omega}+\frac{P}{\pi}\int_{\mu_\pi}^\infty\frac{d\bar\omega'\,\mathrm{Im}\mathbf{t}_{j,1+}{}^\pi(\bar\omega')}{\bar\omega'-\bar\omega} +\frac{P}{\pi}\int_{\mu_\pi}^\infty\frac{d\bar\omega'\,\mathrm{Im}[\mathbf{t}_{j,1+}{}^{\pi,\mathrm{cr}}(\bar\omega')+2\mathbf{t}_{j,1-}{}^{\pi,\mathrm{cr}}(\bar\omega')]}{3(\bar\omega'+\bar\omega)}, \quad (19)$$

where $\bar\omega=W-m_Y$ ($W$ is the total center-of-mass system energy). The constants $\mathfrak{F}_j$, in the $\psi_r-\psi_s$ representation defined by Eqs. (18), are $\mathfrak{F}_{rr}=F_\Lambda^2+F_\Sigma^2$, $\mathfrak{F}_{ss}=-F_\Lambda^2$, and the off-diagonal term $\mathfrak{F}_{rs}$ is zero. From these values of $\mathfrak{F}_j$ it is seen that the state $\psi_s$ cannot resonate but $\psi_r$ is of the resonance type. In the case of global symmetry the state $\psi_r$ is analogous to the $(\frac{3}{2},\frac{3}{2})$ pion-nucleon state.[15] However, it may be seen by comparing the above result with the $\pi-N$ dispersion relations that, for *any* value of $F_\Sigma/F_\Lambda$, the Born term for scattering in the state $\psi_r$ may be obtained from that of the resonant $\pi-N$ amplitude by replacing $m_N$ by $m_Y$ and $F_N^2$ by $\frac{1}{2}(F_\Lambda^2+F_\Sigma^2)$.

In order to see how a resonance in the state $\psi_r$ may appear in $\pi-Y$ production, we must compare the corresponding dispersion relations for $\pi-Y$ scattering and production. The amplitude $t_r$ for production of the state $\psi_r$ is the linear combination $(F_\Sigma^2+2F_\Lambda^2)^{\frac{1}{2}}t_r=\sqrt{2}F_\Lambda t_{1\Lambda,1+}+F_\Sigma t_{1\Sigma,1+}$; the dispersion relation for $t_r$ is the corresponding linear combination of the relations given in Eq. (16). In order to compare the dispersion relations we write the average baryon mass of Eq. (16) in the form $m=\frac{1}{2}(m_Y+m_N)$. Furthermore, since the unitarity condition relates the amplitudes at the same total center-of-mass system energy $W$, we express the $\pi+Y$ production energy parameter $\omega=W-\frac{1}{2}(m_N+m_Y)$ in terms of the $\pi-Y$ scattering energy $\bar\omega=W-m_Y$ of Eq. (19), i.e., $\omega=\bar\omega+\frac{1}{2}(m_Y-m_N)$. The dispersion relation for the production amplitude $t_r$ is then,

$$\mathrm{Re}t_r(\bar\omega)=\frac{2(m_Ym_N)^{\frac{1}{2}}}{3(m_Y+\bar\omega)(2m_N-\bar\omega)^2}\frac{1}{\bar\omega} \times\frac{2F_N(F_\Lambda G_\Lambda-F_\Sigma G_\Sigma)}{(F_\Sigma^2+2F_\Lambda^2)^{\frac{1}{2}}}+\frac{P}{\pi}\int_{\mu_\pi}^\infty\frac{d\bar\omega'\,\mathrm{Im}t_r(\bar\omega')}{\bar\omega'-\bar\omega} +\text{crossing term}. \quad (20)$$

Since the only amplitudes discussed in the remainder of this section and Sec. VII refer to $P_{\frac{3}{2}}$ states, the angular momentum subscripts 1+ will be consistently omitted.

