Deuteron Photodisintegration in the Medium Energy Range*

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The photodisintegration of the deuteron is calculated for the two γ-ray energies in the laboratory of 52.3 Mev and 77.3 Mev corresponding to nucleon-nucleon scattering in the laboratory of 100 Mev and 150 Mev. For the ground state the Hulthén wave function with 4% D-state probability is used. The final state is described by the Signell-Marshak phase shifts. The coupling of the scattered waves is taken into account, as well as the retardation effects and all dipole and quadrupole transitions are included. For unpolarized γ rays the total cross section and the angular distribution are calculated. A comparison with the experiments is made.

1. INTRODUCTION

THE investigation of deuteron photodisintegration can, in principle, give information about the deuteron, the continuum states of the n-p system and the radiative interaction. In the low- and medium-energy range (γ-ray energy less than 100 Mev) we can assume that the interaction mechanism is given on the basis of the gauge invariance of the nonrelativistic Hamiltonian for the two-nucleon system. Neglecting the retardation of the γ-ray momentum we can apply the Siegert theorem for the electric transitions. Then we obtain the usual static electric dipole or quadrupole interaction. On the basis of this interaction several authors have calculated the angular distribution in the disintegration by medium energy photons and have reached more or less satisfactory agreement with the experimental data. To explain these data it is necessary to take into account the transitions from the D state of the deuteron and to describe the final state by phase shifts which correspond to a repulsive long range tensor potential in the triplet odd states.[1–11] In particular de Swart and Marshak[2] got large values, compared to experiment, for the isotropic term of the angular distribution when using for the ground state the Gartenhaus wave function which contains a large percentage of D state (6.7%). The same result has been obtained in references 5, 7, 9, and 11. On the other hand, if a Hulthén wave function with 4% D state is used for the deuteron, the parameters of the angular distribution are considerably different ($\sim$20–30%) from the results for ground states with much higher D-state probability.[2,3,6] Therefore, as soon as accurate experimental values for these parameters are available, one can hope to determine the D-state probability of the deuteron very closely. In the former calculations[3,6] which are based on the 4% D-state admixture, only the electric and magnetic dipole transitions have been taken fully into account and the coupling has been neglected in the final states. The contributions of the electric and magnetic quadrupole transitions have been considered only in so far as they affect the forward asymmetry in the differential cross section. Furthermore, the static interaction has been used.

In order to determine the D-state probability from photodisintegration experiments, it is desirable to know the influence of all of the neglected terms on the cross section. In this paper we present the results of calculations of the angular distribution parameters for two γ-ray energies in the medium-energy range in which the coupling of the final state, all of the dipole and quadrupole transitions, and the γ-ray retardation have been taken into account. Retardation effects have first been considered by Brennan and Sachs[12] and later by Nicholson and Brown[4] and by Hsieh.[7] The phase shifts and coupling parameters have been taken from the work of Signell and Marshak.[13] Since the behavior of the radial wave functions of the final scattering state are not very well defined inside the range of the nuclear forces, we have investigated the influence of different assumptions about their behavior for the transitions which give the main contributions.

In Sec. 2 we list the contributing transitions, give the modification of the amplitudes by the retardation and discuss some approximations which have been applied to the quadrupole transitions. Section 3 contains the results, the influence of the various effects being given

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separately. In Sec. 4 we discuss the comparison with experimental results.

2. ELECTRIC AND MAGNETIC MULTIPOLE TRANSITIONS

A. Electric Transitions

The electric dipole transitions

$$^3S_1+{}^3D_1 \rightarrow {}^3P_0,\ {}^3P_1,\ {}^3P_2+{}^3F_2$$

are the most important transitions for the photodisintegration at medium energies and lead to an angular distribution (in the c.m. system)

$$(d\sigma/d\Omega)_{E1}={}^3a_{E1}+{}^3b_{E1}\sin^2\theta. \tag{1}$$

The electric quadrupole transitions,

$$^3S_1+{}^3D_1 \rightarrow {}^3S_1+{}^3D_1,\ {}^3D_2,\ {}^3D_3+{}^3G_3,$$

are most important through their interference with the electric dipole transitions which causes a forward asymmetry in the angular distribution. However, they also contribute to the isotropic term and to the term proportional $\sin^2\theta$. Their whole contribution is given by:

$$(d\sigma/d\Omega)_{E2}=a_{E2}+b_{E2}\sin^2\theta+c_{E2}\cos\theta+d_{E2}\cos\theta\sin^2\theta +e_{E2}\sin^2\theta\cos^2\theta. \tag{2}$$

