Charge-Exchange Scattering of Negative Pions at 150 Mev*†

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The charge-exchange scattering of negative pions by hydrogen has been measured at a bombarding energy of 150 Mev. The energy distribution of gamma rays from the decay of the neutral pions was measured with a lead glass Čerenkov counter at laboratory angles of 45°, 75°, 105°, and 135°. If the charge exchange differential scattering cross section in the center-of-mass system is expanded as a series of Legendre polynomials, the result is:

$$d\sigma/d\Omega=(1.00\pm0.03)[3.39\pm0.11-(1.54\pm0.29)P_1(\cos\theta')+(3.57\pm0.56)P_2(\cos\theta')]\text{mb-sr}^{-1}.$$

The total cross section for charge exchange, obtained by integration, is then $\sigma_{\text{tot}}(\pi^-\rightarrow\pi^0)=42.6\pm1.9$ mb.

I. INTRODUCTION

THE differential scattering cross section for the charge-exchange reaction

$$\pi^-+p\rightarrow\pi^0+n\rightarrow2\gamma+n, \tag{1}$$

has been studied by the use of a lead glass gamma-ray spectrometer at incident pion energies of 128 Mev,[1] and of 61 and 95 Mev.[2] In the present paper this work is extended to include an incident pion energy of 150 Mev.

The measurement reported here is a direct measurement of the angular distribution and the total cross section. Previous work of Ashkin et al. at this energy[3] consisted of a direct measurement of the angular distribution of the gamma rays from the process of Eq. (1), but their reported value of this cross section is normalized to an indirect measurement of the total cross section for this process.

In addition, the technique of the present experiment, which combined a lead glass Čerenkov counter, utilized as a gamma-ray spectrometer,[4] with a counter telescope and a lead converter, is different from that of reference 3 which used only a counter telescope and a lead converter. Therefore, the possible systematic errors of this experiment are different from those of reference 3.

II. EXPERIMENTAL METHOD

The angular distribution, in the center of mass, of (1) can be expressed as a series of Legendre polynomials; namely,

$$\frac{d\sigma_{\pi^0}}{d\Omega'}=\sum_{l=0}^{n}A_lP_l(\cos\theta'), \tag{2}$$

where θ' is the angle of emission of the neutral pion relative to the direction of the negative pion. Under the assumption that the two-gamma decay of the neutral pion is isotropic in the pion rest frame, then transformation of Eq. (2) into the laboratory system yields the following expression for the energy and angular distribution of the gamma-ray intensity in the laboratory system:

$$\begin{aligned}I(k)&=B\frac{d^2\sigma(k,\alpha)}{d\Omega dk}\\&=\frac{B}{\beta_0\gamma_0k''\gamma(1-\beta\cos\alpha)}\sum_{l=0}^{n}A_lP_l\left[\frac{\cos\alpha-\beta}{1-\beta\cos\alpha}\right]\\&\quad\times P_l\left[\frac{1}{\beta_0}\left(1-\frac{k''}{\gamma_0\gamma k(1-\beta\cos\alpha)}\right)\right].\end{aligned} \tag{3}$$

In this expression B is a constant, $B=[\bar{N}_p\Delta\Omega E_0I]$, in which $\bar{N}_p$ is the average number of target protons traversed by an incident pion, $\Delta\Omega$ is the solid angle of the detector, E_0 is a detection efficiency factor and I is the number of pions traversing the target. β_0 is the velocity of the π^0 in the center-of-mass system and $\gamma_0=(1-\beta_0^2)^{-\frac{1}{2}}$. k'' is the energy of the gamma ray in the rest frame of the neutral pion, 67.5 Mev. β is the velocity of the center of mass and $\gamma=(1-\beta^2)^{-\frac{1}{2}}$.

The summation over l in Eq. (3) is restricted to values of l from 0 to 2 by the assumption that only s and p waves are required to describe the angular distribution of (1).

A lead glass Čerenkov spectrometer was used to measure the gamma-ray energy distribution at four angles. The experimental data are fitted to Eq. (3) after the effects of the converter efficiency and the finite energy resolution of the counter have been folded in. A least-squares fit is used to determine the values of the appropriate A_l and the error associated with these estimates.