In order to illustrate the basic connection between the $\pi+Y$ production and scattering amplitudes, we make the simplifying assumptions that only the terms of lowest order in the $K$ to $\pi$ coupling constant ratios $(G/F)$ need be kept, and that the crossing terms in all the dispersion relations may be neglected. [A correction for the appreciable size of the $(G/F)$ is made in Sec. VII.] The terms of lowest order in $G/F$ of the unitary conditions of Sec. IV for scattering and production in the state $\psi_r$ satisfy the relations, $\mathrm{Im}\mathbf{t}_{rr}{}^\pi=k_\pi{}^3|\mathbf{t}_{rr}{}^\pi|^2$ and $\phi_r=\delta_r{}^\pi$ provided that the $\pi-Y$ scattering is approximately diagonalized. The quantities $\phi_r$ and $\delta_r{}^\pi$ denote the phase of $t_r$ and the real part of the phase shift for the elastic scattering process $\mathbf{t}_{rr}{}^\pi$. The phase condition applies to the $\pi+Y$ production amplitude both in the physical and unphysical regions, so that it is not necessary to distinguish between these two regions in solving the equations for small $(G/F)$. Since the ratio of the inhomogeneous terms (Born terms) of Eqs. (19) and (20) is nearly energy-independent in the low-energy region, it is seen from these equations and the phase condition $\phi_r=\delta_r{}^\pi$ that the ratio of the amplitudes $t_r$ and $\mathbf{t}_{rr}{}^\pi$ is nearly energy-independent and is given approximately by the Born terms. If we set $(m_N/m_Y)^{\frac{1}{2}}\times(2m_Y-\bar\omega)^2/(2m_N-\bar\omega)^2\approx(m_Y/m_N)^{\frac{3}{2}}=1.36$, and make use of the condition $\mathfrak{F}_{rr}=F_\Lambda^2+F_\Sigma^2$, the proportionality relation becomes

$$t_r=\xi\mathbf{t}_{rr}{}^\pi;\quad \xi=1.36\frac{2F_N(F_\Lambda G_\Lambda-F_\Sigma G_\Sigma)}{(F_\Lambda^2+F_\Sigma^2)(F_\Sigma^2+2F_\Lambda^2)^{\frac{1}{2}}}. \quad (21)$$

[15] D. Amati and B. Vitale, Nuovo cimento 9, 895 (1958).

Hence, in this approximation, a scattering resonance in the state $\psi_r$ appears in the $\pi+Y$ production in a direct and simple way. If such a resonance exists, three different types of information concerning the coupling constants can be obtained from the following three types of measurements in the resonance region.

(*A*) *Ratio of resonant* $\Sigma$ *to* $\Lambda$ *production.* It is seen from Eq. (18a) that the relative probabilities of $\pi+\Sigma$ and $\pi+\Lambda$ pairs in the resonant state depend on the coupling constant ratio $F_\Sigma/F_\Lambda$. Hence, if protons are bombarded with $K^-$ at energies in the resonance region, the ratio of $\pi^\pm+\Sigma^\mp$ production to $\pi^0+\Lambda$ production in $P_{\frac{3}{2}}$ states is a measure of $F_\Sigma/F_\Lambda$. (The process $K^-+p\rightarrow\pi^0+\Sigma^0$ corresponds entirely to isotopic spin 0 and thus should show no resonance.)

(*B*) *Width of resonance.* If the energy dependence of either the $\pi^\pm+\Sigma^\mp$ or $\pi^0+\Lambda$ production cross section in the resonant state can be measured, it is easy to use the proportionality of Eq. (21) and the known relations between the center-of-mass momenta of the $\pi+Y$ and $\bar{K}+N$ states to compute the shape of the $\pi-Y$ scattering cross section in the resonance state. The magnitude of the energy at the peak of the resonance cannot be related directly to coupling constants, but if this resonance energy is known, the width of the resonance is a measure of $F_\Lambda^2+F_\Sigma^2$. Measuring the resonance width is equivalent to extrapolating the amplitudes to the zero-energy poles; this could be done by means of an effective range plot, similar to that used for $\pi-N$ scattering.[9]

(*C*) *Height of production resonance.* The size of the $\pi+Y$ production cross section in the resonant state at energies near the peak of the $\pi-Y$ scattering resonance is a measure of the coupling constant ratio of Eq. (21). If the four constants $F_\Lambda$, $F_\Sigma$, $G_\Lambda$ and $G_\Sigma$ are all appreciable, this ratio is very sensitive to the relative signs of the constants.[16] If global and cosmic symmetry were both valid, i.e., $F_\Lambda=F_\Sigma$ and $G_\Lambda=G_\Sigma$, the resonance *would not appear at all* in the $\pi+Y$ production process.[17]

If the $\pi+Y$ production cross section in the resonant state is large, one should include effects of higher order in $G/F$. These effects and the contributions of various crossing terms may be calculated by using the procedure discussed in Sec. V. However, if such corrections are important, the relations between the coupling constants and the experimental data are not as simple as they are in the above illustration.

