Baryon Mass Spectrum*

KATSUMI TANAKA
Argonne National Laboratory, Argonne, Illinois
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It is assumed that the π-baryon interactions are universal and the K-baryon interactions account for the large mass differences between baryons. If further all baryon spins are $\frac{1}{2}$ and K spin is zero, (Σ,Λ) parity is even, the (Σ,Λ) mass difference can be neglected, and the present baryon spectrum and its isotopic spin assignments are correct, then to all orders in the π-baryon coupling constant and to the second order in the K-baryon coupling constants, one can obtain the essential feature of the observed mass spectrum. The magnitudes of the K-coupling constants that yield this mass spectrum are crudely estimated.

1. INTRODUCTION

SEVERAL attempts[1–4] have been made to deduce experimentally verifiable consequences from strong meson-baryon interaction under assumptions stronger than charge independence. Notable among these assumptions is the universal π-baryon interactions in which one assumes that in the absence of all strong K couplings the baryons are completely mass degenerate and then arrives at asymmetries between the K interactions to account for the mass difference among the (isotopic spin) multiplets. There is no understanding of masses of elementary particles within the framework of present field theories. The differences of masses, however, may be explainable on the basis of known interactions with reasonable assumptions.

We assume all strong interactions are charge independent (formally invariant under rotation of isotopic spin) and neglect electromagnetic interactions so that masses within the baryon multiplets are identical. Next, let the π-baryon interactions have universal (global) symmetry so that they do not contribute to the differences between baryon masses. All possible self-mass corrections from the π-baryon interactions give rise to a degenerate baryon mass m_0. The mass differences among the baryon multiplets may only arise from the virtual emission and reabsorption of various numbers of K mesons with their various corrections.

The mass relation that the sum of masses of N and Ξ is equal to that of Λ and Σ [Eq. (16)] is now derived to all orders in the π coupling and to second order in K coupling. This analysis provides the formalism for the estimation of the K-coupling constant.

Since one presumably cannot rely on a perturbation calculation of the K interactions, no attempt was made to explain the details of the mass splitting—not even the (Σ,Λ) mass difference. Such details presumably cannot be explainable until a reliable method of estimating the higher order corrections of K-baryon interactions is known. If inequalities between the K-coupling constants are to account for the mass splits, then one faces the interesting question why baryons with different strangeness have a similar interaction with K mesons but a different value of the coupling constant.

In Sec. 2 an expression for the self-mass of the baryon is obtained. In Sec. 3, a relation among the baryon masses is obtained on the basis of general arguments. In Sec. 4, a complete set of intermediate states is introduced into the expression for the self-mass, and the one-baryon intermediate state and the state with one baryon plus a baryon pair are taken into account in estimating the K-baryon coupling constants. Finally, the result is discussed in Sec. 5.

2. METHOD

The interaction Hamiltonian adopted for π and K mesons is the doublet representation of Gell-Mann[1] and Pais[3] in which $I=\frac{1}{2}$ is assigned to all baryons and $I=0$ to K_+ and K_0; i.e.,

$$H_\pi = iG[\bar{N}_1\boldsymbol{\tau}\gamma_5N_1+\bar{N}_2\boldsymbol{\tau}\gamma_5N_2+\bar{N}_3\boldsymbol{\tau}\gamma_5N_3+\bar{N}_4\boldsymbol{\tau}\gamma_5N_4]\boldsymbol{\pi}, \quad (1)$$

$$H_K = F_{\mathrm{I}}2^{\frac{1}{2}}[(\bar{N}_1\eta_{\mathrm{I}}N_2)K^0+(\bar{N}_1\eta_{\mathrm{I}}'N_3)K^+] + F_{\mathrm{II}}2^{\frac{1}{2}}[(\bar{N}_4\eta_{\mathrm{II}}N_2)\bar{K}^+-(\bar{N}_4\eta_{\mathrm{II}}'N_3)\bar{K}^0]+\text{H.c.}, \quad (2)$$

where H. c. is the Hermitian conjugate,

$$N_1=\begin{pmatrix}p\\ n\end{pmatrix},\quad N_2=\begin{pmatrix}\Sigma^+\\ 2^{-\frac{1}{2}}(\Lambda^0-\Sigma^0)\end{pmatrix},\quad N_3=\begin{pmatrix}2^{-\frac{1}{2}}(\Lambda^0+\Sigma^0)\\ \Sigma^-\end{pmatrix},\quad N_4=\begin{pmatrix}\Xi^0\\ \Xi^-\end{pmatrix}, \quad (3)$$

and the symbol of a particle denotes the field operator that destroys it.

