Pion Multiplicity in Nucleon-Antinucleon Annihilation*

BIPIN R. DESAI
Lawrence Radiation Laboratory, University of California, Berkeley, California
(Received April 1, 1960)

In the annihilation problem we have considered the influence of the Ball-Chew model, according to which, at low energies, only a few of the eigenstates of the nucleon-antinucleon system need be considered. The effect of the selection rules that forbid certain pion multiplicities is thereby examined. The energies considered are 50 Mev, 140 Mev, and 0 Mev in the case of protonium—the bound system of a proton and an antiproton. To obtain the multiplicity, we have used the Fermi statistical model but have introduced Lorentz-invariant phase space, thus defining a new interaction volume. It is found that due to selection rules there is a substantial change in the number distribution of the outgoing pions. At 140 Mev and in the case of protonium the two-pion production is decreased considerably. The zero-prong events for the $p\bar{p}$ annihilation are suppressed by about a factor of two for annihilations at rest in the case of protonium compared to the corresponding events for annihilations in flight. The over-all average multiplicity is unchanged, however. The value of the newly defined interaction volume, in units of Fermi volume, for $p\bar{p}$ and $N\bar{p}$ annihilations (where N denotes an "average" nucleon) should be $\sim$10 in order to fit the observed multiplicities.

INTRODUCTION

MANY calculations[1] have been made of the pion multiplicity in nucleon-antinucleon annihilation according to the Fermi statistical model.[2] We present here the results of one more such calculation. Four recent developments make this new calculation of interest: (a) The success of the meson potential description of the nucleon-antinucleon interaction[3] now makes possible a tentative assignment of relative probabilities to different eigenvalues of angular momentum, parity, isotopic spin, etc., and thus allows the addition of selection rules to the usual elementary statistical considerations.[3,4] (b) A recent calculation[5] has shown that in protonium—the bound system of a proton and an antiproton—the capture occurs predominantly from S states. (c) Some experimental data[6] on annihilation in hydrogen are now available, making worthwhile a calculation of the number distribution of charged pions as well as the over-all average multiplicity. Experiments with complex nuclei are somewhat ambiguous with respect to the number distribution because of the possibility of pion reabsorption. (d) Recently a recursion relation for the phase-space integrals has been published[7] which makes unnecessary any of the approximations used in the early treatments of the annihilation problem.[8]

PHASE-SPACE INTEGRAL

For the phase space associated with each pion, we have used $\mu d^3p\Omega_0/\omega$ rather than $d^3p\Omega_0$ as originally suggested by Fermi,[2] where Ω_0, ω, p, and μ are the interaction volume, energy, momentum, and mass of the pion, respectively. This modification[9] seems plausible on the basis of field theory. The chief reason for adopting the change is the great simplification in numerical evaluation of phase-space integrals that it allows. In view of the crude nature of the Fermi model, such a simple modification is hard to criticize on physical grounds. We thus have in the center-of-mass frame as the phase-space integral at total energy E for annihilation of the nucleon-antinucleon system into n pions

$$(2\mu\Omega_0)^n R_n(E)(2\pi)^{-3n}.$$

Here we have $\hbar=c=1$, and

$$R_n(E)=\int\left[\prod_{i=1}^{n}\frac{d^3p_i}{2\omega_i}\right]\delta(E-\textstyle\sum_i\omega_i)\delta^{(3)}(\textstyle\sum_i \mathbf{p}_i)$$

$$=\int\left[\prod_{i}^{n} d^4q_i\,\delta(q_i^2-\mu^2)\right]\delta^{(4)}(q-\textstyle\sum_i q_i),$$

where $q_i=(\mathbf{p}_i,\omega_i)$ and $q=(0,E)$. For annihilation at rest, we have $E=2m$, where m is the nucleon mass.

With no consideration of selection rules, the transition probability for a state of n pions in a particular isotopic

* This work was done under the auspices of the U. S. Atomic Energy Commission.