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[1] E. L. Garwin, W. J. Kernan, C. O. Kim, and C. M. York, Phys. Rev. **115**, 1295 (1959).
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III. THE APPARATUS

A. 150-Mev Pion Beam

The pions are produced by the 450-Mev proton beam striking a 2-inch beryllium target inside the cyclotron. The negative pions emitted in the forward direction are bent outward by the cyclotron magnetic field and are also focused in the cyclotron fringe field. The 150-Mev negative pion beam enters a vacuum pipe which connects the cyclotron with the experimental area. It then is focused by a quadrupole focusing magnet and traverses the meson channel of the rotary shield to the experimental area. The beam emerges from the vacuum pipe through a thin Mylar window and is deflected and focused by a wedge magnet. Range curves of this beam were measured in aluminum and carbon and gave a pion energy, corrected to the center of the hydrogen target, of 150 ± 1 Mev. The muon and electron contamination of this beam could be estimated from the range curves and was $5.8\pm2.0\%$. The beam distribution at the target position was measured with the aid of a $\frac{1}{4}$ in. cubic scintillation counter. From this it was determined that $99\pm1\%$ of the beam traversed an average of 0.607 g/cm^2 of hydrogen. The beam scan was repeated in the horizontal plane at the end of the experiment to determine whether any drifts in the magnet system had changed the beam distribution. No such change was observed.

The hydrogen target was a cylinder of Mylar 0.003 inch thick, 3.72 inches in diameter and 5 inches high, mounted in a vacuum chamber with an external aluminum wall 0.010 inch thick.[5]

B. Counters and Electronics

All of the scintillation counters used were $\frac{1}{4}$ inch thick and were viewed by 6810 photomultipliers. The incident pion beam was monitored by two of these scintillation counters, A and B in Fig. 1. Counter A was 3 inches square, and counter B was $2\frac{3}{4}$ inches square. The gamma rays were detected by a counter telescope $1+2+\bar{3}$ in front of the Čerenkov counter with a 2-inch diameter lead converter between counters 2 and 3. The converter thickness was 0.317 cm. Counters No. 1, 2, and 3 were disks. The diameter of No. 3 was 6 inches, while the others were 4 inches in diameter.

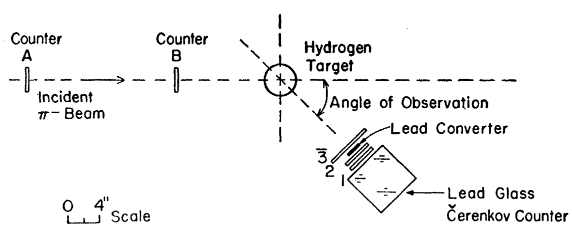

FIG. 1. The experimental arrangement for the charge-exchange scattering experiment.

[5] The author wishes to thank Professor Kruse for the use of his hydrogen target.

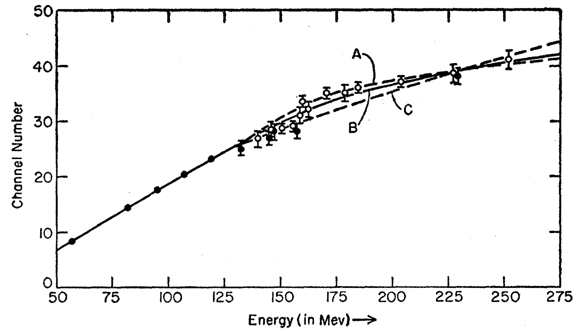

FIG. 2. The results of the pulse height to energy calibration. The points designated by dots were determined with incident electrons contaminating the 70-Mev pion beam and by the electrons contaminating the 128-Mev and 61-Mev negative pion beams. The points designated by circles were estimated from the kinematic limits of the gamma-ray energy from several of the spectra observed in the charge-exchange reaction at bombarding energies of 61, 95, and 150 Mev.

The geometrical arrangement is shown in Fig. 1. Counters 1, 2, and 3, and the Cerenkov counter were mounted in a bracket on a table which rotated about the geometrical center of the hydrogen target.

The electronic circuits used in this experiment are completely new and have been described in detail elsewhere.[4] A coincidence of $(A+B+1+2+\bar{3})$ was used to trigger a linear gate of 20 mμsec duration. Time coincident pulses in the lead glass counter were passed through this gate into a 50-channel pulse-height analyzer.

The efficiency of the counters was determined by placing all five counters in a row in the pion beam. The over-all efficiency of $A+B+1+2$ in this system was 99%. The measured inefficiency of $\bar{3}$ was one part in five thousand.

