Proton-Antiproton Annihilation in Protonium*

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Using the model for the nucleon-antinucleon interaction proposed by Ball and Chew, we have calculated the capture rates for the various eigenstates of protonium—the bound system of a proton and an antiproton. It is found that these rates depend sensitively on spin, isotopic spin, and total angular momentum eigenvalues of protonium, not just on orbital angular momentum, as is usually assumed. The average capture rates for the nS and nP states are $5.3\times10^{18}/n^3$ and $4.3\times10^{14}/n^3$ sec^{-1}, respectively. This P capture rate is two orders of magnitude larger than in the case of the $(K^- - p)$ atom because of the relatively long range of interaction in the Ball-Chew model. The problem of the Stark effect collisions, studied by Day, Snow, and Sucher in connection with the $(K^- - p)$ atom, is therefore re-investigated and at the same time we have considered certain important effects which were not considered by these authors. Rough calculations indicate that for protonium also the capture will take place predominantly from S states.

INTRODUCTION

A SPECIFIC model proposed by Ball and Chew[1] and extended by Ball and Fulco[2] has succeeded in explaining the nucleon-antinucleon interaction at intermediate energies. Using this model, we attempt here to calculate the capture rates from the various eigenstates of protonium—the bound system of a proton and an antiproton. Following Ball and Chew, we employ the WKB approximation even though the energies are low. With these estimates of the capture rates, we then attempt to decide whether the capture takes place predominantly through the S states or the P states. The results of this calculation are used elsewhere in connection with the multiplicity of pions in antiproton annihilation.[3]

An antiproton of low kinetic energy in passing through matter is slowed down principally by ionization. The probability for annihilation in slowing from 50 to zero Mev is very small. At zero energy in hydrogen, the antiproton will be captured by a proton in an orbit of radius approximately $a_0(=5.3\times10^{-9}$ cm), the first Bohr radius of hydrogen. The protonium thus formed will have a large angular momentum, l, and a principal quantum number, n, of about $(m/2m_e)^{\frac{1}{2}}(\sim30)$, where m and m_e are the masses of the proton and the electron, respectively. It will also have a thermal velocity of about 10^5 cm/sec. The protonium will then cascade down to states with lower (n,l) values by radiative transitions or through collisional de-excitations.[4] This process will continue until the antiproton reaches an orbit whose radius is small compared to a_0. The protonium in such a state can pass within the range a_0 of the electric field of nearby protons. While it is within this range, many oscillations will take place between its various states because of the Stark effect. The resulting situation will be similar to the one investigated by Day, Snow, and Sucher in connection with the capture of a K^- meson in hydrogen.[5] These authors showed that radiative transitions as well as P-state captures can be completely ignored while a highly excited $(K^- - p)$ atom undergoes many successive Stark-effect collisions with the protons in hydrogen. Thus they were able to conclude that the K^- meson will be captured predominantly via nS states, with large n.

The capture rates for nP and nS states for protonium will be obtained in the following section. We shall then attempt to decide whether or not the capture takes place primarily from nS states, as in the $(K^- - p)$ atom.

II. CAPTURE RATES

Let $\gamma_c(nl\alpha)$ be the capture rate for protonium in the state n, l, α, where α stands for the remaining quantum numbers—S, the total spin, J, the total angular momentum, and I, the isotopic spin—of protonium.

The capture rates for S and P states are given by[6]

$$\gamma_c(nS\alpha)=(8\pi/m)(\epsilon_{S\alpha}/k)|\psi_{nS}(0)|^2, \tag{1}$$

and

$$\gamma_c(nP\alpha)=(24\pi/m)(\epsilon_{P\alpha}/k^3)|\nabla\psi_{nP}(0)|^2, \tag{2}$$

respectively, where $\epsilon_{S\alpha}/k$ and $\epsilon_{P\alpha}/k^3$ are the imaginary parts of the zero-energy scattering lengths, i.e., the absorption lengths for the corresponding S and P waves, respectively[7]; $\epsilon_{S\alpha}$ and $\epsilon_{P\alpha}$ are the corresponding absorption phase shifts, and k is the relative momentum in the center-of-mass system. Here $\psi_{nS}(\mathbf{r})$ and $\psi_{nP}(\mathbf{r})$ are the undistorted Coulomb wave functions, ignoring α, for the nS and nP states of protonium, respectively.

