The anomalous spectra observed in conducting crystals which is characterized by a large D value as shown in Fig. 5 has not been satisfactorily explained. It may be due to Mn^{++} which is lodged near some defect in the lattice which produces a strong axial field.

After this work was completed, we learned from Professor W. Hayes that parallel and independent work has been carried out at the Clarendon laboratory, Oxford, and is being readied for publication. Results obtained for CdTe are similar to the data reported here.

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Magneto-Attenuation of Sound in Semimetals: Longitudinal Waves*

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The calculation of the magnetic field dependence of ultrasonic attenuation in a semimetal is discussed on a simple model of its band structure. The results are applied to the case where the electron and hole mean free paths are large compared to the wavelength of sound. A series of oscillations and a large peak in the attenuation as a function of magnetic field are derived. The oscillations are geometric resonances of the type previously derived for metals, and the large peak is associated with the presence of density waves in the electron-hole carrier gas. The theoretical results are discussed, compared with experimental data, and found to agree semiquantitatively with the latter.

I. INTRODUCTION

RECENT experiments[1,2] on the attenuation of ultrasonic waves in semimetallic crystals maintained at liquid helium temperatures have revealed a marked magnetic field dependence over a wide range of field strengths. Some features of the field dependence, in particular the occurrence of geometric resonances,[3,4] are shared by metals[5–9] studied under similar circumstances. The amplitude of the geometric resonances as well as the mean level of attenuation in the region of geometric resonances is significantly less in semimetals than in metals. However there exists for semimetals an extremely large increase in the attenuation as the field is increased past the point where the geometric resonances are no longer observed.[1] In some cases a subsequent decrease in the attenuation has also been observed. In metals, on the other hand, the attenuation appears to saturate[4–9] under these conditions. This effect constitutes perhaps the chief difference between the behavior of metals and semimetals.

Blount[10] has given a detailed theory of the attenuation applicable to semimetals in the absence of a magnetic field. However, theoretical studies[4,11–14] of the magnetic field dependence of the attenuation have confined themselves to a discussion of a free electron model of a metal. It is the purpose of the present paper to study the magnetic field dependence of the attenuation in a simple model of a semimetal. The methods used are based on those developed in CHH rather than those of Blount. The model consists of the following. We work in the effective mass approximation, and take both electrons and holes to have isotropic effective masses. We describe the modulation of the energies of these particles by the passing sound wave in terms of deformation potential energies proportional to the local dilation in the lattice. The electron and hole energies are then respectively

$$E_e = E_e^0 + V_{De}\nabla\cdot\mathbf{d},\qquad E_h = E_h^0 + V_{Dh}\nabla\cdot\mathbf{d}, \tag{1.1}$$

where E_e^0 and E_h^0 are the corresponding particle energies in the unstrained crystal and $\mathbf{d}\propto\exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)]$ is the displacement field associated with a sound wave of wave vector $\mathbf{q}$ and frequency ω. (1.1) then defines

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the deformation potential constants V_{De} and V_{Dh}. The position dependence of (1.1) naturally implies the existence of forces on the electrons and holes arising from the acoustic strain in the lattice.

We focus attention on the role of the deformation potentials in determining the acoustic attenuation and take the simplest possible scattering mechanism, neglecting any intervalley scattering effects. When the mean free paths are much larger than the wavelength of sound, we anticipate that the details of the scattering are of secondary importance in marking the distinction between a metal and semimetal compared with the role played by the deformation potentials.

In Sec. II we consider the formal transport theory which provides the basis for studying the ultrasonic attenuation. In Sec. III we explicitly calculate the attenuation in a representative semimetal as a function of magnetic field. In Sec. IV we discuss our results and compare them with existing experimental data. For purposes of continuity and clarity we adhere to the notation and conventions established in CHH.

II. FORMAL THEORY OF THE ATTENUATION

A. The Constitutive Equation

As a model of a semimetal, we adopt a system containing N electrons and N holes per unit volume moving through a uniform neutral background which supports the sound wave. The discreteness of the crystal lattice in real semimetals is unimportant when the sound wavelength greatly exceeds the interatomic separation. A sound wave of propagation vector $\mathbf{q}$ and frequency ω manifests itself as a velocity field $\mathbf{u}(\mathbf{r},t) \propto \exp[i(\mathbf{q}\cdot\mathbf{r}-\omega t)]$ in the neutral background. In the present model, interactions between particles are replaced by interactions of individual particles with a self-consistent electromagnetic field derived from Maxwell's equations. For currents and fields varying as $\exp[i(q\cdot\mathbf{r}-\omega t)]$ the latter may be reduced to

$$\boldsymbol{\mathcal{E}}_{\parallel}=(4\pi/i\omega\epsilon)\mathbf{j}_{\parallel},\quad \boldsymbol{\mathcal{E}}_{\perp}=\frac{(4\pi i/\omega)(v_s/c)^2}{1-\epsilon(v_s/c)^2}\mathbf{j}_{\perp},\tag{2.1}$$

where $\boldsymbol{\mathcal{E}}$ is the electric field and $\mathbf{j}$ is the total current density accompanying the sound wave. ϵ is the dielectric constant of the neutral background continuum. The subscripts $\parallel$ and $\perp$ in (2.1) refer to components parallel and perpendicular to $\mathbf{q}$, respectively, and v_s is the sound velocity.

The total current density contains a contribution from the electrons, $\mathbf{j}_e$, and one from the holes, $\mathbf{j}_h$:

$$\mathbf{j}=\mathbf{j}_e+\mathbf{j}_h.\tag{2.2}$$

Both the electronic and hole currents excited by the sound wave are obtained from the electron and hole distribution functions in the usual manner;

$$\begin{aligned}\mathbf{j}_e(\mathbf{r},t)&=-e\int \mathbf{v}f_e(\mathbf{r},\mathbf{v},t)d^3v,\\ \mathbf{j}_h(\mathbf{r},t)&=+e\int \mathbf{v}f_h(\mathbf{r},\mathbf{v},t)d^3v,\end{aligned}\tag{2.3}$$

where f_e and f_h are the electron and hole distributions, respectively, and e is the absolute magnitude of electronic charge. In the absence of any sound wave the distribution functions reduce essentially to a thermal equilibrium Fermi-Dirac function, $f_0(\mathbf{v},E_F)$, for degenerate particles with Fermi energy E_F, and do not depend explicitly on the static magnetic field H_0 in the semiclassical approximation which we employ (Bohr-van Leeuwen theorem). In the presence of a sound wave the distribution functions are determined from the Boltzmann equations which they satisfy:

$$\begin{aligned}\frac{\partial f_e}{\partial t}+\mathbf{v}\cdot\frac{\partial f_e}{\partial \mathbf{r}}+\frac{\mathbf{F}_e}{m_e}\cdot\frac{\partial f_e}{\partial \mathbf{v}}&=\left.\frac{\partial f_e}{\partial t}\right|_{\text{coll.}}\\ \frac{\partial f_h}{\partial t}+\mathbf{v}\cdot\frac{\partial f_h}{\partial \mathbf{r}}+\frac{\mathbf{F}_h}{m_h}\cdot\frac{\partial f_h}{\partial \mathbf{v}}&=\left.\frac{\partial f_h}{\partial t}\right|_{\text{coll.}}.\end{aligned}\tag{2.4}$$

In (2.4) the forces, $\mathbf{F}_e$ and $\mathbf{F}_h$, experienced by the electrons and holes, respectively, are the sums of the Lorentz force and deformation potential force acting on each type of charge carrier.

$$\begin{aligned}\mathbf{F}_e&=-e[\boldsymbol{\mathcal{E}}+(V_{De}/ei\omega)\mathbf{q}\mathbf{q}\cdot\mathbf{u}+(\mathbf{v}/c)\times\mathbf{H}]\\ \mathbf{F}_h&=+e[\boldsymbol{\mathcal{E}}-(V_{Dh}/ei\omega)\mathbf{q}\mathbf{q}\cdot\mathbf{u}+(\mathbf{v}/c)\times\mathbf{H}],\end{aligned}\tag{2.5}$$

where the magnetic field $\mathbf{H}$ includes a part $\mathbf{H}_1$ associated with the sound wave in addition to the steady field $\mathbf{H}_0$. The quantities m_e and m_h in (2.4) are the electron and hole effective masses, respectively, because it is these which determine the kinetic response of the particles to forces in the effective mass approximation. In (2.5) V_{De} and V_{Dh} are the electron and hole longitudinal deformation potential constants defined above.

