TABLE III. Comparison of the results of this paper with those of reference 3 (Ashkin et al.). The coefficients are for the expansion $d\sigma/d\Omega = a + b\cos\theta + c\cos^2\theta$.

Coefficient	a	b	c
Present paper	1.61±0.26	−1.54±0.30	5.35±0.88
Reference 3 normalized to present total cross sec.[a]	1.89±0.15	−1.65±0.13	4.46±0.31
Reference 3 without renormalization	1.54±0.09	−1.34±0.09	3.63±0.21

[a] The quoted errors on these include a contribution from the uncertainty in the normalization factor.

V. DISCUSSION

A direct comparison of the measurement reported in this paper can be made with the earlier results of Ashkin et al.,[3] which is $\sigma_{tot}(\pi^- \rightarrow \pi^0) = 34.6 \pm 1.2$ mb. The difference is mainly due to the method of calculating $\sigma_{tot}(\pi^- \rightarrow \pi^0)$. Ashkin et al., as was stated previously, normalized their value of the coefficients describing the angular distribution for charge exchange to the difference between their total transmission measurement and the integrated value of their elastic scattering measurement. Kruse and Arnold[8] have recently remeasured both of these cross sections and find a higher value for the total transmission measurement. There is good agreement between the present results and the results of Ashkin et al., if the angular distribution they report is renormalized to agree with the total cross section reported here. This amounts to an increase of 23% for their total cross section and all of their coefficients. The results of this comparison are contained in Table III. The present result agrees well with the result of Kruse and Arnold at 152 Mev. Their result is $\sigma_{tot}(\pi^-, \pi^0) = 40.4$ mb.

[8] U. E. Kruse and R. C. Arnold, Phys. Rev. **116**, 1008 (1959). The value quoted for their result has been calculated by the present author from the angular distribution for elastically scattered pions and the total transmission measurement reported in their paper. No attempt has been made to calculate the error on their values.

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Charge-Exchange Scattering of Negative Pions at 61 Mev and 95 Mev*

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The charge-exchange scattering of negative pions by liquid hydrogen has been measured at 61±1 Mev and 95±2 Mev bombarding energy. The measurements were made with a gamma-ray spectrometer which employs a lead glass Čerenkov counter. If the charge exchange scattering cross section is expanded as a series of Legendre polynomials in the center-of-mass system of the collision, we find that at 61 Mev,

$$d\sigma/d\Omega = (1.00\pm0.05)[0.613\pm0.030 - (0.830\pm0.068)P_1(\cos\theta') + (0.183\pm0.150)P_2(\cos\theta')],$$

and at 95 Mev,

$$d\sigma/d\Omega = (1.00\pm0.03)[1.05\pm0.05 - (1.15\pm0.12)P_1(\cos\theta') + (0.33\pm0.25)P_2(\cos\theta')].$$

The total cross section for charge exchange, obtained by integration, is: $\sigma_{tot}(\pi^- \rightarrow \pi^0) = 7.7\pm0.6$ mb at 61 Mev and $\sigma_{tot}(\pi^- \rightarrow \pi^0) = 13.2\pm0.8$ mb at 95 Mev.

A table summarizing the measurements performed by this group at 61 Mev, 95 Mev, 128 Mev, and 150 Mev is given.

I. INTRODUCTION

THE study of the charge exchange reaction,

$$\pi^- + p \rightarrow \pi^0 + n \rightarrow 2\gamma + n, \qquad (1)$$

which has been carried out for incident pions of 128 Mev with the aid of a lead glass, gamma-ray spectrometer,[1] is extended in the present work to pions of 95 Mev and 61 Mev incident energy. Although some modifications of the apparatus and its mode of operation were made in this work, the technique is essentially the same as that used earlier. A similar measure-

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[1] E. L. Garwin, W. J. Kernan, C. O. Kim, and C. M. York, Phys. Rev. **115**, 1295 (1959).

ment at 150 Mev is described in an accompanying paper.[2]