Substituting the values of $|\psi_{nS}(0)|^2$ and $|\nabla\psi_{nP}(0)|^2$,

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[1] J. S. Ball and G. F. Chew, Phys. Rev. **109**, 1385 (1958).
[2] J. S. Ball and J. R. Fulco, Phys. Rev. **113**, 647 (1959).
[3] Bipin R. Desai, following paper [Phys. Rev. **119**, 1390 (1960)].
[4] A. S. Wightman, thesis, Princeton University, 1949 (unpublished).
[5] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters **3**, 61 (1959).
[6] S. Deser, M. L. Goldberger, K. Bauman, and W. E. Thirring, Phys. Rev. **96**, 774 (1954); J. D. Jackson, D. G. Ravenhall, and H. W. Wyld, Jr., Nuovo cimento **9**, 834 (1958).
[7] The P-wave absorption "length," $\epsilon_{P\alpha}/k^3$, has actually the dimensions of a volume. However, since it occurs as a counterpart of the S-wave absorption length in Eq. (2), we choose to call it a "length."

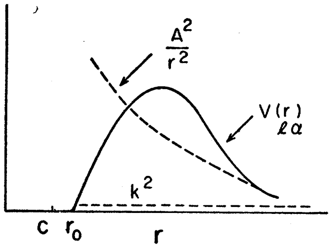

FIG. 1. Typical meson potentials.

we obtain

$$\gamma_c(nS\alpha)=(1/n^3)(8/mb_0^3)(\epsilon_{S\alpha}/k), \tag{3}$$

and

$$\gamma_c(nP\alpha)\simeq(1/n^3)(15/2mb_0^3)(\epsilon_{P\alpha}/k^3), \tag{4}$$

where $b_0=5.7\times10^{-12}$ cm is the first Bohr radius of protonium.

In order to obtain the values of $\epsilon_{S\alpha}/k$ and $\epsilon_{P\alpha}/k^3$, we shall use the Ball-Chew model.[1] The penetration coefficient introduced by Ball and Chew for the case of free proton-antiproton interaction is related to $\epsilon_{l\alpha}$ by

$$T_{l\alpha}=1-e^{-4\epsilon_{l\alpha}}. \tag{5}$$

According to the WKB approximation, we have

$$T_{l\alpha}=\frac{1}{1+\exp(\xi_{l\alpha})}. \tag{6}$$

Here we define

$$\xi_{l\alpha}=\int_{r_0}^{r_1}\left\{4m\left[V_{l\alpha}(r)-\frac{k^2}{m}\right]\right\}^{\frac{1}{2}}dr, \tag{7}$$

where r_0 and r_1 are the turning points and $V_{l\alpha}(r)$ is the effective potential given by Ball and Chew for a free proton-antiproton interaction.[1] For large values of $\xi_{l\alpha}(k\sim0)$ we have

$$\epsilon_{l\alpha}=\tfrac{1}{4}\ln(1+e^{-\xi_{l\alpha}})\sim\tfrac{1}{4}e^{-\xi_{l\alpha}}. \tag{8}$$

The potential $V_{l\alpha}(r)$ contains the centrifugal term in which, as usual, we replace $l(l+1)$ by $(l+\frac{1}{2})^2$.[8]

Typical curves for $V_{l\alpha}(r)$ are given in Figs. 1 and 2, where A^2/r^2 is the centrifugal barrier with $A=(l+\frac{1}{2})/m^{\frac{1}{2}}$. In Fig. 1 the meson potential is strongly attractive so that $V_{l\alpha}(r)$ bends over before reaching the annihilation boundary at $r=c$, the radius of the "black hole" introduced by Ball and Chew.[1] Since we have $k\sim0$, we assume that the turning point r_0 is given by $V_{l\alpha}(r)=0$. In Fig. 2 the potential is repulsive and rises up to the annihilation boundary, thus making $r_0=c$.

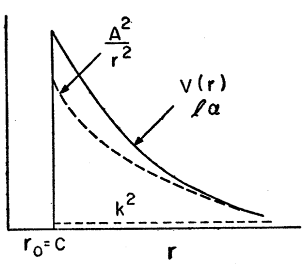

FIG. 2. Typical meson potentials.