Neglecting the retardation, the formulas for a_{E1} and b_{E1} are given in reference 10 where the coupling of the $^3P_2$ and $^3F_2$ waves is included. If the retardation is taken into account, the amplitudes $L_{\lambda j}$ for the $^3P_{0,1,2}+{}^3F_2$ states must be modified in the following way:

$$\begin{aligned}
L_{10}&=\int_0^\infty \alpha v_{10}(kr)\left(S_1(qr)-S_2(qr)\frac{d}{dr}\right)\times\left(u(r)-\sqrt{2}w(r)\right)dr,\\
L_{11}&=\int_0^\infty \alpha v_{11}(kr)\left(S_1(qr)-S_2(qr)\frac{d}{dr}\right)\times\left(u(r)+\frac{1}{\sqrt{2}}w(r)\right)dr,\\
L_{12}&=\cos\epsilon_2\left\{\int_0^\infty \alpha v_{12}{}^1(kr)\left(S_1(qr)-S_2(qr)\frac{d}{dr}\right)\times\left(u(r)-\frac{\sqrt{2}}{10}w(r)\right)dr\right\}\\
&\quad+\frac{\sqrt{3}}{\sqrt{2}}\sin\epsilon_2\,\tfrac{3}{5}\sqrt{2}\left\{\int_0^\infty \alpha v_{32}{}^1(kr)\times\left(S_1(qr)-S_2(qr)\frac{d}{dr}\right)w(r)dr\right\},\\
L_{32}&=\cos\epsilon_2\tfrac{3}{5}\sqrt{2}\left\{\int_0^\infty \alpha v_{32}{}^3(kr)\times\left(S_1(qr)-S_2(qr)\frac{d}{dr}\right)w(r)dr\right\}\\
&\quad-\frac{\sqrt{2}}{\sqrt{3}}\sin\epsilon_2\left\{\int_0^\infty \alpha v_{12}{}^3(kr)\left(S_1(qr)-S_2(qr)\frac{d}{dr}\right)\times\left(u(r)-\frac{\sqrt{2}}{10}w(r)\right)dr\right\}.
\end{aligned}\tag{3}$$

The $L_{\lambda j}$ are amplitudes of the outgoing waves with quantum number λ ($\lambda=j-1,\ j,\ j+1$) and total angular momentum j. M is the nucleon mass, $2\hbar q$ the momentum of the photon, k the relative wave number of the outgoing nucleons and $1/\alpha$ the radius of the deuteron. The normalization of the radial wave functions $u(r)$ and $w(r)$ of the ground state are chosen as usual. The two retardation functions are as following:

$$\begin{aligned}
S_1(qr)&=\left(1-\frac{\hbar q}{2Mc}\right)\frac{3}{2q}\frac{d}{dr}rj_1(qr),\\
S_2(qr)&=(3\hbar/2Mc)j_1(qr),
\end{aligned}\tag{4}$$

where j_1 is the spherical Bessel function. In these expressions for the $L_{\lambda j}$ the contribution of the interaction currents which are present because of the exchange character of the nuclear forces is neglected. Applying the Siegert theorem the exchange currents only contribute to those terms in (3) which contain the function $S_2(qr)$ having an extra factor q. Furthermore, because the currents contain a factor $r^2V(r)$ where $V(r)$ is the n-p-exchange potential, we suppose that their effect is negligible in the considered energy range.[14]

If all contributions which are of the order q^2 compared with the contribution of the usual dipole transitions are to be considered, the spinflip electric dipole transition,

$$^3S_1+{}^3D_1\rightarrow {}^1P_1,$$

must be considered also. The contribution to the cross section of this transition is

$$(d\sigma/d\Omega)_{E1}={}^1a_{E1}+{}^1b_{E1}\sin^2\theta \quad\text{with}\quad b_{E1}=-\tfrac{1}{2}a_{E1}. \tag{5}$$

Because of its minor importance we have neglected the retardation and the coupling for the $E2$-transitions. In this approximation the $E2$-amplitudes have been given in an earlier paper, in which the parameters c_{E2} and d_{E2} have been calculated.[6] With the reported amplitudes of reference 6 we have determined the parameters a_{E2}, b_{E2}, and e_{E2}.

B. Magnetic Transitions

The magnetic dipole spin-flip transitions

$$^3S_1+{}^3D_1\rightarrow {}^1S_0,\ {}^1D_2,$$

[14] The $rj_0(qr)$ part of the function $S_1(qr)$ has been considered by Nicholson and Brown.[4]

TABLE I. $^3a_{E1}$ and $^3b_{E1}$ in μb.