The proposed resonance might also show up in elastic $\bar{K}-N$ scattering. In order to study this possibility we write the dispersion relation for the $I=1$, $P_{\frac{3}{2}}$ $\bar{K}-N$ scattering amplitude $\mathbf{t}_1{}^K$ in the small-momentum approximation, again using the energy variable $\bar{\omega}=W-m_Y$ (rather than the natural $\bar{K}-N$ scattering variable $W-m_N$). The equation is

$$\mathrm{Re}\mathbf{t}_1{}^K(\bar{\omega})=\frac{\mathrm{P}}{\pi}\int_{\mu_\pi}^{\infty}\frac{d\bar{\omega}'\,\mathrm{Im}\mathbf{t}_1{}^K(\bar{\omega}')}{\bar{\omega}'-\bar{\omega}}+\text{crossing term.}\quad(22)$$

The unitarity condition for $\mathbf{t}_1{}^K$, to the lowest order in $G/F$, may be obtained by omitting the $\bar{K}+N$ channel from the sum in Eq. (17). The condition is then the same in the physical and unphysical regions, and may be written $\mathrm{Im}\mathbf{t}_1{}^K=k_\pi{}^3(|t_r|^2+|t_s|^2)$, where $k_\pi$ is the momentum in the $\pi+Y$ states. If the contribution of the nonresonant state $\psi_s$ to this condition and the crossing term of Eq. (22) are neglected, then this equation expresses $\mathbf{t}_1{}^K$ directly in terms of $t_r$.

The Born approximation terms vanish for the amplitude $\mathbf{t}_1{}^K$ as well as for the $P_{\frac{3}{2}}$, $\bar{K}-N$ elastic scattering in the isotopic spin 0 state.[18] Therefore, any low-energy $P_{\frac{3}{2}}$ resonance in the strangeness $(-1)$ states must be "driven" by the pion-hyperon interactions, rather than the $K$ interactions, even if the $\pi$ and $K$ interactions are comparable in strength. This circumstance lends additional support to the general method of approach used in this section and in Sec. V.

## VII. RESONANT PRODUCTION AMPLITUDE FOR SPECIFIC CHOICE OF COUPLING CONSTANTS (MODERATELY SMALL $G/F$)

In this section we assume a $\pi-Y$ scattering resonance does exist in the state $\psi_r$. In order to determine the possible magnitude of $\pi+Y$ production in $\psi_r$, we set the coupling constants equal to the following values, $F_\Sigma=F_\Lambda=F_N=(14)^{\frac{1}{2}}$, $G_\Lambda=-G_\Sigma=(2.2)^{\frac{1}{2}}$. The proportionality constant $\xi$ of Eq. (21) is then equal to 0.62. Since this ratio is not small we will correct the unitarity condition for $\pi-Y$ scattering to include the effects of the $\bar{K}$ channel. The phase condition on the production amplitude will not be corrected, i.e., we continue to assume that the phases of the production and scattering amplitudes are the same. The mass difference $m_\Sigma-m_\Lambda$ and the crossing terms in all dispersion relations are still neglected. The proportionality of Eq. (21) then remains valid, and may be combined with Eq. (17) to give the modified unitarity condition for the scattering,

$$\mathrm{Im}\mathbf{t}_{rr}{}^\pi=k_\pi{}^3|\mathbf{t}_{rr}{}^\pi|^2\beta;\quad\beta=1+\xi^2(k_K/k_\pi)^3\eta(\bar{\omega}),\quad(23)$$

where $\eta(\bar{\omega})$ is the step function defined to be unity at energies above the $\bar{K}+N$ threshold and zero at energies below.

[16] The relative phases of the $\Sigma$, $\Lambda$, and $\bar{K}+N$ states may be chosen so that the signs of $F_\Lambda$, $F_\Sigma$, and $G_\Lambda$ are the same. The sign of $G_\Sigma$ is then fixed, and must be determined by comparison with experiment.

[17] It is also true that the $I=\frac{3}{2}$, $\pi+N\rightarrow K+\Sigma$ amplitude, which is of the same isotopic spin as the $\pi-N$ resonance, vanishes in the case of simultaneous global and cosmic symmetry. This has been pointed out by A. Pais, Phys. Rev. **110**, 574 (1958), Eq. (26). Pais shows that this assumption ($F_\Sigma=F_\Lambda$ and $G_\Sigma=G_\Lambda$) is contradicted by experimental data.

