Microwave Breakdown of Air in Nonuniform Electric Fields

PHILIP M. PLATZMAN† AND ELSA HUBER SOLT
Hughes Research Laboratories, Malibu, California

(Received April 11, 1960)

A new approach to the problem of diffusion-controlled breakdown in nonuniform fields is presented. Two specific cases are considered in detail. The geometrical configurations involved are, respectively, a rectangular microwave cavity with a small hemispherical boss on one of its walls and a similar cavity without a boss. The theoretical predictions of the method are compared with a series of microwave breakdown measurements in air. The theoretical results are found to be in good agreement with the observed experimental data.

INTRODUCTION

THE electrical breakdown of gases at microwave frequencies has been discussed in considerable detail by a number of authors.[1–7] The so-called diffusion theory has been quite successful in describing breakdown so long as the electron mean free path and amplitude of oscillation remain small in comparison with dimensions of the containing vessel. If the electric field can be considered uniform in space, the electron density at breakdown satisfies the well-known scalar Helmholtz equation. The lowest eigenvalue as determined by the boundary conditions on the density is related to the high-frequency ionization coefficient. If the latter is known as a function of electric field, it is then possible to calculate the required breakdown field. However, if the electric field and hence also the ionization vary with position, the differential equation for the density in general is nonseparable and the problem becomes considerably more difficult. Only a few simple cases have been considered,[8,9] in which the treatment consisted of obtaining an approximate solution for the density.

In this paper another approach to the problem of breakdown in nonuniform fields is used. The attempt to solve the boundary-value problem itself is avoided altogether, and a variational principle is set up for the eigenvalue alone. This eigenvalue, together with the high-frequency ionization coefficient obtained from dc data and an appropriately defined equivalent high-frequency field, then determines the breakdown field. The method is applied to a rectangular microwave cavity with a small hemispherical boss on one wall. The strongly divergent electric field in the immediate vicinity of the boss and the more gradual variations across the entire cavity are taken into account. The theoretical results are in good agreement with a series of experimental data for the breakdown of air in several such cavities.

The prediction of possible large deviations from the uniform-field case caused by small nonuniformities on the waveguide surface should prove of interest in high-power microwave work insofar as maximum power-carrying capabilities can be greatly reduced. In addition, the theory is of interest in that it extends the diffusion theory in a relatively straightforward manner to a wider class of problem, heretofore unsolvable.

DIFFUSION-CONTROLLED BREAKDOWN

According to the theory of diffusion-controlled breakdown, the electron density ρ at the breakdown threshold satisfies the equation[1]:

$$\nabla^2\rho+\zeta E^2\rho=0. \tag{1}$$

The high-frequency ionization coefficient ζ is given by

$$\zeta=\nu/DE^2,$$

where D is the electron diffusion coefficient, E the electric field, and ν the net production rate per electron, that is, $\nu=\nu_i-\nu_a$, where ν_i=ionization frequency per electron, and ν_a=attachment frequency per electron.

If E and ν/D are constant in space, the solution of (1) subject to the boundary condition that ρ vanish at the walls of the container yields

$$\zeta E^2=k^2. \tag{2}$$

The eigenvalue k has the dimensions of reciprocal length; the exact form of the diffusion length $\Lambda=1/k$ is determined by the particular geometry in question. If in addition ζ is known as a function of E, the electric field required for breakdown is specified by (2). More generally, if ζ can be expressed as a function of E/p only, p being the gas pressure, then (2) defines a unique relation between E/p and $p\Lambda$ analogous to the Paschen law for dc breakdown.

If E is not constant in space, it can be expressed as

$$E=E'F(x,y,z),$$

where E' is the field at some specified point in the region of interest. The high-frequency ionization coefficient ζ

† Present address Bell Telephone Laboratories, Murray Hill, New Jersey.
1 M. A. Herlin and S. C. Brown, Phys. Rev. **74**, 291 (1948).
2 A. D. MacDonald and S. C. Brown, Phys. Rev. **75**, 411 (1949); **76**, 1629 (1949); **76**, 1634 (1949).
3 W. P. Allis and S. C. Brown, Phys. Rev. **87**, 419 (1952).
4 A. D. MacDonald and D. D. Betts, Can. J. Phys. **30**, 565 (1952); **32**, 812 (1954).
5 F. H. Reder and S. C. Brown, Phys. Rev. **95**, 885 (1954).
6 L. Gould and L. W. Roberts, J. Appl. Phys. **27**, 1162 (1956).
7 D. J. Rose and S. C. Brown, J. Appl. Phys. **28**, 561 (1957).
8 M. A. Herlin and S. C. Brown, Phys. Rev. **74**, 910 (1948); **74**, 1650 (1948).
9 A. D. MacDonald and S. C. Brown, Can. J. Research **A28**, 168 (1950).

