$(pn++-)$, and the remaining two are completely analyzed with missing mass evaluation consistent with single neutral pion emission $(pp+-0)$. The k values of these seven triple production collisions are distributed between 0.5 and unity with a threshold value of 0.42.

It might be expected that the anelasticity characteristic of a nucleon-nucleon collision would rise from a threshold, determined by the pion multiplicity, and exhibit a form dependent on the primary energy. Our results indicate that the characteristics for both double and triple production pass through broad maxima about 50% above the respective thresholds, then fall to a low value as k approaches unity. This similarity occurs even though the c.m. kinetic energy distribution of pions from triple production collisions differs from that for double production, as shown in Fig. 2. Our limited information regarding the velocities and directions of motion of the emitted nucleons enables, for some of the interactions, the determination of Q values of possible isobaric states.

It is evident that the measurements shown in Fig. 2 must be regarded with reserve. This caution is necessary because a fraction of the emergent pions in these experiments is not identified. The reason for this is two-fold—failure to distinguish pions from protons close to the forward direction and emission at large laboratory-system angles when produced in the backward hemisphere. Also, the selection of stars on the basis of analyzable pions might introduce a bias in favor of low values of k. On the other hand, the chance inclusion of a proton-nucleus collision, which otherwise satisfies all the criteria for a p-p collision, might lead to the inference of unduly high anelasticity.

As the available energy in the c.m. system increases, the threshold values of k for production of a few pions drop toward zero. Measurements on jets[16] suggest that at very large energies the maximum of the k distribution may follow the threshold down. The observations made here indicate that this tendency is true even at low energies.

Our present results lead to an average value of k for double and triple production, $k=0.55\pm0.10$, similar to results obtained at higher energies and comprising multiplicities up to six.[17] The widespread nature of the k distribution, however, shows that an average value of k is not very meaningful.

[16] B. Edwards et al., Phil. Mag. 3, 237 (1958).
[17] V. S. Barashenkov et al., Nuclear Phys. 9, 74 (1959).

Implications of the Intermediate Boson Basis of the Weak Interactions: Existence of a Quartet of Intermediate Bosons and Their Dual Isotopic Spin Transformation Properties

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Assuming that all weak interactions are transmitted through an intermediate boson field W, it is shown that the observed $|\Delta I| = \frac{1}{2}$ rule and the small observed mass difference between K_1 and K_2 lead to the conclusion that there exist four W particles: $W^\pm$, W^0, and $\bar{W}^0$. Furthermore, a natural assignment of the isotopic spin transformation property of these W particles follows a dual scheme in which the W's behave sometimes as $I = \frac{1}{2}$ and sometimes as $I = 1$ particles. Various experimental implications are discussed, including neutrino capture experiments, strong collisions exhibiting apparent nonconservation of strangeness, and strong collisions with apparent lepton production.

I. INTRODUCTION

IT is the purpose of this paper to study the consequences of the following three propositions:

(i) All weak interactions are transmitted through an intermediate boson field W.

(ii) The mass difference between K_1 and K_2 is of the order of $\sim 10^{-5}$ ev and not ~ 10 ev. This implies[1] that $\Delta S = \pm 2$ interactions are absent in the usual weak interactions.

(iii) The $|\Delta I| = \frac{1}{2}$ rule holds for the strangeness nonconserving decays of particles, where I is the total isotopic spin of the strongly interacting particles (i.e., baryons and the K and π mesons).

Of these propositions, (iii) has had quite impressive experimental support.[2] Evidence for (ii) has been re-

[1] L. B. Okun and B. M. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1587 (1957) [translation: Soviet Phys.-JETP 5, 1297 (1957)].

[2] See the review article by R. Dalitz, Revs. Modern Phys. 31, 823 (1959). See also F. Crawford et al., Phys. Rev. Letters 2, 266 (1959); J. L. Brown et al., Phys. Rev. Letters 3, 563 (1959).

ported[3] recently. (i) is so far a purely theoretical speculation.

The main conclusions of this paper are: (a) that there must exist at least two neutral W fields, and (b) that the three propositions (i), (ii), and (iii) lead naturally to a quite definite interaction scheme between the W's and the strongly interacting particles which seems to put the $|\Delta I| = \frac{1}{2}$ rule on a less *ad hoc* basis than in various previous discussions.[4] This scheme is first deduced in Secs. IV and V for a specific model from propositions (i), (ii), and (iii). It is then discussed for the general case in the next three sections. The W particles behave in this scheme sometimes as $I=\frac{1}{2}$ and sometimes as $I=1$ particles. For this reason they are referred to as schizons. The usual $|\Delta I|=\frac{1}{2}$ rule is shown to consist of two different *types* of selection rules: one originating from the $I=\frac{1}{2}$ aspect of the schizon, the other from the extent of the *difference* of the $I=\frac{1}{2}$ and $I=1$ aspects of the schizons. It also follows that there are decays and reactions which show a $|\Delta I|=1$ rule originating from the $I=1$ aspect of the schizon. [A possible variation of the scheme is discussed in Sec. VII which allows for an $I=0$ component of the schizons.]

Various experimental implications and therefore tests of the schizon basis of the weak interactions are discussed, especially in Secs. VI, X, XI, and XII.

II. SOME PROPERTIES OF $W^{\pm}$

We first summarize here some immediate consequences of (i). The spin of W is 1 in order to transmit the V and A type of weak interactions. Its mass m_W is $> m_K$ in order to prevent a fast decay $K^{\pm} \to W^{\pm} + \gamma$.

To reconcile[5] the absence of $\mu^{\pm} \to e^{\pm} + \gamma$, it seems necessary to have two sets of two-component neutrino fields ψ_ν and $\psi_{\nu'}$ coupled, respectively, to the e^- and μ^- fields. Both ψ_ν and $\psi_{\nu'}$ represent left-handed ν particles and right-handed $\bar{\nu}$ particles. The charged $W^{\pm}$ particles are coupled to the leptons through the interaction

$$ig_{e\nu}\psi_e{}^{\dagger}\gamma_4(1+\gamma_5)\psi_\nu\phi_\lambda{}^{*}+ig_{\mu\nu}\psi_\mu{}^{\dagger}\gamma_4(1+\gamma_5)\psi_{\nu'}\phi_\lambda{}^{*}$$
$$+\text{Hermitian conjugate}, \quad (1)$$

where ψ_e, ψ_μ, ψ_ν, $\psi_{\nu'}$, and ϕ_λ denote the fields describing e^-, μ^-, ν, ν' and $W_\lambda{}^+$. The operator $\phi_\lambda{}^*$ is related to the Hermitian conjugate field $\varphi_\lambda{}^\dagger$ by[6]

$$\phi_\lambda{}^{*}=\eta_\lambda\phi_\lambda{}^{\dagger},$$

where $\eta_\lambda=+1$ for $\lambda=1, 2, 3$, and $\eta_\lambda=-1$ for $\lambda=4$. The coupling of $W^{\pm}$ to the proton and neutron fields p and n is given by

$$J_\lambda\phi_\lambda{}^{*}+\text{Hermitian conjugate}. \quad (2)$$

The low momentum transfer matrix element of J_λ is related to the transition amplitudes of β decay. Let us write the matrix element between the physical states of a neutron and a proton at rest:

$$\langle p|J_\lambda{}^{*}|n\rangle=ig_{np}{}^{*}u_p{}^{\dagger}\gamma_4\gamma_\lambda(1+a\gamma_5)u_n, \quad (3)$$

where u_p and u_n are the spinor solutions of the free Dirac equations for the proton and the neutron. By suitably choosing the phases of ϕ_λ, ψ_μ and ψ_e we shall make g_{np}, $g_{e\nu}$, and $g_{\mu\nu}$ all real and positive. The β-decay coupling constants G_V and G_A are then given by

$$G_V=\sqrt{2}g_{e\nu}g_{np}(m_W)^{-2}, \quad (4)$$

and

$$G_A=-aG_V. \quad (5)$$

Comparison of the μ-decay rate and the experimental magnitude of $G_V=10^{-5}\ M^{-2}$ where $M=$ nucleon mass shows that[7]

$$g_{np}=g_{\mu\nu}. \quad (6)$$

The ratio of the experimental decay rates[8] $\pi^+ \to e^+ + \nu$ and $\pi^+ \to \mu^+ + \nu'$ leads to the conclusion[7]

$$g_{e\nu}=g_{\mu\nu}. \quad (7)$$

Combining (4), (6), and (7) one obtains

$$g_{e\nu}=g_{\mu\nu}=g_{np}=m_WG_V{}^{\frac{1}{2}}2^{-\frac{1}{4}}. \quad (8)$$

The strength of the lepton-W coupling is measured by

$$(2g_{e\nu})^2/4\pi=(\pi\sqrt{2})^{-1}G_Vm_W{}^2>6.4\times10^{-7}. \quad (9)$$

The $W^{\pm}$ particles are unstable against decays into $e^{\pm}+\nu$, $\mu^{\pm}+\nu$, and 2π, 3π, etc., modes. The decay rates[9] into leptons are given by

$$\lambda_{W\to\mu+\nu}\cong\lambda_{W\to e+\nu}=G_Vm_W{}^3(6\pi\sqrt{2})^{-1}>8\times10^{16}\ \text{sec}^{-1}. \quad (10)$$

