moments quoted in Table I may be regarded as conservative. The error in the relative values of the dipole moment is believed to be less. It will be noted that the dipole moment from observation of the line, 2.986 Debye units, is in good agreement with the dipole moment from the π lines, 2.984 Debye units. We choose as the most probable μ the one obtained by averaging values from the four measured components. The final value,

$$\mu = 2.986 \pm 0.004,$$

is in agreement with the less accurate one obtained by Ghosh, Trambarulo, and Gordy,[1] 3.00 ± 0.02 Debye units. Interestingly it is 0.028 Debye greater than the value, 2.957 ± 0.025 Debye units, obtained by Schulman and Townes[2] for the excited bending vibrational state. A difference of this order might be expected in the moments for the ground and vibrational states.

High Stark precision measurements have earlier been made on OCS in the centimeter wave region.[7,13] This "calibrated" OCS molecule provides a convenient reference standard for measuring electric fields or other dipole moments with centimeter-wave spectroscopy. However, this standard, or others that could be similarly established by centimeter wave measurements, is not satisfactory for the millimeter or submillimeter wave region because of the rapidly decreasing sensitivity of the Stark splitting with increasing J. HCN which has a large dipole moment as well as low J transitions originating in the shorter millimeter wave region provides a convenient millimeter wave measuring standard which can be projected into the submillimeter region. Already our measurements on HCN have been privately communicated to C. A. Burrus who has used them for secondary measurements on several other simple molecules in the 1- to 3-mm wave region.[14]

[13] R. G. Shulman and C. H. Townes, Phys. Rev. **77**, 500 (1950).
[14] C. A. Burrus, J. Chem. Phys. **28**, 427 (1958).

PHYSICAL REVIEW VOLUME 119, NUMBER 1 JULY 1, 1960

Born Cross Sections for Inelastic Scattering of Electrons by Hydrogen Atoms. I. 3*s*, 3*p*, 3*d* States*†

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Born cross sections of all $n=3$ to $n=4$ transitions are calculated at ten incident electron energy values in the range 0.67–1400 ev, and those of strong optically allowed $n=3$ to $n=5$ transitions are calculated at five incident electron energy values in the range 1–10 000 ev. The cross sections obtained are much larger than for comparable transitions from the ground state, and the cross sections for transitions which are optically allowed and in which n and l change in the same sense are larger than those for other transitions. For all strong optically allowed transitions the Bethe (dipole) approximations to the Born cross sections are calculated and comparison shows that the Bethe formula gives a good fit to the Born approximation down to relatively low energies (≈ 10 ev).

INTRODUCTION

MOST calculations of atomic cross sections have involved the ground states of atoms. Since very high temperature gases have recently been produced in the laboratory, and since there is now extensive astrophysical investigation of departures from local thermodynamic equilibrium in such gases, there is a need for inelastic collision cross sections for excited states of atoms.

The cross sections for collisions of electrons with excited hydrogen atoms are not only the easiest among all possible atoms to calculate, but also have immediate application to theories of the solar chromosphere.[1] Apart from cross sections for the ground state,[2] the only other previous calculations involve transitions from the 2*s* state[3] and the 2*p* state[4] calculated in the first Born approximation; Goldstein[5] and Yavorsky[6] wrote down general series expressions for the transition $n \rightarrow n'$, but these do not appear to be useful for numerical calculation of the cross sections.

The present paper discusses transitions from the 3*s*, 3*p*, and 3*d* levels of hydrogen to the $n=4$ and $n=5$

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† Based in part on a thesis submitted by John J. Wahl to the Graduate School of St. John's University, in partial fulfillment of the requirements for the degree of Master of Science.
[1] See the series of papers by R. N. Thomas and collaborators in the *Astrophysical Journal*, and the forthcoming monograph by R. G. Athay and R. N. Thomas, *Physics of the Solar Chromosphere* (Interscience Publishers, Inc., New York, to be published).
[2] R. McCarroll, Proc. Phys. Soc. (London) **A70**, 460 (1957), summarizes Born calculations; for a summary of more extensive calculations see H. S. W. Massey, *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 36, p. 354.
[3] T. J. M. Boyd, Proc. Phys. Soc. (London) **72**, 523 (1958).
[4] H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Oxford University Press, New York, 1952), p. 170.
[5] L. Goldstein, Ann. phys. **19**, 305 (1933).
[6] B. M. Yavorsky, Compt. rend. acad. sci. U.R.S.S. **43**, 151 (1944).