[18] This may be understood from the following argument. The Born approximation terms for any of the amplitudes discussed here are all associated with one-particle intermediate states either of the process itself or of the crossed process. Since the baryon spins are $\frac{1}{2}$, only the intermediate baryons associated with the crossed process can contribute in the case of a $P_{\frac{3}{2}}$ amplitude. However, there are no one-particle intermediate states associated with $K-N$ scattering, which is the crossed process for $\bar{K}-N$ scattering.

For simplicity we neglect terms of order $\bar{\omega}/m$ in the $\pi-Y$ scattering dispersion relation, Eq. (19), and solve this equation and Eq. (23) in the effective range approximation of Chew and Low.[9] The result of this procedure is,

$$\mathbf{t}_{rr}{}^{\pi}(\bar{\omega})=\frac{\Gamma}{k_\pi{}^3(\bar{\omega}_r-\bar{\omega}-i\Gamma\beta)};\ \Gamma=\frac{k_\pi{}^3(F_\Lambda{}^2+F_\Sigma{}^2)\bar{\omega}_r}{6\bar{\omega}m_Y{}^2},\qquad(24a)$$

$$t_r(\bar{\omega})=\xi\mathbf{t}_{rr}{}^{\pi}(\bar{\omega}),\qquad(24b)$$

where $\xi=0.62$, $\beta$ is given in Eq. (23), and $\bar{\omega}_r$ is the resonance energy, assumed to be arbitrary. The cross section for $\pi+Y$ production in the resonant state calculated from Eqs. (24) is shown in Fig. 1, for two different choices of the resonance energy.

The inclusion of the $K$-meson effects ($\xi^2$ term) in Eq. (23) is important because it leads to cross sections that are consistent with unitarity; the predicted elastic cross section is never larger than the maximum consistent with the predicted value of the inelastic cross section. The effect of the moderately strong $K$ coupling has been omitted from the unitary condition on the inelastic amplitude, however. Despite the crude nature of this calculation, it is believed that Eqs. (24) should provide a reasonably accurate indication of the type of energy dependence to be expected if a $P_{\frac{3}{2}}$ resonance in the state $\psi_r$ exists.

### VIII. THE EXPERIMENTAL SITUATION

We continue to assume that orbital angular momenta greater than one may be neglected for $K$ mesons of lab momenta less than 500 Mev/$c$ incident on nucleons. If the target is unpolarized the angular distribution for any $\bar{K}+N\rightarrow\pi+Y$ process is given in terms of the $S$ and $P$ amplitudes by the equation,

$$(k_\pi{}^3k_K)^{-1}d\sigma/d\Omega=|t_0|^2+|t_{1+}|^2+|t_{1-}|^2 -2\,\mathrm{Re}t_{1+}{}^*t_{1-}+\cos\theta[2\,\mathrm{Re}t_0{}^*(2t_{1+}+t_{1-})] +\cos^2\theta[3|t_{1+}|^2+6\,\mathrm{Re}t_{1+}{}^*t_{1-}].\qquad(25)$$

It is seen from this equation that a large $\cos^2\theta$ term in the angular distribution indicates a large $P_{\frac{3}{2}}$ amplitude.

Although the present $\pi+Y$ production data is sparse, the Berkeley hydrogen bubble-chamber experiments provide definite evidence for a large $P$ wave in the processes $K^-+p\rightarrow\pi^\pm+\Sigma^\mp$ at 400 Mev/$c$ lab $K$ momentum.[19] The total $K^-+p$ absorption cross section at this energy ($\sim$38 mb) is much greater than the maximum possible for $S$ waves ($\sim$20 mb). Furthermore, the differential cross sections for both the $\pi^++\Sigma^-$ and the $\pi^-+\Sigma^+$ processes are much larger in the front and back quadrants ($|\cos\theta|>0.5$) than in the central quadrants, indicating large $\cos^2\theta$ terms. In fact, this experimental data is consistent with the assumption that almost all the $\pi^\pm+\Sigma^\mp$ cross section (about 9 mb for each process) occurs in the $P_{\frac{3}{2}}$ state. Although these cross sections are based on a total of only 42 events, it is extremely unlikely that the large $\cos^2\theta$ terms will disappear with the accumulation of further data. Thus, these limited measurements suggest a total $P_{\frac{3}{2}}$ cross section for charged $\pi+\Sigma$ production in the range 10–18 mb. There is a slight hint of smaller $\cos^2\theta$ terms in the $K^-+p\rightarrow\pi^\pm+\Sigma^\mp$ measurements at 240 Mev/$c$.[19] Other than this there is no evidence concerning the energy dependence of the $P_{\frac{3}{2}}$ cross sections.