The existence of W implies a "nonlocality" of a size $\sim m_W{}^{-1}$ for the presently observed weak interactions. For μ decay the Michel parameter ρ is given by[10]

$$\rho-0.75\cong\frac{1}{3}(m_\mu/m_W)^2,$$

which is consistent with the present experimental results.[11]

Furthermore (i) implies that $W^{\pm}$ is also coupled to a strangeness-nonconserving current S (generated by the strongly interacting particles):

$$S_\lambda\phi_\lambda{}^{*}+\text{Hermitian conjugate} \quad (11)$$

[3] F. Muller et al., Phys. Rev. Letters **4**, 418 (1960).

[4] See, e.g., S. B. Treiman, Nuovo cimento **15**, 916 (1960), B. d'Espagnat, J. Prentki, and A. Salam, Nuclear Phys. **5**, 447 (1958).

[5] G. Feinberg, Phys. Rev. **110**, 1482 (1958); J. Schwinger, Ann. Phys. **2**, 407 (1957); M. Gell-Mann, Revs. Modern Phys. **31**, 834 (1959). See also B. Pontecorvo, J. Exptl. Theoret. Phys. (U.S.S.R.) **37**, 1751 (1959) [translation: Soviet Phys.-JETP (to be published)], for possible neutrino experiments to test the existence of ν and ν'. See also S. A. Bludman, Bull. Am. Phys. Soc. **4**, 80 (1959), and Gatlinburg Conference on Weak Interactions, 1958 (unpublished).

[6] Throughout this paper, we use the superscript * to indicate the product of η_λ times the Hermitian conjugation operator.

[7] See R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[8] T. Fazzini et al., Phys. Rev. Letters **1**, 247 (1958). G. Impeduglia et al., Phys. Rev. Letters **1**, 249 (1958).

[9] T. D. Lee and C. N. Yang, Phys. Rev. Letters **4**, 307 (1960).

[10] See T. D. Lee and C. N. Yang, Phys. Rev. **108**, 1611 (1957).

[11] R. J. Plano and A. Lecourtois, Bull. Am. Phys. Soc. **4**, 82 (1959).

to make possible the observed decays,

$$\Lambda \to p + e^- + \bar{\nu}, \tag{12}$$

and

$$K^+ \to \pi^0 + \mu^+ + \nu', \text{ etc.} \tag{13}$$

Such couplings introduce further decay modes of the $W^{\pm}$ particles such as $W^{\pm} \to K^{\pm} + \gamma$, $W^{\pm} \to K^{\pm} + \pi^0$, etc.

III. CONSEQUENCES OF PROPOSITIONS (i) AND (ii)

Reaction (12) implies the existence of the transitions

$$\Lambda \rightleftharpoons p + W^-,$$

and therefore also of

$$\bar{K}^0 \rightleftharpoons \pi^+ + W^-.$$

Proposition (ii) then implies the absence of

$$K^0 \rightleftharpoons \pi^+ + W^-.$$

In other words the current $\mathcal{S}_\lambda$ associated with the annihilation of a W^- must not[12] increase the strangeness of a state by $+1$. One easily concludes that this implies

$$\mathcal{S}_\lambda S = (S+1)\mathcal{S}_\lambda, \tag{14}$$

where S is the strangeness operator. A well-known consequence[12] is that

$$\Sigma^+ \not\to n + e^+ + \nu.$$

Another consequence is, e.g., that

$$\nu' + \text{nucleon} \not\to \mu^- + (\text{system with strangeness} - 1). \tag{15}$$

Thus,

$$\nu' + n \not\to \Sigma^+ + \mu^-,$$
$$\nu' + n \not\to \Lambda + \mu^- + \pi^+.$$

Still another consequence is, e.g.,

$$K^+ \not\to \pi^+ + \pi^+ + \mu^- + \bar{\nu}'. \tag{16}$$

IV. A SIMPLE MODEL

We shall in this and the following section demonstrate conclusions (a) and (b) stated in the introduction. For the sake of clarity of presentation let us consider first a specific model in which J_λ and $\mathcal{S}_\lambda$ each consists of only one term[13]

$$J_\lambda \equiv (\bar{n}p)f_1,$$

and

$$\mathcal{S}_\lambda \equiv (\bar{\Lambda}p)f_2, \tag{17}$$

where f_1 and f_2 are real numerical constants. [The phase of f_2 can be arbitrarily chosen because of strangeness conservation in the strong and electromagnetic

interactions, which leaves arbitrary the choice of the phase of $(\bar{\Lambda}p)$.] Under an isotopic rotation J_λ forms a vector together with $(1/\sqrt{2})[(\bar{p}p) - (\bar{n}n)]f_1$ and $(\bar{p}n)f_1$, while $\mathcal{S}_\lambda$ forms a doublet with $(\bar{\Lambda}n)f_2$. For a strangeness nonconserving decay such as $\Lambda \to p + \pi^-$, after the elimination of the virtual W field the effective matrix element is that of $J_\lambda \mathcal{S}_\lambda^\star$. To satisfy the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule it is clear that the other isotopic partners of J_λ and $\mathcal{S}_\lambda$ will have to enter the picture. Neutral currents[14] and neutral W's will therefore have to be introduced.

We now examine W-J couplings and W-$\mathcal{S}$ couplings so as to generate the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. There seems at this point to be two possibilities:

(A) $\mathbf{I}$ conservation is preserved in the W-J couplings, while $|\Delta \mathbf{I}| = \frac{1}{2}$ is caused by the W-$\mathcal{S}$ couplings. In other words under an isotopic spin rotation the W-J coupling is a scalar while the W-$\mathcal{S}$ coupling is one component of a doublet. Since J behaves like a vector, this arrangement requires that W^+, W^0, and W^- form a triplet (like the pions) to which one assigns the isotopic spin $I = 1$. The W-J coupling is then[15]

$$f_1\{(\bar{n}p)W^\star + (1/\sqrt{2})[(\bar{p}p) - (\bar{n}n)]W^0 + (\bar{p}n)W\}, \tag{18}$$

and the W-$\mathcal{S}$ coupling,

$$f_2\{(\bar{\Lambda}p)W^\star - (1/\sqrt{2})(\bar{\Lambda}n)W^0\}$$
$$+ \text{Hermitian conjugate.} \tag{19}$$

The W^0 term in (19) implies the existence of

$$n \rightleftharpoons \Lambda + W^0; \tag{20}$$

its Hermitian conjugate that of

$$\Lambda \rightleftharpoons n + W^0. \tag{21}$$

Together they give rise to the transition

$$n + n \rightleftharpoons \Lambda + W^0 + n \rightleftharpoons \Lambda + \Lambda \tag{22}$$

in contradiction to proposition (ii).