The assumptions underlying Eqs. (1) and (2) are that all strong interactions are charge independent with π interactions being in addition universal, the present baryon spectrum and its isotopic spin assignments are correct, the baryon spins are $\frac{1}{2}$ and the K spin is zero, the (Σ,Λ) parity is even, and the (Σ,Λ) mass difference can be neglected. Since we carry out a perturbation calculation with respect to the K-baryon interactions

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[1] M. Gell-Mann, Phys. Rev. **106**, 1296 (1957).
[2] J. Schwinger, Phys. Rev. **104**, 1164 (1956); Ann. Physik **2**, 407 (1957).
[3] A. Pais, Phys. Rev. **110**, 574 (1958).
[4] B. d'Espagnat, J. Prentki, and A. Salam, Nuclear Phys. **3**, 446 (1957).

whose coupling constants are large compared to (say) the electromagnetic interactions, we cannot expect to explain the (Σ,Λ) mass difference which is the smallest baryon mass difference.

In Eq. (1), the $\eta_{\rm I}$, $\eta_{\rm I}'$, $\eta_{\rm II}$, and $\eta_{\rm II}'$ stand for 1 or $i\gamma_5$. If the (K^0,K^+) parity is even we have $\eta_{\rm I}=\eta_{\rm I}'$, $\eta_{\rm II}=\eta_{\rm II}'$, and $\eta_{\rm I}=1(i\gamma_5)$ corresponds to even (odd) parity of (N_1,Λ,K). Further, if the (Ξ,N) parity is even, then $\eta_{\rm I}=\eta_{\rm II}$; and it it is odd, then $\eta_{\rm I}=1$, $\eta_{\rm II}=i\gamma_5$ or $\eta_{\rm I}=i\gamma_5$, $\eta_{\rm II}=1$. If (K^0K^+) parity is odd, we have $i\eta_{\rm I}'\gamma_5=\eta_{\rm I}$ and $i\eta_{\rm II}\gamma_5=\eta_{\rm II}'$. These factors η are suppressed in the following.

If the (K^0,K^+) parity is odd, then there is an additional possible strong interaction of the type $KK\pi$ whose coupling constant is presumably very small.[5] Neither this interaction nor other possible πK interactions are considered here.

It should be mentioned as a danger signal that H_K given by Eq. (2) with $\eta_{\rm I}=\eta_{\rm I}'$, $\eta_{\rm II}=\eta_{\rm II}'$ is incompatible with associated-production experiments.[3]

The total Hamiltonian of our system of interacting baryons, π mesons and K mesons is given as

$$H=H_{N\pi}+H_{0K}+H_K, \tag{4}$$

where $H_{N\pi}$ is the Hamiltonian of the interacting baryons and π-meson field including counter terms for mass renormalization and four-meson divergence, H_{0K} is the free Hamiltonian for the K-meson field, and H_K is the Hamiltonian for the K-baryon interactions given by Eq. (2). Equation (1) is a part of $H_{N\pi}$.

An arbitrary Schrödinger operator is transformed as[6]

$$O(t)=\exp[i(H_{N\pi}+H_{0K})t]O\exp[-i(H_{N\pi}+H_{0K})t], \tag{5}$$

where we have taken $\hbar=c=1$. Inserting the K-meson field K and its current operator j_K (which is a bilinear product of baryon field operators) into O of Eq. (5), one finds that the $K(t)$ is the bare operator and $j_K(t)$ is the dressed operator by the π-baryon interactions.

In this interaction picture, the S_K operator is given as

$$S_K=\sum_{n=0}^{\infty}\frac{(-i)^n}{n!}\int dx_1\cdots dx_n(H_K(x_1)\cdots H_K(x_n))_+, \tag{6}$$

where the subscript$+$designates the chronological P product of the parenthesis. The initial and final states of the S_K matrix are eigenstates of the Hamiltonian $H_{N\pi}+H_{0K}$, i.e.,

$$(H_{N\pi}+H_{0K})|n\rangle=E_n|n\rangle, \tag{7}$$

so $|n\rangle$ can be represented as the product of the eigenstates of the free K-meson Hamiltonian H_{0K} and those of the Hamiltonian $H_{N\pi}$ which includes the π-baryon interactions. In the following expressions the K-meson state is suppressed since it can be factored out.

