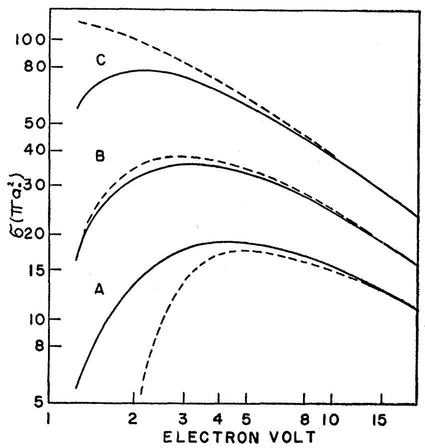

FIG. 4. Born (solid lines) and Bethe (broken lines) total cross sections for electron scattering by hydrogen atoms. A, $3s-5p$; B, $3p-5d$; C, $3d-5f$.

It is of interest to note that the Bethe formula gives a good fit to the Born approximation down to relatively low energies, which suggests that it is a satisfactory and much simpler substitute, provided that the energy is not too low. However, this requires a knowledge of κ_c which can be determined accurately only by evaluating

TABLE III. Values of $\kappa_c a_0$ and estimates of the percentage deviation of the Bethe approximation from the Born approximation.

	$\kappa_c a_0$	$E(1\%)$(ev)	$E(10\%)$(ev)
$3s-4p$	0.06314	30	8.4
$3p-4d$	0.07819	20	4.2
$3d-4f$	0.10593	7	1.2
$3s-5p$	0.09478	14	4.5
$3p-5d$	0.12918	12	2
$3d-5f$	0.21450	10	3.3

the Born cross section at one high energy. For more approximate work, a very simple method of evaluating κ_c has recently been found.[12]

As stated above, the question of the range of validity of the first Born approximation for these transitions is the subject of current investigations.

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We should like to thank Dr. R. N. Thomas for pointing out the need for these cross sections in chromospheric problems, and for his continued encouragement; Dr. M. J. Seaton and Dr. Wade Fite also helped by discussions of scattering theory and experiments.

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Born Cross Sections for Inelastic Scattering of Electrons by Hydrogen Atoms. II. $4s$, $4p$, $4d$, $4f$ States*†

LEONARD FISHER, S. N. MILFORD, AND FRANK R. POMILLA
St. John's University, Jamaica, New York

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The Born total cross sections are calculated for the inelastic scattering of electrons by hydrogen atoms for the strong optically allowed transitions from $n=4$ to $n'=5$. The nine incident energies considered range from 0.546 ev to 1361 ev. In addition, the $4s$ to $6p$ and $4f$ to $6g$ transitions are considered. Bethe (multipole) cross sections are also calculated and found to reproduce the Born results down to low energies.

INTRODUCTION

THIS paper extends the recent calculations[1] of the Born cross sections for inelastic scattering of electrons by hydrogen atoms to the $4s$, $4p$, $4d$, and $4f$ states. The four optically allowed transitions from $n=4$ to $n'=5$ are considered as well as the transitions $4s$ to $6p$ and $4f$ to $6g$.

FORMULATION

In Paper I it was shown that the Born differential cross section for the transition from (nl) to $(n'l')$ is given by

$$\sigma_{nl,n'l'}(\gamma)=B_{nl,n'l'}\frac{\pi}{k^2(1+\gamma^2)^{2(n+n')}}\times(a_{-1}\gamma^{-1}+a_1\gamma+a_3\gamma^3+\cdots+a_t\gamma^t), \quad (1)$$

where $t=23$ for $n=4$, $n'=5$ and $t=27$ for $n=4$, $n'=6$; B and a_i are constants determined by the particular transition.

The total cross section has been averaged over m and summed over m'; $\gamma=\kappa a_0 nn'/(n+n')$, where $\hbar\mathbf{k}$ is the initial momentum, and $\hbar\boldsymbol{\kappa}$ the change of momentum of the scattered electron; a_0 is the Bohr radius. The total cross section is

$$\sigma_{nl,n'l'}(k)=B_{nl,n'l'}[\pi a_0^2/(ka_0)^2]\times(a_{-1}I_q^{-1}+a_1I_q+a_3I_q^3+\cdots+a_tI_q^t), \quad (2)$$

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† Based in part upon a thesis to be submitted by Leonard Fisher to the Graduate School of St. John's University in partial fulfillment of the requirements for the degree of Master of Science.

[1] G. McCoyd, S. N. Milford, and J. Wahl, preceding paper [Phys. Rev. 119, 149 (1960)], hereafter referred to as Paper I.