The fact that the $P_{\frac{3}{2}}$ parts of the charged $\Sigma$ cross sections appear to be about equally large is consistent with an $I=1$ resonance, since the $I=1, \pi+\Sigma$ state occurs with 50% probability in each of the $\pi^++\Sigma^-$ and $\pi^-+\Sigma^+$ states. If these cross sections result from a resonance in the state $\psi_r$ [see Eq. (18a)], the $K^-+p\rightarrow\pi^0+\Lambda$ angular distribution should also contain a large $\cos^2\theta$ term, while no such term should occur for the $I=0$, $K^-+p\rightarrow\pi^0+\Sigma^0$ process. The angular distributions for these two processes are not known, but the total cross section at 400 Mev/$c$ is about $7\frac{1}{2}$ mb for $\pi^0+\Lambda$ production, and about 6 mb for $\pi^0+\Sigma^0$ production. This ratio of $\pi^0+\Lambda$ to $\pi^0+\Sigma^0$ events is in strong contrast to the corresponding ratio of ($\sim\frac{1}{4}$) that exists at threshold.[19]

The angular distribution for $K^--p$ elastic scattering at 400 Mev/$c$ also appears to contain a large $\cos^2\theta$ term, and is consistent with the assumption that most of the ($\sim$50 mb) cross-section results from scattering in the $P_{\frac{3}{2}}$ state.[19] This is additional evidence for a resonance effect. However, since the resonance proposed in this paper is "driven" by the $\pi-Y$ interactions (see Sec. VI) such a large $K^--p$ elastic scattering cross sec-

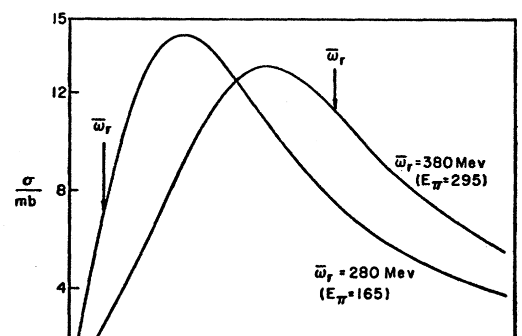


FIG. 1. Total calculated cross section for production of the resonant $P_{\frac{3}{2}}$, $I=1$, $\pi+Y$ state $\psi_r$ from $K^-$-mesons incident on protons, as a function of the $K$ lab momentum. The cross section is calculated from Eqs. (27) and the relation $\sigma=4\pi k_K k_\pi{}^3\times|t_r|^2$. The quantity $E_\pi$ denotes the assumed resonant energy in terms of the pion *kinetic* energy in the hyperon rest frame in Mev. The peak of the $E_\pi=165$ curve is shifted to the right of the indicated resonance position because of the $k_K$ dependence of the cross section, while the $E_\pi=295$ curve is shifted to the left of the resonance position because of the $k_\pi$ and $\beta$ dependence.

[19] L. W. Alvarez, Proceedings of the 1959 International Conference on Physics of High-Energy Particles at Kiev, July, 1959 (to be published).

tion is consistent with the proposed resonance only if the resonant $K^-+p\rightarrow\pi+Y$ cross section is quite large, on the order of 15 mb or larger.

It is seen from the above discussion that the existing data is insufficient for one to conclude that the front-back peaking of the $\pi^\pm+\Sigma^\mp$ production data results from the resonance process predicted here. In order to test the resonance hypothesis, one needs to know the approximate energy dependence of the $P_{\frac{3}{2}}$ cross section. If future measurements verify that this energy dependence is of the general type predicted by Eqs. (24) then information concerning the coupling constants can be gained. Of the three measures of the coupling constants discussed in Sec. VI, the relation between the resonance width and the sum $F_\Sigma^2+F_\Lambda^2$ probably is the least sensitive to some of the effects neglected in Secs. VI and VII, such as the contribution of the state $\psi_s$ and the $I=0$ state to the $P_{\frac{3}{2}}$, $\pi^\pm+\Sigma^\mp$ amplitudes, the effect of the $\Sigma-\Lambda$ mass difference, and the contributions of the crossing terms in the dispersion relations. Thus, measurements of the energy dependence are crucial, not only to verify the resonance hypothesis, but to make it useful. More accurate angular distribution measurements of the $K^-+p\rightarrow\pi^\pm+\Sigma^\mp$ processes for $K$ lab momenta in the range 200–500 Mev/$c$ are especially needed. It would also be interesting to see if the postulated large $P_{\frac{3}{2}}$ cross section were present for the process $K^-+p\rightarrow\pi^0+\Lambda$.