	Retardation	Coupling	Final state wave function
I	neglected	neglected	square well potential with hard core
II	neglected	neglected	square well potential without hard core
III	neglected	neglected	approximation (7)
IV	taken into acc.	neglected	square well potential with hard core
V	neglected	taken into acc.	square well potential with hard core
VI	neglected	taken into acc.	approximation (7)
VII	taken into acc.	taken into acc.	approximation (7)
VIII	taken into acc.	taken into acc.	square well potential with hard core

	I		II		III		IV		V		VI		VII		VIII	
E_γ (Mev)	a	b	a	b	a	b	a	b	a	b	a	b	a	b	a	b
52.3	3.52	11.6	3.50	11.8	4.07	11.1	3.38	11.5	4.11	11.4	4.91	10.9	4.75	10.8	4.05	11.4
77.3	2.62	4.81	2.87	4.91	3.35	4.42	2.45	4.71	3.20	4.79	4.24	4.28	4.05	4.15	3.11	4.68

which lead to the cross section

$$(d\sigma/d\Omega)_{M1}={}^1a_{M1}+{}^1b_{M1}\sin^2\theta \tag{6}$$

have been considered in reference 3. It was found that this transition also gives a considerable contribution to the parameter b for the higher energies. We have recalculated the parameter $^1a_{M1}$ and $^1b_{M1}$, using for the $^1S_0$ amplitude the values published in reference 9, instead of the values calculated for square well potentials as the final state interaction in reference 3 which we suppose to be less accurate in the case of higher energies. The magnetic dipole singlet transition can interfere with the electric dipole singlet transition. This interference contributes to the parameter c only. The magnetic dipole triplet transition

$$^3S_1+{}^3D_1\rightarrow{}^3S_1+{}^3D_1,\ {}^3D_2$$

was found to give a negligible contribution in the considered energy range.

The magnetic quadrupole singlet transitions

$$^3S_1+{}^3D_1\rightarrow{}^1P_1,\ {}^1F_2$$

interfere with the magnetic dipole transitions, and this contribution has been calculated in reference 6. This interference contributes to the parameters c and d. The effect on parameters a, b, and e was found to be negligible.

The magnetic quadrupole triplet transitions

$$^3S_1+{}^3D_1\rightarrow{}^3P_1,\ {}^3P_2+{}^3F_2$$

contribute to the parameters a, b, and e. Furthermore, an interference is possible between these transitions and the electric dipole transitions which lead to the same final states. This interference only contributes however, to the parameters a and b ($^3a_{E1,M2}$, $^3b_{E1,M2}$). The parameter $^3b_{E1,M2}$ is given by the relation: $^3b_{E1,M2}=-\frac{3}{2}a_{E1,M2}$, so this interference does not change the total cross section.

Ignoring coupling, the radial wave functions $v_{lj}{}^\lambda(kr)$ of the final P states have been calculated from square well potentials with a radius $R=1.55\times10^{-13}$ cm. Their depths have been determined by the Signell-Marshak phase shifts and the central part consists of a hard core with radius $r_c=1/10\alpha=0.4316\times10^{-13}$ cm. This approximation of the triplet odd state potential was discussed earlier.[3] To see the influence of the central hard core we have also calculated the $v_{lj}{}^\lambda(kr)$ for square well potentials with the same radius but without any hard core.

De Swart and Marshak have proposed the following approximation for the radial wave functions in the P state[2]:

$$v_{lj}{}^\lambda(kr)=kr\begin{cases}\cos\delta_{\lambda j}j_l(kr)-\sin\delta_{\lambda j}n_l(kr) & \text{for}\quad r\geqslant R\\ \cos\delta_{\lambda j}j_l(kr) & \text{for}\quad r<R\end{cases}, \tag{7}$$

where j_l and n_l are, respectively, the spherical Bessel and Neumann functions. These authors got nearly the same results for the amplitudes $L_{\lambda j}$ with this approximation as in the exact treatment under the condition that the radius R be chosen to be $R=1.41\times10^{-13}$ cm^2.

We have chosen this approximation with the radius $R=1.55\times10^{-13}$ cm and have found that this small change of R affects the amplitudes very little. Therefore we can consider the results with this approximation as characteristic of the model of Signell and Marshak. Because these potentials have long tails we consider them to be more realistic than the square well potentials. In particular, all coupling effects are calculated applying this approximation.

For the ground-state wave functions Hulthén wave

TABLE II. $^1a_{E1}$, a_{E2}, b_{E2}, and e_{E2} in μb.

E_γ (Mev)	$^1a_{E1}$	a_{E2}	b_{E2}	e_{E2}
52.3	0.08	0.03	0.07	0.67
77.3	0.12	0.03	0.07	0.39

TABLE III. $^1a_{M1}$, $^1b_{M1}$, and $^1c_{E1,M1}$ in μb.