II. EXPERIMENTAL METHOD

The reaction in Eq. (1) can be studied by assuming that the neutral pions are emitted in the center-of-mass system of the collision with an angular distribution expressed in terms of Legendre polynomials. The expansion is limited to three terms, corresponding to s and p wave emission of the neutral pions. The differential cross section can then be written:

$$\frac{d\sigma(\theta')}{d\Omega'}=\sum_{l=0}^{2} A_l P_l(\cos\theta'), \qquad (2)$$

where θ' is the angle of emission of the neutral pion, measured relative to the direction of the negative pion. The spectral intensity of the gamma rays produced by the neutral pion decay and detected in the laboratory, can be shown to be:

$$I(k)=B\frac{d^2\sigma}{d\Omega dk}$$

$$=\frac{B}{\beta_0\gamma_0 k''\gamma(1-\beta\cos\alpha)}\sum_{l=0}^{2} A_l P_l\left[\frac{\cos\alpha-\beta}{1-\beta\cos\alpha}\right]$$

$$\times P_l\left[\frac{1}{\beta_0}\left(1-\frac{k''}{k\gamma_0\gamma(1-\beta\cos\alpha)}\right)\right], \qquad (3)$$

where k is the gamma-ray energy in the laboratory in Mev; B is a constant, $B=(\bar{N}_p\Delta\Omega E_0 I)$, with $\bar{N}_p$ the average number of target protons traversed by an incident pion, $\Delta\Omega$ the solid angle of the detector, I the number of pions traversing the target, and E_0 a detection efficiency factor. β_0 is the velocity of the π^0 in the center-of-mass system and $\gamma_0=(1-\beta_0^2)^{-\frac{1}{2}}$. k'' is the energy of the gamma ray in the rest frame of the neutral pion (=67.5 Mev). β is the velocity of the center-of-mass system and $\gamma=(1-\beta^2)^{-\frac{1}{2}}$. α is the angle of observation in the laboratory system.

The method consists of measuring the absolute number of gamma rays and their energy distribution for several angles of observation, α. The results are used to determine the parameters A_l which in turn specify the angular distribution of the emitted neutral pions in Eq. (2). However, before the A_l can be determined, the spectra in Eq. (3) must have the spectrometer resolution and detection efficiency folded into them. This folding operation has been carried out numerically. The gamma-ray spectra measured in this experiment receive a small contribution from the reaction

$$\pi^-+p\rightarrow n+\gamma. \qquad (4)$$

[2] W. J. Kernan, preceding paper [Phys. Rev. **119**, 1092 (1960)].

TABLE I. The number of counts in each 25-Mev interval of the experimental data and the expected number in that interval, calculated from the result of the least-squares fit.

Pion energy (Mev)	Laboratory angle	Gamma energy (Mev)	Number of counts Experimental	Calculated
61	45°	75–100	98±18	72
		100–125	60±13	50
		125–150	34±10	35
		150–175	7± 8	17
		175–200	0±15	3
	75°	75–100	118±18	110
		100–125	100±15	102
		125–150	86±20	83
		150–175	24±15	34
		175–200	0±12	3
	90°	75–100	166±44	131
		100–125	130±17	132
		125–150	118±27	105
		150–175	21±18	35
		175–200	0±10	2
	105°	75–100	133±40	153
		100–125	151±19	162
		125–150	122±32	20
		150–175	40±20	31
		175–200	27±18	2
	135°	75–100	225±50	192
		100–125	195±20	211
		125–150	145±33	128
		150–175	45±17	21
		175–200	9±12	1
95	45°	50–75	145±48	138
		75–100	109±32	113
		100–125	100±25	96
		125–150	106±25	86
		150–175	73±26	78
		175–200	46±27	57
		200–225	0±18	18
	75°	50–75	157±43	154
		75–100	152±35	153
		100–125	157±29	147
		125–150	126±30	143
		150–175	96±30	127
		175–200	73±16	63
		200–225	27±34	8
	105°	50–75	189±38	168
		75–100	217±35	196
		100–125	182±31	211
		125–150	248±58	215
		150–175	172±33	154
		175–200	89±30	38
		200–225	50±32	2
	135°	50–75	218±38	182
		75–100	214±35	235
		100–125	216±35	269
		125–150	259±65	262
		150–175	210±41	132
		175–200	126±61	17
		200–225	41±33	0

This contribution can be calculated,[2] and a suitable correction applied to the data. The A_l were then determined from the corrected data by the method of least squares.

TABLE II. A summary of the systematic correction factors and their uncertainties.