For the collision terms on the right-hand side of (2.4) we make the relaxation time ansatz

$$\begin{aligned}\left.\frac{\partial f_e}{\partial t}\right|_{\text{coll.}}&=-\frac{f_e-f_{se}}{\tau_e},\\ \left.\frac{\partial f_h}{\partial t}\right|_{\text{coll.}}&=-\frac{f_h-f_{sh}}{\tau_h}.\end{aligned}\tag{2.6}$$

It is here that our previously mentioned assumption of no intervalley scattering becomes explicit. This ansatz is likely to have far reaching consequences, but we expect that the main features of the magnetic field dependence of the attenuation will not depend strongly on the form of the collision operator. In (2.6) τ_e and τ_h

are the electron and hole relaxation times, respectively; f_{se} and f_{sh} are the corresponding particle distribution functions after scattering. Since the impurities are moving with velocity $\mathbf{u}$, f_{se} and f_{sh} are local equilibrium distribution functions centered in velocity space about the impurity velocity $\mathbf{u}$. Thus we have

$$\begin{aligned} f_{se}(\mathbf{r},\mathbf{v},t)&=f_0[\mathbf{v}-\mathbf{u}(\mathbf{r},t),\, E_{Fe}(\mathbf{r},t)],\\ f_{sh}(\mathbf{r},\mathbf{v},t)&=f_0[\mathbf{v}-\mathbf{u}(\mathbf{r},t),\, E_{Fh}(\mathbf{r},t)] \end{aligned} \tag{2.7}$$

for the distributions toward which f_e and f_h are locally relaxing, where the Fermi energies $E_{Fe}(\mathbf{r},t)$ and $E_{Fh}(\mathbf{r},t)$ are directly determined by the electron and hole densities, $N_e(\mathbf{r},t)$ and $N_h(\mathbf{r},t)$.

The transport problem involved in the calculation of $\mathbf{j}_e$ and $\mathbf{j}_h$ is now completely specified. After solving the Boltzmann equations (2.4) with the collision terms given by (2.6) and (2.7), we may calculate the current densities according to (2.3). This type of problem has been considered and solved in CHH. The result of the calculation is an expression for $\mathbf{j}_e$ and $\mathbf{j}_h$ in the form of a constitutive equation. We may take over this result essentially unchanged.

$$\begin{aligned} \mathbf{j}_e&=\boldsymbol{\sigma}_e\cdot[\boldsymbol{\mathcal{E}}+(V_{De}/ei\omega)\mathbf{q}\mathbf{q}\cdot\mathbf{u}-m\mathbf{u}/e\tau_e]-\mathbf{R}_eN_e'ev_s,\\ \mathbf{j}_h&=\boldsymbol{\sigma}_h\cdot[\boldsymbol{\mathcal{E}}-(V_{Dh}/ei\omega)\mathbf{q}\mathbf{q}\cdot\mathbf{u}+m\mathbf{u}/e\tau_h]+\mathbf{R}_hN_h'ev_s. \end{aligned} \tag{2.8}$$

In (2.8) $\boldsymbol{\sigma}_e$ and $\boldsymbol{\sigma}_h$ are the electron and hole magnetoconductivity tensors, respectively, for frequency ω and wave vector $\mathbf{q}$. They characterize the response of the two carriers to the force fields accompanying the incident sound wave in the presence of the steady field $\mathbf{H}_0$. The vectors $\mathbf{R}_e$ and $\mathbf{R}_h$ characterize the diffusion of the nonuniformly distributed carriers; $N_e'=N_e(\mathbf{r},t)-N$, and $N_h'=N_h(\mathbf{r},t)-N$ are the deviations of the carrier densities from their equilibrium values. Collisions with moving impurities have the effect of adding an apparent drag $m(\mathbf{u}/\tau_e)$ and $m(\mathbf{u}/\tau_h)$ to the force acting on the carriers. It is the free electron mass, m, which appears here because the actual momentum transfer to the lattice in a collision involves the change in the expectation value of the linear momentum operator, which is just the product of the free electron mass and the velocity operator. The magnetic field $\mathbf{H}_1$ associated with the sound wave is not explicitly present in (2.8).

We now choose a right-handed coordinate system in which $\mathbf{q}$ lies along the 1-axis, and $\mathbf{H}_0$ lies along the 3-axis. From the equations of continuity for electrons and holes we obtain

$$N_e'=-j_{e1}/ev_s,\quad N_h'=j_{h1}/ev_s. \tag{2.9}$$

The diffusive terms in (2.8) can therefore be written as

$$\mathbf{R}_e j_{e1}=\mathsf{R}_e\cdot\mathbf{j}_e,\quad \mathbf{R}_h j_{h1}=\mathsf{R}_h\cdot\mathbf{j}_h, \tag{2.10}$$

where R_e and R_h are tensors which in our coordinate systems have the components

$$(\mathsf{R}_e)_{ij}=R_{ei}\delta_{1j},\quad (\mathsf{R}_h)_{ij}=R_{hi}\delta_{1j}. \tag{2.11}$$

We can now simplify the constitutive equations (2.8) to

$$\begin{aligned} \mathbf{j}_e&=\sigma_e^0\boldsymbol{\sigma}_e'\cdot[\boldsymbol{\mathcal{E}}+(V_{De}/ei\omega)\mathbf{q}\mathbf{q}\cdot\mathbf{u}-m\mathbf{u}/e\tau_e],\\ \mathbf{j}_h&=\sigma_h^0\boldsymbol{\sigma}_h'\cdot[\boldsymbol{\mathcal{E}}-(V_{Dh}/ei\omega)\mathbf{q}\mathbf{q}\cdot\mathbf{u}+m\mathbf{u}/e\tau_h] \end{aligned} \tag{2.12}$$

by use of (2.10); where

$$\begin{aligned} \boldsymbol{\sigma}_e'&=[\mathsf{I}-\mathsf{R}_e]^{-1}\cdot\boldsymbol{\sigma}_e/\sigma_e^0,\\ \boldsymbol{\sigma}_h'&=[\mathsf{I}-\mathsf{R}_h]^{-1}\cdot\boldsymbol{\sigma}_h/\sigma_h^0 \end{aligned} \tag{2.13}$$

are effective conductivity tensors which include diffusion and are measured in units of the dc conductivities of the respective electron and hole bands.

$$\sigma_e^0=Ne^2\tau_e/m_e,\quad \sigma_h^0=Ne^2\tau_h/m_h. \tag{2.14}$$

We employ (2.12) in formulating an expression for the acoustic attenuation, and take the required components of $\boldsymbol{\sigma}_e'$ and $\boldsymbol{\sigma}_h'$ in any particular case of interest from CHH.