is assumed to be a known function of the field, and we call ζ' its value at the point where $E=E'$. We may therefore write

$$\zeta E^2=\zeta' E'^2\left(\frac{\zeta}{\zeta'}\right)\left(\frac{E}{E'}\right)^2=k'^2\left(\frac{\zeta}{\zeta'}\right)\left(\frac{E}{E'}\right)^2, \quad (3)$$

where

$$k'^2=\zeta' E'^2. \quad (4)$$

Substitution of (3) in (1) yields

$$\nabla^2\rho+k'^2\left(\frac{\zeta}{\zeta'}\right)\left(\frac{E}{E'}\right)^2\rho=0. \quad (5)$$

The problem thus reduces to finding the eigenvalue k', which through (4) determines the field E' at breakdown.[10]

The validity of the foregoing as it applies to breakdown in nonuniform fields rests on the assumption that at a given point the ionization, and therefore the electron energy, are defined by the field at that point. For this to be true, it is necessary that both (a) the distance through which an electron diffuses during a time equal to the energy relaxation time and (b) the electron oscillation amplitude must be small compared with the distance over which the field changes appreciably. Estimates of the relaxation distance and oscillation amplitude can be made and indicate that both requirements are well satisfied for the experimental conditions to be considered.

THE VARIATIONAL METHOD[11]

If either ρ or its normal derivative vanish on the boundary, then with the use of Green's theorem (5) may be written

$$k'^2=\int(\nabla\rho)^2dv\bigg/\int\rho^2\left(\frac{\zeta}{\zeta'}\right)\left(\frac{E}{E'}\right)^2dv. \quad (6)$$

In the standard manner it may be shown that $\delta(k'^2)=0$ for ρ satisfying (5) and either of the boundary conditions specified above. Furthermore, if ζ/ζ' is nowhere negative, k'^2 will have an absolute minimum corresponding to the lowest eigenvalue. Larger values of k' for which $\delta(k'^2)=0$ represent the higher eigenvalues.

The variational method then involves the following procedure. A trial function is chosen for ρ having a general form suggested by the physical situation and containing one or more arbitrary parameters. k'^2 as given by (6) is minimized with respect to the parameters, and the value of k' corresponding to the minimum then represents an upper bound on the lowest eigenvalue.

[10] It is to be pointed out that Eq. (1) is not strictly valid for nonuniform fields. For this case, since the diffusion coefficient D is a function of electron energy and hence of position, Eq. (1) is more correctly written as

$$\nabla^2\rho+(\nabla D)\cdot(\nabla\rho)/D+\zeta E^2\rho=0.$$

However, D is a slowly varying function of energy, and its variation with position will be neglected in the subsequent analysis except insofar as it is included in the variation of ζ. The effect of thermal diffusion is also assumed to be negligible.

[11] P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill Book Company, Inc., New York, 1953), p. 1106.

The accuracy of the method of course depends on how well the trial function approximates the actual solution. However, it is generally the case that even a very crude function usually results in a fairly good estimate of the eigenvalue.

HEMISPHERICAL BOSS IN RECTANGULAR WAVEGUIDE CAVITY

The geometry for which the breakdown problem is to be solved is shown in Fig. 1, and consists of a resonant microwave cavity of sides L, b, and c with a hemispherical boss of radius a centered on one wall. The cavity is assumed to be excited in the TE_{011} mode such that in the absence of the hemisphere the field is given by

$$E=E_1\sin(\pi y/b)\sin(\pi z/c), \quad (7)$$

where E_1 is the maximum field at the center of the cavity. The hemisphere is centered about the point $x=0$, $y=b/2$, $z=c/2$.

The experimental results to be presented in a later section suggested that over one portion of the range of experimental parameters the boss had no effect on the breakdown threshold for the cavity, while in the remaining portion its effect was all-important. We therefore shall consider two limiting cases separately. In the first case, we solve the breakdown problem for the cavity alone, without taking into account any distortion of the field by the boss. In the second case, we obtain a solution for a hemispherical boss on one of a pair of infinite parallel plates and thus neglect the field variation caused by the side walls of the cavity. In principle, both variations could be taken into account simultaneously; in practice, however, the numerical complexity of the analysis would then be greatly increased. It will be seen that for the situation to be considered here, the transition between the two limiting cases is extremely sharp. Therefore, no real "intermediate region" exists, and the more general treatment will lead to no new understanding of the problem.