levels. The Born approximation is used for two reasons. (1) For these states an enormous amount of labor would be required even to make a moderate improvement of the Born approximation. (2) Plausibility arguments suggest that the Born approximation is valid down to much lower energies for these transitions (with threshold energies of 0.66 ev, and 0.97 ev, respectively) than for the transition $1s-2p$ (threshold 10.2 ev) for which experimental evidence is available. Current work is directed at reinforcing these plausibility arguments by direct calculation.

FORMULATION

For electron collisions with hydrogen atoms, the Born approximation differential cross section for the transition $nlm \rightarrow n'l'm'$ is given by[7]

$$\sigma'_{nlm,n'l'm'}(\kappa)=8\pi(ka_0)^{-2}\kappa^{-3}|\epsilon_{nlm,n'l'm'}(\kappa)|^2, \quad (1)$$

where $\boldsymbol{\kappa}=\mathbf{k}-\mathbf{k}'$, and $\hbar\mathbf{k}$, $\hbar\mathbf{k}'$, $\hbar\boldsymbol{\kappa}$ are the initial, final, and change of momentum of the electron, respectively. The proton is assumed at rest throughout the collision; a_0 is the Bohr radius, and ϵ is given by

$$\epsilon_{nlm,n'l'm'}(\kappa)=\int e^{i\kappa z}\psi_{nlm}(\mathbf{r})\psi^*_{n'l'm'}(\mathbf{r})d\mathbf{r}. \quad (2)$$

Introducing the normalized hydrogen atom wave functions $\psi=R_{nl}(r)Y_{lm}(\theta,\phi)$ and expanding $e^{i\kappa z}$, Eq. (2) becomes[8]

$$\epsilon_{nlm,n'l'm'}(\kappa)=\sum_{p=|l-l'|}^{l+l'}\mathcal{Y}_{p,lm,l'm'}\mathcal{R}_{p,nl,n'l'}(\kappa), \quad (3)$$

where

$$\begin{aligned}\mathcal{Y}_{p,lm,l'm'}&=i^p[4\pi(2p+1)]^{\frac{1}{2}}\\&\quad\times\int Y_{p0}(\theta,\phi)Y_{lm}(\theta,\phi)Y^*_{l'm'}(\theta,\phi)d\omega\\&=i^p(2p+1)(2l+1)^{\frac{1}{2}}(2l'+1)^{-\frac{1}{2}}\\&\quad\times C(pll';0mm')C(pll';000), \quad (4)\end{aligned}$$

with the Clebsch-Gordan notation C as used by Rose.[9] Also

$$\begin{aligned}\mathcal{R}_{p,nl,n'l'}(\kappa)&=\mathcal{R}_{p,nl,n'l'}(\gamma)\\&=\int_0^\infty R_{nl}(r)R^*_{n'l'}(r)j_p(\kappa r)r^2dr\\&=A_{nl,n'l'}\int_0^\infty x^\beta e^{-x}j_p(\gamma x)\\&\quad\times L_{n+l}^{2l+1}(cx)L_{n'+l'}^{2l'+1}(c'x)dx, \quad (5)\end{aligned}$$

where j_p are spherical Bessel functions, L_n^m are associated Laguerre polynomials, and

$$\begin{aligned}A_{nl,n'l'}&=(2/na_0)^l(2/n'a_0)^{l'}\\&\quad\times[a_0nn'/(n+n')]^{l+l'+3}N_{nl}N^*_{n'l'},\\N_{nl}&=-[(2/na_0)^3(n-l-1)!(2n)^{-1}\{(n+l)!\}^{-3}]^{\frac{1}{2}}, \quad (6)\\\beta&=l+l'+2; \qquad c'=2[1+(n'/n)]^{-1};\\c&=2[1+(n/n')]^{-1}; \quad \gamma=\kappa a_0nn'/(n+n').\end{aligned}$$