[5] A. Pais, Phys. Rev. **112**, 624 (1958).

[6] An analogous situation has been discussed in S. Sunakawa and K. Tanaka, Phys. Rev. **115**, 754 (1959). The main steps are included here for the sake of completeness.

To the second order in the K-coupling constants, the mass shift of the baryon is due to the virtual emission and reabsorption of a K meson by the (physical) baryon. In the rest system of the baryon, the S matrix is given by the expression

$$S_K^{(2)}=\left\langle n_i\left|\frac{(-i)^2}{2!}\int dx\int dy(H_K(x)H_K(y))_+\right|n_i\right\rangle$$
$$-\left\langle 0\left|\frac{(-i)^2}{2!}\int dx\int dy(H_K(x)H_K(y))_+\right|0\right\rangle, \tag{8}$$

where $|n_i\rangle$ is an eigenstate of Eq. (7) with one baryon n_i and no K mesons.[7] The letter n_i represents any member of the baryon multiplet i (for example, proton or neutron for $i=1$). The baryons n_1, n_2, n_3, and n_4 have the degenerate baryon mass m_0 which is already modified by the self-mass arising from π interactions.

This is exactly what would have been obtained from a term of the form[8] $-\delta m_i\bar{N}_iN_i$ in the interaction Hamiltonian H_K. According to Eq. (6), we thus get a term

$$i\delta m_i\left[\left\langle n_i\left|\int dx\,\bar{N}_i(x)N_i(x)\right|n_i\right\rangle\right.$$
$$\left.-\left\langle 0\left|\int dx\,\bar{N}_i(x)N_i(x)\right|0\right\rangle\right],$$

which eliminates Eq. (8) so that we have,[9,10] after substituting Eq. (2) into Eq. (8) and changing variables to $z=x-y$,

$$\delta m_i\bar{u}_iu_i$$
$$=-i\frac{1}{2}\int dz\langle T(K(z)K^\dagger(0))\rangle_0$$
$$\times\langle n_i|T[2F_{\rm I}^2(\bar{N}_1N_2\bar{N}_2N_1+\bar{N}_2N_1\bar{N}_1N_2)$$
$$+2F_{\rm I}^2(\bar{N}_1N_3\bar{N}_3N_1+\bar{N}_3N_1\bar{N}_1N_3)$$
$$+2F_{\rm II}^2(\bar{N}_4N_2\bar{N}_2N_4+\bar{N}_2N_4\bar{N}_4N_2)$$
$$+2F_{\rm II}^2(\bar{N}_4N_3\bar{N}_3N_4+\bar{N}_3N_4\bar{N}_4N_3)]|n_i\rangle. \tag{9}$$

The expression (9) for the self-mass can be represented graphically by Fig. 1.

[7] The S_K matrix element is defined by $\langle p|S_K|p\rangle/\langle 0|S_K|0\rangle$. The expectation value of the S_K operator with respect to the vacuum $\langle 0|S_K|0\rangle$ gives rise to the second term on the right-hand side of Eq. (8). We shall suppress this term as it will not affect our arguments in any way, but will restore it at the appropriate place later.

[8] This is based on the fact that $S_K^{(2)}$ is a constant when the baryon is a free particle (with the π meson cloud) in the initial and final states. The subscripts i are not to be summed.

[9] The cross terms (whose coefficient is $F_{\rm I}F_{\rm II}$) are of the type $\sum_n\langle n_i|\bar{N}_1N_2|n\rangle\langle n|\bar{N}_4N_2|n_i\rangle$ after introducing a complete set of states. As can be seen from Sec. 3, all such terms vanish.

[10] For the proton and neutron self-masses a related expression has been obtained in G. C. Wick, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, Inc., New York, 1957), p. 1–34; R. A. Sorensen (to be published).

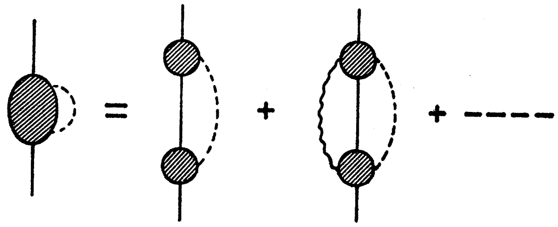

FIG. 1. Baryon self-mass diagram. The solid line represents baryons. The dotted line represents K mesons and the wavy lines represent π mesons. The shaded area represents all possible corrections arising from virtual mesons and baryon pairs.

