Final-State Interactions and $|\Delta \mathbf{I}| = \frac{1}{2}$ Selection Rule*

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We consider the effect of various isotopic spin selection rules and final-state interactions between two outgoing pions upon the K_{e4}/K_{e3} branching ratio.

Many features of strange-particle decays are consistent with the so-called $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. However, the real origin of this selection rule has not yet been known. One of the striking facts that suggests the approximate validity of the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule is the branching ratio

$$\frac{W(K^+ \rightarrow \pi^+ + \pi^0)}{W(K_1{}^0 \rightarrow 2\pi)} \approx \frac{1}{400}. \tag{1}$$

At the moment, however, there seems to be three different standpoints to explain this branching ratio. They are[1]:

(I) No $|\Delta \mathbf{I}| = \frac{1}{2}$ selection rule but a strong attractive final-state pion-pion interaction in $I=0$, $J=0$ state. For a weak-interaction Hamiltonian which does not satisfy the $|\Delta \mathbf{I}| = \frac{1}{2}$ hypothesis, Okubo and Marshak[2] have indicated the possibility of a strong final-state pion-pion interaction in the $I=0$, $J=0$ state by which the rate of $K_1{}^0 \rightarrow 2\pi$ decay is enhanced (by two orders of magnitude) as compared to that of $K^+ \rightarrow \pi^+ + \pi^0$ decay.[3]

(II) Strict $|\Delta \mathbf{I}| = \frac{1}{2}$ rule and a strong attractive pion-pion interaction in the $I=2$, $J=0$ state.[4] If we assume a strict $|\Delta \mathbf{I}| = \frac{1}{2}$ rule for the primary weak interactions, the only way of making the decay mode $K^+ \rightarrow \pi^+ + \pi^0$ allowed is by means of the electromagnetic corrections. It is, however, usually believed that the electromagnetic effects are too small (by about two orders of magnitude). However, if we assume a strong attraction only in the $I=2$, $J=0$ state [contrary to the standpoint (I)] between the two final pions, the decay rate of $K^+ \rightarrow \pi^+ + \pi^0$ mode may be enhanced to agree again with (I).

(III) Usual $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. This is the assumption that the structure of primary weak interactions is such that an approximate $|\Delta \mathbf{I}| = \frac{1}{2}$ rule holds (the $|\Delta \mathbf{I}| = \frac{3}{2}$ part which is not of electromagnetic origin may be about 5–10%) and that the final-state interaction does not play an essential role as in the case (I) or (II).

The first point we wish to make in this paper is to show that the standpoint (I) leads to some difficulties for the decay modes K_{e4} and $K_{\mu 4}$. Let us consider, for example, the mode

$$K^+ \rightarrow \pi^{+,0} + \pi^{-,0} + e^+ + \nu.$$

As has been shown earlier,[5] a reasonable estimate leads to the following branching ratio

$$\frac{W(K^+ \rightarrow \pi^+ + \pi^- + e^+ + \nu)}{W(K^+ \rightarrow \pi^0 + e^+ + \nu)} \approx 10^{-3}. \tag{2}$$

In the K_{e4} modes the system of two pions should contain an appreciable amount of $I=0$ component.[6] Therefore, if there exists a strong attraction in the $I=0$, $J=0$ state between outgoing pions as assumed in (I), the ratio (2) will be enhanced by two orders of magnitude leading to a value $\sim 10^{-1}$. The fact that not even a single definite K_{e4} event has yet been observed compared with more than a hundred K_{e3}-decay events seems rather to disfavor the standpoint (I). Accumulation of more experimental information is, however, certainly desirable.

Our second point is that if the standpoint (II) is correct, then from the absence of any single K_{e4} event we may conclude that the leptonic decays of K meson proceed through an approximate $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. The reason is as follows: Let us consider again the K_{e4} decay modes and let us assume that the relative

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[1] It should be noted that the final two-pion system can only have $I=2$ in the K^+ decay whereas the $K_1{}^0$ decay may go through the $I=0$ as well as the $I=2$ channel.

[2] S. Okubo and R. Marshak, Phys. Rev. **100**, 1809 (A) (1955).

[3] As regards Λ decay, S. Okubo et al., Phys. Rev. **113**, 944 (1959), have also proposed a mechanism which does not need the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule. They have shown that even if we start from the strangeness-nonconserving nonleptonic interaction $(\bar{\Lambda}p)(\bar{p}n)$ +H.c. which contains an appreciable amount of $|\Delta \mathbf{I}| = \frac{3}{2}$ component as well as $|\Delta \mathbf{I}| = \frac{1}{2}$ component, nearly the same result as due to the strict $|\Delta \mathbf{I}| = \frac{1}{2}$ rule may be obtained for the Λ decay if we consider a certain set of diagrams. See, however, S. Oneda, J. C. Pati, and B. Sakita, Phys. Rev. **119**, 482 (1960).

[4] M. L. Good and W. G. Holladay, Phys. Rev. Letters **4**, 138 (1960). This possibility was also pointed out by S. Oneda, Nuclear Phys. **3**, 97 (1957). The asymmetry of the energy spectrum of pions in the $K^+ \rightarrow \pi^+ + \pi^+ + \pi^-$ decay could also be attributed to the strong pion-pion interactions in the $I=2$, $J=0$ state. See B. S. Thomas and W. G. Holladay, Phys. Rev. **115**, 1329 (1959).