Type of correction	Correction factor 95 Mev	Correction factor 61 Mev
1. Contamination of pion beam.	0.864±0.015	0.483±0.020
2. Fraction of the beam traversing the target.	0.845±0.016	0.982±0.002
3. "Empty" target filled with hydrogen gas at liquid hydrogen temperature.	0.98 ±0.00	0.98 ±0.00
4. Losses of gamma rays due to conversion in the target walls.	1.00 ±0.00	1.00 ±0.00
5. Energy of the incident beam.	1.00 ±0.00	1.00 ±0.00
6. Counter telescope inefficiency.	0.99 ±0.00	0.99 ±0.00
7. Uncertainty in the absolute gamma-ray detection efficiency.	1.00 ±0.02	1.00 ±0.02
Resultant	0.708±0.023	0.460±0.021

III. APPARATUS

A. Pion Beams

The pion beams were monitored by two scintillation counters, A and B, indicated in Fig. 1 of the accompanying paper. The energy of each beam was determined by measuring the range of the pions in carbon. The energies so determined were: 61±1 Mev and 95±2 Mev. From the range curves it was possible to determine the contamination of the beam by mu-mesons and electrons. This contamination was 51.7±2.0% in the 61-Mev beam and 13.6±1.5% in the 95-Mev beam. The intensity distributions of both beams were measured with the aid of a $\frac{1}{4}$-in. cubic scintillation counter. From this it was determined that 98.2±1.0% of the 61-Mev beam traversed an average of 0.597 g/cm^2 of liquid hydrogen and that 84.5±1.6% of the 95-Mev beam traversed 0.580 g/cm^2 of hydrogen.

The target was a cylinder of Mylar 0.003 inch thick mounted in a vacuum chamber with an external aluminum wall 0.010 inch thick.[3]

B. Electronics

The electronic circuits used in the present work are entirely new and have been described in detail elsewhere.[4] A coincidence of $(A+B+1+2+\bar{3})$ was used to trigger a linear gate of 20 mμsec duration. Time coincident pulses in the lead glass counter were passed through this gate into a 50-channel pulse-height analyzer. An over-all counter efficiency of 99% was determined with all five counters placed directly in the pion beam. The anticoincidence counter had a measured inefficiency of one part in five thousand in this arrangement.

[3] The authors wish to express their gratitude to Professor Kruse for the loan of his hydrogen target and the associated equipment.

[4] R. Gabriel, E. L. Garwin, and C. M. York, Nuclear Instrs. Methods **5**, 1 (1959).

C. Calibration of the Lead Glass Counter

The lead glass counter was calibrated in the same way that it had been done earlier. A detailed description of the procedure is given in the accompanying paper.[2]

IV. THE EXPERIMENTAL DATA AND RESULTS

The number of counts observed in each 25-Mev interval of the spectra obtained at various angles of observation are summarized in Table I. It is to be noted that the data of the two energies were taken for different numbers of monitor counts. The errors assigned to the experimental points include both the statistical uncertainty of the number of counts combined with the uncertainty of the size of a 25-Mev interval as determined from the calibration curve in Fig. 2 of the accompanying paper.[2]

A least-squares fit of the data to the expression of Eq. (3) gives:

at 61 Mev,

$$d\sigma/d\Omega=(1.00\pm0.05)\times[(0.613\pm0.030)-(0.830\pm0.068)P_1(\cos\theta')+(0.183\pm0.150)P_2(\cos\theta')]; \quad (6)$$

at 95 Mev,

$$d\sigma/d\Omega=(1.00\pm0.03)\times[(1.05\pm0.05)-(1.15\pm0.12)P_1(\cos\theta')+(0.33\pm0.25)P_2(\cos\theta')]. \quad (7)$$

In these equations we have written the coefficient outside of the bracket to indicate the systematic error which is independent of angle, while the errors attached to the coefficients inside the bracket result from the least squares fit and are statistical in origin. The integrated values of the cross sections given in Eq. (6) and Eq. (7) are:

at 61 Mev,

$$\sigma_{\text{tot}}(\pi^-\rightarrow\pi^0)=4\pi A_0=7.7\pm0.6 \text{ mb}, \quad (8)$$

TABLE III. Error matrices for the angular distributions in units of $10^3\times$(millibarns per steradian)^2.

(A) For 95 Mev

	A_0	A_1	A_2
A_0	2.592	−2.312	4.185
A_1	···	14.181	−2.684
A_2	···	···	62.080

$\chi^2=23.3$. Degrees of freedom = 25. Level of confidence ~55%.

(B) For 61 Mev

	A_0	A_1	A_2
A_0	0.882	−1.332	1.809
A_1	···	4.564	−4.117
A_2	···	···	22.413

$\chi^2=13.3$. Degrees of freedom = 27. Level of confidence > 98%.

TABLE IV. Summary of the data on charge exchange scattering by this group. Cross sections expressed as $d\sigma/d\Omega = A_0 + A_1P_1(\cos\theta) + A_2P_2(\cos\theta)$, in the center-of-mass system.