If we assume that future measurements will verify the existence of a large $P_{\frac{3}{2}}$, $\pi^\pm+\Sigma^\mp$ production cross section with a resonance-type energy dependence, then the present experimental data suggest the following conclusions. These conclusions are related to the effects ($A$) and ($C$) discussed in Sec. VI.

*Ratio of* $F_\Sigma$ *to* $F_\Lambda$. The ratio of the $\pi+\Sigma$ and $\pi+\Lambda$ contributions in the resonance state $\psi_r$ depends on the coupling constant ratio $F_\Sigma/F_\Lambda$. In fact, Eq. (18a) implies that the experimental ratio $[\sigma(\Sigma^+)+\sigma(\Sigma^-)]/\sigma(\Lambda)$ is equal to $\frac{1}{2}F_\Sigma^2/F_\Lambda^2$, for $\pi+Y$ production in the resonant state. If this relation were taken at face value, the experimental indication that the $P_{\frac{3}{2}}$, charged $\pi+\Sigma$ production cross section is greater than the total $\pi+\Lambda$ cross section at 400 Mev/$c$ would imply that $F_\Sigma^2>2F_\Lambda^2$. Such a quantitative conclusion is not justified, however, because the ratio of charged $\Sigma$ to $\Lambda$ production cross sections may be influenced greatly by effects we have neglected. Nevertheless, we feel that a large resonance-type $P_{\frac{3}{2}}$, $\pi^\pm+\Sigma^\mp$cross section implies that $|F_\Sigma|$ is not small compared to $|F_\Lambda|$. (We recall from Sec. VI and CN that if $F_\Sigma$ is small, the states $\pi+\Sigma$ and $\pi+\Lambda$ nearly diagonalize the $I=1$, $P_{\frac{3}{2}}$ part of the scattering matrix, and the $\pi+\Sigma$ state is characterized by a repulsive interaction.) This conclusion concerning $F_\Sigma$ is significant in view of the fact that the existence of hyperfragments, which is the one solid piece of evidence we have for strong pion-hyperon interactions, tells us essentially nothing about $F_\Sigma$.

*The K to* $\pi$ *coupling ratio* (*magnitude of* $\pi+Y$ *production at resonance*). The magnitude of the resonant $\pi+Y$ production near the resonance peak is a measure of the coupling constant ratio of Eq. (21). The expressions of Eq. (24) are too crude to be used for a quantitative estimate of this ratio, even if the size and shape of the resonance were well known. Nevertheless, the apparent large $P_{\frac{3}{2}}$, $\pi^\pm+\Sigma^\mp$ cross sections are evidence for a large value of $F_\Lambda G_\Lambda-F_\Sigma G_\Sigma$. If $F_\Lambda$ and $F_\Sigma$ are of comparable magnitude, and if $G_\Lambda$ and $G_\Sigma$ are of comparable magnitude, a large resonance effect in the production of $\pi+Y$ pairs in the state $\psi_r$ implies that one of the coupling constants has the opposite sign from the others.[16]

The experimental determination of the $P_{\frac{3}{2}}$ amplitudes is difficult at $K$ momenta less than 400 Mev/$c$, because of the large $S$-wave contributions to the absorption processes. In the cases of $\Sigma^+$ and $\Lambda$ production, the separation of the $S$, $P_{\frac{1}{2}}$, and $P_{\frac{3}{2}}$ amplitudes may be facilitated by polarization measurements on the hyperons, since the decay asymmetries of these particles provide direct measures of their polarizations.

The formalism and conclusions of this paper depend upon the assumption that the $\pi\Sigma\Lambda$, $KN\Lambda$, and $KN\Sigma$ parities are all odd. If this parity condition is violated by the $K$ interactions, but not by the $\pi\Sigma\Lambda$ interaction, the $P_{\frac{3}{2}}$ resonance in the state $\psi_r$ could just as easily exist, but might be difficult to detect in the $\bar{K}+N\rightarrow\pi+Y$ process, since $D_{\frac{3}{2}}$ waves of the $\bar{K}+N$ system would be involved.

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