B. The Attenuation Coefficient

The sound wave supplies both kinetic and potential energy to the particles as it propagates. The electrons and holes dissipate this energy to the neutral background through collisions. When the collisions suffered by the particles are local and occur without any net change of potential energy, it is possible to show by straightforward manipulation based on the Boltzmann equation that the average rate per unit volume, W, at which particles lose kinetic energy in collisions is given by

$$W=\tfrac{1}{2}\,\mathrm{Re}\{\mathbf{j}_e^*\cdot[\boldsymbol{\mathcal{E}}+(V_{De}/ei\omega)\mathbf{q}\mathbf{q}\cdot\mathbf{u}] +\mathbf{j}_h^*\cdot[\boldsymbol{\mathcal{E}}-(V_{Dh}/ei\omega)\mathbf{q}\mathbf{q}\cdot\mathbf{u}]\}, \tag{2.15}$$

where we have again used complex quantities for convenience. Not all of this kinetic energy transfer is irreversibly dissipated as heat; a part is coherently fed back into the sound wave. Since the average electron and hole velocities, $\langle\mathbf{v}_e\rangle$ and $\langle\mathbf{v}_h\rangle$, respectively, before collision in general differ from those after collision, $\mathbf{u}$, there are net forces exerted by the electrons and holes on unit volume of the moving neutral background. These are

$$\begin{aligned} \mathfrak{F}_e&=(Nm/\tau_e)(\langle\mathbf{v}_e\rangle-\mathbf{u}),\\ \mathfrak{F}_h&=(Nm/\tau_h)(\langle\mathbf{v}_h\rangle-\mathbf{u}), \end{aligned} \tag{2.16}$$

where $\mathfrak{F}_e$ is the force exerted by the electrons and $\mathfrak{F}_h$ is that exerted by the holes. In (2.16) m is the free electron mass because the actual momentum transfer to the moving lattice in a collision is again identical to the change in the expectation value of the linear momentum operator. Energy is coherently returned to the sound wave at an average rate per unit volume $P=\langle(\mathfrak{F}_e+\mathfrak{F}_h)\cdot\mathbf{u}\rangle_{\mathrm{av}}$. The net power dissipated per unit volume is then $Q=W-P$. Noting that $\mathbf{j}_e=-eN\langle\mathbf{v}_e\rangle$ and $\mathbf{j}_h=eN\langle\mathbf{v}_h\rangle$, and that $\mathrm{Re}(\mathbf{j}^*\cdot\boldsymbol{\mathcal{E}})$ vanishes by virtue of Maxwell's equations (2.1) we may write after some manipulation

$$Q=\tfrac{1}{2}\,\mathrm{Re}\{\mathbf{u}^*\cdot((i/e\omega)\mathbf{qq}\cdot[\mathbf{j}\tfrac{1}{2}(V_{De}-V_{Dh})+(\mathbf{j}_e-\mathbf{j}_h)\tfrac{1}{2}(V_{De}+V_{Dh})]+(m/e)\times[\mathbf{j}\tfrac{1}{2}(1/\tau_e-1/\tau_h)+(\mathbf{j}_e-\mathbf{j}_h)\tfrac{1}{2}(1/\tau_e+1/\tau_h)]+(1/\tau_e+1/\tau_h)Nm\mathbf{u})\}. \quad (2.17)$$

The quantity of direct experimental interest is the attenuation coefficient α, which gives the exponential decay of sound intensity with distance. α is the power density dissipated per unit incident energy flux, or

$$\alpha=Q/\tfrac{1}{2}\rho|\mathbf{u}|^2v_s, \quad (.218)$$

where ρ is the mass density of the semimetal being represented by our simple model.

Our goal is now to render the calculation of currents and fields self-consistent by combining (2.1), (2.2), and (2.12), and to use (2.17) in investigating the possible kinds of dependence of α on magnetic field and frequency under the condition that the mean free paths are much larger than the sound wavelength.

C. Special Cases

We consider the special case where $m_e=m_h=m^*$, and $\tau_e=\tau_h=\tau$. Under these conditions $\boldsymbol{\sigma}_e'=\boldsymbol{\sigma}_h'$. While this assumption allows considerable formal simplification, the special symmetry thereby established between electrons and holes is likely to effect profoundly our final result. Nevertheless we anticipate that effects characteristic of semimetals will not depend qualitatively on relative values of effective mass and relaxation time.

For the present we leave the values of V_{De} and V_{Dh} unspecified, and obtain self-consistent expressions for the total current, $\mathbf{j}=\mathbf{j}_e+\mathbf{j}_h$, and for the difference of the two currents, $\mathbf{j}_e-\mathbf{j}_h$. In the coordinate system which we have adopted (2.1) can be written in matrix form

$$\boldsymbol{\varepsilon}=\mathbf{F}\cdot\mathbf{j}, \quad (2.19)$$

where $\mathbf{F}$ is a diagonal matrix given by

$$\mathbf{F}=\begin{bmatrix}-1/\epsilon & 0 & 0\\ 0 & (v_s/c)^2 & 0\\ 0 & 0 & (v_s/c)^2\end{bmatrix}\frac{4\pi i}{\omega} \quad (2.20)$$

after the $\epsilon(v_s/c)^2$ in the denominator of (2.1) has been neglected. From (2.19), (2.2), and the assumption of equal effective masses and relaxation times we may transform (2.12) into

$$\mathbf{j}_e=\sigma^0\boldsymbol{\sigma}'\cdot[\mathbf{F}\cdot\mathbf{j}+(V_{De}/ei\omega)\mathbf{qq}\cdot\mathbf{u}-m\mathbf{u}/e\tau], \quad (2.21)$$
$$\mathbf{j}_h=\sigma^0\boldsymbol{\sigma}'\cdot[\mathbf{F}\cdot\mathbf{j}-(V_{Dh}/ei\omega)\mathbf{qq}\cdot\mathbf{u}+m\mathbf{u}/e\tau],$$

where now $\boldsymbol{\sigma}'=\boldsymbol{\sigma}_e'=\boldsymbol{\sigma}_h'$, and $\sigma^0=Ne^2\tau/m^*$ is the dc conductivity of either band of carriers by itself. Adding the two currents in (2.21) and solving for the total current, we obtain

$$\mathbf{j}=[\mathbf{I}-\boldsymbol{\sigma}'\cdot(2\sigma^0\mathbf{F})]^{-1}\cdot\boldsymbol{\sigma}'\cdot\frac{\sigma^0(V_{De}-V_{Dh})}{ei\omega}\mathbf{qq}\cdot\mathbf{u}, \quad (2.22)$$

where $\mathbf{I}$ is the unit matrix. Subtracting the two currents in (2.21) we obtain

$$\mathbf{j}_e-\mathbf{j}_h=\sigma^0\boldsymbol{\sigma}'\cdot\left[\frac{(V_{De}+V_{Dh})}{ei\omega}\mathbf{qq}-\frac{2m}{e\tau}\mathbf{I}\right]\cdot\mathbf{u}. \quad (2.23)$$

Equation (2.22) reveals that when the effective masses and relaxation times are equal the total current induced by the sound wave responds only to the difference between the electron and hole deformation potentials. The magnitudes and relative phase of V_{De} and V_{Dh} will therefore have a profound effect on the resultant attenuation. It is convenient to consider two extreme cases. First we take $V_{De}=V_{Dh}=V_D$, which corresponds to the electron and hole energy bands moving in opposition to each other, with variable overlap at any given point in the crystal. Equation (2.22) shows that $\mathbf{j}=0$, and consequently no electric field develops in the system by virtue of (2.19). Because of this, as we shall see, the ultrasonic attenuation may become quite large and exhibit a marked magnetic field dependence. Secondly, we take $V_{De}=-V_{Dh}=V_D$. Here the electron and hole energy bands move up and down together, with constant overlap at any given point in the crystal. There exists a tendency for the mutual separation of positive and negative carriers. Thus a finite $\mathbf{j}$ and $\boldsymbol{\varepsilon}$ may develop in the system, and therefore the attenuation remains small for all values of magnetic field.