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spin state $I=0$ or $I=1$ is then given by

$$S_n(I)=A\frac{g_n(I)}{n!}\frac{(2\mu\Omega_0)^n}{(2\pi)^{3n}}R_n(E),$$

where A is a constant independent of n, and $g_n(I)$ is the isotopic-spin weight factor given in Table I.[1]

Srivastava and Sudarshan[7] have shown that because of the Lorentz invariance of $R_n(\mathbf{p},E)$ the following recurrence relation holds[10]:

$$R_{n+1}(E)=\int\frac{d^3p_{n+1}}{2\omega_{n+1}}R_n[(E^2-2E\omega_{n+1}+\mu^2)^{\frac{1}{2}}].$$

It is convenient to introduce dimensionless quantities $x=\omega/E$, $y=\mu/E$, and $F_n(y)=E^{4-2n}R_n(E)$ so that the recurrence relation becomes

$$F_{n+1}(y)=2\pi\int_y^{x_0}dx(x^2-y^2)^{\frac{1}{2}}(1-2x+y^2)^{n-2}\times F_n\left[\frac{y}{(1-2x+y^2)^{\frac{1}{2}}}\right],$$

TABLE I. Values of $g_n(I)$ and of $F_n(y)$ for annihilation at rest.

n	$g_n(0)$	$g_n(1)$	$F_n(\mu/2m)$
2	1	1	1.553321
3	1	3	0.986864
4	3	6	0.174194
5	6	15	0.011323
6	15	36	0.000302
7	36	91	0.000003

where

$$x_0=\tfrac{1}{2}[1-(n^2-1)y^2] \quad\text{and}\quad F_2(y)=\tfrac{1}{2}\pi(1-4y^2)^{\frac{1}{2}}.$$

For annihilation at rest, we have $y=\mu/2m=0.07437$. The corresponding values of $F_n(y)$ are given in Table I. The curves for $(10)^nF_n(y)$ for different n values are given in Fig. 1. Since the present model approaches the conventional Fermi model for y values near threshold, one can use for these y values the expression for the phase-space integrals in the nonrelativistic approximation given by Lepore and Stuart.[8] Thus near threshold, we have

$$F_n(y)\simeq\frac{2^{(n-3)/2}\pi^{(3n-3)/2}}{n^{\frac{3}{2}}\Gamma(\frac{1}{2}(3n-3))}y^{(n-3)/2}(1-ny)^{(3n-5)/2}.$$

Let us write the interaction volume Ω_0 in units of the Fermi volume (i.e., that of a sphere of radius $1/\mu$):

$$\Omega_0=\lambda(4\pi/3)(1/\mu^3).$$

Then the probability for n-pion annihilation with no consideration of selection rules may be calculated from

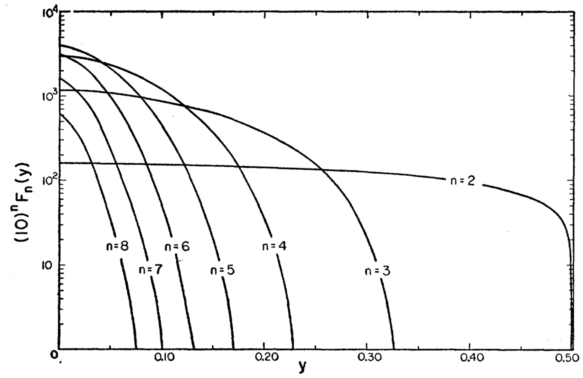

FIG. 1. Curves for $(10)^nF_n(y)$.

Table I using the formula

$$S_n(I)=B\frac{g_n(I)}{n!}\left(\frac{\lambda}{3\pi^2y^2}\right)^nF_n(y),$$

where B is a constant independent of n.

SELECTION RULES

If one takes seriously a meson-potential description of the nucleon-antinucleon interaction such as proposed by Ball and Chew,[3] it is possible to add selection rules to the above statistical considerations. In the Ball-Chew approximation, a given eigenstate has a definite probability of contributing to the annihilation process, and at low energies only a few eigenstates need be considered. Thus, the selection rules, which forbid certain pion multiplicities in each eigenstate, might be expected to be important.

According to Ball and Fulco,[3] annihilation in the $I=0$ state at 50-Mev laboratory energy occurs only in the $^1S_0$, $^3S_1$, $^3P_0$, and $^3P_2$ states, while at 140 Mev, the $^3D_3$ state also contributes. For $I=1$, the 50-Mev contributors are $^1S_0$, $^3S_1$, and $^3P_1$, with $^1P_1$ and $^3P_2$ contributing at 140 Mev.