Explicit dependence on the upper limit r_1 can be eliminated if we write $\xi_{l\alpha}$ as follows:

$$\xi_{l\alpha}=(4m)^{\frac{1}{2}}\left\{\int_{r_0}^{r_1}\left(\frac{A^2}{r^2}-\frac{k^2}{m}\right)^{\frac{1}{2}}dr\right.$$
$$\left.+\int_{r_0}^{r_1}\left[\left(V_{l\alpha}(r)-\frac{k^2}{m}\right)^{\frac{1}{2}}-\left(\frac{A^2}{r^2}-\frac{k^2}{m}\right)^{\frac{1}{2}}\right]dr\right\}. \tag{9}$$

We note that, since we have $k\sim0$, the value of r_1 is very large. However, at large distances $V_{l\alpha}(r)$ approaches the centrifugal term A^2/r^2. Thus, the upper limit r_1 in the first integral above is given by $A^2/r^2=k^2/m$. And in the second integral r_1 can be replaced by ∞.

Hence we can write

$$\int_{r_0}^{r_1}\left(\frac{A^2}{r^2}-\frac{k^2}{m}\right)^{\frac{1}{2}}dr=A\ln\left(\frac{2A}{e}\frac{m^{\frac{1}{2}}}{kr_0}\right)$$
$$=\frac{(l+\frac{1}{2})}{\sqrt{m}}\ln\left(\frac{2l+1}{e}\frac{1}{kr_0}\right), \tag{10}$$

TABLE I. Values of ρ_0 and $\Delta_{l\alpha}$ for S states.

State	$I=0$ ρ_0	$I=0$ $\Delta_{S\alpha}$	$I=1$ ρ_0	$I=1$ $\Delta_{S\alpha}$
$^3S_1$	2.16	−0.06	1.67	−0.07
$^1S_0$	3.20	−0.15	1.17	0.16

and

$$\int_{r_0}^{r_1}\left\{\left[V_{l\alpha}(r)-\frac{k^2}{m}\right]^{\frac{1}{2}}-\left[\frac{A^2}{r^2}-\frac{k^2}{m}\right]^{\frac{1}{2}}\right\}dr\sim\frac{\Delta_{l\alpha}}{(4m)^{\frac{1}{2}}}, \tag{11}$$

where

$$\Delta_{l\alpha}=(4m)^{\frac{1}{2}}\int_{r_0}^{\infty}\left\{[V_{l\alpha}(r)]^{\frac{1}{2}}-\frac{A}{r}\right\}dr.$$

Substituting the above integrals in Eq. (9), we get

$$\xi_{l\alpha}=(2l+1)\ln\left(\frac{2l+1}{e}\cdot\frac{1}{kr_0}\right)+\Delta_{l\alpha}, \tag{12}$$

where e is the base of the natural logarithm.

From Eq. (8) we obtain

$$\epsilon_{l\alpha}=\tfrac{1}{4}[e/(2l+1)]^{2l+1}(kr_0)^{2l+1}e^{-\Delta_{l\alpha}}, \tag{13}$$

or

$$\epsilon_{l\alpha}/k^{2l+1}=\tfrac{1}{4}[er_0/(2l+1)]^{2l+1}e^{-\Delta_{l\alpha}}. \tag{14}$$

Using Eq. (14), we can immediately write for the values of $\gamma_c(nS\alpha)$ and $\gamma_c(nP\alpha)$ given in Eq. (3) and Eq. (4), respectively,

$$\gamma_c(nS\alpha)=2.52[\rho_0\exp(-\Delta_{S\alpha})/n^3]\times10^{18}\ \text{sec}^{-1}, \tag{15}$$

[8] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 1101.

and

$$\gamma_c(nP\alpha)=3.80[\rho_0^3\exp(-\Delta_{P\alpha})/n^3]\times 10^{14}\ \mathrm{sec}^{-1}, \quad (16)$$

where $\rho_0=\mu r_0$, μ being the mass of the pion.

From $V_{S\alpha}$, $V_{P\alpha}$, and ρ_0 for different values of α,[9] the values of $\Delta_{S\alpha}$ and $\Delta_{P\alpha}$ have been calculated and are given in Tables I and II, together with the corresponding ρ_0.