Since the individual transitions $m\rightarrow m'$ are not of particular interest at present, it is convenient to average σ over m and sum over m' (here $m=m'$). Using Eqs. (1) and (3), the resulting cross section is

$$\begin{aligned}\sigma_{nl,n'l'}(\kappa)&=(2l+1)^{-1}\textstyle\sum_m\sum_{m'}\sigma'_{nlm,n'l'm'}(\kappa)\\&=8\pi(ka_0)^{-2}\kappa^{-3}(2l+1)^{-1}\textstyle\sum_m\sum_p\sum_{p'}\mathcal{R}_{p,nl,n'l'}(\kappa)\\&\quad\times\mathcal{R}^*_{p',nl,n'l'}(\kappa)\mathcal{Y}_{p,lm,l'm}\mathcal{Y}^*_{p',lm,l'm}. \quad (7)\end{aligned}$$

Using the properties of the Clebsch-Gordan coefficients[9]

$$\textstyle\sum_m\mathcal{Y}_{p,lm,l'm}\mathcal{Y}^*_{p',lm,l'm}=(2p+1)(2l+1)|C(pll';000)|^2\delta_{pp'},$$

which simplifies Eq. (7) to

$$\begin{aligned}\sigma_{nl,n'l'}(\kappa)&=8\pi(ka_0)^{-2}\kappa^{-3}\textstyle\sum_p(2p+1)\\&\quad\times|C(pll';000)|^2|\mathcal{R}_{pnln'l'}(\gamma)|^2. \quad (8)\end{aligned}$$

When the Laguerre functions are expressed as polynomials, the radial integrals in Eq. (5) reduce to sums of integrals which integrate directly in terms of hypergeometric functions[10]:

$$\begin{aligned}\int_0^\infty x^\beta e^{-x}j_p(\gamma x)dx&=\frac{\Gamma(\frac{1}{2})\Gamma(p+\beta+1)}{2^{p+1}\Gamma(p+\frac{3}{2})}\frac{\gamma^p}{(1+\gamma^2)^{\frac{1}{2}(p+\beta+1)}}\\&\quad\times F\left(\frac{p+\beta+1}{2},\frac{p+1-\beta}{2},\frac{2p+3}{2},\frac{\gamma^2}{1+\gamma^2}\right). \quad (9)\end{aligned}$$

The hypergeometric functions in Eq. (9) are expressed as polynomials directly when possible, i.e., when in $F(a,b,c,z)$ a or b is a negative integer; otherwise the function is expanded about the singularity at $z=1$ instead of $z=0$, again giving a polynomial. Equations (5), (8), and (9) then give the differential cross section

$$\begin{aligned}\sigma_{nl,n'l'}(\gamma)&=B'_{nl,n'l'}\pi k^{-2}(1+\gamma^2)^{-2(n+n')}(a_{-1}\gamma^{-1}+a_1\gamma\\&\quad+a_3\gamma^3+\cdots+a_{19}\gamma^{19}), \quad (10)\end{aligned}$$

where B', a_i are constants determined by the particular transition.

The integrals involved in the total cross section

$$\sigma(k)=\int_{\kappa_1}^{\kappa_2}\sigma(\kappa)d\kappa=\left[\frac{(n+n')}{nn'a_0}\right]\int_{\gamma_1}^{\gamma_2}\sigma(\gamma)d\gamma \quad (11)$$

[7] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), 2nd ed., p. 224.

[8] A similar analysis has been given by I. C. Percival and M. J. Seaton, Proc. Cambridge Phil. Soc. **53**, 654–62 (1957).

[9] M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957).

[10] This integral is given in P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 1575 but with an error in one of the gamma function arguments.

TABLE I. Born differential cross-section coefficients.