C. Calibration of the Lead Glass Counter

The general method of calibrating the lead glass counter has been described earlier.[1] A negative pion beam with a large electron contamination was used. The energy of the electrons incident on the glass counter could be varied from 50 to 150 Mev by adjusting the current in a wedge magnet. The response of the system was linear from 50 to 130 Mev. At higher energies, there were significant deviations from a simple linear relation. Because the highest energy gamma ray that could be observed at the angles used in this experiment was 252 Mev for spectra observed at 45° in the laboratory, it was necessary to know the behavior of the counter in this nonlinear region. There were two different methods of obtaining information about this region of the calibration curve. First, the electrons which contaminate the 128-Mev and the 61-Mev π^- beams could be separated from the pions by pulse-height analysis with the glass counter directly in the beam. This yielded two points on the calibration curve, shown in Fig. 2, one at 148 Mev and the other at 228 Mev. The second method of determining points at high energy was to observe the maxima of the gamma-ray spectra at various angles in this experiment and in the charge-exchange reactions at 61 Mev and 95 Mev.[2] The bombarding energy and angle of observation determine the kinematic limit of the gamma-ray energy which can be observed. This kinematic limit is independent of any assumption as to the angular distributions involved. Care was taken to allow for the finite resolution of the counter and errors of reasonable size were assigned to the estimates. The results are shown in Fig. 2. The pulse height as a function of energy is shown for the incident electrons as well as the upper limits of the gamma-ray spectra. It is easily seen that where both types of estimate were available, the two are in fair agreement. The straight line portion of the curve is a least-squares fit to the experimental points in that region. The equation for the straight line is

$$C=(0.236\pm0.004)E-(5.0\pm0.4), \quad (4)$$

where C is the channel number in the pulse-height analyzer and E is the energy in Mev.

The solid curve indicates the relation used to convert pulse height to energy over the entire energy region. The two dashed curves represent estimates of the extreme limits which can be placed on the pulse height to energy relation. Since these curves intersect near 225 Mev, this was used as the cutoff point for the interpretation of the data. This is also necessary because there is only one point at significantly higher energies so that the behavior of the curve in this region is not well defined.

The errors on the spectra in the nonlinear region of this curve were set by grouping the data alternately using each of the three curves. The deviations of each energy bin using curves A and C in Fig. 2 from the same energy bin determined by using curve B, were combined incoherently with the statistical error of the point using curve B, and this combined total is assigned as the experimental error on that point.

The nonlinear effects discussed above are thought to be the result of the operation of the electronics and are not to be associated with any shower losses from the glass counter. The most probable explanation is that the timing of the signal pulse in this rather short gate (20 mμsec) was a function of pulse height and that this change in the timing of the gate introduced the observed nonlinearity.

Figure 3 shows the width, σ, of the Gaussian fitted to the pulse-height distributions obtained with incident electrons, plotted against the incident electron energy. These widths are consistent with a constant value within the experimental errors. It has been assumed that the width is independent of energy over the entire energy region of interest. In fact, the final cross sections are very insensitive to the resolution. This has been checked by fitting the experimental data with three different values of the resolution. The best value of the resolution, as determined from Fig. 3, was varied by $\pm10\%$, and the data were reanalyzed with each of these values for the resolution. The coefficients, A_l, of the angular distribution changed by less than 1% as a result of a 10% change in the resolution. The total cross section varied by about 1% with these variations of the resolution.

A precision pulser was used to check on the stability of the electronic system at frequent intervals during the course of the experiment. Each time the angle of observation of the spectrometer was changed, it was placed directly in the meson beam and a pulse-height spectrum recorded. This gave added assurance that within any given set of data, there was no change in the spectrometer performance.

The gamma-ray detection efficiency of the system has been determined in exactly the manner described earlier.[1]

IV. THE EXPERIMENTAL DATA AND RESULTS

Pulse-height spectra were taken with hydrogen in and out of target, and with the converter in and out of position. The four types of spectra were alternated throughout the experiment. The signal to background ratio was greater than 10 to 1 at all angles. From the net of this data, the counts due to the process

$$\pi^-+p\rightarrow\gamma+n, \quad (5)$$

were subtracted. This was accomplished by calculating the number of gamma rays per steradian due to the process in Eq. (5) by detailed balance from the inverse process

$$\gamma+n\rightarrow\pi^-+p. \quad (6)$$

The cross section for this process is related to the reaction,

$$\gamma+p\rightarrow\pi^++n, \quad (7)$$

by the ratio of negative to positive pion photoproduction on deuterium

$$\frac{\gamma+d\rightarrow p+p+\pi^-}{\gamma+d\rightarrow n+n+\pi^+}. \quad (8)$$