III. CALCULATION OF THE ATTENUATION

We substitute Eqs. (2.22) and (2.23) into Eq. (2.17), and recalling that we have now set $\tau_e=\tau_h=\tau$, $m_e=m_h=m^*$, we obtain

$$Q=\tfrac{1}{2}\,\mathrm{Re}\left\{\mathbf{u}^*\cdot\left[\frac{i}{e\omega}\mathbf{qq}\cdot\left(\frac{\sigma^0(V_{De}-V_{Dh})^2}{2ei\omega}[\mathbf{I}-\boldsymbol{\sigma}'\cdot(2\sigma^0\mathbf{F})]^{-1}\cdot\boldsymbol{\sigma}'\cdot\mathbf{qq}+\frac{\sigma^0(V_{De}+V_{Dh})}{2}\boldsymbol{\sigma}'\cdot\left[\frac{(V_{De}+V_{Dh})}{ei\omega}\mathbf{qq}-\frac{2m}{e\tau}\mathbf{I}\right]\right)+\frac{\sigma^0m}{e\tau}\boldsymbol{\sigma}'\cdot\left(\frac{(V_{De}+V_{Dh})}{ei\omega}\mathbf{qq}-\frac{2m}{e\tau}\mathbf{I}\right)+\frac{2Nm}{\tau}\mathbf{I}\right]\cdot\mathbf{u}\right\}. \quad (3.1)$$

Equation (3.1) may be transformed by elementary manipulation into

$$Q=(Nm|\mathbf{u}|^2/\tau)\hat{u}\cdot\mathbf{S}\cdot\hat{u}, \quad (3.2)$$

where $\hat{u}$ is a unit vector in the direction of polarization and

$$\mathbf{S}=\mathrm{Re}\left\{\frac{i\tau\sigma^0}{2e^2\omega Nm}\mathbf{qq}\cdot\left[\frac{(V_{De}-V_{Dh})^2}{2i\omega}[\mathbf{I}-\boldsymbol{\sigma}'\cdot(2\sigma^0\mathbf{F})]^{-1}\cdot\boldsymbol{\sigma}'\cdot\mathbf{qq}+\frac{(V_{De}+V_{Dh})}{2}\boldsymbol{\sigma}'\cdot\left(\frac{(V_{De}+V_{Dh})}{i\omega}\mathbf{qq}-\frac{2m}{\tau}\mathbf{I}\right)\right]\right.$$
$$\left.+\frac{\sigma^0}{2Ne^2}\boldsymbol{\sigma}'\cdot\left(\frac{(V_{De}+V_{Dh})}{i\omega}\mathbf{qq}-\frac{2m}{\tau}\mathbf{I}\right)+\mathbf{I}\right\}. \quad (33.)$$

Since $\hat{u}$ is polarized along the 1-axis in the present discussion of longitudinal sound waves, we need only the 11 component of $\mathbf{S}$ in order to obtain the power dissipation,

$$Q=(Nm|\mathbf{u}|^2/\tau)S_{11}. \quad (3.4)$$

It is instructive at this point to investigate the orders of magnitude of the various parameters entering the expression for the attenuation coefficient. From (2.18) and (3.4) we obtain

$$\alpha=(2Nm/\rho v_s\tau)S_{11}. \quad (3.5)$$

α, by its nature, is the reciprocal of the mean free path L of the sound wave in the semimetal,

$$L=(\rho v_s\tau/2Nm)(1/S_{11}). \quad (3.6)$$

For example,in bismuth,[15] N is 5.5×10^{17}, and with a τ of 10^{-10}, $(\rho v_s\tau/2Nm)$ is about 10^5, so that $L\approx10^5/S_{11}$ for bismuth. Ease of measurement and the general order of magnitude of background attenuation require that L be of order 1 cm and perhaps somewhat less. This implies that we must have $S_{11}\gtrsim10^5$ for the attenuation to be readily observable in a typical semimetal. We employ this estimate in subsequent discussion of out final results.

We now consider the case where $V_{De}=V_{Dh}=V_D$. From (3.3) and the relation $\sigma^0=Ne^2\tau/m^*$ we then obtain after straightforward manipulation

$$S_{11}=1-(m/m^*)\,\mathrm{Re}\left\{\sigma_{11}'\left(1+\frac{iV_D\tau}{\omega m}q^2\right)^2\right\}. \quad (3.7)$$

The tensor component σ_{11}' in the coordinate system which we have adopted is given in CHH. For magnetic fields such that the classical orbit radius is of the order of or smaller than the sound wavelength one has

$$\sigma_{11}'=\frac{-3i\omega\tau(1-i\omega\tau)(1-g_0(X))}{q^2l^2[1-i\omega\tau-g_0(X)]}, \quad (3.8)$$

where $X=ql/\omega_c\tau$; $l=v_F\tau$ is the mean free path for particles with the Fermi velocity, v_F, and $\omega_c=eH/m^*c$ is the cyclotron frequency. The function g_0 is given by

$$g_0(X)=\int_0^{\pi/2}J_0^2(X\sin\theta)\sin\theta d\theta, \quad (3.9)$$

where J_0 is the Bessel function of order zero; $g_0(X)$ is tabulated in CHH.

From (3.7) and (3.8) we calculate S_{11} and obtain

$$S_{11}=1+\frac{3(v_s/v_F)^2(g_0-1)(m/m^*)}{(g_0-1)^2+(\omega\tau)^2}\times\left[g_0(1-(V_D/mv_F^2)^2(v_F/v_s)^2(ql)^2)+2\left(\frac{V_D}{mv_F^2}\right)\left(\frac{v_F}{v_s}\right)^2(1-g_0+\omega^2\tau^2)\right]. \quad (3.10)$$

Further approximation is based on the comparison which we ultimately wish to make with bismuth. Experiments indicate[16] that $mv_F^2\approx0.35$ ev for bismuth if we assume an average mass ratio $(m/m^*)=10$. The deformation potential, V_D, has not been measured in bismuth, but we may assume it is the same order of magnitude as that measured in germanium,[17] and possibly even larger. As a reasonable estimate we take V_D to be the order of 10 ev and set $(V_D/mv_F^2)=20$ as a value pertinent to a discussion of bismuth.[10] Then, neglecting terms of order (V_D/mv_F^2) in comparison with terms of order $(V_D/mv_F^2)^2(ql)^2$ in (3.10) we obtain in the limit $(ql)\gg1$

$$S_{11}=1+3\left(\frac{V_D}{mv_F^2}\right)^2(ql)^2\left(\frac{m}{m^*}\right)\frac{g_0(1-g_0)}{(1-g_0)^2+(\omega\tau)^2}. \quad (3.11)$$

Equation (3.11) is plotted in Fig. 1 versus $qR(=X)$ as abscissa, where $R=p_Fc/eH$ is the orbit radius of a particle at the Fermi surface. As the field is increased the attenuation goes through a series of oscillations, and then rises to a rather large maximum. For the present choice of $\omega\tau=0.1$, Fig. 1 indicates that the maximum occurs at about $qR\approx0.5$. The amount of rise is more than an order of magnitude greater than the amplitude of the oscillations occurring at lower field values. As the field is increased still further the attenuation begins to drop, and continues to do so throughout the high field region, in which quantum effects render the present theory inadequate.