	$3s-4s$	$3s-4p$	$3s-4d$	$3s-4f$	$3p-4s$	$3p-4p$	$3p-4d$	$3p-4f$
B	1145.769	79.76968	2351.741	11915.48	5.307429	1315.765	101.7519	1688.500
a_{-1}	0	1	0	0	1	0	1	0
a_1	1	−32.00000	1	0	−60.00000	1	−18.00000	1
a_3	−10.43961	337.0689	−10.17940	1	1065.626	−10.57369	141.2041	−3.532468
a_5	43.20486	−1357.526	37.28712	−2.653465	−5098.330	45.23079	−469.4694	8.880756
a_7	−90.50426	2620.195	−59.88477	2.176061	10766.79	−86.31133	703.0000	−7.310508
a_9	102.2245	−2615.258	42.33049	−0.5517106	−11405.95	79.14854	−443.1429	1.944510
a_{11}	−62.45405	1333.355	−11.11736	0.04323106	6067.905	−33.85977	105.0000	
a_{13}	20.57359	−313.4915	0.9540292		−1464.781	5.683966		
a_{15}	−3.430352	26.91778			127.8639			
a_{17}	0.2267364							

	$3d-4s$	$3d-4p$	$3d-4d$	$3d-4f$	$3s-5p$	$3p-5d$	$3d-5f$
B	25.72690	1.808923	800.0606	167.4545	3.978130(−4)[a]	1.278685(−2)	4.795067(−3)
a_{-1}	0	1	0	1	3.422500(4)	1.225000(3)	3.675000(3)
a_1	1	−74.00000	1	−4.000000	1.232100(5)	1.421000(4)	7.203000(4)
a_3	−28.71429	1746.918	−5.435216	11.71428	−6.064041(6)	−9.575500(4)	2.290050(5)
a_5	245.5561	−7914.041	13.70764	−6.285714	1.880636(6)	−5.446600(5)	−1.289820(6)
a_7	−573.0816	15109.86	−16.06645	22.71428	2.942334(8)	6.871835(6)	3.351285(6)
a_9	489.1531	−10615.71	11.63123		−1.185096(9)	−1.979887(7)	−2.962050(6)
a_{11}	−138.0000	2625.000			1.928470(9)	2.307918(7)	4.090275(6)
a_{13}	12.25000				−1.554436(9)	−1.126020(7)	
a_{15}					6.332861(8)	2.058000(6)	
a_{17}					−1.195776(8)		
a_{19}					8.294400(6)		

[a] The numbers in parenthesis indicate the powers of ten by which the tabular numbers are to be multiplied, e.g., 3.978130(−4) = 3.978130×10^{-4}.

can be integrated analytically[11]:

$$I_q^r=\int_{\gamma 1}^{\gamma 2}\gamma^r(1+\gamma^2)^{-q}d\gamma,$$

$$I_q^r(r\geq 1)=\left[\sum_{\nu=0}^{\frac{1}{2}(r-1)}(-1)^{\nu+1}\binom{\frac{1}{2}[r-1]}{\nu}\times\{2(q-1-\tfrac{1}{2}[r-1]+\nu)\times(1+\gamma^2)^{q-1-[(r-1)/2]+\nu}\}^{-1}\right]_{\gamma 1}^{\gamma 2}, \quad (12)$$

$$I_q^{-1}=\left[\frac{1}{2}\sum_{\nu=1}^{q-1}\frac{1}{(q-\nu)(1+\gamma^2)^{q-\nu}}+\frac{1}{2}\ln\left(\frac{\gamma^2}{1+\gamma^2}\right)\right]_{\gamma 1}^{\gamma 2}.$$

The total cross section can then be written in the form

$$\sigma_{nl,n'l'}(k)=B_{nl,n'l'}(ka_0)^{-2}(a_{-1}I_q^{-1}+a_1I_q^1+a_3I_q^3+\cdots+a_{19}I_q^{19})\pi a_0^2, \quad (13)$$

where $q=2(n+n')$.

RESULTS

All twelve transitions between the $n=3$ and $n=4$ states, and the three strong optically allowed transitions between the $n=3$ and $n=5$ states were considered.