A calculation[5] based on the Ball-Chew model[3] has recently been made to obtain capture rates for the various eigenstates of protonium—the bound system

TABLE II. Allowed and forbidden multiplicities in S, P, and D states for $I=0$.

State	n^a 2	3	4	5	6	7	$2J+1$
$^1S_0$	f	f	a	f	a	f	1
$^3S_1$	f	a	f	a	f	a	3
$^1P_1$	f	a	f	a	f	a	3
$^3P_0$	a	f	a	f	a	f	1
$^3P_1$	f	f	a	f	a	f	3
$^3P_2$	a	f	a	f	a	f	5
$^1D_2$	f	f	a	f	a	f	5
$^3D_1$	f	a	f	a	f	a	3
$^3D_2$	f	a	f	a	f	a	5
$^3D_3$	f	a	f	a	f	a	7

[a] Allowed multiplicities are denoted by a, forbidden by f.

[10] Such a relation could not be derived for the original Fermi phase-space integrals.

TABLE III. Allowed and forbidden multiplicities in S, P, and D states for $I=1$.

State	n^{a} 2	3	4	5	6	7	$2J+1$
$^1S_0$	f	a	f	a	f	a	1
$^3S_1$	a	f	a	f	a	f	3
$^1P_1$	f	f	a	f	a	f	3
$^3P_0$	f	f	f	a	f	a	1
$^3P_1$	f	a	f	a	f	a	3
$^3P_2$	f	a	f	a	f	a	5
$^1D_2$	f	a	f	a	f	a	5
$^3D_1$	a	f	a	f	a	f	3
$^3D_2$	f	f	a	f	a	f	5
$^3D_3$	a	f	a	f	a	f	7

[a] Allowed multiplicities are denoted by a, forbidden by f.

of a proton and an antiproton. We assume that this bound system is formed by the capture of an antiproton in an outer Bohr orbit about a proton in liquid hydrogen. The result of the above calculation is that the capture will take place predominantly from S states.

Tables II and III show the allowed and forbidden multiplicities in S, P, and D states.[4]

TRANSITION PROBABILITY

Without selection rules, the transition probability for annihilation of a nucleon-antinucleon system into n pions is given by

$$S_n = \tfrac{1}{2}S_n(0) + \tfrac{1}{2}S_n(1),$$

for $p\bar{p}$ annihilation and

$$S_n = \tfrac{1}{4}S_n(0) + \tfrac{3}{4}S_n(1),$$

for $N\bar{p}$ annihilation, where N denotes an "average" nucleon, 50% proton and 50% neutron.

With selection rules, the transition probability for annihilation of a nucleon-antinucleon system at energy E into n pions is given by

$$S_n = \sum_{\beta(I=0)} P_\beta(E)R_\beta(n) + \sum_{\beta(I=1)} P_\beta(E)R_\beta(n),$$

where $\sum_\beta$ denotes a sum over states characterized by the angular momentum l, total angular momentum J, spin S, and isotopic spin I; $P_\beta(E)$ is the probability of annihilation of the nucleon-antinucleon system in the state β at energy E; and $R_\beta(n)$ is the probability for the production of n pions in the state β.

For annihilation in flight ($E\neq 0$), we have

$$P_\beta(E) \sim (2J_\beta+1)P_I T_\beta(E),$$

where $P_I=\frac{1}{2}$ for both $I=0$ and $I=1$ in $p\bar{p}$ annihilation, $P_I=\frac{1}{4}$ for $I=0$ and $\frac{3}{4}$ for $I=1$ in $N\bar{p}$ annihilation, and $T_\beta(E)$ is the probability of annihilation of the state β at energy E, to be calculated here according to the Ball-Chew model.[3] Table IV gives the Ball-Chew values of $T_\beta(E)$ at 50 Mev and at 140 Mev.

TABLE IV. Values of $T_\beta(E)$ at 50 and 140 Mev (from Ball *et al.*[a]).

State	$E=50$ Mev $I=0$	$I=1$	$E=140$ Mev $I=0$	$I=1$
$^1S_0$	1	1	1	1
$^3S_1$	1	1	1	1
$^1P_1$	0	0	0	1
$^3P_0$	1	0	1	0
$^3P_1$	0	1	0	1
$^3P_2$	1	0	1	1
$^1D_2$	0	0	0	0
$^3D_1$	0	0	0	0
$^3D_2$	0	0	0	0
$^3D_3$	0	0	1	0

[a] See reference 3.