The meson potentials for the states ${}^3P_1{}^3$, ${}^3P_1{}^1$, ${}^3P_0{}^3$, and ${}^1P_1{}^1$ rise up to the annihilation boundary (see Fig. 2) which in the present calculation was set at about a third of a pion Compton wavelength. A change $\Delta\rho_0=\pm 0.1$ for these states causes the corresponding $\gamma_c(nl\alpha)$ to change by almost 100%. Because of this sensitive dependence on the radius of the "black hole," we can believe the capture rates from these four states only up to their orders of magnitude. The over-all conclusions to be arrived at, however, depend only on the average rates. The rates for the above four states will be quite small compared to the rest for any reasonable choice of ρ_0 and will, therefore, contribute very little to the average.

The values of $\gamma_c(1S\alpha)$ and $\gamma_c(2P\alpha)$ are given in Tables III and IV. From these values, $\gamma_c(nS\alpha)$ and

TABLE II. Values of ρ_0 and $\Delta_{l\alpha}$ for P states.

State	$I=0$ ρ_0	$I=0$ $\Delta_{P\alpha}$	$I=1$ ρ_0	$I=1$ $\Delta_{P\alpha}$
${}^3P_2$	1.17	-0.48	0.54	-0.79
${}^3P_1$	$\sim$0.3	3.52	$\sim$0.3	-1.77
${}^3P_0$	1.83	-0.93	$\sim$0.3	3.26
${}^1P_1$	$\sim$0.3	0.77	0.71	-0.39

$\gamma_c(nP\alpha)$ can be obtained directly. It is interesting to note that the above capture rates depend sensitively on the spin, isotopic-spin, and total-angular-momentum eigenvalues of protonium, not just on the orbital angular momentum, as is usually assumed.

The average capture rate, $\gamma_c(nl)$, of the (nl)th quantum state, is obtained as follows:

$$\gamma_c(nl)=\frac{\sum_\alpha(2J_\alpha+1)\gamma_c(nl\alpha)}{\sum_\alpha(2J_\alpha+1)}.$$

Thus for nS and nP states we have

$$\gamma_c(nS)=5.3\times 10^{18}/n^3\ \mathrm{sec}^{-1}, \quad (17)$$

$$\gamma_c(nP)=4.3\times 10^{14}/n^3\ \mathrm{sec}^{-1}. \quad (18)$$

These rates can be compared to the rates estimated qualitatively by Bethe and Hamilton.[10] For a protonium in an (n,l) state described by an undistorted Coulomb wave function, they assumed the capture rate to be proportional to the probability that the antiproton is within an interaction range $\sim\lambda 10^{-13}$ cm from the proton. The constant of proportionality was taken to be the typical nuclear annihilation frequency 10^{23} [$\sim$velocity of light/nuclear radius ($\sim 10^{-13}$)]. This rate, of course, depends crucially on λ. In order to reproduce our result (17) for $\gamma_c(nS)$ it is necessary to choose $\lambda\sim 2$. Bethe and Hamilton would then find a P-state capture rate slightly smaller than ours, but only by a factor ~ 4.

TABLE III. Values of the capture rates for S states.

State	$\gamma_c(1S\alpha)$ (sec^{-1}) $I=0$	$\gamma_c(1S\alpha)$ (sec^{-1}) $I=1$
$1\,{}^3S_1$	5.8×10^{18}	4.5×10^{18}
$1\,{}^1S_0$	9.3×10^{18}	2.5×10^{18}

III. COMPARISON WITH $(K^- - p)$ RATES

Before comparing the above rates with those for the $(K^- - p)$ atom, we should note that unlike the $(\bar{p}-p)$ case, where the Ball-Chew[1] model works quite well, the $(K^- - p)$ interaction has not yet been described by any specific model. It becomes necessary, therefore, in the $(K^- - p)$ case, either to use experimental information or to make a plausible guess.

Experiments show that at low energies the absorption cross section is predominantly S wave.[11,5] From this information one can obtain the S-wave absorption length, which from a formula similar to Eq. (1) gives the S-state capture rate. This rate turns out to be $6\times 10^{17}/n^3$ sec^{-1}, only a factor of 8 smaller than our calculated rate for protonium.[12] For P-state capture of K^- no experimental information is yet available. It is conventional to estimate the capture rate from a formula similar to Eq. (2) by assuming the P-wave absorption "length" to be equal to the S-wave absorption length times the square of the K-meson Compton wave-

TABLE IV. Values of the capture rates for P states.