Since the mass difference between charged and neutral K mesons is neglected, the T products of all the K mesons are identical so that the subscript may be dropped and these products factored out. The arguments of the baryon field operators in all terms in the bracket on the right-hand side of Eq. (9) have been suppressed because it is obvious that the argument of the first two field operators is z and that of the last two is the null vector.

For further discussion, we set

$$-i\frac{1}{2}\int dz\langle T(K(z)K^{\dagger}(0))\rangle_0 \times\langle n_i|\,T(\bar{N}_1N_2\bar{N}_2N_1+\bar{N}_2N_1\bar{N}_1N_2)\,|n_i\rangle =(N_1N_2,n_i). \quad (10)$$

The remaining terms are defined by a similar expression. Then, the self-masses are expressible as

$$\delta m_i=2F_{\mathrm{I}}^2\{(N_1N_2,n_i)+(N_1N_3,n_i)\} +2F_{\mathrm{II}}^2\{(N_4N_2,n_i)+(N_4N_3,n_i)\}. \quad (11)$$

The subscripts i designate the particles: $i=1$ corresponds to N, $i=2$, 3 to Σ, Λ, and $i=4$ to Ξ.

Equation (11) is our basic expression from which a relation among the self-masses will be obtained. It is noted again that all baryons that appear in the brackets have the same degenerate baryon mass m_0 and are distinguished by their strangeness quantum number. The first two letters within the brackets will be called the first argument and the last letter the second argument. The brackets are symmetric with respect to the subscripts within the first argument so that, for instance,

$$\begin{aligned}(N_1N_2,n_1)&=(N_1N_2,n_2),\\ (N_4N_2,n_2)&=(N_4N_2,n_4).\end{aligned} \quad (12)$$

3. SELF-MASS RELATION

The self-masses of the baryons are given by Eq. (11). There are two kinds of brackets that appear here. The first kind is that in which the subscript of the second argument of the bracket is identical to one subscript of the first argument such as (N_1N_2,n_1). The second kind is that in which the subscript of the second argument is different from the subscripts of the first argument such as (N_4N_2,n_1) and (N_1N_3,n_2). We shall now show that brackets of the latter kind vanish.

In order to examine the bracket (N_1N_3,n_2) further, we need to consider the T product of the baryon field operators of Eq. (10). We take the case $z_0>0$ (as the case $z_0<0$ can be handled similarly) and introduce a complete set of states of the Hamiltonian $H_{N\pi}$:

$$\begin{aligned}\langle n_2|\bar{N}_1N_3\bar{N}_3N_1+\bar{N}_3N_1\bar{N}_1N_3|n_2\rangle &=\textstyle\sum_n \langle n_2|\bar{N}_1N_3|n\rangle\langle n|\bar{N}_3N_1|n_2\rangle\\ &\quad+\textstyle\sum_n \langle n_2|\bar{N}_3N_1|n\rangle\langle n|\bar{N}_1N_3|n_2\rangle. \end{aligned}\quad (13)$$

Because of conservation of baryons the intermediate states n consist of states with one baryon, one baryon plus an arbitrary number of π mesons, and baryon-antibaryon pairs.

The state vectors and baryon field operators are clothed by the π-baryon interactions. When a π meson interacts with baryons, it does not change the kind of baryon as can be seen from Eq. (1) so that the baryon-antibaryon pairs have no net strangeness, i.e., $S=0$. This means that the baryon that appears in any possible state of the first term on the right-hand side of Eq. (13) should have strangeness $S=-2$ because $\bar{N}_1N_3$ increases S by one unit and n_2 has $S=-1$, so that the state should consist of a Ξ particle (corresponding to n_4) plus π mesons and baryon pairs so as to conserve charge and strangeness.

After separating out the interaction Hamiltonian Eq. (1) with its counter terms between the π mesons and baryons from $H_{N\pi}$, we get for the matrix elements[6] of the first factor of the first term of Eq. (13)

$$\begin{aligned}\langle n_2|\bar{N}_1N_3|n\rangle&=\langle n_2|\bar{N}_1N_3|n_4\ldots\rangle\\ &=(n_2|\,(S\bar{N}_1{}^{\mathrm{I}}N_3{}^{\mathrm{I}})_+|n_4\ldots),\end{aligned} \quad (14)$$