[5] K. Chadan and S. Oneda, Phys. Rev. Letters **3**, 292 (1959). See also V. S. Mathur, Nuovo cimento **14**, 1322 (1959). The K_{e4} decay has also been discussed by Okun et al.: see summary talk by R. E. Marshak, Proceedings of the Ninth International Conference on High-Energy Physics, Kiev, 1959 (unpublished), see reference 29.

[6] Of course, this could be avoided if the leptonic decay modes of the K meson satisfy the $|\Delta \mathbf{I}| = \frac{3}{2}$ rule. However, presently available experiments seem to favor the $|\Delta \mathbf{I}| = \frac{1}{2}$ rule.

angular momentum of the two-pion system is $J=0$ (S wave), or at least that the S wave is dominant. If we had an appreciable amount of $|\Delta \mathbf{I}|=\frac{3}{2}$ part in the transition amplitude, the system of final pions could reach the state $I=2$, $J=0$. The enhancement factor (E.F.) in this state as obtained by the Fermi method and given in reference 4 is

$$\text{E.F.}=\frac{\cot^2\delta}{1+\cot^2\delta}\left[1+\frac{1}{\alpha k\cot\delta}\right]^2;\quad k\cot\delta=\frac{1}{a_{20}},$$

where δ is the pion-pion phase shift in the $I=2$, $J=0$ state (a_{20} being the corresponding scattering length), α is the radius of interaction of the outgoing particles, and k the relative momentum of the pions. With the same value of parameters chosen in reference 4, this formula leads to

$$75\leqslant \text{E.F.}\leqslant 225,$$
$$k_{\max}\leftarrow k\rightarrow 0,$$

where $k_{\max}=m_\pi c/0.7\hbar$. Perhaps it is worth observing that the E.F. is larger at low momenta, when the S wave is more likely to be dominant, than at high momenta. Combining these results with the pion spectrum given by Mathur,[5] we see that the ratio (2) may be enhanced by two orders of magnitude leading again to a value $\sim 10^{-1}$. Therefore, the fact that the K_{e4} decay has not yet been observed implies in this case that the $|\Delta \mathbf{I}|=\frac{3}{2}$ part plays an insignificant role in K_{e4} modes. So if we insist on the standpoint (II), the leptonic decay modes of the K meson must satisfy the approximate, if not strict, $|\Delta \mathbf{I}|=\frac{1}{2}$ rule.

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Some Remarks on the Vertex Function

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It is noted that the integral representations for matrix elements derived in three papers, (1) Dyson, (2) Deser, Gilbert, and Sudarshan, and (3) Faïnberg, do not give the most general functions satisfying the conditions stated. It is shown that in (1) and (2) the convergence of certain integrals was not treated rigorously, while in (3) there was a misunderstanding of the Jost-Lehmann-Dyson representation.

WE wish to point out that the integral representations for matrix elements derived in three papers[1–3] do not give the most general functions satisfying the conditions stated.

Dyson's representation for the double commutator is

$$\langle 0|[A(x),[B(0),C(-y)]]|0\rangle =\int_0^\infty ds\int_0^\infty dt\int_0^1 d\lambda\,\phi(s,t,\lambda)\delta(y^2-t^2)\epsilon(y_0)\times\delta((x+\lambda y)^2-s^2)\epsilon(x+\lambda y).\quad (1)$$

This is not the most general form for a function with the properties of the double commutator. For, Eq. (1) implies

$$\langle 0|A(x)[B(0),C(-y)]|0\rangle =\int_0^\infty ds\int_0^\infty dt\int_0^1 d\lambda\frac{\delta(y^2-t^2)\epsilon(y_0)\phi(s,t,\lambda)}{(x+\lambda y)^2-s^2},\quad (2)$$

showing this to be, for fixed $y^2>0$, a regular function of x^2, $(x+y)^2$ except "to the right" of the S curve, as shown with horizontal shading in Fig. 1. But the most general form[4] for (2) is

$$\langle 0|A(x)[B(0),C(-y)]|0\rangle =\int_0^\infty \psi(s,t,\lambda;k)\times\Delta_k(y)\Delta^{(+)}(x,x+y;s,t,\lambda)dsdtd\lambda dk,$$

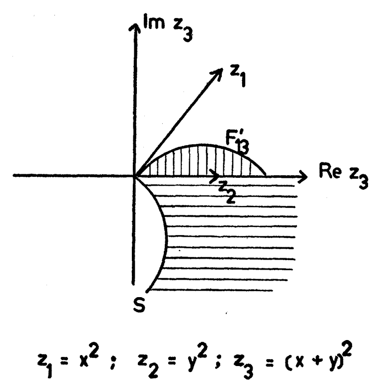

FIG. 1. Diagram showing singularities of the function $\langle 0|A(x)[B(0),C(-y)]|0\rangle$.

[1] F. J. Dyson, Phys. Rev. 111, 1717 (1958).
[2] S. Deser, W. Gilbert, and E. Sudarshan, Phys. Rev. 115, 731 (1959).
[3] V. Ya. Faïnberg, Zhur. Eksp. i Teoret. Fiz. 36, 1503 (1959) [Translation: Soviet Phys.—JETP 36(9), 1066 (1959)].
[4] R. F. Streater, Proc. Roy. Soc. (to be published).