This possibility therefore does not work out in the simple form described above.[16]

(B) $\mathbf{I}$ is conserved in W-$\mathcal{S}$ couplings, while $|\Delta \mathbf{I}| = \frac{1}{2}$ is caused by the W-J couplings. To satisfy $\mathbf{I}$ conservation in W-$\mathcal{S}$ couplings it is necessary to have W^+ and W^0 form an isotopic doublet. Therefore $\bar{W}^0$ and W^- also form a doublet. Since W^0 and $\bar{W}^0$ have different isotopic rotation properties, they cannot be the same particle. The four W's thus form a quartet very similar

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[12] Such possibilities have been discussed extensively in the literature in connection with the proposal that all weak interactions originate from couplings of the form (current)×(current). See, in particular, R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[13] From here on we drop the index λ. Notice that $(\bar{n}p) \equiv i\bar{\psi}_n \gamma_\lambda (1+\gamma_5)\psi_p$ and $(\bar{n}p)^\star = (\bar{p}n) = i\bar{\psi}_p \gamma_\lambda (1+\gamma_5)\psi_n$ where $\bar{\psi} \equiv \psi^\dagger \gamma_4$.

[14] The possible existence of neutral currents has also been discussed in the literature. See, in particular, S. Treiman, Nuovo cimento **15**, 916 (1960).

[15] From here on, we use for convenience W to represent ϕ_λ which annihilates a W^+ particle. Thus, for example, $(\bar{n}p)W^\star$ represents $i\bar{\psi}_p \gamma_\lambda (1+\gamma_5)\psi_n \phi_\lambda^\star$. Similarly [see (25) and (26)], $(\bar{p}p)W_a^0$ and $(\bar{p}p)W_b^0$ represent, respectively, $i\bar{\psi}_p \gamma_\lambda (1+\gamma_5)\psi_p (\phi_a^0)_\lambda$ and $i\bar{\psi}_p \gamma_\lambda (1+\gamma_5)\psi_p (\phi_b^0)_\lambda$ where $(\phi_a^0)_\lambda = -(\phi_\lambda^0 + \phi_\lambda^{0\star})/\sqrt{2}$ and $(\phi_b^0)_\lambda = i(\phi_\lambda^0 - \phi_\lambda^{0\star})/\sqrt{2}$. As another example, in (29) and (30) $JW^\star$ and $\mathcal{S}^0 W^{0\star}$ represent, respectively, $J_\lambda \phi_\lambda^\star$ and $\mathcal{S}_\lambda^0 (\phi_\lambda^0)^\star$.

[16] As we shall see in the next section, actually possibility (B) can lead to a final result which is expressible as possibility (A) plus an additional neutral W field whose effect is to cancel out the process (22).

to the quartet of K particles. The W-S coupling is

$$f_2\{(\bar\Lambda p)W^\star+(\bar\Lambda n)W^{0\star}\}+f_2\{(\bar n\Lambda)W^0+(\bar p\Lambda)W\}. \quad (23)$$

The W-J coupling is now one component of an isotopic doublet. Thus it is

$$f_1\{(\bar n p)W^\star-\tfrac12[(\bar p p)-(\bar n n)]W^{0\star}\}$$
$$+\text{Hermitian conjugate}. \quad (24)$$

The interactions (1), (23), (24) taken together with the strong and electromagnetic interactions clearly are consistent with propositions (i), (ii), and (iii).

V. A SIMPLE MODEL (CONTINUED)

We have seen in the last section that propositions (i), (ii), and (iii) lead to the existence of W^0 and $\bar W^0$ forming with $W^\pm$ a quartet of two isotopic doublets. We shall now write the W-J interaction (24) in the following form[15]

$$f_1\{(\bar n p)W^\star+(1/\sqrt2)[(\bar p p)-(\bar n n)]W_a^0+(\bar p n)W\}, \quad (25)$$

where

$$W_a^0=(-W^0-W^{0\star})/\sqrt2. \quad (26)$$

In this form it closely resembles the rejected expression (18), and demonstrates the following fact:

If one regards W^+, W_a^0, and W^- as forming an isotopic vector then the W-J interaction conserves $\mathbf{I}$. [The difficulty discussed under (A) does not now arise because the field

$$W_b^0\equiv i(W^0-W^{0\star})/\sqrt2 \quad (27)$$

describes another neutral particle W_b^0 and the process

$$n+n\rightleftharpoons\Lambda+W_b^0+n\rightleftharpoons\Lambda+\Lambda$$

exactly cancels $n+n\rightleftharpoons\Lambda+W_a^0+n\rightleftharpoons\Lambda+\Lambda$. See reference 16.]

The picture that emerges is as follows:

The four W fields are coupled to the strongly interacting particles by W-J and W-S interactions which are roughly comparable in strength. Each of these interactions taken separately with the strong interactions satisfy $\mathbf{I}$ conservation. For the W-J interaction, $\mathbf{I}$ conservation is satisfied with the assignment that W^+, W_a^0, W^- form an isotopic triplet. For the W-S interaction, $\mathbf{I}$ conservation is satisfied with the assignment that W^+, W^0 and $\bar W^0$, W^- form two isotopic doublets. Violation of $\mathbf{I}$ conservation only occurs when the mixed effects of W-J and W-S interactions are observed. In such cases, to the order of the strength of the usual weak interactions (i.e., amplitude $\propto G_V$) the violation of $\mathbf{I}$ conservation satisfies $|\Delta\mathbf{I}|=\tfrac12$ since that represents the extent of the difference between the two isotopic spin transformation properties of the W particles.

VI. THE W PARTICLES AS SCHIZONS

The reasonings and conclusions of the last two sections are obviously not restricted to the specific model

discussed. One can conclude in general that propositions (i), (ii), and (iii) lead[17] to the existence of $W^\pm$, W^0, and $\bar W^0$ as transmitters of weak interactions. The W's are generated by charge-current densities formed by the leptons, and by the strongly interacting particles in strangeness conserving motions and in strangeness non-conserving motions. A natural possibility is that these charge-current densities have the same transformation properties as those discussed in the model above. We shall now discuss these properties explicitly.

One may write the interaction Lagrangian density in the following form:

$$\mathcal{L}_{\text{strong}}+\mathcal{L}_\gamma+\mathcal{L}_{Wl}+\mathcal{L}_{WJ}+\mathcal{L}_{WS}, \quad (28)$$

where $\mathcal{L}_\gamma$ denotes the electromagnetic interactions, $\mathcal{L}_{Wl}$ denotes the W-lepton interaction (1) [neutral lepton currents will be discussed in Sec. VIII], and $\mathcal{L}_{WJ}$ and $\mathcal{L}_{WS}$ are given by

$$\mathcal{L}_{WJ}=JW^\star+J_a^0W_a^0+J^\star W, \quad (29)$$

and

$$\mathcal{L}_{WS}=\{SW^\star+S^0W^{0\star}\}+\text{Hermitian conjugate}. \quad (30)$$

Here W and W^0 represent the fields for the W particles,[15] W_a^0 is defined in (26). S and S^0 represent currents for which[18] $\Delta N=0$, $\Delta S=-1$, where $N=$ number of baryons. Thus both satisfy (14) and

$$SN-NS=0. \quad (31)$$

J, J_a^0, and $J^\star$ represents currents for which $\Delta N=0$, $\Delta S=0$. Under an isotopic rotation, J, J_a^0, $J^\star$ transform[19] like an isotopic vector and S, S^0 an isotopic doublet. One also has the additional condition[6,13]

$$(J^0)^\star=J^0. \quad (32)$$

Under an isotopic rotation, $\mathcal{L}_{\text{strong}}+\mathcal{L}_{WJ}$ is invariant if W^+, W_a^0, and W^- transform like an isotopic vector[20] (and are therefore considered to have $S=0$), while $\mathcal{L}_{\text{strong}}+\mathcal{L}_{WS}$ is invariant if W^+, W^0 and $\bar W^0$, W^- transform[21] like two isotopic doublets (and are therefore considered to have strangenesses 1, 1, -1, and -1, respectively).

The dual isotopic spin transformation property of the W particles gives rise to an integrated view of many interesting characteristics of the weak interactions, such

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[17] It is possible to have more W fields than these four. E.g., it may be that the neutral lepton currents $(\bar e e)$, etc., generate additional neutral W fields. To have more W fields, however, is contrary to the spirit of proposition (i). For reasons of economy of the number of fields we shall not further discuss such possibilities.