The existence of the maximum becomes more transparent if the Bessel function J_0 is expanded in a Taylor series and the lowest nonvanishing power of $X=(ql/\omega_c\tau)$

[15] G. Smith, Phys. Rev. **115**, 1561 (1959).

[16] D. Shoenberg, Trans. Roy. Soc. (London) **A245**, 1 (1952).

[17] H. Fritzsche, Phys. Rev. **114**, 336 (1959).

is retained in the expression for g_0. This leads to an approximate form for S_{11} when $X \ll 1$.

$$S_{11} \simeq 1+\left(\frac{V_D}{mv_F^2}\right)^2\left(\frac{v_F}{v_s}\right)^2(ql)^2\left(\frac{m}{m^*}\right) \times \frac{(\omega_c\tau)^2}{(\omega_c\tau)^4+\frac{1}{9}(ql)^2(v_F/v_s)^2}. \quad (3.12)$$

Equation (3.12) is not quite valid for the case illustrated in Fig. 1, but is a fair representation of S_{11} in the in the neighborhood of the maximum when it occurs at a value X somewhat less than 0.5. Equation (3.12) leads to the following approximate relation determining the position of the large maximum in S_{11}.

$$(\omega_c\tau)^2=(ql/3)(v_F/v_s). \quad (3.13)$$

We now consider the case where $V_{De}=-V_{Dh}=V_D$. It is first convenient to introduce the diagonal matrix $\mathbf{B}=(2\sigma^0\mathbf{F})^{-1}$ with diagonal elements $B_{11}=i\gamma$, $B_{22}=B_{33}=-i\beta$, where

$$\gamma=\omega\epsilon/8\pi\sigma^0, \quad \beta=\omega c^2/8\pi\sigma^0 v_s^2. \quad (3.14)$$

Employing a matrix identity we may then write

$$[\mathbf{I}-\boldsymbol{\sigma}'\cdot(2\sigma^0\mathbf{F})]^{-1}\cdot\boldsymbol{\sigma}'=[[\mathbf{I}-\boldsymbol{\sigma}'\cdot\mathbf{B}^{-1}]^{-1}-\mathbf{I}]\cdot\mathbf{B}. \quad (3.15)$$

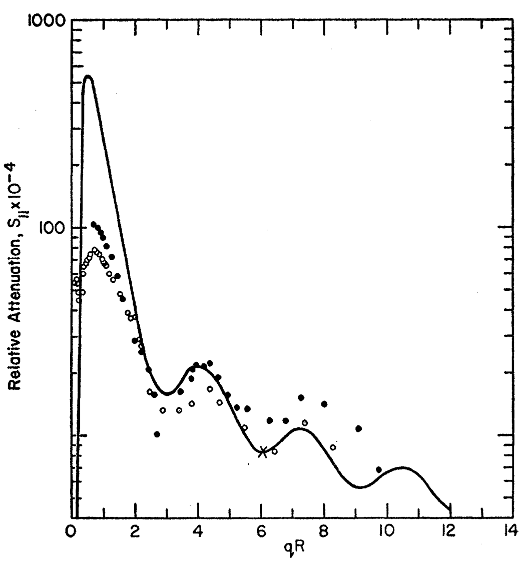

FIG. 1. The solid curve represents the relative attenuation as given by Eq. (3.11) for the parameter values $(ql)=10$, $(m/m^*)=10$, $\omega\tau=.1$, and $(V_D/mv_F^2)=20$, where $(V_{De}+V_{Dh})=2V_D$. The hollow circles represent experimental data[1] on the attenuation of a 60 megacycle per second longitudinal mode propagated along an x axis of a single crystal of bismuth, with the magnetic field directed along the corresponding y axis. The solid circles represent data[1] on the attenuation of a 60 megacycle per second longitudinal mode propagated along an x axis, with the magnetic field directed along the z axis of a single crystal of bismuth. Both sets of data are normalized at the crossed point on the theoretical curve, and the other points plotted accordingly.

From (3.3), (3.14) and the relation $\sigma^0=Ne^2\tau/m^*$, we find that in the present case S_{11} is given by

$$S_{11}=1+\mathrm{Re}\left\{\left(\frac{V_D}{mv_F^2}\right)^2(ql)^2\left(\frac{v_F}{v_s}\right)^2\left(\frac{m}{m^*}\right)B_{11} \times([\mathbf{I}-\boldsymbol{\sigma}'\cdot\mathbf{B}^{-1}]^{-1}-\mathbf{I})_{11}-\left(\frac{m}{m^*}\right)\sigma_{11}'\right\}. \quad (3.16)$$

For the present orientation of magnetic field and choice of coordinate system, the structure of $\boldsymbol{\sigma}'$ is given in CHH as

$$\boldsymbol{\sigma}'=\begin{bmatrix}\sigma_{11}' & \sigma_{12}' & 0\\ -\sigma_{12}' & \sigma_{22}' & 0\\ 0 & 0 & \sigma_{33}'\end{bmatrix}. \quad (3.17)$$

Using (3.17) and the expressions for the diagonal components of $\mathbf{B}$ in terms of the γ and β of (3.14) we find after some manipulation that (3.16) may be transformed into

$$S_{11}=1+\left(\frac{m}{m^*}\right)\mathrm{Re}\left\{\left(\frac{V_D}{mv_F^2}\right)^2(ql)^2\left(\frac{v_F}{v_s}\right)^2 \times\frac{\gamma[(\sigma_{11}'\sigma_{22}'+\sigma_{12}'^2)+i\beta\sigma_{11}']}{[i(\sigma_{11}'\sigma_{22}'+\sigma_{12}'^2)-\beta\sigma_{11}'+(\sigma_{22}'+i\beta)\gamma]}-\sigma_{11}'\right\}. \quad (3.18)$$

We must now investigate the orders of magnitude of γ, β and the components of $\boldsymbol{\sigma}'$ in order to establish a basis for approximate evaluation of (3.18). For a typical semimetal of the highest purity which is in practice obtainable, we may take $\sigma^0\sim10^{15}$ cgs. With $\epsilon\sim10$, which is characteristic of bismuth,[18] $v_s\sim10^5$ cm/sec and a sound frequency of 50 megacycles per second we get $\gamma\sim10^{-7}$, and $\beta\sim10^2$ upon evaluation of the defining expressions (3.14). We can make β smaller only by decreasing the frequency and hence the wave number, q. But such a decrease would in practice lead to a violation of the requirement that $ql>1$, which corresponds to the case of greatest interest to us. For instance, in the case of bismuth, if we take a relaxation time of $\tau=10^{-10}$ sec and an average Fermi velocity $v_F=10^7$ cm/sec, we obtain $l=v_F\tau=10^{-3}$ cm. With a frequency of 50 megacycles per second and a sound velocity of $v_s\simeq10^5$ cm/sec we have $q\simeq3.0\times10^3$ cm^{-1}. Thus we obtain $ql\simeq3.0$, and any decrease in frequency by several orders of magnitude in an attempt to make $\beta<1$ surely results in the unwanted consequence that $ql<1$. In light of the above considerations we evaluate Eq. (3.18) in the case that $\beta\gg1$, keeping in mind the representative values $\beta\sim10^2$ and $\gamma\sim10^{-7}$.