The calculations were performed on ten figure desk calculators. The constants in the differential and total cross sections are given in Table I, and the final cross sections are given in Table II. Note that the calculations were extended to low energies, where the Born approximation is certainly invalid, in connection with the development of simple approximate cross sections.[12] Each number in the tables is believed to be accurate to an error of at most one in the last figure.

The cross sections are presented graphically in Figs. 1 to 4.

DISCUSSION

The cross sections are much larger than for comparable transitions from the ground state; this is true at energies where the Born approximation is certainly

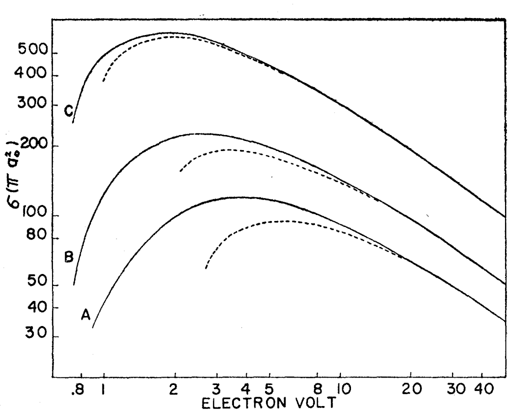

FIG. 1. Born (solid lines) and Bethe (broken lines) total cross sections for electron scattering by hydrogen atoms. *A*, $3s-4p$; *B*, $3p-4d$; *C*, $3d-4f$.

[11] W. Grobner and N. Hofreiter, *Integraltafel Erster Teil Unbestimmte Integral* (Wien, Springer-Verlag Vienna, 1950).

[12] S. N. Milford, Astrophys. J. **131**, 407 (1960).

TABLE II. Born total cross sections (units of πa_0^2).

Electron volts	0.7391	0.7809	0.8384	0.9675	2.704	3.256	13.89	59.38	120.8	1361
γ_1	0.27	0.25	0.23	0.2	0.1	0.09	0.0417392	0.02	0.014	0.00416717
$3s-4s$	64	80	96	116	89.6	78.0	21.5	5.191	2.564	0.2288
$3s-4p$	3(1)	3(1)	34	39.8	116	119	77.5	31.02	18.41	2.598
$3s-4d$	1.2(2)	1.6(2)	191	235	183	159	43.84	10.61	5.239	0.4674
$3s-4f$	2.7(2)	2.9(2)	301	290	118	98.8	23.32	5.456	2.681	0.2381
$3p-4s$	24.6	25.4	24.8	21.9	10.9	10.2	5.312	2.079	1.230	0.1732
$3p-4p$	83	1.0(2)	122	145	107	93.0	25.43	6.145	3.035	0.2707
$3p-4d$	50	65	83.0	118	226	221	119.0	44.50	25.91	3.532
$3p-4f$	2.9(2)	3.4(2)	369	394	226	193	50.16	11.99	5.910	0.5265
$3d-4s$	3.3	3.8	4.17	4.22	2.21	1.91	0.5119	0.1235	0.06099	0.005440
$3d-4p$	25	26	26.9	25.4	10.7	9.19	3.094	1.005	0.5647	0.07193
$3d-4d$	1.0(2)	1.2(2)	132	146	90.9	78.1	20.58	4.936	2.434	0.2169
$3d-4f$	2.5(2)	3.1(2)	378	479	575	540	243.7	84.77	48.35	6.321
Electron volts	1.243	3.193	4.697	20.12	9675					
γ_1	0.3	0.15	0.12	0.0555	0.0025					
$3s-5p$	5.6	18.4	19.2	10.89	0.08163					
$3p-5d$	16.5	36.2	33.6	15.68	0.1007					
$3d-5f$	57	72.0	60.7	23.54	0.1259					

valid. This agrees with the trend suggested by the results for transitions from the $n=2$ level.

By examining the numerical values in Table II, it is seen that (see optical transitions[13]) the cross sections are larger for transitions (a) which are optically allowed and (b) in which n and l change in the same sense.