State	$\gamma_c(2P\alpha)$ (sec^{-1}) $I=0$	$\gamma_c(2P\alpha)$ (sec^{-1}) $I=1$
$2\,{}^3P_2$	1.0×10^{14}	1.3×10^{13}
$2\,{}^3P_1$	5.8×10^{10}	1.2×10^{13}
$2\,{}^3P_0$	6.4×10^{14}	7.6×10^{10}
$2\,{}^1P_1$	1.0×10^{12}	2.0×10^{13}

[9] I am indebted to J. S. Ball, Lawrence Radiation Laboratory, Berkeley, for supplying me with the values of different meson potentials.

[10] H. Bethe and J. Hamilton, Nuovo cimento 4, 1 (1956).

[11] P. Nordin, A. H. Rosenfeld, F. Solmitz, R. Tripp, and M. Watson, Bull. Am. Phys. Soc. 4, 24 (1959); A. H. Rosenfeld, Bull. Am. Phys. Soc. 3, 363 (1958); M. F. Kaplon, *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 171.

[12] G. F. Frye, Phys. Rev. 113, 688 (1959). The rate obtained here is $4\times 10^{17}/n^3$ sec^{-1}. However, I am told by G. F. Frye, Lawrence Radiation Laboratory, Berkeley, that a better value is obtained by using the absorption lengths given by R. H. Dalitz and S. F. Tuan, Ann. Phys. 8, 100 (1959). This value is $6\times 10^{17}/n^3$ sec^{-1}. The conclusions of Day *et al.*[5] are not altered by the above change.

length.[7] The rate then turns out to be roughly $10^{12}/n^3$ sec^{-1},[13] much smaller than the rate we have calculated for protonium. Such a large difference in the two P-state capture rates may be attributed to the relatively long range of the interaction in the Ball-Chew[1] model which is associated with the pion Compton wavelength. If new information on the $(K^- - p)$ interaction shows a long-range interaction there as well, the difference will be reduced.

Day *et al.* used the above rates to show that the S-state capture process will dominate for the $(K^- - p)$ atom.[5,12] For protonium, however, our rather large value of the P-state capture rate may mean that the P-state capture process will become significant. It is, therefore, necessary to re-investigate the problem of the Stark-effect collisions for protonium.

IV. STARK-EFFECT COLLISIONS

Since the Stark-effect collisions are quite complicated, we shall confine ourselves to rough calculations. However, we shall consider, at the same time, certain important effects ignored by Day *et al.*[5]

The interaction of a protonium with the screened electric field of a proton in hydrogen, can be described by time-dependent perturbation theory with the proton as a fixed source. The error due to the finite mass of the proton will be insignificant in our very crude calculation.

The interaction Hamiltonian $H'(t)$ for the Stark effect of the screened electric field, with a screening factor taken as $\exp[-R(t)/a_0]$, is given by

$$H'(t) = e^2 \frac{\mathbf{R}(t)\cdot\mathbf{r}}{R^3(t)} \exp[-R(t)/a_0], \tag{19}$$

where $\mathbf{R}(t)$ is the distance of the external proton from the protonium center of mass, $\mathbf{r}$ is the distance of the antiproton from the protonium center of mass, and e is the elementary charge.

Let $\gamma_s(nl)$ denote the matrix element $\langle n, l-1|H'(t)|n, l\rangle$, which is the same as $\langle n, l|H'(t)|n, l-1\rangle$. This matrix element will be time-dependent, since the electric field experienced by protonium is time-dependent. In particular, the interaction (19) leads to

$$\gamma_s(nP) \sim n^2\, 4.2\times10^{13}[a_0/R(t)]^2 \exp[-R(t)/a_0]. \tag{20}$$

Let λ denote the ratio of the radius of protonium to the Bohr radius of hydrogen. Clearly, the Stark-effect collisions cannot take place unless $\lambda<1$. We expect, however, that by the time λ reaches the value of about $\frac{1}{4}$ (therefore, n is about 15) the Stark-effect collisions will already be of considerable importance. We shall thus limit our discussion to n values between 5 and 20. The values of $\gamma_s(nl)$ for different nl values will then be less than the above matrix element and will differ from each other by not more than an order of magnitude.