[18] We use the following convention: $\Delta S=j$ if $\langle b|S|a\rangle\neq0$ only for $S_b-S_a=j$.

[19] To be specific, we adopt the convention that the fields $\psi_p^\dagger$ and $\psi_n^\dagger$ transform under an isotopic rotation like $|\tfrac12,\tfrac12\rangle$ and $|\tfrac12,-\tfrac12\rangle$ where we use the notations of A. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957). Then $J^\star$, $-J_a^0$, and $-J$ transform like $|1,1\rangle$, $|1,0\rangle$ and $|1,-1\rangle$, and S^0 and $-S$ like $\tfrac12,\tfrac12\rangle$ and $|\tfrac12,-\tfrac12\rangle$.

[20] To be more precise, in the convention of footnote 19, $\phi^\star$, $-\phi_a^0$ and $-\phi$ transform like $|1,1\rangle$, $1,0\rangle$ and $|1,-1\rangle$.

[21] To be more precise, in the convention of footnote 19, $\phi^\star$, $\phi^{0\star}$ transform like $|\tfrac12,\tfrac12\rangle$ and $|\tfrac12,-\tfrac12\rangle$. So do ϕ^0, $-\phi$.

as the $|\Delta\mathbf{I}| = \frac{1}{2}$ rule, and the so-called $\Delta Q = \Delta S$ rule discussed in Sec. III. Because of this dual property the W particles will be called schizons. [One may mention that in fact the transformation property of W under a space inversion (*without* charge conjugate) also manifests a dual character, because J and S both contain vector and axial vector parts.]

The reactions that are caused by the W interactions are classifiable into the following classes (cases where the electromagnetic processes are important will not be considered here):

(α) Those in which one real (not virtual) W particle is involved, e.g.,

$$\pi^- + p \rightarrow \Lambda^0 + W^0,$$

and

$$\pi^- + p \rightarrow p + W^-.$$

These involve transition amplitudes of the first order of either $\mathcal{L}_{WJ}$ or $\mathcal{L}_{WS}$. This class of reactions is characterized by the strength $\sim g^2/4\pi \sim 10^{-6}$. In these reactions $\mathbf{I}$ and S are conserved ($\mathcal{L}_{WJ}$ and $\mathcal{L}_{WS}$ terms do not interfere with each other) provided the W particles receive the proper $\mathbf{I}$ and S assignments stated above. However, because of the short lifetimes of the W particles, "apparent" violation of $\mathbf{I}$ conservation and S conservation may occur. This will be discussed more in detail in Sec. X.

(β) Those in which four leptons and no (real particles) W are involved, e.g., μ decay. This and the subsequent classes are characterized by the strength $(g^2/4\pi)^2 \sim 10^{-13}$.

(γ) Those in which two leptons and no (real particles) W are involved, and in which there is no change in strangeness among the strongly interacting particles, e.g., β decay. For this class, the leptons interact through a W particle. The interaction of this W with the baryons and bosons is described by $\mathcal{L}_{WJ}$ and therefore conserves $\mathbf{I}$ and S with the proper assignments.

Examples of this class of reactions are the decays[22]

$$\Sigma^+ \rightarrow \Lambda^0 + e^+ + \nu, \tag{33}$$

$$\Sigma^- \rightarrow \Lambda^0 + e^- + \bar{\nu}. \tag{34}$$

It is easy to prove that they have the same rate except for the phase space factor due to the difference between $\Sigma^\pm$ masses. The identity of their rates is a consequence of the requirement that J, $J_a{}^0$ and $J^\star$ form an isotopic vector, which in turn is an essential feature of the present schizon interpretation of the weak interactions. Intensity rules such as these can be described (in analogy with the usual $|\Delta\mathbf{I}| = \frac{1}{2}$ rule) as given by $|\Delta\mathbf{I}| = 1$.

Still another type of reactions of this class are found in the neutrino capture reactions.[23,9] These will be discussed later in Sec. XI.

(δ) Those in which two leptons and no (real particle)

W are involved, and in which there is a change of strangeness $\Delta S = \pm 1$ among the strongly interacting particles. This is similar to the above case except that the interaction between the strongly interacting particles and the virtual W is described by $\mathcal{L}_{WS}$.

One example of this class of reactions is the leptonic decay[24] mode of K. The $\mathbf{I}$ conservation property of $\mathcal{L}_{WS}$ implies here that for the strongly interacting particles $|\Delta\mathbf{I}| = \frac{1}{2}$. Consequences of this rule have been explored before.[24] Another consequence is, e.g., (16). (It is important to remember here that $\mathcal{L}_{Wl}$ does not seem to involve W^0. See Sec. VIII.) Further examples will be discussed in Sec. XI.

(ϵ) Those in which no leptons and no (real particles) W are involved. The transition amplitudes are proportional to some elements of $\mathcal{L}_{WJ}{}^2$, $\mathcal{L}_{WS}{}^2$ or ($\mathcal{L}_{WJ}\mathcal{L}_{WS}$). Those proportional to $\mathcal{L}_{WJ}{}^2$ and $\mathcal{L}_{WS}{}^2$ observe $\mathbf{I}$ and S conservations, and are therefore of no experimental interest since they are thoroughly masked by the strong interactions. Those proportional to ($\mathcal{L}_{WJ}\mathcal{L}_{WS}$) satisfy $|\Delta\mathbf{I}| = \frac{1}{2}$, and therefore $\Delta S = \pm 1$. This is so, because (29) can also be written as [in analogy with (24)]

$$\mathcal{L}_{WJ} = \{JW^\star - (1/\sqrt{2})J^0W^{0\star}\} + \text{Hermitian conjugate},$$

showing that if $W^\star$ and $W^{0\star}$ are taken to be a doublet, $\mathcal{L}_{WS}$ causes $\Delta\mathbf{I} = 0$ and $\mathcal{L}_{WJ}$ causes $|\Delta\mathbf{I}| = \frac{1}{2}$.

The $\Delta S = \pm 1$ rule leads directly to proposition (ii). (See Sec. VII about electromagnetic corrections.) The nonleptonic decay modes of K and of hyperons are examples of this class of reactions. The $|\Delta\mathbf{I}| = \frac{1}{2}$ rule for these reactions is due to the dual aspects of the isotopic spin properties of the W particles (just as in the model discussed in Secs. IV and V). In contrast, the $|\Delta\mathbf{I}| = \frac{1}{2}$ rule for reactions of class (δ) is due to the fact that for those reactions W behaves like a particle with $I = \frac{1}{2}$.

VII. REMARKS

We make a few general remarks here about the latitude allowed in the interaction scheme described in the last section.

1. In (29) a $W_b{}^0$ interaction was not included. It is clear that it may be included if it involves a neutral current $J_b{}^0$ that is Hermitian and is an isotopic scalar:

$$\mathcal{L}_{WJ} = JW^\star + J_a{}^0W_a{}^0 + J^\star W + J_b{}^0W_b{}^0. \tag{35}$$

Also $J_b{}^0$ must satisfy[18]

$$\Delta N = 0, \quad \Delta S = 0.$$

Inclusion of this term does not change any of the considerations of the last section. A possible form for $J_b{}^0$ is,

$$(\bar{p}p) + (\bar{n}n), \tag{36}$$

or

$$2(\bar{\Lambda}\Lambda) - (\bar{p}p) - (\bar{n}n). \tag{37}$$

[22] See Appendix for a more detailed analysis of these $\Sigma^\pm$ decays. See also S. Treiman, Nuovo cimento **15**, 916 (1960).

[23] M. Schwartz, Phys. Rev. Letters **4**, 306 (1960).

[24] S. Okubo, R. E. Marshak, E. C. G. Sudershan, W. B. Teutsch, and S. Weinberg, Phys. Rev. **112**, 665 (1958).

However, the introduction of (36) or (37) or both would lead to the violation of time reversal invariance. (It is of course, possible to construct more complicated form for $J_b{}^0$ which satisfies time reversal invariance.)

A remark about time reversal invariance is in order here. It has been pointed out by Dalitz[2] that in a theory in which $|\Delta I| = \frac{1}{2}$ is satisfied, there is little existing experimental verification of time reversal invariance other than that contained in neutron decay measurements which, of course, is completely unrelated to the couplings of the neutral $W_b{}^0$.