where S is the scattering operator of the π-baryon interactions and $|\ )$ designates bare states, and dots designate an arbitrary number of π mesons and baryon pairs. Since the S operator does not change one kind of baryon into another kind, all such matrix elements as well as those of the second term of Eq. (13) vanish so that $(N_1N_3,n_2)=0$. Similarly, one can show that all the brackets of the second kind vanish. From this fact, and from Eqs. (11) and (12) the baryon self-masses are given as

$$\left.\begin{aligned}\delta m_1&=2F_{\mathrm{I}}^2\{(N_1N_2,n_1)+(N_1N_3,n_1)\},\\ \delta m_2&=2F_{\mathrm{I}}^2(N_1N_2,n_1)+2F_{\mathrm{II}}^2(N_4N_2,n_2),\\ \delta m_3&=2F_{\mathrm{I}}^2(N_1N_3,n_1)+2F_{\mathrm{II}}^2(N_4N_3,n_3),\\ \delta m_4&=2F_{\mathrm{II}}^2\{(N_4N_2,n_2)+(N_4N_3,n_3)\},\end{aligned}\right\} \quad (15)$$

from which it follows that

$$\delta m_1+\delta m_4=\delta m_2+\delta m_3.$$

The physical masses of the baryons are defined as

$$m_i=m_0+\delta m_i,\qquad i=1,\,2,\,3,\,4,$$

which immediately yields the observable baryon mass relation[11]

$$m_1+m_4=m_2+m_3. \qquad (16)$$

If the nucleon mass is taken as unity and the center of gravity of the masses of the Λ and Σ's is taken for m_2, the experimental result is

$$m_1=1,\quad m_2=m_3=1.248,\quad \text{and}\quad m_4=1.408,$$

so that Eq. (16) is valid to 4%.

Equation (16) is correct for even (Σ,Λ) and (K^0,K^+) parities and is independent of the (Ξ,N) parity. Although we have taken direct type interactions of K mesons and baryons in Eq. (2), Equation (16) is also valid when derivative type interactions are taken. This is a powerful result because the essential feature of the spectrum of baryon masses is reproducible to all orders in the π-baryon coupling constant and to the second order in the K-baryon coupling constant, once it is assumed that the (Σ,Λ) and (K^0,K^+) parities are even. Relations similar to Eq. (16) have been discussed before under more special conditions such as to the second order in the K-baryon coupling constants and explicitly in the zeroth[1] or second order[12] in the π-baryon coupling constant.

Let us consider in the remainder of the paper that the (K^0,K^+) parity is even. Then the Hamiltonians $H_{N\pi}$, H_{0K}, and H_K are invariant under the following combined interchanges[3,5]:

$$N_2\rightarrow N_3,\ K^+\rightarrow K^0,\ -\bar{K}^0\rightarrow \bar{K}^+. \qquad (17)$$

The T product of the K-meson field operators in Eq. (10) is invariant under the interchanges (17). The remaining part of Eq. (10) does not depend on the K-meson field operators so that we need to be concerned only with the interchanges between the baryon field operators.

Let U be the unitary transformation which generates the interchanges (17). Since this commutes with the Hamiltonian $H_{N\pi}$ that defines the states $|n\rangle$, it leaves the vacuum state invariant and generates interchanges in the one-particle states and field operators so that, for instance, inserting $UU\dagger=1$ in Eq. (10) leads to

$$\begin{aligned}(N_1N_3,n_2)&=-i\frac{1}{2}\int dz\langle T(K(z)K^\dagger(0))\rangle_0\langle n_2|UU^\dagger(\bar{N}_1N_3\bar{N}_3N_1\\&\qquad+\bar{N}_3N_1\bar{N}_1N_3)UU^\dagger|n_2\rangle\\&=-i\frac{1}{2}\int dz\langle T(K(z)K^\dagger(0))\rangle_0\\&\qquad\times\langle n_3|T(\bar{N}_1N_2\bar{N}_2N_1+\bar{N}_2N_1\bar{N}_1N_2)|n_3\rangle\\&=(N_1N_2,n_3). \qquad (18)\end{aligned}$$

From Eq. (15) and rules given in Eq. (18), we finally obtain

$$\begin{aligned}\delta m_1&=4F_{\mathrm{I}}^2A,\\ \delta m_2=\delta m_3&=2F_{\mathrm{I}}^2A+2F_{\mathrm{II}}^2B, \qquad (19)\\ \delta m_4&=4F_{\mathrm{II}}^2B,\end{aligned}$$

where

$$A=(N_1N_2,n_1)\quad\text{and}\quad B=(N_4N_2,n_2).$$

Equation (19) leads to $m_1+m_4=2m_2$ but we have not said anything about the relative magnitudes of m_1 and m_4. This point will be discussed in the next section. We have explicitly obtained the relation (19) that can also be found in a perturbation theory of the π interactions in order to provide the formalism for further analysis.