[13] Robert Karplus, Lawrence Radiation Laboratory (private communication).

We further note that the reciprocal of the time of transit through the range a_0 of the electric field is $\sim10^{13}$ sec^{-1}. From Eqs. (17), (18), and (20) the following inequalities hold for the above range of n values:

$$\text{(a)}\quad \gamma_s(nl) \gg 10^{13}\ \text{sec}^{-1},\quad \text{within the range } a_0,\text{ i.e., for } R(t)\lesssim a_0, \tag{21}$$

$$\text{(b)}\quad \gamma_c(nS) > 10^{13}\ \text{sec}^{-1},$$

$$\text{(c)}\quad \gamma_c(nP) < 10^{13}\ \text{sec}^{-1}.$$

We shall ignore the level shifts due to Coulomb and nuclear interactions.[14] In other words, we consider different angular-momentum states for a fixed n to be completely degenerate. Further, we consider the magnetic quantum number m to be an adiabatic invariant within the range a_0,[15] with the z axis along the slowly changing direction of the electric field.

A protonium outside the range a_0 of the electric field will have a definite l value for a given n. Within the range a_0, however, because of the Stark effect the protonium will oscillate continually between all its degenerate angular-momentum states with a frequency roughly given by

$$[\gamma_s^2(nl)+\gamma_s^2(nl_1)+\gamma_s^2(nl_2)+\cdots]^{\frac{1}{2}},$$

where $l_1, l_2, \ldots$, in addition to l, are the angular momenta for the given n.[16]

Consider a protonium, with $m=0$, within the electric field. Its wave function will contain an S part, i.e., the S state will be among the various angular-momentum states between which the protonium oscillates. The protonium will decay, therefore, with a rate that depends on how rapid the oscillations are compared to the capture rate of the S state. Since the Stark-effect matrix element, $\gamma_s(nP)$, goes like n^2 while the S-state capture rate, $\gamma_c(nS)$, goes like $1/n^3$, there will be a critical n value when the oscillation frequency equals $\gamma_c(nS)$. This n value is ~10. For $n\gtrsim10$, the oscillation frequency will be $\gtrsim\gamma_c(nS)$ and therefore the decay rate of the protonium will be $\sim\gamma_c(nS)/n$, the factor $1/n$

[14] The Coulomb level shifts will be less than 10^{13} sec^{-1}. (The level shifts for positronium are given by H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two- Electron Atoms* (Academic Press, Inc., New York, 1957). The nuclear level shifts for the P and the S states will presumably be of the same order as $\frac{1}{2}\gamma_c(nP)$ and $\frac{1}{2}\gamma_c(nS)$, respectively. Thus, we believe, the Coulomb as well as the nuclear level shifts will not affect the Stark-effect oscillations.

[15] The approximate condition for the adiabatic invariance of m is

$$\gamma_s^2(nl) \gg \langle nlm|\partial H'(t)/\partial t|nl'm'\rangle$$

or, approximately,

$$\gamma_s(nl) \gg 10^{13}\ \text{sec}^{-1}.$$

Because of inequality (21a), this condition is satisfied when the protonium is within the range a_0.

[16] It should be noted that we cannot speak of a "transition" that goes in one direction, viz., $n, l\rightarrow n, l-1$; we have oscillations between all angular momentum states. The matrix element $\gamma_s(nl)$, therefore, does not determine in any sense a "transition" rate but controls the oscillation of the entire state. This characteristic feature of the present problem is due to the degeneracy of different angular-momentum states.

being the probability with which the S state occurs in the protonium wave function. Obviously, this rate decreases as n increases. However, for $10<n<20$, it is $\gtrsim 10^{13}$ sec^{-1}, the reciprocal of the time of transit through the range a_0. For n values less than 10, the decay rate will, of course, be $<\gamma_c(nS)/n$, but since for $n=10$, $\gamma_c(nS)/n$ is already as large as 5×10^{14} sec^{-1}, it is very plausible that even down to $n=5$ the decay rate will be $\gtrsim 10^{13}$ sec^{-1}. Hence, a protonium within the range a_0 will be captured via an S state, if m is 0 and n is between 5 and 20. For a protonium with $m=\pm 1$, no captures will occur within the range a_0, since $\gamma_c(nP)$ for the above range of n values is much smaller than 10^{13} sec^{-1}. However, as this protonium emerges from the electric field, its wave function will be partly in a P state, and hence, there is a possibility of direct capture from the P state. Our final task, therefore, is to compare the two processes: (a) Stark captures via the S state, and (b) direct captures from the P state.[17]