2. In the scheme discussed in Sec. VI the electromagnetic interactions introduce corrections to the selection rules and intensity rules. However, since $\mathcal{L}_\gamma$ commutes with I_z, the strangeness selection rule holds intact. An important consequence is the following: The amplitude for the transiton $K^0 \to \bar{K}^0$ (for which $\Delta S = -2$), as discussed under class (ϵ) of the last section, vanishes in the order $(\mathcal{L}_{WJ} + \mathcal{L}_{WS})^2$ because of the strangeness selection rule $\Delta S = \pm 1$. Electromagnetic correction to this therefore also vanishes to all orders of $(e^2/\hbar c)$. The matrix element for $K^0 \to \bar{K}^0$ only becomes nonvanishing in the order $\mathcal{L}_{WJ}{}^2 \mathcal{L}_{WS}{}^2 \sim g^4 \sim 10^{-13}$. This is consistent with proposition (ii).

$|\Delta I| = \frac{1}{2}$ selection rules are, however, corrected by the electromagnetic interaction. The correction introduces $|\Delta I| = \frac{3}{2}$ and $|\Delta I| = \frac{5}{2}$ components with comparable strengths, and higher $|\Delta I|$ values only in higher orders of $e^2/\hbar c$. If experiments on the branching ratio of $K_1{}^0$ decays become more accurate, it may be possible to obtain a lower limit to the amplitude of the $|\Delta I| = \frac{5}{2}$ component in K decay.

3. The conserved current hypothesis[7] is consistent with the schizon interactions discussed in the last section. It is equivalent to the statement that the vector part of the interaction $\mathcal{L}_{WJ}$ describes the vector field W as originating from a source J which is the isotopic spin density-current of the strongly interacting particles, in complete analogy with the generation of the vector electromagnetic field A_λ from the electric charge density-current. If the conserved vector current hypothesis is correct, a pertinent question would be the interpretation of the generation of W through the term $\mathcal{L}_{WS}$.

VIII. LEPTON COUPLINGS OF W^0

The lepton coupling $\mathcal{L}_{Wl}$ in (28) should in general include, in addition to (1) which represents $W^\pm$ couplings to the leptons, also lepton couplings with W^0 and $\bar{W}^0$. We write these neutral couplings as

$$[g_{\mu\mu}(\bar{\mu}\mu) + g_{ee}(\bar{e}e) + g_{\nu\nu}(\bar{\nu}\nu) + g_{\nu'\nu'}(\bar{\nu}'\nu')]W^0$$
$$+\text{Hermitian conjugate.} \quad (38)$$

Comparison of (38) with (1) shows that the ratio of the rates of $K^+ \to \pi^+ + e^+ + e^-$, $K^+ \to \pi^+ + \text{neutrinos}$ and $K^+ \to \pi^0 + e^+ + \nu$ are

$$R(K^+ \to \pi^+ + e^+ + e^-)/R(K^+ \to \pi^0 + e^+ + \nu)$$
$$\cong 2|g_{ee}|^2/|g_{e\nu}|^2, \quad (39)$$

$$R(K^+ \to \pi^+ + \text{neutrinos})/R(K^+ \to \pi^0 + e^+ + \nu)$$
$$\cong 2\{|g_{\nu\nu}|^2 + |g_{\nu'\nu'}|^2\}/|g_{e\nu}|^2. \quad (40)$$

A cursory survey of the experimental limits[25,26] on the absence of $K^+ \to \pi^+ + e^+ + e^-$ and $K^+ \to \pi^+ + \text{neutrinos}$ indicates

$$[|g_{ee}|^2/|g_{e\nu}|^2] < [\sim \tfrac{1}{2} \times 10^{-2}], \quad (41)$$

$$[(|g_{\nu\nu}|^2 + |g_{\nu'\nu'}|^2)/|g_{e\nu}|^2] < [\sim \tfrac{1}{6}]. \quad (42)$$

To set an experimental upper limit on $g_{\mu\mu}$ let us first consider the absence of $K_2{}^0 \to \mu^+ + \mu^-$. The state of $\mu^+ + \mu^-$ in $K^0 \to \mu^+ + \mu^-$ is an eigenstate of CP with eigenvalue -1. If time reversal invariance holds for W interactions, this state is also the decay product of $K_2{}^0 \to \mu^+ + \mu^-$. The rate of this last process is then

$$\frac{R(K_2{}^0 \to \mu^+ + \mu^-)}{R(K^+ \to \mu^+ + \nu')} = 4\frac{|g_{\mu\mu}|^2 m_K{}^3 (m_K{}^2 - 4m_\mu{}^2)^{\frac{1}{2}}}{|g_{\mu\nu}|^2 (m_K{}^2 - m_\mu{}^2)^2}, \quad (43)$$

where m_K and m_μ are the masses of K and μ, respectively. Experimentally[27] this ratio is $< 10^{-3}$. Thus

$$|g_{\mu\mu}|^2/|g_{\mu\nu}|^2 < (2.5) \times 10^{-4}. \quad (44)$$

If time reversal invariance is not assumed, an upper limit can be set on $g_{\mu\mu}$ by considering the absence of $K^+ \to \pi^+ + \mu^+ + \mu^-$. This process is theoretically similar to $K^+ \to \pi^0 + \mu^+ + \nu'$, with an amplitude ratio of $\sqrt{2}g_{\mu\mu}:g_{\mu\nu}$, except for kinematical differences. The Q values for the two processes are 143 Mev and 241 Mev, respectively. A conservative estimate then gives

$$\frac{R(K^+ \to \pi^+ + \mu^+ + \mu^-)}{R(K^+ \to \pi^0 + \mu^+ + \nu')} > [\sim \tfrac{1}{2}|g_{\mu\mu}|^2/|g_{\mu\nu}|^2].$$

Experimentally $K^+ \to \pi^+ + \mu^+ + \mu^-$ resembles a τ decay which has been extensively analyzed. It is safe to conclude that the ratio is less than 10^{-3}, giving

$$|g_{\mu\mu}|^2/|g_{\mu\nu}|^2 \lesssim 2 \times 10^{-3}.$$

The absence of W^0 and $\bar{W}^0$ couplings to the leptons makes it difficult to understand (8) in terms of a "universal" W interaction. It is to be emphasized, however, that this particular difficulty is not a consequence of the schizon theory, but rather is inherent in the experimental absence of neutral leptonic decay modes and the experimental rule $|\Delta I| = \frac{1}{2}$.

One may also set an upper limit on the strength $g_{e\mu}$ of the W^0 coupling to $(e\mu)$. One has

$$\frac{R(K^+ \to \pi^+ + \mu^+ + e^-)}{R(K^+ \to \pi^0 + \mu^+ + \nu')} \cong \frac{2|g_{e\mu}|^2}{|g_{\mu\nu}|^2}.$$

This is experimentally $\lesssim 10^{-3}$.

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[25] F. Anderson, G. Lawlor, and T. E. Nevin, Nuovo cimento 2, 608 (1955). See also R. Dalitz, Proc. Phys. Soc. (London) A69, 527 (1956).
[26] R. W. Birge et al., Nuovo cimento 4, 834 (1956).
[27] M. Bardon et al., Ann. Phys. 5, 156 (1958).

IX. DECAY OF THE W PARTICLES

The leptonic decay modes of the $W^{\pm}$ were mentioned in Sec. II. Those of W^0 and $\bar{W}^0$ are absent as discussed in the last section. It is important to notice that decay modes such as

$$W \rightarrow \mu + \nu' + \text{pions}$$

occur with an amplitude smaller than $\sim ge^2$, and are therefore negligible.

The nonleptonic modes of decay include various channels: 2π, 3π, $K+\pi$, $\pi+\gamma$, $K+\gamma$, etc. To discuss the selection and intensity rules we shall neglect electromagnetic correction terms, but shall include the $J_b{}^0 W_b{}^0$ term of (35).