Case 1. Breakdown in the Cavity Alone

With the electric field in the cavity as specified by (7), we choose as the trial function for the electron

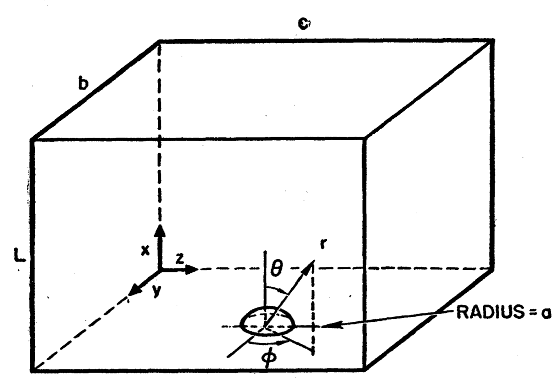

FIG. 1. Microwave cavity with hemispherical boss.

density

$$\rho \approx f = A \sin k_0 x \sin^m(\pi y/b) \sin^m(\pi z/c), \tag{8}$$

where $k_0 = \pi/L$ is the eigenvalue for a uniform field between infinite parallel plates separated by L, and A is a constant which will cancel on taking the ratio given by (6). The properties of this trial function are that it vanishes on the walls of the cavity; the variation in the x direction is the same as in the uniform-field case; in the y and z directions it is peaked more sharply at the center of the cavity than if the field were uniform, because the field, and to a greater extent the ionization, are peaked at the center. The parameter m determines how strongly the density is peaked. For this case we take $E' = E_1$ so that the eigenvalue k_1 we determine will be defined in terms of the ionization and electric field at the center of the cavity.

Substitution of (7) and (8) into the integrals of (6) then yields

$$\int (\nabla f)^2 dv = A^2\left(\frac{Lbc}{2}\right) k_0^2 G_1(m),$$

$$\int f^2\left(\frac{\zeta}{\zeta'}\right)\left(\frac{E}{E'}\right)^2 dv = A^2\left(\frac{Lbc}{2\pi^2}\right) S_1(m),$$

$$\frac{k_1^2}{k_0^2} = \frac{\pi^2 G_1(m)}{S_1(m)}, \tag{9}$$

where

$$G_1(m) = \left[1 + \frac{m^2}{2m-1}\frac{L^2}{\lambda^2}\right]\left[\frac{1\cdot 3\cdot 5\cdots(2m-1)}{2\cdot 4\cdot 6\cdots(2m)}\right]^2, \quad m \text{ an integer},$$

$$\frac{1}{\lambda^2} = \frac{1}{b^2} + \frac{1}{c^2},$$

$$S_1(m) = \int_0^\pi \int_0^\pi \left(\frac{\zeta}{\zeta_1}\right) \sin^{2m+2} y' \sin^{2m+2} z' dy' dz'.$$

Case 2. Breakdown Around the Hemisphere

We now consider the case in which breakdown is governed by the nonuniform field in the vicinity of a hemispherical boss on one of a pair of infinite parallel plates. We assume that the radius of the hemisphere a is much less than the plate separation L, so that the other plate can be assumed to be at infinity. The field distribution may be obtained from the well-known solution of the potential problem for a conducting sphere in a uniform field. The result is

$$E = E_1[(1 + 2a^3/r^3)^2 \cos^2\theta + (1 - a^3/r^3)^2 \sin^2\theta]^{\frac{1}{2}}, \tag{10}$$

where E_1 is the uniform field at large r.

The trial function is now taken to be

$$\rho \approx g = B(r-a)e^{-nr/a} \cos\theta, \tag{11}$$

where n is the parameter to be varied and B is a constant. The function g vanishes at the boundaries and is peaked in the neighborhood of the boss, its maximum occurring at $r_{\max} = a + a/n$. For this case we take $E' = E_2 = 3E_1$ so that the eigenvalue k_2 will be defined in terms of the maximum field on the surface of the hemisphere.

The results of substituting (10) and (11) into (6) are

$$\int (\nabla g)^2 dv = B^2\left(\frac{\pi a^3}{6}\right) e^{-2n} G_2(n),$$

$$\int g^2\left(\frac{\zeta}{\zeta_2}\right)\left(\frac{E}{E_2}\right)^2 dv = B^2\left(\frac{2\pi a^5}{9}\right) e^{-2n} S_2(n),$$

$$k_2^2 a^2 = 3G_2(n)/4S_2(n), \tag{12}$$

where

$$G_2(n) = \frac{1}{n} + \frac{1}{n^2} + \frac{3}{n^3},$$

$$S_2(n) = \int_0^1 \int_1^\infty \left(\frac{\zeta}{\zeta_2}\right)(u-1)^2 e^{-2n(u-1)} u^2 v^2 \times\left[\left(\frac{6}{u^3} + \frac{3}{u^6}\right) v^2 + \left(1 - \frac{2}{u^3} + \frac{1}{u^6}\right)\right] du dv.$$

HIGH-FREQUENCY IONIZATION COEFFICIENT AND EQUIVALENT ELECTRIC FIELD

Before the integrals in (9) and (12) can be evaluated, the variation of the high-frequency ionization coefficient ζ with electric field must be determined. The manner in which this can be done by using dc data has essentially been outlined in references 6 and 7, and the result is

$$\zeta = \frac{3}{2} \frac{\alpha/p - \beta/p}{u_{\mathrm{av}}(E_{\mathrm{dc}}/p)}, \tag{13}$$

where p = gas pressure, α = number of ionizing collisions/cm per electron, β = number of attaching collisions/cm per electron, u_{av} = electron average energy, and E_{dc} = dc electric field. Experimental data for α/p, β/p, and u_{av} as functions of E_{dc}/p are available for a number of gases, and therefore ζ as a function of E_{dc}/p can be determined.