For annihilation at rest ($E=0$) in the case of protonium, we have

TABLE V. Values of S_n/S_2 for different values of λ for the $p\bar{p}$ annihilation.

n	$\lambda=1$ W^{a}	50 Mev	140 Mev	0 Mev	$\lambda=4$ W	50 Mev	140 Mev	0 Mev
2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3	2.6	1.6	4.4	3.1	10.4	6.8	18.1	18.5
4	1.6	1.6	2.3	3.7	25.1	22.7	29.4	62.7
5	0.3	0.2	0.5	0.4	18.6	12.5	34.1	37.7
6					5.1	4.0	5.6	11.8
7					0.4	0.2	0.2	0.5
$\bar{n}^{\pm}$	2.1	2.2	2.1	2.3	2.8	2.8	2.8	2.9
$\bar{n}$	3.2	3.2	3.3	3.4	4.3	4.3	4.3	4.3

n	$\lambda=8$ W	50 Mev	140 Mev	0 Mev	$\lambda=10$ W	50 Mev	140 Mev	0 Mev
2	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
3	20.7	11.1	29.7	35.0	26.0	12.5	33.0	35.0
4	100.5	84.3	106.6	255.0	160.0	120.0	150.7	341.7
5	148.5	82.3	224.0	284.0	300.0	145.0	394.2	475.0
6	81.9	57.7	78.3	205.0	170.0	128.5	172.7	425.0
7	13.1	6.0	16.3	20.0	35.0	17.5	47.5	56.7
$\bar{n}^{\pm}$	3.2	3.2	3.2	3.3	3.3	3.3	3.3	3.4
$\bar{n}$	4.9	4.8	4.9	4.9	5.0	5.0	5.1	5.1

n	$\lambda=12$ W	50 Mev	140 Mev	0 Mev
2	1.0	1.0	1.0	1.0
3	31.1	14.0	38.0	46.7
4	226.1	196.5	246.0	553.3
5	501.1	283.0	769.5	926.6
6	414.7	303.5	404.0	980.0
7	99.5	49.0	133.0	160.0
$\bar{n}^{\pm}$	3.5	3.4	3.5	3.5
$\bar{n}$	5.2	5.2	5.2	5.2

[a] Here W means without selection rules.

$$P_\beta(E) \sim (2J_\beta+1)Q_I,$$

for S states and

$$P_\beta(E) = 0,$$

for other states,[5] where $Q_I=\frac{1}{2}$ for both $^3S_1{}^3$ and $^3S_1{}^1$ states, $Q_I=\frac{1}{5}$ for the $^1S_0{}^3$ state, and $Q_I=\frac{4}{5}$ for the $^1S_0{}^1$ state.[11]

[11] Note that due to the Coulomb field there is a continual oscillation between the states with $I=0$ and $I=1$ with a frequency of about $10^{19}/n^2$ sec^{-1}, the atomic frequency of protonium. The capture rates calculated in reference 5 for $^3S_1{}^3$, $^3S_1{}^1$, $^1S_0{}^3$, and $^1S_0{}^1$ are $(4.5\times10^{18})/n^3$, $(5.8\times10^{18})/n^3$, $(2.5\times10^{18})/n^3$, and $(9.3\times10^{18})/n^3$ sec^{-1}, respectively, and are, therefore, much smaller than the

The quantities $R_\beta(n)$ may be expressed as $r_\beta(n)/\sum_{n'} r_\beta(n')$, where $r_\beta(n)=\phi_\beta(n)S_n(I)$. Here we have $\phi_\beta(n)=1$ if the n-pion state is allowed and $\phi_\beta(n)=0$ if the n-pion state is forbidden according to the selection rules (see Tables II and III).[4]

$p\bar{p}$ ANNIHILATION

From the results given in the previous sections, the values of the average charged-pion multiplicity, $\bar{n}^{\pm}$, and the average total multiplicity, $\bar{n}$, will be obtained for different values of λ. The values of the probabilities of the different charged-prong multiplicities will also be obtained. A comparison will then be made with the existing experimental data.