E_γ	$^1a_{M1}$	$^1b_{M1}$	$^1c_{E1,M1}$
52.3	0.03	0.61	−0.19
77.3	0.08	0.41	−0.29

TABLE IV. ${}^3a_{E1,M1}$, ${}^3a_{M2}$, ${}^3b_{M2}$, and ${}^3e_{M2}$ in μb.

E_γ (Mev)	${}^3a_{E1,M2}$ (1)	(2)	${}^3a_{M2}$	${}^3b_{M2}$	${}^3e_{M2}$
52.3	0.44	0.55	0.05	−0.02	−0.04
77.3	0.50	0.66	0.03	−0.01	−0.03

TABLE V. a, b, c, d, and e in μb.

E_γ (Mev)	(1) a	(1) b	(2) a	(2) b	c	d	e
52.3	4.68	11.4	5.49	10.6	0.37	6.20	0.64
77.3	3.87	4.34	4.97	3.57	0.28	2.92	0.35

functions with 4% D-state admixture and a hard core radius $r_c = 1/10\alpha$ are used.[15]

3. RESULTS

The calculations have been performed for γ-ray energies E_γ of 52.3 Mev and 77.3 Mev, corresponding to n-p scattering at 100 Mev and 150 Mev in the laboratory system. In calculating these γ-ray energies the relativistic formulas of reference 8 have been applied. All P-state amplitudes have been calculated by numerical methods. In order to see the influence of the retardation, of the coupling in the final state and of the two different descriptions of the radial wave functions we give in Table I the results for the parameter ${}^3a_{E1}$ and

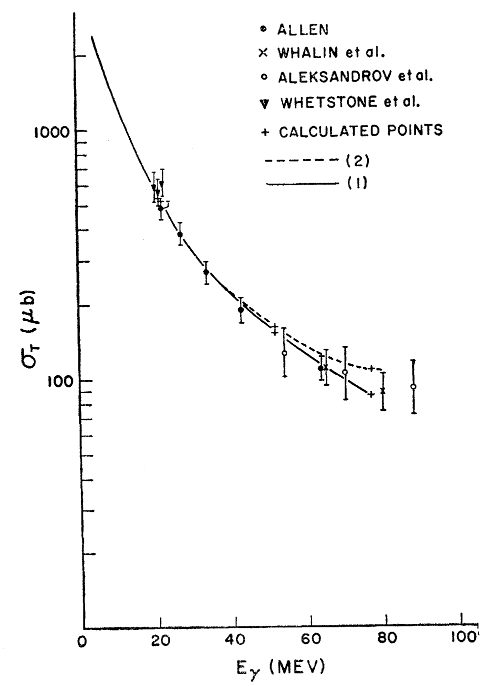

FIG. 1. Total cross section as function of γ-ray energy in the lab system.

[15] L. Hulthén and M. Sugawara, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 110 ff.

${}^3b_{E1}$ under the specified assumptions concerning retardation, coupling and final state wave functions.

From Table I it can be seen that the effect of the retardation is very small. The reason is that the correction of the unretarded amplitudes in the form of the function $S_1(qr)$ is of opposite sign from the correction given by the integrals which contain the function $S_2(qr)$. The coupling of the final states is important and raises the value of the isotropic term and lowers the value of the $\sin^2\theta$ term. This effect was also found in references 5, 8, and 9. These terms are changed even more strongly in the same direction by the use of approximation (7) for the final state wave functions over and against the square well approximation. As mentioned before, the use of the wave functions (7) seems to be more realistic.

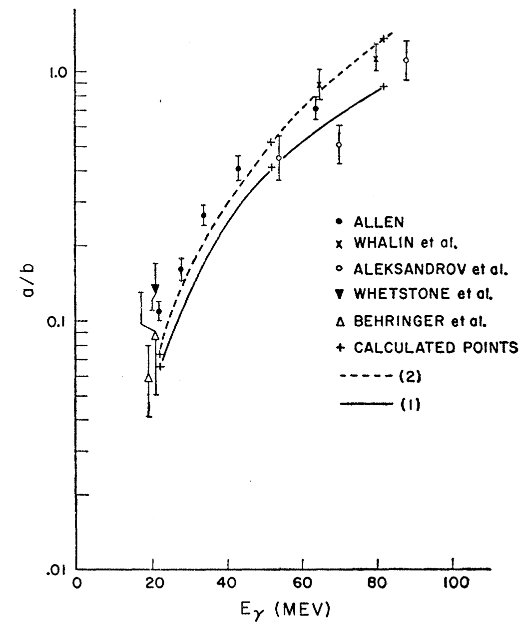

FIG. 2. Ratio a/b as function of γ-ray energy in the lab system.