With elementary manipulation we can write Eq. (3.18) as

[18] W. S. Boyle and A. D. Brailsford, Report on International Conference on Electronic Properties of Metals at Low Temperature, Geneva, New York, 1958 (unpublished), p. 58.

$$S_{11}=1+\left(\frac{m}{m^*}\right)\mathrm{Re}\left[\left(\frac{V_D}{mv_F^2}\right)^2(ql)^2\times\left(\frac{v_F}{v_s}\right)^2\frac{A\gamma^2}{(A\gamma)^2+(1+B\gamma)^2}-\sigma_{11}'\right], \quad (3.19)$$

where A and B are real quantities defined by

$$\frac{(\sigma_{22}'+i\beta)}{[\sigma_{11}'\sigma_{22}'+\sigma_{12}'^2+i\beta\sigma_{11}']}=A+iB. \quad (3.20)$$

At moderate magnetic fields, such that $\omega_c\tau\approx ql$, the components of the dimensionless tensor $\boldsymbol{\sigma}'$ do not differ from unity in absolute value by more than a few orders of magnitude. Since Eq. (3.20) is of degree zero in β, the quantities A and B also cannot differ from unity by more than a few orders of magnitude. Further, since $\gamma\sim10^{-7}$ for conditions of practical interest, we may take $(A\gamma)^2+(1+B\gamma)^2\sim1$ in (3.19). The first term in the bracket of (3.19) is then proportional to γ^2. Using Eq. (3.8) as an expression for σ_{11}' when $\omega_c\tau\approx ql$ we may then write down an estimate for (3.19).

$$S_{11}\sim1+\left(\frac{m}{m^*}\right)\left[\left(\frac{V_D}{mv_F^2}\right)^2(ql)^2\left(\frac{v_F}{v_s}\right)^2A\gamma^2-3\left(\frac{v_s}{v_F}\right)^2\frac{g_0(1-g_0)}{[(1-g_0)^2+\omega^2\tau^2]}\right]. \quad (3.21)$$

For $\omega\tau=0.1$ and $(v_s/v_F)=10^{-2}$, the maximum value of the second term in the bracket of (3.21) is approximately 10^{-3}. With the above values for (v_s/v_F) and $\omega\tau=(v_s/v_F)ql$, and a reasonable value for (V_D/mv_F^2) such as that previously employed, say 20, the value of the first term in the bracket of Eq. (3.21) is approximately 10^{-6} if we take $A\sim1$ and $\gamma\sim10^{-7}$. With a representative mass ratio $(m/m^*)=10$, the value of S_{11} under our present assumptions concerning deformation potentials and magnetic field range is therefore only of order of magnitude unity, $S_{11}\sim1$. This result gives rise to an exceedingly small attenuation, since as we have observed above, we must have $S_{11}\sim10^5$ for semimetallic values of N in order to have a readily observable attenuation.

However, in view of the result of CHH that the sound attenuation in metals undergoes an enormous increase at very high magnetic fields, we explore the behavior of the attenuation in our model of a semimetal at magnetic fields sufficiently high to realize the condition $\omega_c\tau\gg ql$. This is the only limit we need consider in detail for the case where $V_{De}=-V_{Dh}=V_D$. In this limit the components of $\boldsymbol{\sigma}'$ are given in CHH as

$$\begin{aligned}\sigma_{11}'&=\frac{1-i\omega\tau}{(\omega_c\tau)^2+i\frac{1}{3}(ql)^2/\omega\tau},\\ \sigma_{22}'&=\frac{1}{(\omega_c\tau)^2}\left[\frac{2}{5}\frac{(ql)^2}{1-i\omega\tau}+1-i\omega\tau\right]+\frac{i\frac{1}{3}(ql)^2/\omega\tau}{[(\omega_c\tau)^2+i\frac{1}{3}(ql)^2/\omega\tau](1-i\omega\tau)},\\ \sigma_{12}'&=-\sigma_{21}'=\frac{-(\omega_c\tau)}{(\omega_c\tau)^2+i\frac{1}{3}(ql)^2/\omega\tau}.\end{aligned} \quad (3.22)$$

We substitute Eqs. (3.22) into Eq. (3.18) and make the approximations $ql\gg1$, $(\omega_c\tau)\gg ql$, and $ql\gg\omega\tau$. In the process of making our approximations, we employ the representative values $\omega\tau\gtrsim0.1$, $ql\gtrsim10$, $\gamma\sim10^{-7}$, and $\beta\sim10^2$. After a lengthy but straightforward calculation we obtain

$$S_{11}=1+\left(\frac{m}{m^*}\right)\left[\left(\frac{V_D}{mv_F^2}\right)^2(ql)^2\left(\frac{v_F}{v_s}\right)^2\frac{(\gamma\beta)^2(\omega_c\tau)^2}{\beta^2+[1+\beta\omega\tau+\gamma\beta(\omega_c\tau)^2]^2}-\frac{(\omega_c\tau)^2}{(\omega_c\tau)^4+\frac{1}{9}(ql)^4/(\omega\tau)^2}\right]. \quad (3.23)$$

Inspection of Eq. (3.23) shows that in the present case S_{11} indeed exhibits a maximum at sufficiently high magnetic fields in addition to a slight minimum at the same value of magnetic field where a large maximum occurred in the case $V_{De}=V_{Dh}$. However further inspection of (3.23) using the same values for all parameters as previously quoted again leads to the conclusion that for all values of field S_{11} is here too small for the attenuation to be observable. The interpretation of this result is deferred for later discussion.

It is well to note at this point that our procedure of giving separate consideration to the two cases, $V_{De}=V_{Dh}$, and $V_{De}=-V_{Dh}$, was more in the interest of convenience than necessity. We obtain a general expression for S_{11} for arbitrary values of V_{De} and V_{Dh} by substituting Eq. (3.15) into Eq. (3.3) and using the relation $\sigma^0=Ne^2\tau/m^*$. After some manipulation we find

$$S_{11}=1+\left(\frac{m}{m^*}\right)\mathrm{Re}\left[(ql)^2\left(\frac{v_F}{v_s}\right)^2\left(\frac{V_{De}-V_{Dh}}{2mv_F^2}\right)^2B_{11}([\mathbf{I}-\boldsymbol{\sigma}'\cdot\mathbf{B}^{-1}]^{-1}-\mathbf{I})_{11}-\sigma_{11}'\left(1+\frac{i\tau(V_{De}+V_{Dh})}{2m\omega}q^2\right)^2\right]. \quad (3.24)$$

Our detailed discussion of the two separate cases indicates that for reasonable values of the deformation potential constants the second term in the bracket of (3.24) dominates the first term by many orders of magnitude, provided $(V_{De}+V_{Dh})\neq0$. We may therefore neglect the contribution of the first term in calculating the attenuation. Figure 1 is then an accurate representation of the total attenuation under the condition $(V_{De}+V_{Dh})/mv_F^2>1$, which we expect is satisfied for bismuth. The significance of the far greater importance of the term containing the sum $(V_{De}+V_{Dh})$ is that to a good approximation both charge neutrality and vanishing electric current are achieved.