TABLE I. Born cross-section coefficients.[a]

	4s–5p	4p–5d	4d–5f	4f–5g	4s–6p	4f–6g
B	4.206584962(−11)	2.596063863(−6)	1.846089858(−6)	1.770475788(−4)	1.809481993(−9)	1.727259709(−6)
a_{-1}	4.599352052(12)	8.344822500(7)	1.714608000(8)	2.702700000(6)	1.758673822(10)	2.432430000(7)
a_1	−2.869995680(14)	−3.705101190(9)	−4.526565120(9)	−2.270268000(7)	−2.110408587(10)	7.005398400(8)
a_3	6.122854744(15)	6.838687156(10)	5.102020224(10)	1.005867720(8)	−8.581928897(12)	2.453557392(9)
a_5	−5.471266734(16)	−5.777012524(11)	−2.704058381(11)	−1.514541600(8)	4.759146813(13)	−3.924876384(10)
a_7	2.549573425(17)	2.555106418(12)	7.889743008(11)	2.382237000(8)	9.532171516(14)	1.599308053(11)
a_9	−6.866321694(17)	−6.279591959(12)	−1.351071187(12)	−8.694972000(7)	−1.027551108(16)	−2.426301821(11)
a_{11}	1.119365884(18)	8.859532206(12)	1.403990703(12)	4.306044600(8)	4.260995116(16)	2.995101252(11)
a_{13}	−1.123839178(18)	−7.191961789(12)	−7.944350400(11)		−9.515174093(16)	−2.820506832(11)
a_{15}	6.950166537(17)	3.247123992(12)	1.908358704(11)		1.259857195(17)	3.139106513(11)
a_{17}	−2.592376934(17)	−7.434593870(11)			−1.023911690(17)	
a_{19}	5.525101224(16)	6.835659862(10)			5.141188286(16)	
a_{21}	−6.000058597(15)				−1.565378239(16)	
a_{23}	2.541865827(14)				2.741951092(15)	
a_{25}					−2.467446680(14)	
a_{27}					8.720947266(12)	

[a] The numbers in parentheses represent powers of 10.

TABLE II. Born total cross sections (in units of πa_0^2).[a]

Energy (ev)[b]	γ_1	4s–5p	4p–5d	4d–5f	4f–5g	4s–6p	4f–6g
0.546	0.1500	2(2)	4(2)	921	2938		
0.708	0.1250	28(1)	544.6	1188	3296		
0.850	0.2000					4(1)	360.7
1.20	0.0900	43(1)	707.3	1400	3245		
1.67	0.0750	473	721.8	1362	2945		
1.76	0.1250					7(1)	337.8
2.60	0.1000					7(1)	280.4
3.56	0.0500	440	602.7	1053	2053		
9.69	0.0500					47	117.9
17.37	0.0222...	201	250.8	405.8	714.3		
21.4	0.0200	176	217.6	350.0	611.2		
1360.65	0.0025	6.760	7.902	12.06	19.54		
3779.40	0.0025					0.46	0.7568

[a] The numbers in parentheses represent powers of 10.
[b] The γ_1 values are exact. The relation between γ_1 and E is

$$E=\frac{13.6050}{4}\left(\frac{1}{n}+\frac{1}{n'}\right)^2(\gamma_1+\gamma_2)^2.$$

where $q=2(n+n')$, and

$$I_q{}^r=\int_{\gamma_1}^{\gamma_2}\frac{\gamma^r}{(1+\gamma^2)^q}d\gamma,\quad \gamma_1\gamma_2=\frac{n'-n}{n'+n}. \tag{3}$$

RESULTS

The total cross sections for the four optically allowed transitions between $n=4$ and $n'=5$ were calculated for electrons with nine different incident energies in the range 0.546 ev (threshold energy is 0.306 ev) to 1361 ev. For the 4s to 6p and 4f to 6g transitions calculations of the total cross sections were made for five different energies in the range 0.850 ev (threshold energy is 0.472 ev) to 3800 ev.

The constants which appear in the differential and total cross sections are given in Table I, and they should be accurate to the number of figures given in the table.

Because of the heavy cancellation in the $I_q{}^r$ and in Eq. (2), the final cross sections in Table II are accurate only to several figures (the calculations were begun with ten significant figures). In view of the nature of the Born approximation, this is more than adequate.

If the $I_q{}^r$ for $r\geq 3$ are evaluated by Beta functions, agreement to two figures can be obtained with the results in Table II down to $E=3.56$ ev in the 4–5 transitions and down to $E=9.69$ ev in the 4–6 transitions.

The graphs of the total cross sections appear in Figs. 1 and 2.

DISCUSSION

Previous work[1,2] on the $n=2$ to $n'=3$ and $n=3$ to $n'=4$ transitions indicated that the largest cross sec-

TABLE III. Approximate energies (ev) at which the Bethe and Born cross sections differ by 1% and 10%. Momentum cutoff values.[a]

	E(1%)	E(10%)	$\kappa_c a_0$
4s–5p	17	4	0.03704
4p–5d	16	2.8	0.04318
4d–5f	15.5	2	0.05084
4f–5g	3	0.85	0.06684
4s–6p	20	3	0.05764
4f–6g	8	2	0.15516

[a] a_0 is the Bohr radius.