The values of S_n/S_2 for different values of λ are given in Table V. For a given λ, the first column gives S_n/S_2 without selection rules. The second and third columns give S_n/S_2 with selection rules at 50 Mev and 140 Mev, respectively. The fourth column gives S_n/S_2 with selection rules for annihilation at rest ($E=0$) in the case of protonium. From this, $\bar{n}^{\pm}$ and $\bar{n}$ are calculated and shown in the bottom row.

TABLE VI. Probability ratios of 2-, 4-, and 6-charged-prong to zero-charged-prong events for $\lambda=8$.

Ratio	W[a]	$\lambda=8$ 50 Mev	140 Mev	0 Mev
r_2	18.7	11.9	15.3	33.0
r_4	25.5	15.8	21.2	45.1
r_6	2.6	1.6	1.8	4.8
s_0	2.1	3.3	2.5	1.2

[a] Here W means without selection rules.

In a recent hydrogen bubble chamber experiment, the values observed for $\bar{n}^{\pm}$ and $\bar{n}$ were 3.21 ± 0.12 and 4.94 ± 0.31, respectively.[6] There were 81 ± 1 events recorded, out of which 6 ± 2 annihilations occured in flight at an average laboratory energy of 50 Mev. In a recent propane bubble chamber experiment, the $\bar{n}^{\pm}$ and $\bar{n}$ values for the $\bar{p}$-H annihilations were 3.06 ± 0.12 and 4.7 ± 0.5, respectively.[12] There were 139 $\bar{p}$-H annihilation events recorded at an average laboratory energy of 80 Mev.

From Table V we see that $\lambda\sim10$ gives values of $\bar{n}^{\pm}$ and $\bar{n}$ about the same as the experimental values given above. Further, we observe that the selection rules change significantly the number distribution of the outgoing pions. For annihilation at rest and at 140 Mev, the two-pion production is considerably decreased. The change in the average multiplicity is, however, quite insignificant. Note that the results at 260 Mev would be the same as at 140 Mev if, according to Ball and Fulco,[3] we ignore partial transmission in ${}^3D_3{}^3$ and ${}^3F_4{}^1$ states.

TABLE VII. Values of S_n/S_2 for different values of λ for the $N\bar{p}$ annihilation.

n	W[a]	$\lambda=1$ 50 Mev	140 Mev	W	$\lambda=10$ 50 Mev	140 Mev
2	1.0	1.0	1.0	1.0	1.0	1.0
3	3.2	2.3	5.7	32.0	22.8	55.2
4	1.8	1.7	3.3	180.0	145.2	219.0
5	0.3	0.3	0.6	300.0	247.2	610.6
6				205.0	173.6	279.8
7				42.0	30.0	74.0
$\bar{n}$	3.2	3.2	3.3	5.0	5.1	5.1
n	W	$\lambda=13$ 50 Mev	140 Mev	W	$\lambda=n$ 50 Mev	140 Mev
2	1.0	1.0	1.0	1.0	1.0	1.0
3	42.0	29.5	72.5	21.8	16.8	40.4
4	309.4	263.5	394.0	117.2	99.2	147.8
5	773.3	594.5	1467.5	275.0	213.6	527.2
6	654.0	536.5	856.0	267.1	217.8	347.4
7	185.6	120.0	296.0	102.9	69.0	170.2
$\bar{n}$	5.3	5.3	5.3	5.4	5.4	5.4

[a] Here W means without selection rules.

Table VI gives the ratios of the probability of occurrence of multiple charged-prong events to that of a zero-prong event for $\lambda=8$.[13] These ratios are indicated by r_2, r_4, and r_6, respectively, and are not sensitive to small changes in λ. The quantity s_0 indicates the % ratio of zero-prong events to the total number of events.

We note that for annihilations in flight the zero-prong events are about 2 or 3% of the total number of events, while at rest they are only about 1% of the total events. Thus there is a significant difference, by about a factor of two, in the probability of zero-prong events when one compares annihilations in flight with those at rest. The reason is clear if one notices that protonium annihilation occurs predominantly from S states whereas for annihilation in flight more states are available. For the ${}^3S_1$ states, for both $I=0$ and $I=1$, zero-prong events are forbidden due to charge conjugation,[4] and since these states have a higher statistical weight than the ${}^1S_0$ states, the zero-prong events at rest are considerably reduced compared to those in flight. Notice also that for S states no neutral pions are produced at all for $n=2$, and that for ${}^1S_0$ states due to G conjugation only even (odd) numbers of pions are produced in $I=0$ ($I=1$) states.[4]