IV. DISCUSSION

We interpret the oscillations in S_{11} coming from the term containing $(V_{De}+V_{Dh})$ as the geometric resonances observed by Reneker in bismuth. Their physical origin may be understood along the same qualitative lines which are useful in understanding the effect in metals. The geometric resonances in the attenuation enter the theory through the Bessel functions appearing in the conductivity tensor. These have to do with the strength of interaction between individual orbits and the force fields in the system, rather than with the resonant absorption of energy. Inspection of Fig. 1 shows that the troughs in the attenuation are very nearly spaced π apart, in units of qR. The spacing departs most from π the higher the magnetic field and the closer to the first geometric resonance peak. The location of the minima are closely given by $qR=n\pi$. In the lower field region, where $qR\gtrsim 1$, the $(\omega\tau)^2$ in the denominator of Eq. (3.11) can be neglected so that we may write

$$S_{11}=1+3\left(\frac{V_D}{mv_F^2}\right)^2(ql)^2\left(\frac{m}{m^*}\right)\frac{g_0.}{1-g_0} \qquad (4.1)$$

Differentiation of Eq. (4.1) shows that the maxima and minima in S_{11}, and therefore in α, correspond to those values of $X=qR$ for which g_0' vanishes; these are listed in Table I of CHH.

Since an ellipsoid can always be transformed into a sphere by a coordinate transformation, the present work can readily be generalized to apply directly to actual semimetals. Such features as locations of extrema would remain essentially unchanged by this generalization. Thus the magneto-attenuation should be a powerful tool for exploring the band structures of semimetals, as has already been demonstrated by Reneker's work.

It is instructive to consider the ultrasonic attenuation in semimetallic crystals from a simplified physical viewpoint in order to obtain a qualitative understanding of the phenomena involved. In the presence of an applied sound wave electrons and holes experience deformation potential forces measured by the parameters V_{De} and V_{Dh}, respectively. In the steady state a total current $\mathbf{j}$ is set up which depends on these parameters only through their difference, $(V_{De}-V_{Dh})$, as we see from Eq. (2.22). However, the discussion in Sec. III implies that the total current, and hence the self-consistent electric field, is very small indeed. This result corresponds to the frequently invoked assumption of charge neutrality. It is the difference current, $\mathbf{j}_e-\mathbf{j}_h$, which responds to the sum $(V_{De}+V_{Dh})$, as we see from Eq. (2.23). The sum of deformation potential parameters essentially represents an unscreened external force acting on the system without modification by long range electric fields. This is why we can observe acoustic attenuation in semimetals despite the low carrier concentration. Each carrier responds to this force symbolically as $\mathbf{j}_e\sim\sigma_e\cdot\frac{1}{2}(V_{De}+V_{Dh})$ and $\mathbf{j}_h\sim\sigma_h\cdot\frac{1}{2}(V_{De}+V_{Dh})$. Thus the power dissipation will schematically be given by $Q\propto\sigma(V_{De}+V_{Dh})^2$, which may be compared with Eq. (3.7), or more generally with Eq. (3.24). Since relative extrema in Q and therefore in α then correspond to extrema in σ, the attenuation varies as the response of the system to the unscreened part of the deformation potentials, namely $(V_{De}+V_{Dh})$. For example, if the magnetic field has a value such that the orbit radii are given by $2R=n\lambda$, then the total momentum communicated to a particle around an orbit averages to zero. The response to the external force represented by $(V_{De}+V_{Dh})$ is therefore a minimum, and the attenuation should pass through a minimum. This qualitative result agrees with the calculated attenuation as presented in Fig. 1, the condition $2R=n\lambda$ being equivalent to $qR=n\pi$.

An analogous discussion can be given of the term in the attenuation containing the difference combination $(V_{De}-V_{Dh})$, which is the quantity giving rise to the current. The procedure of rendering the fields and currents self-consistent now implies that the force represented by $(V_{De}-V_{Dh})$ is very well screened. The effect of the residual long range electric field is to reduce the efficacy of $(V_{De}-V_{Dh})$ by the factor $\gamma=\omega\epsilon/8\pi\sigma^0$, as can be inferred from Eqs. (2.22), (3.15), and the definition of $\mathbf{B}$. The corresponding power dissipation and contribution to the attenuation are then proportional to γ^2 at all except the highest fields, and consequently are very small. At very high magnetic fields, where $(\omega_c\tau)^2\gtrsim 1/\gamma$, inspection of Eq. (3.23) indicates that the contribution to the attenuation arising from $(V_{De}-V_{Dh})$ becomes of degree zero in γ while decreasing with field like $1/(\omega_c\tau)^2$ from an unobservably small and broad maximum. The screening of $(V_{De}-V_{Dh})$ is in fact reduced in the presence of very high fields, but the increase in the corresponding part of the attenuation is insufficient to make any perceptible contribution.

One can obtain some insight into the physical significance of the great peak appearing in S_{11} by following a direct kinetic approach. When complete screening is realized $\boldsymbol{\mathcal{E}}=0$, and the only forces acting on a given particle species are the magnetic force and a deformation potential force $\mathbf{F}$. We multiply the Boltzmann equation by velocity, then integrate over velocity, and after some manipulation obtain an equation of motion for the average velocity $\mathbf{v}$ at a point when the cyclotron orbits are much smaller than a wavelength;

$$\frac{\partial \mathbf{v}}{\partial t}=-\frac{\mathbf{v}-\mathbf{u}}{\tau}-\omega_c\mathbf{v}\times\hat{z}+\frac{iq^2V_D}{m\omega}u\hat{x}-i\frac{v_F^2}{3}\frac{q^2}{\omega}v_x\hat{x}, \quad (4.2)$$

where u is the lattice velocity amplitude of a longitudinal disturbance of wave number q and frequency ω moving in the direction of the unit vector $\hat{x}$. The magnetic field is taken perpendicular to $\hat{x}$ and along the direction of the unit vector $\hat{z}$. In obtaining Eq. (4.2) we neglect for simplicity the distinction between the true electron mass and its effective mass in the crystal. Further, we do not yet necessarily require that $\omega=qv_s$, as would be the case for an applied sound wave of velocity v_s. The first term on the right hand side of Eq. (4.2) represents the frictional force experienced by the average velocity and the second term is of course the Lorentz force due to the magnetic field. The third term represents a deformation potential force, and the fourth term is a hydrodynamic contribution arising from the nonuniform carrier density. The latter term is in fact equal to $-\nabla p/mN$, where N is the local density and p the scalar pressure in a degenerate Fermi gas of particles with mass m whose density fluctuates harmonically in position and time with wave number q and frequency ω, respectively.

Let us now neglect collisions by sending $\tau\rightarrow\infty$, and take the deformation potential force to be zero, $V_D=0$. The resulting equation of motion possess a solution of the form $\mathbf{v}\sim e^{i(qx-\omega_r t)}$ provided

$$\omega_r=[(v_F^2/3)q^2+\omega_c^2]^{\frac{1}{2}}. \quad (4.3)$$

We interpret Eq. (4.3) as giving the natural resonant frequency of a sound wave of wave number q in the carrier gas in the presence of a magnetic field. For $H=0$ this collective oscillation becomes a pure longitudinal mode of wave number q, and corresponds to a wave of density fluctuation in the electron-hole gas. The resonant frequency and wave number are then related by

$$\omega_r=(v_F/\sqrt{3})q. \quad (4.3)$$

When H is finite, transverse motion of the electrons and holes occurs as well.