The results are applicable to the high-frequency case if in place of E_{dc} an equivalent high-frequency field is used that would result in the same net ionization as a dc field of the same magnitude. So long as the relaxation time for the electron energy is appreciably longer than the rf period, the equivalent field is the same as the so-called effective field[1,12] defined as

$$E_e^2 = \frac{E_{\mathrm{rms}}^2}{1 + (\omega/\nu_c)^2}, \tag{14}$$

where E_e = effective field, E_{rms} = rms value of the high-

[12] W. P. Allis, in *Handbuch der Physik*, edited by S. Flügge (Springer-Verlag, Berlin, 1956), Vol. 21, p. 383.

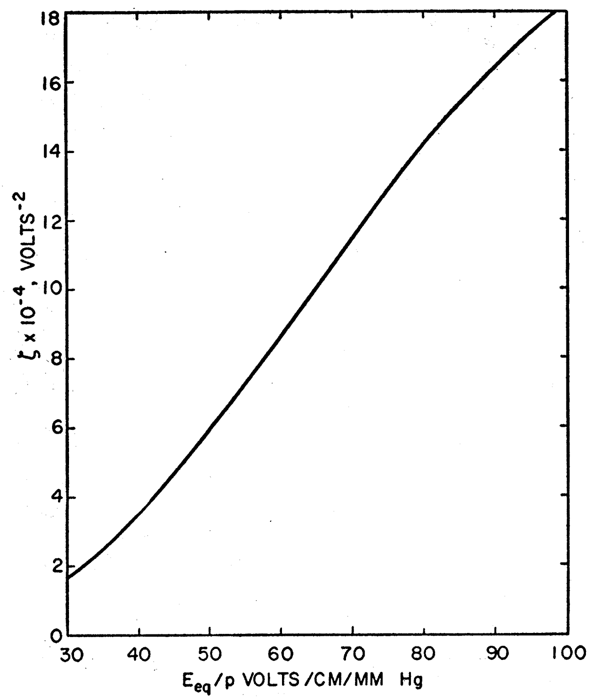

FIG. 2. High-frequency ionization coefficient ζ as a function of E_{eq}/p for air, as computed from dc data. E_{eq} is the equivalent high-frequency field resulting in the same ionization as a dc field of the same magnitude.

frequency field, ω= angular frequency of the high-frequency field, and ν_c= electron-molecule collision frequency. At high pressures, however, the energy relaxation time can become comparable with or shorter than the rf period, and the effect of electron energy modulation[6,12] must also be taken into account, with the result that the equivalent field is slightly higher than the effective field as defined above. That is,

$$E_{eq}/p = E_e/p + \Delta, \tag{15}$$

where E_{eq} is the equivalent field and Δ represents the energy modulation effect.

In terms of this equivalent field, and with ζ' expressed as a function of E_{eq}'/p, the breakdown criterion (4) may be written as

$$\frac{k'^2}{p^2} = \frac{1}{(p\Lambda')^2} = \zeta'\left(\frac{E_{eq}'}{p}\right)^2. \tag{16}$$

This relationship between $p\Lambda'$ and E_{eq}'/p thus represents a universal breakdown curve for a given gas, so long as Λ' and E_{eq}' are appropriate to the geometry in question.

Results of the theoretical analysis are compared with experimental data for air, and Fig. 2 shows ζ versus E_{eq}/p for this gas as computed from Eq. (13). The data of Harrison and Geballe[13] were used for $\alpha/p - \beta/p$, and those of Crompton, Huxley, and Sutton,[14] and of Healey and Reed[15] were used for u_{av}. Below E_{dc}/p of about 31.5 volt/cm/mm Hg attachment exceeds ionization and ζ actually becomes negative; however, since reliable measurements of $\alpha/p - \beta/p$ are not available much below this value, ζ is taken as zero for E_{dc}/p less than 31.5. For any reasonable assumptions regarding the behavior of ζ in the region of negative net ionization, it can be shown that our final results are not greatly affected by the foregoing approximation.

Measured breakdown fields were in all cases reduced to equivalent fields as defined by (15). The analysis and results of Gould and Roberts[6] were used to determine the effect of electron energy modulation, and a collision frequency ν_c of $6\times10^9 p$, with p in mm Hg, was used in computing the effective field. This value for ν_c/p is slightly higher than values used previously.[6,7] However, the choice is to a certain extent arbitrary in that ν_c is not measured directly but must be estimated in an indirect fashion, and that furthermore ν_c is not constant as implied by (13) but varies somewhat with E/p. The value used here means that the average ν_c is being weighted slightly in favor of higher E/p (that is, lower pressures), where the effective field differs most from the actual field. Perhaps more important, it can also be demonstrated[16] that the present value fits experimental breakdown data over a wider range of variables than the lower values used heretofore.