[2] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Oxford University Press, New York, 1952), pp. 170–171.

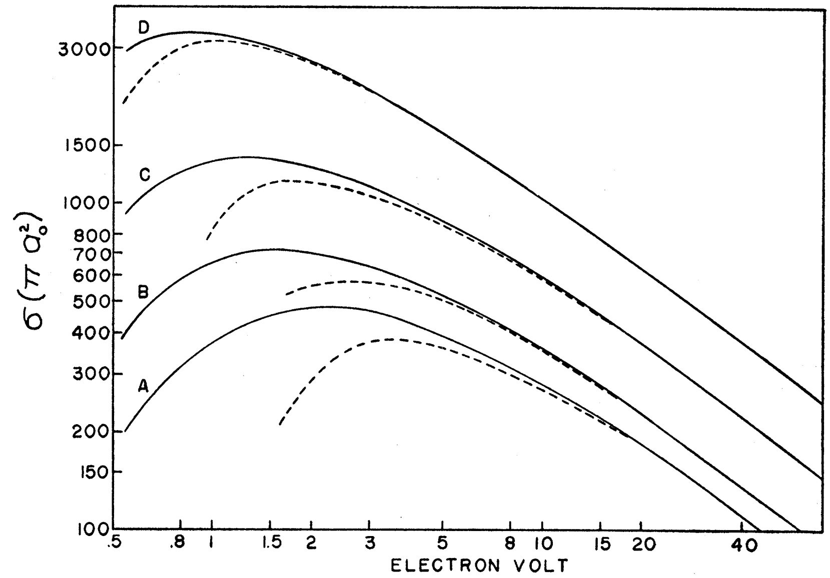

FIG. 1. Born (solid curves) and Bethe (broken curves) total cross sections for the 4–5 transitions of H atoms. (*A*) 4*s*–5*p*. (*B*) 4*p*–5*d*. (*C*) 4*d*–5*f*. (*D*) 4*f*–5*g*.

tions would be for optically allowed transitions in which n and l change in the same sense; these are the ones calculated for the $n=4$, $n'=5$ case. For the $n=4$, $n'=6$ case, only 4*s* to 6*p* and 4*f* to 6*g* were calculated with the hope that 4*p* to 6*d* and 4*d* to 6*f* could be obtained by interpolation.

The Bethe (dipole) approximation for allowed transitions is given in Paper I:

$$\sigma^D{}_{nl,n'l'}(k)=\int_{\kappa_1}^{\kappa_c}\sigma(\kappa)d\kappa=\frac{8}{3}\frac{\pi a_0^2}{(ka_0)^2}|C(1ll';000)|^2\times\left|\frac{J_{nl,n'l'}}{a_0}\right|^2\ln\left(\frac{\kappa_c}{\kappa_1}\right), \quad (4)$$

where $C(1ll'000)$ is the Clebsch-Gordan coefficient[3] and the squares of the radial integrals, $|J_{nl,n'l'}/a_0|^2$, can be found in the tables of Green *et al.*[4] κ_c is the usual cutoff replacing the upper limit κ_2. The cutoff κ_c for each transition was determined by setting the Bethe cross section [Eq. (4)] equal to the Born cross section at the highest energy calculated; $\kappa_c a_0$ is given in Table III. These values of κ_c were then used in Eq. (4) to compute the Bethe cross sections which are shown in Figs. 1 and 2. The energies $E(1\%)$ and $E(10\%)$ at which the Bethe and Born cross sections differ by 1% and 10%, respectively, were estimated and are listed in Table III.

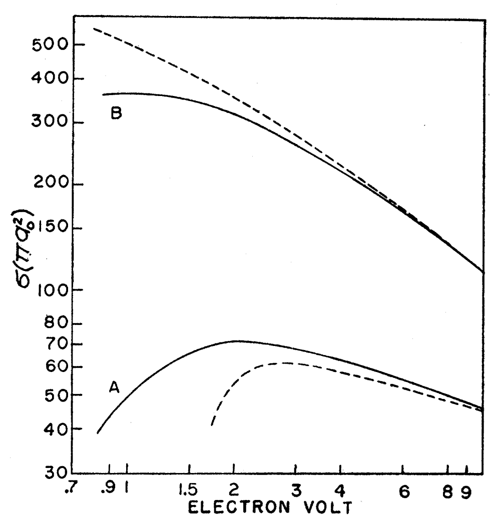

FIG. 2. Born (solid curves) and Bethe (broken curves) total cross sections for the 4–6 transitions of H atoms. (*A*) 4*s*–6*p*. (*B*) 4*f*–6*g*.

ACKNOWLEDGMENTS

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[3] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

[4] L. C. Green, P. P. Rush, and C. D. Chandler, Suppl. Astrophys. J. **3**, 37 (1957).