Such a sound wave may be excited by a longitudinal perturbation of the same wave number, q, whose frequency is some ω. Suppose we consider H to be fixed and imagine that ω can be arbitrarily varied. Then there will be a certain frequency dependent absorption of energy by the carriers. In the absence of collisions, the absorption is a δ function centered about ω_r. If we now suppose that collisions occur, the absorption versus ω curve will broaden, and the frequency of peak absorption may shift away from ω_r. Thus far we have considered only fixed field H and have discussed the shape of the absorption versus ω curve for fixed H. Let us now consider the dependence of the shape of these curves on H. The resonant frequency ω_r increases as H increases, and in general we anticipate a change of shape of the absorption versus ω curve. Let us now cease to suppose that the relation between q and ω is arbitrary and make the perturbation specifically a sound wave in the background medium with $\omega=qv_s$. Then for any H and ω in the megacycle range we shall be sitting on the low-frequency tail of the absorption versus frequency curve. The resonant frequency given by Eq. (4.3) is far higher than any applied ultrasonic frequency of the same wave number q.

Let H now increase and consider what the dependence of the absorption on field might be at the fixed frequency $\omega=qv_s$. One can discern two possible effects. First, as the magnetic field begins to increase the absorption versus frequency curve can either broaden or increase in over-all height or both without a major shift of the line center. This would account for the initial increase in the attenuation occurring as qR becomes less than unity. Second, as the field becomes indefinitely large, the position of peak absorption corresponding to the line center shifts to higher frequencies further away from the applied frequency $\omega=qv_s$. When this shift begins to dominate the effects of broadening and enhancement which increased H brings about, the attenuation will finally decrease with increasing field. Thus the field at which the absorption or attenuation is greatest for fixed $\omega=qv_s$ has nothing directly to do with the resonance frequency for the type of collective motion described by Eq. (4.2). Instead, it relates to the magnetic field dependence of the low-frequency tail of the general absorption versus frequency curve associated with the existence of the collective mode. Because the tail of a resonance is involved, we infer that relaxation effects will play an important role.

The above remarks may be compared with the results of direct calculation based on Eq. (4.2). In the presence of an applied sound wave, $\omega=qv_s$, we seek a solution of (4.2) of the form $\mathbf{v}\sim e^{-i\omega t}$. The velocity amplitude in the direction of propagation is then given by

$$v_x=\frac{(1+i\omega\tau V_D/mv_s^2)(1-i\omega\tau)u}{[1-i\omega\tau+(\omega_c\tau)^2+(v_F^2/3v_s^2-1)(i+\omega\tau)\omega\tau]}. \quad (4.5)$$

This result for v_x can be inserted into

$$Q=\tfrac{1}{2}\,\mathrm{Re}\{Nv_x^*F-Nm[(v_x-u)^*/\tau]u\}, \quad (4.6)$$

where $F=iq(V_Du/v_s)$ is the deformation potential force, and S_{11} can be calculated directly in the range of interest, $qR<1$. The result is

$$S_{11}=1+\frac{(ql)^2(v_F/v_s)^2(V_D/mv_F^2)^2(\omega_c\tau)^2}{[(\omega_c\tau)^2+\frac{1}{3}(ql)^2]^2+\frac{1}{9}(v_F/v_s)^2(ql)^2}, \quad (4.7)$$

where we have made approximations employing the inequalities $(v_F/v_s)\gg1$, $\omega\tau/ql=v_s/v_F\ll1$, and $(V_D/mv_F^2)>1$ which characterize the crystal, and also the inequalities $ql>1$, $\omega_c\tau\gg1$, and $qR<1$ which characterize the values of wavelength and magnetic field in the region of interest. Neglecting the $(ql)^2/3$ compared to $(\omega_c\tau)^2$ in the first term of the denominator of Eq. (4.7) we obtain Eq. (3.12) for the case $m=m^*$. Thus, from the above derivation of the attenuation coefficient in the range $qR<1$ we see that the high field peak represented in Fig. 1 is a manifestation of a collective motion in the carrier gas, and is associated with a wave of density fluctuation induced by the applied sound wave. Differentiation of Eq. (4.7) shows that its maximum occurs at a value of magnetic field given by

$$(\omega_c\tau)^2=\tfrac{1}{3}ql[(v_F/v_s)^2+(ql)^2]^{\frac{1}{2}}. \qquad (4.8)$$

From Eqs. (4.7) and (4.8) we see that the collision time or mean free path dominates both the position of the peak and its shape in a complicated way. Since, for practical purposes, $ql\ll(v_F/v_s)$, Eq. (4.8) may be replaced by

$$(\omega_c\tau)^2=(ql/3)(v_F/v_s), \qquad (4.9)$$

which is identical to Eq. (3.13). However, if the collision time were so long that $ql\gg(v_F/v_s)$, the effects of relaxation would be negligible and Eq. (4.8) would imply a peak at

$$\omega_c\tau=ql/\sqrt{3}, \quad \text{or} \quad \omega_c=qv_F/\sqrt{3}. \qquad (4.10)$$

We have seen above in Eq. (4.4) that $(1/\sqrt{3})qv_F$ is precisely the natural resonant frequency, ω_r, of the carrier gas for a density fluctuation of wave number q in the absence of collisions, magnetic field, and applied sound wave. Although the case of such a long mean free path is unattainable in practice, it is of some physical interest to note the connection between the value of ω_c at peak absorption and the value of ω_r at zero field when collisions are negligibly rare.

The general level of ultrasonic attenuation in semimetals is less than that in metals and the geometric oscillations are less marked, for given ql. However, although their amplitude is relatively lower than in metals, the detection of the oscillations is facilitated by using a field modulation technique, such as was employed by Reneker. This would be difficult in high purity metals, for, because of their high conductivity, one cannot get a suitably modulated magnetic field to penetrate them at low temperatures except at very low modulation frequencies.

The behavior of S_{11} as presented in Fig. 1 bears considerable resemblance to the measured attenuation of 60 megacycle longitudinal sound waves in bismuth.[1] The measurements show a series of relatively weak oscillations (geometric resonances) in the attenuation as the field is increased, followed by a large rise whose magnitude is an order of magnitude greater than the amplitude of the low field oscillations. The data indicate, though perhaps not conclusively, a subsequent decrease in the attenuation. For the one orientation at which the onset of the rise took place at a significantly lower field, there is definite evidence of a subsequent decline in the attenuation as the field is further increased. Extended measurements at higher fields are clearly desirable for a variety of orientations. Experimentally, the large increase begins in bismuth at about $H\sim100$ gauss. For $\tau=10^{-9}$, $ql=10$, and $(m/m^*)=10$, which were the values used in constructing Fig. 1, the large maximum in S_{11} occurs at a value of qR corresponding to $H\sim125$ gauss. The values of relaxation time and frequency employed are intended to typify the values attained in the experimental work. Since the effective mass ratio $(m/m^*)=10$ is roughly comparable to an average mass ratio for electrons and holes in bismuth, we tentatively identify the large increase in the attenuation observed in bismuth with the high field peak in S_{11} here derived.

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