EXPERIMENTAL METHOD

Three microwave cavities of the configuration shown in Fig. 1 were used in the experiment, all having $L = 0.400$ inch, $b = 0.900$ inch, and $c = 0.981$ inch, with resonant frequencies of about 8850 Mc/sec in the

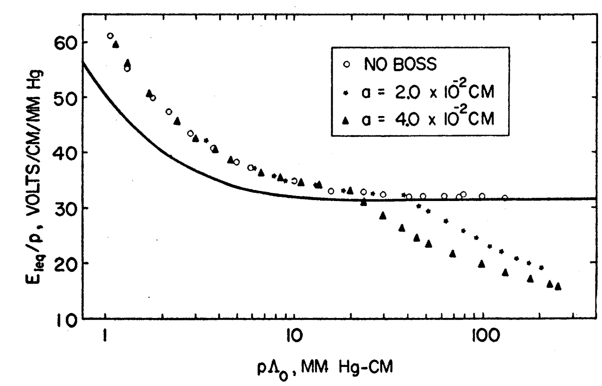

FIG. 3. Results of breakdown measurements in air at 8850 Mc/sec in the three experimental cavities. $E_{1\,eq}$ is the equivalent field at the center of the cavity. The theoretical curve (solid line) would apply if breakdown were occurring in a uniform field of magnitude $E_{1\,eq}$ between infinite parallel plates of separation $L = \pi\Lambda_0$.

[13] M. A. Harrison and R. Geballe, Phys. Rev. **91**, 1 (1953).

[14] R. W. Crompton, L. G. H. Huxley, and D. J. Sutton, Proc. Roy. Soc. (London) **A218**, 507 (1953).

[15] R. H. Healey and J. W. Reed, *The Behavior of Slow Electrons in Gases* (Amalgamated Wireless of Australasia, Ltd., Sydney, 1941).

[16] E. H. Solt (to be published).

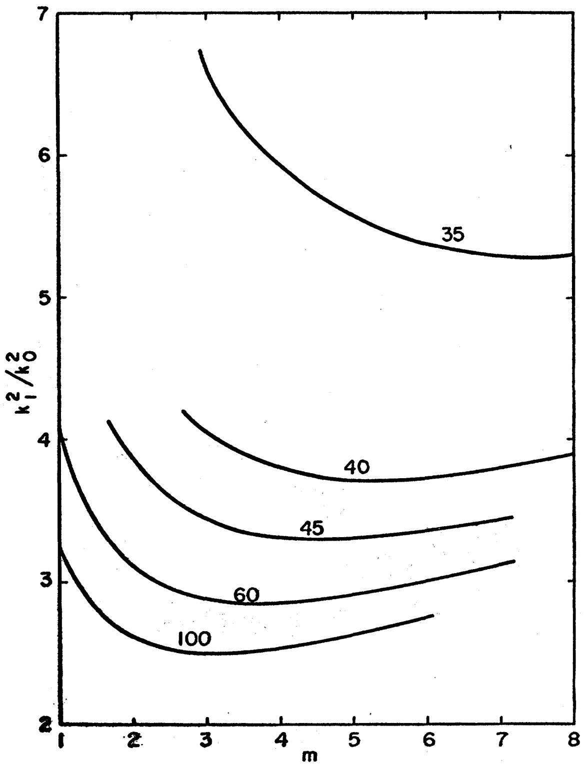

FIG. 4. k_1^2/k_0^2 as a function of the parameter m and $E_{1\,\mathrm{eq}}/p$ as computed from Eq. (9) for a rectangular cavity with $L^2/\lambda^2=0.364$. k_1 is the eigenvalue for breakdown in the nonuniform field of the cavity, the effect of the hemispherical boss having been neglected.

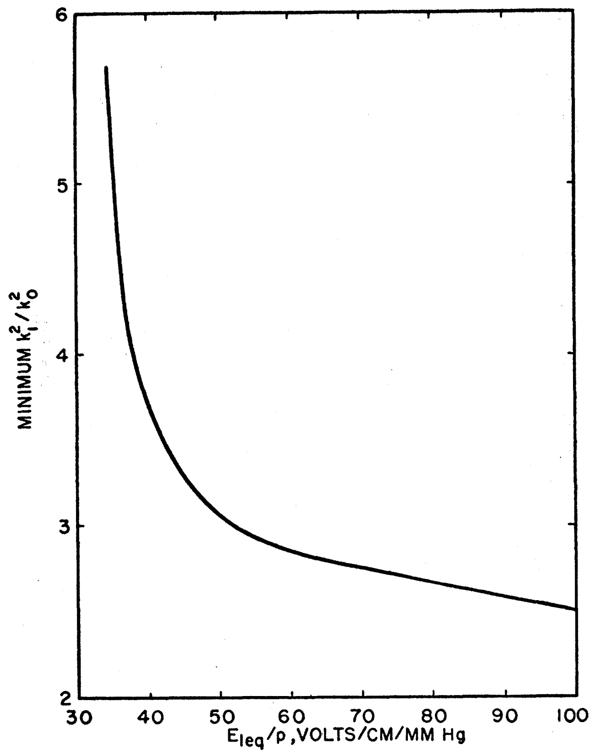

FIG. 5. Minimum k_1^2/k_0^2 as a function of $E_{1\,\mathrm{eq}}/p$ as obtained from Fig. 4.