For optically allowed transitions, the Bethe (dipole) approximations to the Born formulas are obtained by taking the $p=1$ terms only, and setting $j_1(\gamma x)\approx\gamma x/3$ in the Born formulas (5), (8), (11):

$$\mathcal{R}_{1,nl,n'l'}(\kappa)=(\kappa/3)\int_0^\infty R_{nl}(r)R^*_{n'l'}(r)r^3dr$$

$$=(\kappa/3)J_{nl,n'l'}, \qquad (14)$$

$$\sigma^D{}_{nl,n'l'}(k)=(8/3)|C(1ll';000)|^2|J_{nl,n'l'}/a_0|^2 \times(ka_0)^{-2}\ln(\kappa_c/\kappa_1)\pi a_0^2, \qquad (15)$$

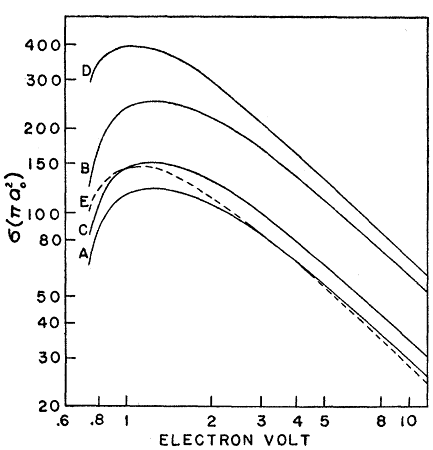

FIG. 2. Born total cross sections for electron scattering by hydrogen atoms. A, $3s-4s$; B, $3s-4d$; C, $3p-4p$; D, $3p-4f$; E, $3d-4d$.

[13] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Berlin, 1957), p. 267.

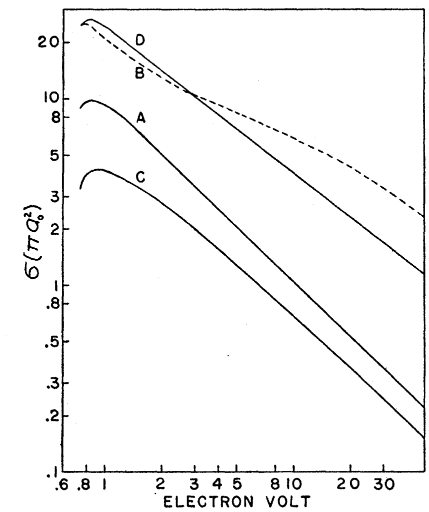

FIG. 3. Born total cross sections for electron scattering by hydrogen atoms. A, $3s-4f$ (actual values equal 30× plotted values); B, $3p-4s$; C, $3d-4s$; D, $3d-4p$.

where the upper limit κ_2 has been replaced by the cutoff κ_c in the usual way.[14]

The squares of the radial integrals, $|J/a_0|^2$ are tabulated by Green *et al.*[15] for $n, n'=1$ to 20. The cutoff $\kappa_c a_0$ for each transition was determined by setting the Bethe cross section [Eq. (15)] equal to the Born cross section at the highest energy calculated; $\kappa_c a_0$ is given in Table III. These values of $\kappa_c a_0$ were then used in Eq. (15) to compute the Bethe cross sections which are shown in Figs. 1 and 4. The energies $E(1\%)$ and $E(10\%)$ at which the Bethe and Born cross sections differ by 1%, 10%, respectively, were estimated and are listed in Table III.

[14] D. R. Bates, A. Fundaminsky, J. W. Leech, and H. S. W. Massey, Trans. Roy. Soc. (London) A243, 93 (1950).

[15] L. C. Green, P. P. Rush, and C. D. Chandler, Suppl. Astrophys. J. 3, 37–50 (1957).