The decay of W^0 and $\bar{W}^0$ resembles the corresponding situation in the decay of K^0 and $\bar{K}^0$. In the present schizon interaction scheme, through $\mathcal{L}_{WJ}$ the particle $W_a{}^0$ and $W_b{}^0$ can make transitions into pion channels. These channels have, however, isotopic spins 1 and 0 for $W_a{}^0$ decay and for $W_b{}^0$ decay, respectively. [See (35).] There is therefore no interference between them.

Using the notations of Lee et al.,[28] contributions to the decay matrix $\Gamma + iM$ from $\mathcal{L}_{WS}$ are proportional to the unit matrix. It follows from these considerations that $W_a{}^0$ and $W_b{}^0$ are the eigenstates ψ_+ and ψ_-, so that each follows a single exponential decay law with respect to the time. Their mass difference is ~ 10 ev. These conclusions are independent of CP invariance.

The nonleptonic decays of W^+, $W_a{}^0$, $W_b{}^0$, and W^- into particles with total strangeness $S=0$ thus obeys $\mathbf{I}$ conservation, with the assignment $I=1$ for W^+, $W_a{}^0$, and W^-, and the assignment $I=0$ for $W_b{}^0$. The nonleptonic decays of these particles into particles with total strangeness $S=+1$ is not possible for W^-. It is possible for W^+ and for the W^0 part of $W_a{}^0$ and $W_b{}^0$. Furthermore, $\mathbf{I}$ conservation is observed for W^+ and W^0 decay, with W^+, W^0 forming an isotopic doublet. Similar conclusions hold for decays into particles with total $S=-1$. Some detailed examples of these intensity and selection rules will now be given.

For the 2π modes we have the following equalities

$$R(W^+ \rightarrow \pi^+ + \pi^0) = R(W_a{}^0 \rightarrow \pi^+ + \pi^-)$$
$$= R(W^- \rightarrow \pi^- + \pi^0),$$
$$R(W_a{}^0 \rightarrow 2\pi^0) = R(W_b{}^0 \rightarrow 2\pi^0)$$
$$= R(W_b{}^0 \rightarrow \pi^+ + \pi^-) = 0. \quad (45)$$

For the 3π modes, if barrier penetration factors play an important role,

$$R(W_a{}^0 \rightarrow 3\pi^0) \approx 0,$$
$$R(W_b{}^0 \rightarrow 3\pi) \approx 0,$$
$$R(W^+ \rightarrow \pi^+ + \pi^+ + \pi^-) \cong R(W^+ \rightarrow \pi^0 + \pi^0 + \pi^+) \quad (46)$$
$$\cong \tfrac{1}{2}R(W_a{}^0 \rightarrow \pi^+ + \pi^- + \pi^0).$$

Furthermore the density distribution in a Dalitz plot[29]

[28] T. D. Lee, R. Oehme, and C. N. Yang, Phys. Rev. **106**, 340 (1957).

[29] R. Dalitz, Phil. Mag. **44**, 1068 (1953); Phys. Rev. **94**, 1046 (1954); E. Fabri, Nuovo cimento **11**, 479 (1954).

for the last three processes are the same and are proportional to p^2 where p is the momentum of the π^-, π^+, and π^0 in the three cases, respectively.

For the $K+\pi$ modes one has the following relations:

$$R(W^+ \rightarrow K^+ + \pi^0)$$
$$= R(W^- \rightarrow K^- + \pi^0) = \tfrac{1}{2}R(W^+ \rightarrow K^0 + \pi^+)$$
$$= \tfrac{1}{2}R(W^- \rightarrow \bar{K}^0 + \pi^-) = 2R(W_a{}^0 \rightarrow K^0 + \pi^0)$$
$$= 2R(W_a{}^0 \rightarrow \bar{K}^0 + \pi^0) = R(W_a{}^0 \rightarrow K^+ + \pi^-)$$
$$= R(W_a{}^0 \rightarrow K^- + \pi^+), \quad (47)$$

where the subscript $\alpha = a$ or b.

The decay of $W_b{}^0$ into 2π is forbidden and into 3π is hindered by barrier penetration factors, as shown by (45) and (46). If, therefore, $m_W < m_K + m_\pi$, the decay modes

$$W_b{}^0 \rightarrow \pi^0 + \gamma,$$
$$W_b{}^0 \rightarrow \pi^+ + \pi^- + \gamma, \text{ etc.} \quad (48)$$

become important. If further the $J_b{}^0 W_b{}^0$ coupling is absent in $\mathcal{L}_{WJ}$, the decay modes

$$W_b{}^0 \rightarrow K^0 + \gamma \text{ and } W_b{}^0 \rightarrow \bar{K}^0 + \gamma, \quad (49)$$

which have equal rate, become important.

It is important to notice that the W particles in general are polarized when produced through either neutrino capture experiments (see Sec. XI) or collisions between strongly interacting particles. The spin states of W can be easily analysed by measuring the angular distributions of its decay products.

X. "APPARENT" NONCONSERVATION OF STRANGENESS

In the usual theory in a collision between pions and nucleons the probability of a reaction exhibiting a strangeness change $\Delta S = \pm 1$ is $\sim 10^{-12}$ compared with that of the strong processes. That of a reaction showing a strangeness change $\Delta S = \pm 2$ is, by proposition (ii), $\sim 10^{-24}$ compared with that of the strong processes. In the present theory these conclusions remain true. However, in a process in which a real W particle is emitted, its short lifetime causes its immediate disintegration, and the disintegration products would exhibit apparent strangeness changes $\Delta S = 0$, ± 1 for the charged $W^{\pm}$ particles, and $\Delta S = 0$, ± 1, ± 2 for the neutral W's. For collisions with enough energy to produce a real W, the probability of such processes is $\sim 10^{-6}$ of the strong processes.

We give some examples below:

1.
$$\pi^+ + p \rightarrow W^+ + p,$$
$$W^+ \rightarrow K^+ + \pi^0. \quad (50)$$

Apparent process:

$$\pi^+ + p \rightarrow p + K^+ + \pi^0 \quad (\Delta S = 1) \quad (51)$$

2.
$$K^+ + Z \rightarrow W^0 + \text{nucleons and pions},$$
$$W^0 \rightarrow \text{all decay products of } W_a{}^0 \text{ and } W_b{}^0.$$

For the decay mode $W_\alpha{}^0 \to K^- + \pi^+ (\alpha = a,b)$ the apparent process becomes

$$K^+ + Z \to K^- + \pi^+ + \text{(nucleons and pions)}$$
$$(\Delta S = -2) \quad (52)$$

Detection and positive identification of such phenomena, which occur with a cross section 10^{-6} times that of the strong processes, is of course very difficult. If one thinks in terms of counter experiments, a source of difficulty is the competing apparent change of strangeness involved in the decay of the $K^0 - \bar{K}^0$ complex. One way to avoid this difficulty is to do an experiment below the threshold of strange particle production, such as (51) at a pion energy above the threshold for W^+ production but below the threshold of

$$\pi^+ + p \to K^+ + \Sigma^+.$$

This is feasible only if $m_K + 135 \text{ Mev} < m_W < m_K + 250$ Mev. If $m_W < m_K + 135$ Mev, the apparent process

$$\pi^+ + p \to p + K^+ + \gamma$$

can occur, but with a probability only $\sim 10^{-8}$ times that of the strong processes. It seems worthwhile to explore these and other possibilities for a detection of an apparent strangeness violation. In any case it is desirable to improve the present experimental limit of strangeness nonconservation in a collision process involving only strongly interacting particles.

XI. NEUTRINO CAPTURE EXPERIMENTS

It has already been pointed out[9] that the creation of the pair of particles $\mu^- + W^+$ in the Coulomb field of a nucleus by a neutrino has a relatively high cross section:

$$\nu' + Z \to Z + \mu^- + W^+. \quad (53)$$

It seems[23] that high-energy neutrino experiments may be quite feasible in the near future. We shall in this section discuss some implications of the schizon interaction scheme for those neutrino capture reactions in which no W particle is emitted.