Consider n^2 antiprotons distributed statistically, i.e., with a $(2l+1)$ distribution in $l=0$ to $l=n-1$ levels, with principal quantum number $n(5<n<20)$. Let these antiprotons enter the electric field of a proton at time $t=0$. This will be the first Stark-effect collision.[18] From the arguments, given above, all antiprotons with $m=0$ will be annihilated via nS states. The remaining n^2-n antiprotons emerging after the first Stark-effect collision will still be distributed statistically to a good approximation, since we have assumed complete degeneracy between various angular-momentum states for a fixed n. A similar situation will prevail for all subsequent collisions. At the end of each collision, the number of antiprotons will be reduced, and the number of collisions required to reduce the total number of antiprotons to $n^2(1/e)$ would be approximately n. Thus the rate, $\omega_c(nS)$, of capture of antiprotons via nS states due to the Stark effect will be given by

$$\omega_c(nS)\sim N\sigma v/n, \tag{22}$$

where N is the number of hydrogen atoms per cm^3, $\sigma=\pi a_0^2=0.88\times 10^{-16}$ cm^2, and v is the thermal velocity of protonium$=10^5$ cm/sec. For liquid hydrogen, with $N=4\times 10^{22}$ H atoms per cm^3, we have

$$\omega_c(nS)\sim 3\times 10^{11}/n \text{ sec}^{-1}. \tag{23}$$

[17] We can safely ignore captures from higher angular-momentum states.

[18] The collisional de-excitations, primarily due to the Auger effect, are ignored in this discussion.[4] The reason for this will be clear later.

TABLE V. Values of $\omega_c(nS)$ and $\omega_c(nP)$ for protonium.

n	$\omega_c(nS)$ (sec^{-1})	$\omega_c(nP)$ (sec^{-1})
20	1.5×10^{10}	3.4×10^{8}
15	2.0×10^{10}	1.4×10^{9}
10	3.0×10^{10}	$\sim 10^{10}$
5	6.0×10^{10}	2.4×10^{10}

In order to obtain the rate, $\omega_c(nP)$, of direct capture from the nP states, we note that for a given n, the upper limit for the ratio of antiprotons captured directly from P states to those captured via S states is $2/n$. This limit is attained for $n<10$, i.e., for $\gamma_c(nP)>10^{11}$ sec^{-1}, the reciprocal of the time between two collisions. Therefore for $n<10$ we have

$$\omega_c(nP)\sim (2/n)\omega_c(nS)=6\times 10^{11}/n^2 \text{ sec}^{-1}. \tag{24}$$

For $n>10$, however, we have $\gamma_c(nP)<10^{11}$ sec^{-1} and, therefore,

$$\omega_c(nP)\sim (2/n^2)\gamma_c(nP)=1.1\times 10^{15}/n^5 \text{ sec}^{-1}. \tag{25}$$

Values of $\omega_c(nS)$ and $\omega_c(nP)$ for different values of n are given in Table V.

We thus see that the P-state capture becomes comparable to the S-state capture only for $n<10$. However, as remarked earlier, we expect that by the time an antiproton reaches a state with $n\sim 15$, the Stark-effect collisions will already be of considerable significance. It seems, therefore, that for protonium, the capture will take place predominantly from S states.[19]

V. ACKNOWLEDGMENTS

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[19] Our conclusion is not changed if we include Auger transitions in our discussion. The reasons are the following: (a) Auger effect is important only for high n values ($n>10$). (b) Each Auger-effect collision will increase the population of antiprotons with $m=0$. The number of collisions required to reduce the number of antiprotons to $(1/e)n^2$ due to the S-state Stark captures is then about $n[1-(1/e)]$ instead of n. The rate, $\omega_c(nS)$, is therefore increased. (c) Because the antiprotons start undergoing Auger transitions from $n\sim 30$, by the time they reach a state with $n\sim 10$ where P state capture becomes important, a substantial number of them will already be Stark captured via the S state.