Reference	Energy (Mev)	A_0 (mb-sr^{-1})	A_1 (mb-sr^{-1})	A_2 (mb-sr^{-1})	$\sigma(\pi^- \to \pi^0)$ mb
a	61	0.613±0.043	−0.830±0.080	0.183±0.152	7.7±0.6
a	95	1.05 ±0.06	−1.15 ±0.13	0.33 ±0.25	13.2±0.8
b, c	128	2.00 ±0.10	−1.58 ±0.14	1.40 ±0.24	25.1±1.3
d	150	3.39 ±0.19	−1.54 ±0.30	3.57 ±0.59	42.6±1.9

[a] Present paper.
[b] Garwin et al., reference 1.
[c] A 2% systematic correction has been applied to the previously reported values given in reference 1. This was due to a re-evaluation of the efficiency of the lead converter used.
[d] W. J. Kernan, reference 2 (accompanying paper).

and at 95 Mev,

$$\sigma_{tot}(\pi^- \to \pi^0) = 4\pi A_0 = 13.2 \pm 0.8 \text{ mb}. \tag{9}$$

Table II provides a summary of the uncertainties in the absolute value of the cross sections. Table III gives the elements of the error matrices for the least-squares fit at the two energies, together with the confidence level of the fit as determined from the "chi-squared" test.

Table IV presents a summary of the four sets of measurements carried out at 61, 95, 128, and 150 Mev with the aid of the lead glass spectrometer described above.

V. DISCUSSION

A direct comparison of the measurements reported in this paper can be made with the earlier results of Bodansky et al.,[5] who worked at 65 Mev, and with those of Edwards et al.[6] at 98 Mev. The former report $\sigma_{tot}(\pi^- \to \pi^0) = 12.1 \pm 1.5$ mb. If one subtracts approximately 0.5 mb from this to compensate for the difference in energy, a comparison with the result of Eq. (8) can be made. It is seen that the two results do not agree within the quoted errors, the value of Bodansky et al. being somewhat larger than that of the present work. Considerably better agreement is obtained with the 98-Mev data of Edwards et al. One can compute from their work that $\sigma_{tot} = 15.5 \pm 0.5$ mb. Again a subtraction must be made to compensate for the energy difference. If this is taken as approximately 0.8 mb, the result agrees well with the value of Eq. (9) above. To facilitate these comparisons and make it possible to compare our results at higher energies with those of other workers, Fig. 1 has been prepared. At 128 Mev and 150 Mev the comparison is made with the results of Kruse and Arnold.[7] For completeness, the work of Tinlot and Roberts[8] at 40 Mev is included. The various results show a very satisfactory selfconsistency.

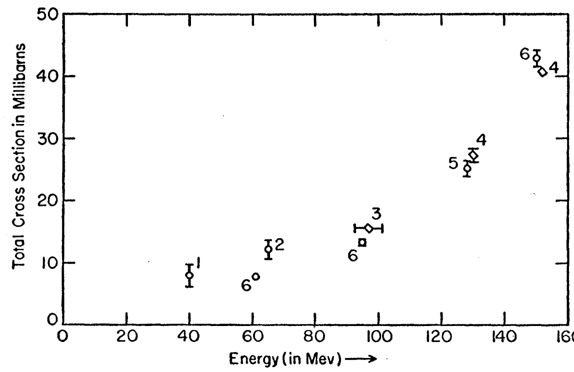

FIG. 1. A summary of the available data on the experimental values of $\sigma_{tot}(\pi^- \to \pi^0)$. Those points indicated with a diamond were calculated from inelastic scattering and transmission measurements, while those with circles were determined from direct gamma-ray measurements. 1. Tinlot and Roberts, see reference 8; 2. Bodansky et al., see reference 5; 3. Edwards et al., see reference 6; 4. Kruse and Arnold, see reference 7; 5. Garwin et al., see reference 1; and 6. Present work (including the accompanying paper).

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[5] D. Bodansky, A. M. Sachs, and J. Steinberger, Phys. Rev. **93**, 1367 (1954).
[6] D. N. Edwards, S. G. F. Frank, and J. R. Holt, Proc. Phys. Soc. (London) **73**, 856 (1959).
[7] U. E. Kruse and R. C. Arnold, Phys. Rev. **116**, 1008 (1959).
[8] J. Tinlot and A. Roberts, Phys. Rev. **95**, 137 (1954).