4. ESTIMATE OF K-BARYON COUPLING CONSTANT

The expression $4F_{\mathrm{I}}^2A$ that appears in the self-mass relations (19) can be written with the aid of Eq. (10), after the second term of Eq. (8) is restored, as

$$\begin{aligned}4F_{\mathrm{I}}^2A=-i\int dz\langle T(K(z)K^\dagger(0))\rangle_0&\\ \times[\langle n_1|(j_K(z)j_K{}^\dagger(0))_+|n_1\rangle&\\ -\langle 0|j_K(z)j_K{}^\dagger(0)|0\rangle], &\qquad (20)\end{aligned}$$

where the baryon current is

$$j_K(z)=\sqrt{2}F_{\mathrm{I}}\bar{N}_1(z)\eta N_2(z);$$

The remaining term again vanishes for the reasons given in Sec. 3. The baryon current is clothed by the π-baryon interactions. The matrix elements of Eq. (20) are unknown so that again a sum over a complete set of states $|n\rangle$ of the Hamiltonian $H_{N\pi}$ is introduced on the right-hand side of Eq. (20), and the T product of the K-meson field operators is replaced by

$$\langle T(K(z)K^\dagger(0))\rangle_0=\tfrac{1}{2}D_F(z);$$

$$\begin{aligned}4F_{\mathrm{I}}^2A=-i\int_0^\infty dzD_F(z)\sum_n[\langle n_1|j_K(z)|n\rangle\langle n|j_K{}^\dagger(0)|n_1\rangle&\\ -\langle 0|j_K(z)|n\rangle\langle n|j_K{}^\dagger(0)|0\rangle]. &\qquad (21)\end{aligned}$$

A factor of 2 has been inserted on the right-hand side of Eq. (21) because of restricting the integral to the region $z_0>0$ since it can be proved that the contribution from the region $z_0>0$ is idnetical to that from $z_0<0$.

In order to make some estimate, it is advantageous to treat the one-baryon intermediate state on an equal footing with the intermediate state with one baryon plus a baryon-antibaryon pair because a part of the latter gives rise to the contribution corresponding to the negative-energy state of the intermediate nucleon in perturbation theory. The one-baryon approximation would amount to retaining only the first diagram on the right-hand side of Fig. 1.

[11] The equality $m_2=m_3$, which has been assumed, imposes the restriction that the (K_0,K_+) parity should be even.

[12] H. Katsumori, Progr. Theoret. Phys. (Kyoto) **19**, 342 (1958); **20**, 578 (1958).

The one-baryon state and the state of one baryon plus a baryon-antibaryon pair are taken from the first term of Eq. (21), and the state of a baryon-antibaryon pair is taken from the second term. The one-baryon intermediate state is characterized by the momentum variable $q[\mathbf{q},(\mathbf{q}^2+m_0^2)^{\frac{1}{2}}]$, the other internal variables being suppressed. It is a physical state in the absence of K-baryon interaction.

Let n_1 be a proton[13] whose momentum variable is $p(\mathbf{p}=0,\ m_0)$. It follows from the conservation of charge and strangeness (and the fact that j_K increases the strangeness quantum number by one unit) that the intermediate baryon state is a Σ^+ hyperon state. The further analysis may be carried out in a way analogous to that in reference 6 so that it will not be repeated here. The result is

$$4F_{\mathrm{I}}^2A=-i\int_0^\infty dz\,D_F(z)\times\sum_q\left[\langle Pp|j_K(z)|\Sigma q\rangle\langle\Sigma q|j_K{}^\dagger(0)|Pp\rangle-\langle 0|j_K(z)|\Sigma p,\bar{P}q\rangle\langle\Sigma p,\bar{P}q|j_K{}^\dagger(0)|0\rangle\right].\quad(22)$$