The numbers of 0-, 2-, 4-, and 6-prong events in the hydrogen bubble chamber[6] were observed to be 2 ± 1, 33, 41, and 5, respectively, where annihilations occurred predominantly at rest. In the propane bubble chamber[12] for the $\bar{p}$-H annihilations the numbers of events were 8, 54, 67, and 6, respectively, where annihilations occurred at an average energy of 80 Mev. Hence, the

above frequency, Hence, the values of Q_I for different I-spin states with a given J value are proportional to the corresponding capture rates. Thus roughly we have $Q_I=\frac{1}{2}$ for both ${}^3S_1{}^3$ and ${}^3S_1{}^1$ states, $Q_I=\frac{1}{5}$ for the ${}^1S_0{}^3$ state, and $Q_I=\frac{4}{5}$ for the ${}^1S_0{}^1$ state.

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[13] Dr. Gerson Goldhaber of Lawrence Radiation Laboratory kindly provided me with the relevant Clebsch-Gordan coefficients given in Table VI which were calculated by Dr. Donald Stork.

zero-prong events ar rest are about $(2.5\pm1.2)\%$ and at 80 Mev about 6% of the total number of events. With improved statistics and a better resolution of the π^0 events, we believe the above theoretical estimates can be checked more correctly.

$N\bar{p}$ ANNIHILATION

For $N\bar{p}$ annihilation, the values of S_n/S_2 for different values of λ are given in Table VII. The values of $\bar{n}$ thus determined are also given. As in the $p\bar{p}$ annihilation, the selection rules change significantly the number distribution of the outgoing pions without changing the average multiplicity. If, as remarked earlier, we ignore partial transmission in ${}^3D_3{}^3$ and ${}^3F_4{}^1$ states, then the results at 140 and 260 Mev would be identical.

In the collaboration emulsion experiment,[14] the value of $\bar{n}$ was observed to be 5.3 ± 0.4. Here 35 events were recorded out of which 21 annihilations occurred in flight at an average laboratory energy of 140 Mev. In another recent emulsion experiment,[15] $\bar{n}$ was observed to be 5.36 ± 0.3. There were 221 events recorded out of which 95 events occurred in flight at an average laboratory energy of 140 Mev. In the propane bubble-chamber experiment, the $\bar{n}$ value was observed to be 4.7 ± 0.5.[12] Here there were 337 $\bar{p}$C events recorded out of which 166 occurred in flight at an average laboratory energy of 80 Mev.

We see that for $\lambda\sim10$ a good agreement with experiment is obtained. It is interesting to note that $\lambda=n$ also gives the multiplicity close to the experimental values. This might suggest that there is a strong pion-pion interaction in the final state.[16]

ACKNOWLEDGMENTS

I wish to thank Professor Geoffrey F. Chew for suggesting this problem and for his guidance. I would also like to thank Professor E. H. Wichmann for his helpful discussions about the phase-space integrals.

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PHYSICAL REVIEW VOLUME 119, NUMBER 4 AUGUST 15, 1960

Threshold Effects in Three-Body Channels

LUCIANO FONDA*
Institute for Advanced Study, Princeton, New Jersey

AND

ROGER G. NEWTON
Department of Physics, Indiana University, Bloomington, Indiana

(Received April 5, 1960)

The possibility of obtaining threshold anomalies in reactions leading to three-particle channels is studied in detail. It is found that a threshold cusp or rounded step exists in reactions whose final three-body channels have at least one particle in common. The effect appears as a function of the momentum of the common particle while the total energy is fixed.

I. INTRODUCTION

THE anomalous energy dependence of a scattering or reaction cross section at the threshold of a new inelastic process (the so-called "Wigner cusp"[1]) has been investigated in a number of recent theoretical papers.[2–9] The analysis of this effect, apart from the information one can obtain about scattering phase shifts, proves to be particularly useful for the determination of parities and spins of the reaction products.[2–13]

It is now well understood that the physical reason for the infinite energy derivative of old cross sections at the threshold of a new channel is the sudden removal of flux from the incident beam due to the opening of a new cross section which starts with an infinite slope. There is consequently no such cusp (or rounded step)

* On leave of absence from University of Trieste, Trieste, Italy.

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