1. Some implications were already mentioned in Sec. III. [See especially (15).] Some others result from the fact that in $\mathfrak{L}_{WS}$ S transforms like an isotopic doublet. Thus, e.g. the cross sections for

$$\bar{\nu}' + n \to \mu^+ + \Sigma^-,$$
and
$$\bar{\nu}' + p \to \mu^+ + \Sigma^0 \quad (54)$$

are in the ratio of 2 to 1 and have the same angular distribution. The same holds for the pair

$$\bar{\nu}' + n \to \mu^+ + \Lambda^0 + \pi^-,$$
and
$$\bar{\nu}' + p \to \mu^+ + \Lambda^0 + \pi^0. \quad (55)$$

These implications can all be summarized by the rule that $|\Delta I| = \frac{1}{2}$ for the strongly interacting particles.

2. Another type of implication can be summarized by the rule that $|\Delta I| = 1$ for the strongly interacting

particles. These result from the fact that in $\mathfrak{L}_{WJ}$ the current J transforms like an isotopic vector. One consequence is, e.g., that if the differential cross sections for

$$\nu' + n \to \mu^- + n + \pi^+,$$
$$\nu' + n \to \mu^- + p + \pi^0, \quad (56)$$
$$\nu' + p \to \mu^- + p + \pi^+$$

are denoted by σ_1, σ_2, σ_3, respectively, then $(\sigma_1)^{\frac{1}{2}}$, $(2\sigma_2)^{\frac{1}{2}}$ and $(\sigma_3)^{\frac{1}{2}}$ satisfy the triangular inequalities

$$(\sigma_1)^{\frac{1}{2}} + (2\sigma_2)^{\frac{1}{2}} \geq (\sigma_3)^{\frac{1}{2}},$$
$$(2\sigma_2)^{\frac{1}{2}} + (\sigma_3)^{\frac{1}{2}} \geq (\sigma_1)^{\frac{1}{2}}, \quad (57)$$
$$(\sigma_3)^{\frac{1}{2}} + (\sigma_1)^{\frac{1}{2}} \geq (2\sigma_2)^{\frac{1}{2}}.$$

Another consequence is, e.g., found in the reactions

$$\nu' + n \to \mu^- + \Gamma, \quad (58)$$
$$\bar{\nu}' + p \to \mu^+ + \Gamma', \quad (59)$$

where Γ and Γ' are complexes of strongly interacting particles with total strangeness $= 0$. The strongly interacting particles contribute factors $\langle \Gamma | J^\star | n \rangle$ and $\langle \Gamma' | J | p \rangle$ to the matrix elements for the transitions. The fact that J and $J^\star$ transform into each other under an I rotation means that these two factors are identical for pairs of states Γ and Γ' which are isotopic spin partners of each other. The contribution of $\mathfrak{L}_{Wl}$ to the matrix element consists of factors that can be explicitly computed in terms of the momenta and spins of the leptons in the reactions (58) and (59). The result of such an analysis is that the differential cross sections for (58) and (59) can both be expressed[9] in terms of certain structure functions, and that the structure functions for (58) and (59) are related to each other. More explicitly, the differential cross section for (58), is of the form

$$d\sigma(\nu' \to \mu_L^- + \Gamma)$$
$$= dk_\mu d(\cos\theta)(4\pi k_\nu)^{-1}$$
$$\times k_\mu [(k_\mu + k_\nu)^2 - P^2]^{\frac{1}{2}} (1 + v_\mu) D$$
$$\times [xA_+ + x^{-1}A_- + yB_+ + y^{-1}B_- + C], \quad (60)$$
$$d\sigma(\nu' \to \mu_R^- + \Gamma)$$
$$= dk_\mu d(\cos\theta)(4\pi k_\nu)^{-1}$$
$$\times k_\mu [(k_\mu + k_\nu)^2 - P^2]^{\frac{1}{2}} (1 - v_\mu) D$$
$$\times [xB_+ + x^{-1}B_- + yA_+ + y^{-1}A_- - C], \quad (61)$$

and that for (59) is of the form

$$d\sigma(\bar{\nu}' \to \mu_R^+ + \Gamma')$$
$$= dk_\mu d(\cos\theta)(4\pi k_\nu)^{-1}$$
$$\times k_\mu [(k_\mu + k_\nu)^2 - P^2]^{\frac{1}{2}} (1 + v_\mu) D$$
$$\times [xA_- + x^{-1}A_+ + yB_+ + y^{-1}B_- + C], \quad (62)$$
$$d\sigma(\bar{\nu}' \to \mu_L^+ + \Gamma')$$
$$= dk_\mu d(\cos\theta)(4\pi k_\nu)^{-1}$$
$$\times k_\mu [(k_\mu + k_\nu)^2 - P^2]^{\frac{1}{2}} (1 - v_\mu) D$$
$$\times [xB_+ + x^{-1}B_- + yA_- + y^{-1}A_+ - C]. \quad (63)$$

The notations in these formulas are defined as follows: $\mu_L{}^- = \mu^-$ with left-handed helicity, etc., $\mathbf{k}_\mu$, $\mathbf{k}_\nu$ = momenta of μ and ν' (or $\bar{\nu}'$) in the laboratory system, k_μ, $k_\nu = |\mathbf{k}_\mu|$, $|\mathbf{k}_\nu|$, θ = angle between $\mathbf{k}_\mu$ and $\mathbf{k}_\nu$, $\mathbf{P} = \mathbf{k}_\nu - \mathbf{k}_\mu$, v_μ = velocity of $\mu^\pm$, $x = (k_\mu + k_\nu + P)^{-1}(k_\mu + k_\nu - P)$, $y = (P + k_\nu - k_\mu)^{-1}(P - k_\nu + k_\mu)$, $E_\mu = (m_\mu{}^2 + k_\mu{}^2)^{\frac{1}{2}}$ = total energy of $\mu^\pm$ in the laboratory system, $D = [P^2 - (k_\nu - E_\mu)^2]^{-1}[P^2 - (k_\nu - k_\mu)^2]$, and A_+, A_-, B_+, B_-, and C are structure functions depending only on the state Γ and on the magnitude of the momentum transfer P and the energy transfer $k_\nu - E_\mu$ from the leptons to the strongly interacting particles.

One notices that in the forward direction, though $y^{-1} = \infty$, Dy^{-1} = finite. If the mass of μ is negligible, $D = 1$, $v_\mu = 1$, y = functions of P and $k_\nu - E_\mu$ and (60)–(63) reduce to Eqs. (16) and (21) of reference 9.

XII. CONCLUDING REMARKS

It is seen above that from propositions (i), (ii), and (iii) stated in the Introduction one is quite naturally led to the schizon interaction scheme. This scheme gives rise to a rather integrated picture of the various $|\Delta \mathbf{I}| = \frac{1}{2}$ rules, and of the so-called $\Delta Q = \Delta S$ rule.

To test the existence of the W particles and the validity of such a scheme four types of experiment seem worth considering:

(a) Neutrino capture with the production of $\mu^- + W^+$. This was touched upon in reference 9 and in Sec. XI above.

(b) Chamber type experiment of W production in pion-nucleon or nucleon-nucleon collisions. The main difficulty here is of course the fact that one can only have one W production event in millions of interactions.

(c) Counter experiment on apparent nonconservation of strangeness. This was discussed in Sec. X.