TE_{011} mode. One of the cavities had no boss, while for the other two the radius of the boss was 1/128 inch and 1/64 inch, respectively. The cavities were of copper, and were constructed by electroforming over a mandrel on which a hemispherical indentation could be accurately made and centered. Connection to a glass vacuum system was made at one end of the cavity through a Kovar-to-glass seal. The other end was connected to the microwave line via a coupling hole and a vacuum window attached with an O-ring seal.

The cavities were isolated from the vacuum pumps and mercury manometers by means of a dry ice-acetone trap, which served also to remove water vapor. Data were obtained for air at pressures ranging from a few mm Hg to one atmosphere. Measurements were performed with both room air and tank air with no difference in results. A fresh gas sample was admitted for each breakdown measurement, the system being evacuated and flushed before each filling to avoid contamination by products of previous breakdowns.[7]

The microwave circuit and the procedure used for measuring the breakdown field as a function of gas pressure were similar to those described in references 1–7, except that the tunable cw magnetron furnishing the microwave power operated in the X-band region instead of about 3000 Mc/sec as in the earlier experiments. The waveguide line from magnetron to cavity incorporated a power divider for varying the power incident on the cavity and a directional coupler leading to a thermistor and Hewlett-Packard Model 430C power meter for measuring a fraction of the incident power. The power-measuring section was calibrated in an absolute fashion with an Elliott Brothers torque vane wattmeter. Power reflected from the cavity was monitored by means of a second directional coupler and a crystal detector. The magnetron was kept tuned to the cavity resonant frequency by keeping the reflected power at a minimum; a sudden large increase in reflected power as the incident power was gradually increased indicated that breakdown of the gas in the cavity had occurred. A γ source consisting of 2 millicuries of cobalt-60 was used to insure an adequate supply of initial ionization when the breakdown threshold was reached. The measured breakdown power can be related to the electric field at breakdown through the cavity Q and the known field configuration.[17] The unloaded Q was determined from standing-wave measurements by standard techniques.[18] Possible calibration errors result in an estimated uncertainty of 5% in the absolute values of the measured breakdown fields, although reproducibility of the data was considerably better than this.

[17] D. J. Rose and S. C. Brown, J. Appl. Phys. **23**, 719 (1952).
[18] S. C. Brown and D. J. Rose, J. Appl. Phys. **23**, 711 (1952).

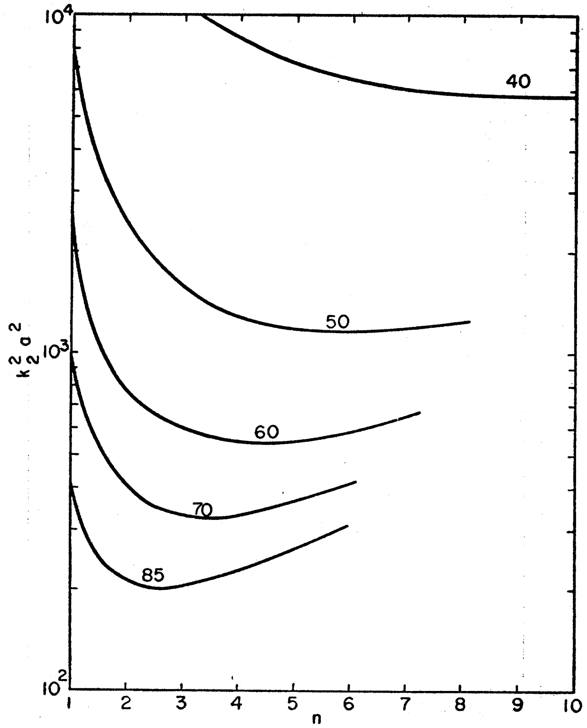

FIG. 6. $k_2^2a^2$ as a function of the parameter n and $E_{2\,\mathrm{eq}}/p$ as computed from Eq. (12) for a hemispherical boss of radius a. $E_{2\,\mathrm{eq}}$ is the maximum equivalent field at the surface of the hemisphere, and k_2 is the eigenvalue for breakdown in the nonuniform field near the boss, nonuniformities due to the side walls of the cavity having been neglected.