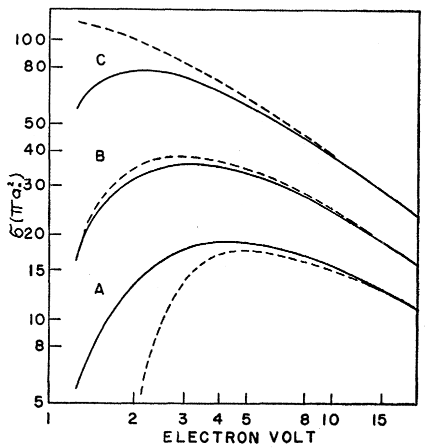

FIG. 4. Born (solid lines) and Bethe (broken lines) total cross sections for electron scattering by hydrogen atoms. A, $3s-5p$; B, $3p-5d$; C, $3d-5f$.

It is of interest to note that the Bethe formula gives a good fit to the Born approximation down to relatively low energies, which suggests that it is a satisfactory and much simpler substitute, provided that the energy is not too low. However, this requires a knowledge of κ_c which can be determined accurately only by evaluating the Born cross section at one high energy. For more approximate work, a very simple method of evaluating κ_c has recently been found.[12]

As stated above, the question of the range of validity of the first Born approximation for these transitions is the subject of current investigations.

TABLE III. Values of $\kappa_c a_0$ and estimates of the percentage deviation of the Bethe approximation from the Born approximation.

	$\kappa_c a_0$	$E(1\%)$(ev)	$E(10\%)$(ev)
$3s-4p$	0.06314	30	8.4
$3p-4d$	0.07819	20	4.2
$3d-4f$	0.10593	7	1.2
$3s-5p$	0.09478	14	4.5
$3p-5d$	0.12918	12	2
$3d-5f$	0.21450	10	3.3

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Born Cross Sections for Inelastic Scattering of Electrons by Hydrogen Atoms. II. 4s, 4p, 4d, 4f States*†

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(Received January 20, 1960)

The Born total cross sections are calculated for the inelastic scattering of electrons by hydrogen atoms for the strong optically allowed transitions from $n=4$ to $n'=5$. The nine incident energies considered range from 0.546 ev to 1361 ev. In addition, the $4s$ to $6p$ and $4f$ to $6g$ transitions are considered. Bethe (multipole) cross sections are also calculated and found to reproduce the Born results down to low energies.

INTRODUCTION

THIS paper extends the recent calculations[1] of the Born cross sections for inelastic scattering of electrons by hydrogen atoms to the $4s$, $4p$, $4d$, and $4f$ states. The four optically allowed transitions from $n=4$ to $n'=5$ are considered as well as the transitions $4s$ to $6p$ and $4f$ to $6g$.

FORMULATION

In Paper I it was shown that the Born differential cross section for the transition from (nl) to $(n'l')$ is given by

$$\sigma_{nl,n'l'}(\gamma)=B_{nl,n'l'}\frac{\pi}{k^2(1+\gamma^2)^{2(n+n')}}\times(a_{-1}\gamma^{-1}+a_1\gamma+a_3\gamma^3+\cdots+a_t\gamma^t), \quad (1)$$

where $t=23$ for $n=4$, $n'=5$ and $t=27$ for $n=4$, $n'=6$; B and a_i are constants determined by the particular transition.

The total cross section has been averaged over m and summed over m'; $\gamma=\kappa a_0 nn'/(n+n')$, where $\hbar\mathbf{k}$ is the initial momentum, and $\hbar\boldsymbol{\kappa}$ the change of momentum of the scattered electron; a_0 is the Bohr radius. The total cross section is

$$\sigma_{nl,n'l'}(k)=B_{nl,n'l'}[\pi a_0^2/(ka_0)^2]\times(a_{-1}I_q^{-1}+a_1I_q+a_3I_q^3+\cdots+a_tI_q^t), \quad (2)$$

* This work was supported in part by the Office of Naval Research and in part by the Air Force Office of Scientific Research.

† Based in part upon a thesis to be submitted by Leonard Fisher to the Graduate School of St. John's University in partial fulfillment of the requirements for the degree of Master of Science.

[1] G. McCoyd, S. N. Milford, and J. Wahl, preceding paper [Phys. Rev. 119, 149 (1960)], hereafter referred to as Paper I.