(d) Counter experiment on apparent lepton production in pion-nucleon or nucleon-nucleon collisions, such as

$$\pi^+ + p \rightarrow W^+ + p \rightarrow \begin{Bmatrix} \mu^+ + \nu' \\ e^+ + \nu \end{Bmatrix} + p. \qquad (64)$$

Such processes occur with a probability of $\sim 10^{-6}$ of the strong interactions, provided the threshold of W production is exceeded. The difficulty here is to separate these events from the background of μ^+ and e^+ produced in $\pi^+ \rightarrow \mu^+ + \nu'$, $\pi^0 \rightarrow e^+ + e^- + \gamma$, and K decays. To achieve this separation suppose one measures the momenta $\mathbf{P}_\pi$, $\mathbf{P}_p$, and $\mathbf{P}_l$ of the incoming π, the final p and the outgoing lepton. One then describes the observed process as

$$\pi^+ + p \rightarrow l^+ + p + X, \qquad (65)$$

where X is not detected, and is in general a complex of particles. By energy and momentum conservation one easily computes the energy m_X of X in its center-of-mass system, and the energy m_{X+l} of the complex $X+l$

in its center-of-mass system:

$$m_X{}^2 = (E_\pi + m_p - E_l - E_p)^2 - (\mathbf{P}_\pi - \mathbf{P}_l - \mathbf{P}_p)^2, \qquad (66)$$

and

$$m_{X+l}{}^2 = (E_\pi + m_p - E_p)^2 - (\mathbf{P}_\pi - \mathbf{P}_p)^2, \qquad (67)$$

where E_π, E_l, and E_p are the energies of the incoming pion, the lepton and the final proton. Process (64) is uniquely determined by the specifications

$$m_X = 0, \qquad (68)$$

$$m_{X+l} = m_W \text{ (which is } > m_K). \qquad (69)$$

To discuss the sensitivity of such a separation let us take, say, the example of a π^+ beam with good momentum resolution on a target of liquid hydrogen and detect μ^+ and p in coincidence. The background μ mesons In this case come mainly from

$$\pi^+ + p \rightarrow \pi^+ + p \rightarrow \mu^+ + \nu' + p, \qquad (70)$$

and

$$\pi^+ + p \rightarrow n\pi + \pi^+ + p \rightarrow n\pi + \mu^+ + \nu' + p, \quad (n \geq 1), \qquad (71)$$

and, e.g.,

$$\pi^+ + p \rightarrow K^+ + \Sigma^+ \rightarrow \mu^+ + \nu' + \pi^0 + p. \qquad (72)$$

Reaction (70) is identifiable by the conditions,

$$m_{X+l} = m_\pi, \quad m_X = 0, \qquad (73)$$

and (71) and (72) by the condition that m_X and m_{X+l} both have continuous spectra with the lower limits:

$$m_X > m_\pi, \quad m_{X+l} \geq 2m_\pi. \qquad (74)$$

The residual background is then due to the imperfect separation of processes (70)–(72), and due to chance coincidence. Amidst such background the desired events (64) constitute a peak in both m_X and m_{X+l} at the values of 0 and m_W, respectively. The identification and separation of (64) from the background in a counter experiment may thus be feasible.

APPENDIX

In this appendix we study the decays

$$\Sigma^- \rightarrow \Lambda^0 + e^- + \bar{\nu}, \qquad (A.1)$$

and

$$\Sigma^+ \rightarrow \Lambda^0 + e^+ + \nu. \qquad (A.2)$$

Their rates are unfortunately very small (see below). For completeness, however, we analyze in some detail these decays as an illustration of the $|\Delta \mathbf{I}| = 1$ rule in a decay process.

Throughout the Appendix we shall neglect the mass of the electron and consider only the decays of unpolarized $\Sigma^\pm$. Let $\mathbf{k}$ and $\mathbf{q}$ be, respectively, the momenta of $e^\pm$ and Λ^0 in the rest system of $\Sigma^\pm$. The Λ^0 particle would, in general, be longitudinally polarized. We define $P_L{}^-(q,k)dqdk$ and $P_R{}^-(q,k)dqdk$ to be the rates for the decay (A.1) of Σ^- in which the final Λ^0 has a helicity (i.e., spin component along its direction of motion) $= -\frac{1}{2}$

and $+\frac{1}{2}$, respectively. Similarly, let $P_L{}^+(q,k)dqdk$ and $P_R{}^+(q,k)dqdk$ be the corresponding rates for the decay (A.2) of Σ^+.

By using the Lagrangian (28), the dependence of $P_L{}^\pm$ and $P_R{}^\pm$ on k can be calculated explicitly. The following theorem can be readily established:

Theorem

$$P_L{}^\pm(q,k)=A_L[q\pm(Q-2k)]^2+B_L[q^2-(2k-Q)^2], \quad \text{(A.3)}$$

and

$$P_R{}^\pm(q,k)=A_R[q\mp(Q-2k)]^2+B_R[q^2-(2k-Q)^2], \quad \text{(A.4)}$$

where

$$Q=m_\Sigma-(m_\Lambda{}^2+q^2)^{\frac{1}{2}}, \quad \text{(A.5)}$$

and A_L, A_R, B_L, B_R are *functions of q only*. In (A.5) m_Λ is the mass of Λ^0 and m_Σ is the appropriate mass of Σ^+ or Σ^-.

It is important to notice that the explicit dependence of $P_\alpha{}^\pm$ ($\alpha=L,R$) on k follows from the special form of lepton current in $\mathcal{L}_{Wl}$ [Eq. (1)]. In $\mathcal{L}_{WJ}$, $J_\mu{}^\star$ and J_μ belong to the same isotopic spin multiplet. Consequently, $J_\mu{}^\star$ and J_μ are related by a 180° rotation along the y-axis in the isotopic spin space

$$J_\mu{}^\star=-e^{-i\pi I_y}J_\mu{}^{i\pi I_y}, \quad \text{(A.6)}$$

which leads to the result that in (A.4) and (A.5) the same structure functions A_L, A_R, B_L, B_R occur in both Σ^+ decay and in Σ^- decay. In terms of the matrix elements of J_μ these structure functions are given by

$$A_L=(8\pi^3q)^{-1}(Q^2-q^2)\,|\langle\Lambda_\downarrow|J_x|\Sigma_\uparrow\rangle|^2\Delta,$$

$$A_R=(8\pi^3q)^{-1}(Q^2-q^2)\,|\langle\Lambda_\uparrow|J_x|\Sigma_\downarrow\rangle|^2\Delta,$$

$$B_L=(8\pi^3q)^{-1}|Q\langle\Lambda_\downarrow|J_z|Z_\downarrow\rangle-iq\langle\Lambda_\downarrow|J_4|\Sigma_\downarrow\rangle|^2\Delta,$$

and

$$B_R=(8\pi^3q)^{-1}|Q\langle\Lambda_\uparrow|J_z|\Sigma_\uparrow\rangle-iq\langle\Lambda_\uparrow|J_4|\Sigma_\uparrow\rangle|^2\Delta, \quad \text{(A.7)}$$

where the z axis is parallel to $\mathbf{q}$ and $\uparrow$, $\downarrow$ indicate the appropriate spin states of Σ and Λ with respect to the z axis, and Δ is related to the coupling constant g_{ev} in $\mathcal{L}_{W-l}$ and the propagator of the $W^\pm$ particle by

$$\Delta=|g_{ev}|^2[q^2+m_W{}^2-Q^2]^{-2}. \quad \text{(A.8)}$$

We may expand Δ and the matrix elements of J_μ in powers of q and neglect terms that are proportional to either $(q/m_W)^2$ or (q/m_Λ). Similar to the case of neutron decay, we find that in such a nonrelativistic limit A_α and B_α ($\alpha=L,R$) depend only on two constans C_1 and C_2:

$$A_L=A_R=(16\pi^3q)^{-1}(Q^2-q^2)\,|C_2|^2, \quad \text{(A.9)}$$

$$B_L=(16\pi^3q)^{-1}|C_2Q-C_1q|^2, \quad \text{(A.10)}$$

and

$$B_R=(16\pi^3q)^{-1}|C_2Q+C_1q|^2. \quad \text{(A.11)}$$

It is interesting to notice that in this nonrelativistic approximation, if we sum over the helicity of Λ^0, the spectrum $P_L{}^+(q,k)+P_R{}^+(q,k)$ for Σ^+ decay is the same as that for Σ^- decay except for the mass difference between Σ^+ and Σ^-. Using the known masses of $\Sigma^\pm$ we find that the total rates R for these decays are given by

$$\frac{R(\Sigma^-\to\Lambda^0+e^-+\bar{\nu})}{R(\Sigma^+\to\Lambda^0+e^++\nu)}\cong1.57, \quad \text{(A.12)}$$

and

$$\frac{R(\Sigma^-\to\Lambda^0+e^-+\bar{\nu})}{R(\Sigma^-\to n+\pi^-)}\cong(2\times10^{-4})\eta, \quad \text{(A.13)}$$

where

$$\eta=\frac{|C_1|^2+3|C_2|^2}{|G_V|^2+3|G_A|^2},$$

and G_V, G_A are the Fermi and Gamow-Teller coupling constants in neutron decay.