The vertex of the K-baryon interaction that appears in the first term on the right-hand side of Eq. (22) can be expressed as

$$\langle Pp|j_K(z)|\Sigma q\rangle=(m_0^2/q_0p_0)^{\frac{1}{2}}\sqrt{2}F_{\mathrm{I}}\bar{u}(p)\eta_{\mathrm{I}}u(q)\times G[(p-q)^2]e^{-i(p-q)\cdot z},\quad(23)$$

where the form factor G is normalized such that $G(-\mu_K^2)=1$, and is dependent on a three-dimensional momentum transfer. The matrix elements in the second term on the right-hand side of Eq. (22) can be written as

$$\langle 0|j_K(z)|\Sigma p,\bar{P}q\rangle=-(m_0^2/q_0p_0)^{\frac{1}{2}}\sqrt{2}F_{\mathrm{I}}\bar{v}(q)\eta_{\mathrm{I}}u(p)\times G[(p+q)^2]e^{i(p+q)\cdot z}.\quad(24)$$

The factor $\Lambda_+(q)=(-iq_\mu\gamma_\mu+m_0)/2m_0$ is a projection operator and $\eta_{\mathrm{I}}=1$ or $i\gamma_5$, depending on whether the (K,Λ) parity is even or odd.

The substitution of Eqs. (23) and (24) and the relation $D_F(z)=2iD^{(+)}(z)(z_0\rangle 0)$ into Eq. (22) leads to

$$4F_{\mathrm{I}}^2A=4F_{\mathrm{I}}^2\int_0^\infty dz\,D^{(+)}(z)\int_0^\infty d^3q[m_0/(2\pi)^3q_0]\times\{\bar{u}(p)\eta_{\mathrm{I}}\Lambda_+(q)\eta_{\mathrm{I}}u(p)e^{-i(p-q)\cdot z}G^2[(p-q)^2]+\bar{u}(p)\eta_{\mathrm{I}}\Lambda_+(-q)\eta_{\mathrm{I}}u(p)e^{i(p+q)\cdot z}G^2[(p+q)^2]\}.\quad(25)$$

Carrying out the integrations in Eq. (25) as in reference 6 and taking $m_0=1$, we obtain

$$4F_{\mathrm{I}}^2A=-\frac{F_{\mathrm{I}}^2}{4\pi}\frac{2}{\pi}\int_0^\infty dk\frac{k^2}{\omega E}\left(\frac{E\pm1}{\omega+E-1}G^2[2(E-1)]-\frac{E\mp1}{\omega+E+1}G^2[-2(E+1)]\right),\quad(26)$$

where $E=(k^2+1)^{\frac{1}{2}}$ and $\omega=(k^2+\mu_K{}^2)^{\frac{1}{2}}$. The μ_K is the mass of the K meson. The upper and lower signs of Eq. (26) refer to the cases $\eta_{\mathrm{I}}=1$ and $\eta_{\mathrm{I}}=i\gamma_5$, respectively.

It is appropriate to note the relation between Eq. (26) and the more familiar lowest order perturbation result without a cutoff factor.[14] The latter result is denoted by A'. The self-mass that a nucleon has by virtue of a virtual emission and reabsorption of a K meson is given to the second order by

$$4F_{\mathrm{I}}^2A'=4F_{\mathrm{I}}^2\frac{i}{(2\pi)^4}\bar{u}\int\frac{d^4k}{k^2-i\epsilon}\times\frac{\eta_{\mathrm{I}}[-i(p_\mu-k_\mu)\gamma_\mu+m_0]\eta_{\mathrm{I}}}{[(p-q)^2+m_0^2-i\epsilon]}u.\quad(27)$$

Integration of Eq. (27) with respect to k_0, and rearrangement, gives

$$4F_{\mathrm{I}}^2A'=-\frac{F_{\mathrm{I}}^2}{4\pi}\frac{2}{\pi}\int_0^\infty dk\frac{k^2}{\omega E}\left[\frac{E\pm1}{\omega+E-1}-\frac{E\mp1}{\omega+E+1}\right].\quad(28)$$

Comparison of Eq. (26) with Eq. (28) shows that the present result is similar to the lowest order perturbation result but that it is multiplied by a natural cutoff which is a manifestation of the strong π-baryon interaction modifying the vertex operator of K mesons and baryons. Equation (26) is essentially a consequence of retaining the state with one baryon and the state with one baryon plus a baryon-antibaryon pair. The first term on the right-hand side of Eq. (26) gives the contribution from the intermediate positive-energy baryon state and the second gives that from the intermediate negative-energy baryon state. Some differences between this method and the lowest order perturbation should be noted here. Since Eq. (28) may be written in many different ways by adding the same term to both the first and second terms on the right-hand side, the contributions from the intermediate baryon states (with positive and negative energy) are not unique. Further, the self-mass is logarithmically divergent in perturbation theory so that some arbitrary cutoff must be employed. The present method avoids these difficulties so far as lowest order perturbation theory is concerned. However, an infinite number of other terms to order

[13] One can just as well take a neutron, in which case the only difference would be that one has an intermediate state with a different charge. The internal variables such as spin, isotopic spin and strangeness are suppressed.