In Table II we give the results for ${}^1a_{E1}$ and for the contribution of the quadrupole transitions.

Table III contains the results for the recalculated magnetic dipole singlet transition and the contribution of the $M1$-$E1$ interference to c. The contribution of the $E1$-$M2$ interference depends only on the amplitudes of the ${}^3P$ state and the ${}^3F_2$ state. We have calculated ${}^3a_{E1,M2}$ for the two kinds of radial wave functions, (1) calculated from square well potentials and (2) calculated from Eq. (7). In both calculations the coupling of the final states has been included, the retardation, however, being neglected. The values are presented in Table IV, together with the results for ${}^3a_{M2}$, ${}^3b_{M2}$, and ${}^3e_{M2}$. From the last table it can be seen, that only the $E1$-$M2$ interference gives an appreciable contribution to a and b.

In Table V we give the sum of all contributions including the results for c and d obtained earlier in reference 6. a and b are given for two cases: (1) the final-state radial wave functions are calculated from square well potentials with hard core; (2) they are calculated using the approximation by Eq. (7). For the contributions of the electric dipole transitions these two cases correspond to (VIII) and (VII), respectively, of Table I.

4. DISCUSSION

First we compare the total cross section σ_t,

$$\sigma_t = 4\pi[a+\tfrac{2}{3}b+(2/15)e], \qquad (8)$$

calculated from the values of Table V, with the measured total cross sections for various energies[16–20] (Fig. 1). The results for $E_\gamma = 22.2$ Mev of references 3 and 6 are included. Inside the large experimental errors the agreement is satisfactory. From the various parameters of the angular distribution only the ratio a/b is known experimentally. The comparison with the data of other authors[16–21] is made in Fig. 2. In contrast to our earlier calculations with a deuteron wave containing a 4% D-state admixture the agreement can be considered to be satisfactory. The inclusion of the $E1$-$M2$ interference and coupling in the final states is chiefly responsible for this improvement. Both effects increase a and decrease b by nearly the same amount. Of course, to determine the D-state probability definitely much better experimental data are needed. But we like to point out that such a low D-state probability as 4% seems to be large enough to explain the experimental data of the angular distribution. This means that the tensor potential of the triplet even state should be weaker than obtained from meson theoretical calculations.[22–24] On this basis the magnetic moment of the deuteron can be explained without the inclusion of large mesonic and relativistic corrections.[25] To see the influence of the calculated forward asymmetry parameters c and d, we have plotted the angular distributions for the two γ-ray energies, $E_\gamma = 52.3$ Mev and $E_\gamma = 77.3$ Mev and compared with the experimental data of reference 17 and 16, respectively in Fig. 3 and Fig. 4.[26] No conclusion can be drawn about c and d from this comparison.

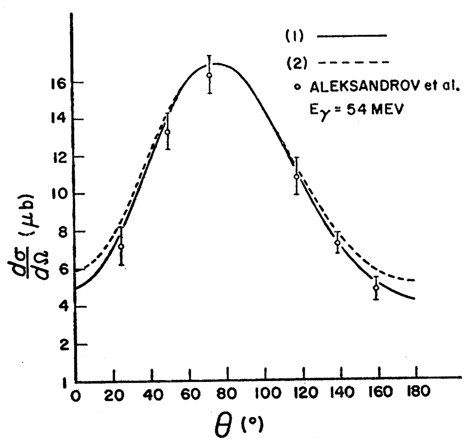

FIG. 3. Angular distribution for $E_\gamma = 52.3$ Mev.

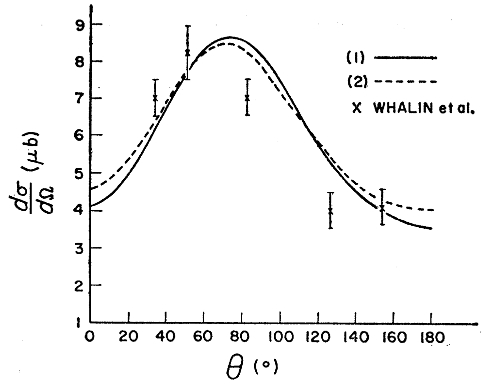

FIG. 4. Angular distribution for $E_\gamma = 77.3$ Mev.

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[25] L. Hulthén and M. Sugawara, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 69 ff.

[26] The measured values of reference 18 are raised by 20% (see Fig. 1).