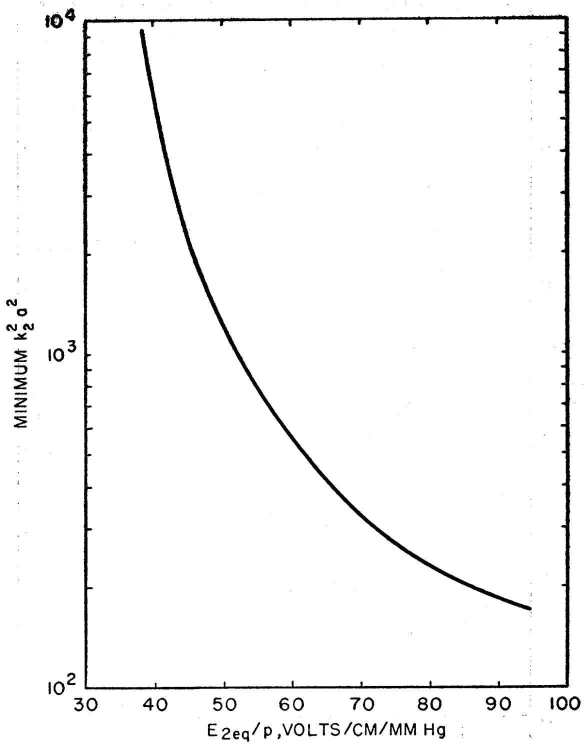

FIG. 7. Minimum $k_2^2a^2$ as a function of $E_{2\,\mathrm{eq}}/p$ as obtained from Fig. 6.

DISCUSSION OF RESULTS

The experimental results for the three cavities are shown in Fig. 3, and in this figure are compared with a theoretical curve in such a way as to point up the deviations from uniform-field breakdown. $E_{1\,\mathrm{eq}}$ is the rms electric field at the center of the cavity at breakdown, reduced to an equivalent field as described in an earlier section. The data have been plotted in terms of $\Lambda_0=1/k_0=L/\pi$. The theoretical curve was obtained from the universal expression (16) and the ζ versus E_{eq}/p plot of Fig. 2, with $\Lambda'=\Lambda_0$ and $E_{\mathrm{eq}}'=E_{1\,\mathrm{eq}}$. This curve would be applicable if breakdown were occurring in a uniform field of magnitude $E_{1\,\mathrm{eq}}$ between infinite parallel plates of separation L, and deviations of the data from the theoretical curve in this form give a measure of the effect of the nonuniform fields in the cavity. It is to be noted that below a certain pressure all three cavities give essentially identical results, so apparently in this region the boss has no effect on breakdown and deviations from the theoretical curve arise from the nonuniform fields of the cavity alone. At higher pressures, however, the effects of the boss are readily apparent and result in a very marked lowering of the breakdown threshold of the cavity from the uniform field case.

To determine the effective diffusion lengths applicable to the two cases of interest, Eqs. (9) and (12) were evaluated as functions of the parameters and of E/p. Integration of $S_1(m)$ and $S_2(n)$ was performed numerically on a high-speed digital computer, with ζ/ζ_1 or ζ/ζ_2 obtained from the curve of Fig. 2. For case 1, Fig. 4 shows how k_1^2/k_0^2 varies with m at different $E_{1\,\mathrm{eq}}/p$ for the cavities of the experiment. As one might expect, the value of m required to minimize k_1^2/k_0^2 decreases with increasing $E_{1\,\mathrm{eq}}/p$, that is, the density distribution is peaked more sharply at the center of the cavity at the lower values of $E_{1\,\mathrm{eq}}/p$. The minimum k_1^2/k_0^2 is shown as a function of $E_{1\,\mathrm{eq}}/p$ in Fig. 5. Similar curves for case 2 are shown in Figs. 6 and 7. It is to be noted that in both cases, unlike the situation for uniform fields, the eigenvalues and therefore the effective diffusion lengths vary with E/p. The variation is particularly strong for k_2, presumably because a higher degree of nonuniformity in the field occurs near the hemisphere than in other parts of the cavity.

In Fig. 8, theory and experiment are compared for the limit in which the boss has no effect on the breakdown. All the data for the cavity without a boss are shown, but for the other two cavities, only the points corresponding to pressures below the transition have been included. In this case $\Lambda_1=1/k_1$ is the effective diffusion length to be used. For each experimental point the appropriate Λ_1 from Fig. 5 was computed, and the theoretical curve obtained from (16) now relates $p\Lambda_1$ and $E_{1\,\mathrm{eq}}/p$, where $E_{1\,\mathrm{eq}}$ refers to the equivalent field at the center of the cavity. The agreement appears to be excellent, in fact better than could have been expected in view of the various approximations that have been made.