[14] The comparison of A with the lowest order perturbation result with a cut-off function is obtained by substituting $i\gamma_5G$ for J_μ in Eq. (47) of reference 6.

$F_I^2/4\pi$ have been left out, notable among them the state with one baryon and one meson. The consequent error is very difficult to estimate.

Returning to Eq. (26), we have no information about the form factor G. For the sake of illustration, let us take a Yukawa model:

$$G=\Lambda^2/(\Lambda^2+q^2),$$

with a rms radius $a^2=6/\Lambda^2=(0.566\times 10^{-13})^2$ cm^2; and let us evaluate Eq. (26) in steps of 0.2 from $k=0$ to 2 and in steps of 2 from $k=2$ to 10 by use of Simpson's rule.

The first form factor that appears on the right-hand side of Eq. (26) depends on a space-like momentum so that it is in the experimental region, whereas the second form factor depends on a time-like momentum so that it is in the nonexperimental region. For the latter we assume a form factor that agrees with the former in the experimental region. In the evaluation of Eq. (26), the previously mentioned singularity in the form factor[6] does not occur for the rms radius given above.

The numerical result of $4\pi A$ is $4\pi A_e=-0.078$ for $\eta_I=1$ and $4\pi A_0=-0.0064$ for $\eta_I=i\gamma_5$. If the (Ξ,N) parity is even, then we have $B_e=A_e\, B_0=A_0$ from which according to Eq. (19), it follows that

$$\delta m_2-\delta m_1=m_2-m_1=-2(F_I^2/4\pi-F_{II}^{2}-/\pi)4\pi A=0.2,$$

so that

$$\begin{aligned} F_I^2/4\pi-F_{II}^2/4\pi &= 1.3, \qquad \eta_I=1\\ &=15.6, \qquad \eta_I=i\gamma_5. \end{aligned} \tag{29}$$

If (Ξ,N) parity is odd, then we have $A=A_e$, $B=A_0$ or $A=A_0$, $B=A_e$ so that

$$12.2F_I^2/4\pi-F_{II}^2/4\pi=15.6,$$

or

$$F_I^2/4\pi-12.2F_{II}^2/4\pi=15.6. \tag{30}$$

In addition, if $F_I^2=F_{II}^2=F^2$ then $F^2/4\pi\approx 1.4$ satisfies $m_4>m_1$ for the upper case of Eq. (30).

5. RESULTS

We have shown that the essential feature of the spectrum of baryon masses, namely $m(\Xi)+m(N)=2m(\Sigma)$, can be explained on the basis of universal π interactions, reasonable spin assignments to the particles, and even (Σ,Λ) and (K^0,K^+) parities with asymmetries between the K interactions to account for the large mass splits. The result is correct to all orders in the π-baryon coupling constant but only to the second order in the K-baryon coupling constants.

The crude estimate of the K-baryon coupling constants, Eqs. (29) and (30), shows that if the (Ξ,N) and (K^0,K^+) parities are even and if the (K,Λ) parity is even, then the (N_1N_2K) coupling constant should be $F_I^2/4\pi>1.3$ and if it is odd $F_I^2/4\pi>15.6$ to reproduce the observable mass differences between baryons. These values of the coupling constants depend on the assumed K-meson form factor so that they should be regarded only as an illustration.

So far we have assumed the universality of the π interactions without any valid reason. There is also a possibility that the K interactions are universal and the π interactions are asymmetric so as to be able to account for the baryon mass splits.[15] For this case also one can readily see that carrying out a calculation similar to that previously discussed (to all orders in the K-coupling constant and second order in the π-coupling constants) would lead to the baryon mass splits on the basis of inequalities of the coupling constants because one must assume that the parities of (Σ,Λ) and (Ξ,N) are even. This is to be contrasted with the case of universal π interactions in which the mass spectrum can be explained on the basis of inequalities of the coupling constants or relative parities or both.

In the event that neither the universality of π interactions nor that of K interactions is valid, then it would be very difficult for present field theories to make any reliable statements about baryon mass differences if the mass differences are a manifestation of the breakdown of symmetries of the strong interactions.

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[15] J. J. Sakurai, Phys. Rev. **113**, 1679 (1959).