Comparison between theory and experiment for the limit in which breakdown occurs around the hemisphere is made in Fig. 9. Data at pressures above the transition are shown for the two cavities containing a boss. $\Lambda_2=1/k_2$ is now the appropriate effective diffusion length, and was computed for each experimental point from the curve of Fig. 7. The theoretical curve still has the universal form (16), but now relates $p\Lambda_2$ and $E_{2\,\mathrm{eq}}/p$, where $E_{2\,\mathrm{eq}}$ refers to the maximum field at the surface of the boss. The qualitative agreement is quite satisfactory, particularly in view of the rapid variation of k_2 with $E_{2\,\mathrm{eq}}/p$ indicated by Fig. 7. Quantitatively, the agreement is not as good as in the preceding case, but nevertheless allows prediction of breakdown fields for a nonuniformity of this nature to better than 20%.

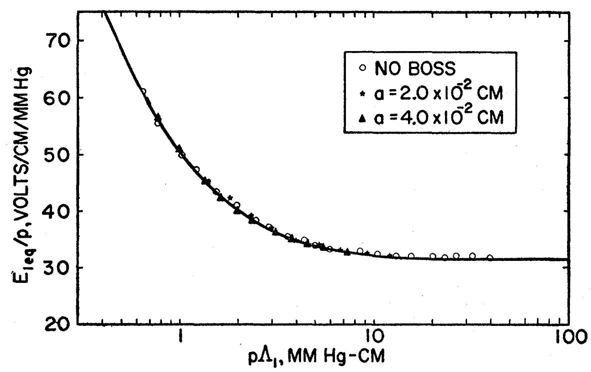

FIG. 8. Comparison of theory (solid curve) and experimental points for the region in which the boss has no effect on the cavity breakdown threshold. $\Lambda_1=1/k_1$ for each experimental point was obtained from Fig. 5.

The fact that the theoretical curve falls somewhat above the experimental one is consistent with the fact that the variational method only gives an upper bound to the eigenvalue.

The transition from one limiting case to the other, in terms of the curves of Fig. 3, can be understood qualitatively in the following manner. At high values of $E_{1\,\mathrm{eq}}/p$ (such that E_{eq}/p is greater than 31.5 volts/cm/mm Hg throughout most of the cavity) the effect of the boss is negligible because the intensified field in the vicinity of the boss extends over only a small fraction of the total volume and this region contributes only a small fraction of the total ionization. As the value of $E_{1\,\mathrm{eq}}/p$ decreases, the ionization around the boss becomes increasingly more important until finally, at the point that $E_{1\,\mathrm{eq}}/p$ reaches the value 31.5 volt/cm/mm Hg, no electrons at all are being produced except in the intensified field near the hemisphere. Clearly then the

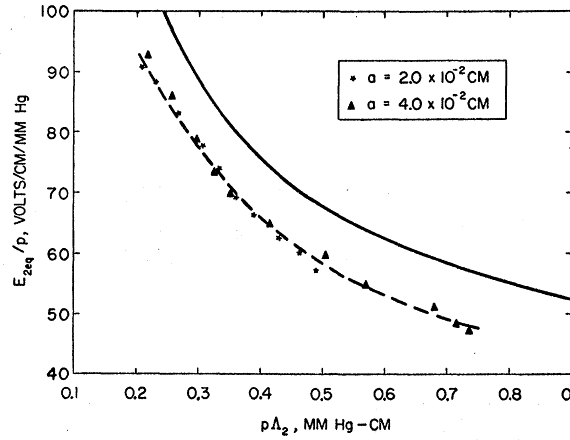

FIG. 9. Comparison of theoretical curve (solid) and experiment (dashed) for breakdown around the hemisphere. $\Lambda_2=1/k_2^2$ for each experimental point was obtained from Fig. 7.

breakdown must occur near the boss. In the intermediate region both parts of the volume contribute to the net ionization. However, for the present experimental conditions, the transition occurs in a region where the slope of the $E_{1\,\mathrm{eq}}/p$ versus $p\Lambda_1$ curve is small. The flatness of the curve measures in effect the relative importance of the ionization in the region of the boss to that in the remainder of the cavity. If the curve is quite flat, it takes only a slight change in $E_{1\,\mathrm{eq}}/p$ to cause a very large change in the ratio, and as a result the break is sharp. For the case of a nonattaching gas, however, we would not expect such a sharp transition since the net ionization frequency in the cavity remains finite down to very low values of $E_{1\,\mathrm{eq}}/p$. Thus the relative importance of the two regions of ionization is never as great as in the case of an attaching gas. Because of the small fraction of the total volume represented by the nonuniformity around the hemisphere, one would therefore expect, for the case of a nonattaching gas, that the effect of the boss in reducing the overall breakdown threshold of the cavity would be very small. We have taken a limited number of measurements in nitrogen and hydrogen to determine whether this is indeed true. The measurements indicate that the effect. if present at all, was so small as to be indistinguishable from experimental scatter or the effects of slight differences in gas purity from one run to the next.

ACKNOWLEDGMENTS

The authors are indebted to A. Kleider for his assistance in construction of the cavities and many of the experimental measurements, and to L. G. Komai for helpful discussions regarding the numerical computations.