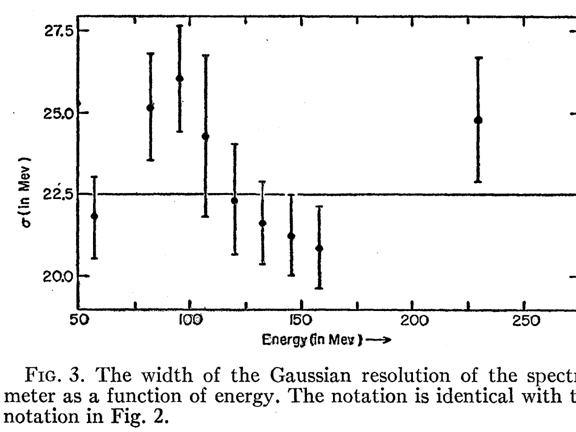

FIG. 3. The width of the Gaussian resolution of the spectrometer as a function of energy. The notation is identical with the notation in Fig. 2.

The measured values for the processes Eq. (7)[6] and Eq. (8)[7] were used to determine the number of gamma rays due to the reaction of Eq. (5) at each laboratory angle. This number was multiplied by the calculated detection efficiency for gamma rays in each appropriate energy interval at each angle, and the resolution of the counter was folded in. The resulting distribution of gamma rays was then subtracted from the observed spectrum at each angle. These spectra are attributed to gamma rays from the charge exchange reaction of Eq. (1).

A least-squares fit of the constants of Eq. (1) to the corrected data has been carried out. Under the assumption that only s and p wave scattering contribute to the cross section, the result is

$$d\sigma/d\Omega=(1.00\pm0.03)[3.39\pm0.11-(1.54\pm0.29)P_1(\cos\theta') + (3.57\pm0.57)P_2(\cos\theta')]\text{mb-sr}^{-1}. \quad (9)$$

If, however, the possibility of d wave scattering is considered, the number of Legendre polynomials must be increased. For a complete analysis of d wave scattering, the analysis would have to be extended to include $P_4(\cos\theta')$. However, the effects of d wave scattering are not thought to be an important contribution at incident pion energies of less than 200 Mev. Therefore, if an effect is present at this energy, it is expected to be small and will be more evident in the cross terms than in the term in $P_4(\cos\theta')$. For this reason, the analysis was carried out again using the Legendre polynomials up to $l=3$, and the result is

TABLE I. A summary of the systematic correction factors and their uncertainties.

Type of correction	Correction Factor
1. Contamination of pion beam.	0.94±0.02
2. Fraction of the beam traversing the target.	0.99±0.01
3. "Empty" target filled with hydrogen gas at liquid hydrogen temperature.	0.98±0.00
4. Losses of gamma rays due to conversion in the target walls.	1.00±0.00
5. Energy of the incident beam.	1.00±0.00
6. Counter telescope inefficiency.	0.99±0.00
7. Uncertainty in the absolute gamma-ray detection efficiency.	1.00±0.02
Resultant	0.90±0.03

$$d\sigma/d\Omega=(1.00\pm0.03)\times[(3.43\pm0.12)-(1.35\pm0.32)P_1(\cos\theta') + (3.72\pm0.57)P_2(\cos\theta') + (1.36\pm1.00)P_3(\cos\theta')]\text{mb-sr}^{-1}. \quad (10)$$

This is not a significantly better fit to the experimental data than the fit of Eq. (9), and so it may be concluded that, if any effects of d-wave scattering exist at this energy, they are smaller than the present experiment can detect.

TABLE II. Error matrix for the angular distribution in units of (millibarns per steradian)^2.

	A_0	A_1	A_2
A_0	0.01306	0.00283	0.02619
A_1	...	0.08183	0.03481
A_2	...	...	0.31119

$\chi^2=16.8$. Degrees of freedom = 25.
Level of confidence ~88%.

In these equations, the coefficient outside of the bracket has been written to indicate the uncertainties which are independent of angle, while the errors attached to the coefficients inside the bracket result from the least-squares fit and are statistical in origin.

The integrated value of the cross section in Eq. (9) gives:

$$\sigma_{\text{tot}}(\pi^- \rightarrow \pi^0)=4\pi A_0=42.6\pm1.9 \text{ mb}. \quad (11)$$

Table I provides a summary of the uncertainties in the absolute value of the cross section. Table II gives the elements of the error matrix for the least-squares fit, together with the confidence level of the fit as determined from the "chi-squared" test.

Figure 4 shows the energy distribution of the charge exchange gamma rays at each of the four angles of observation. The experimental values with their attached errors are shown together with the histograms which indicate the numbers predicted for each energy interval by the parameters obtained in the fitting procedure.

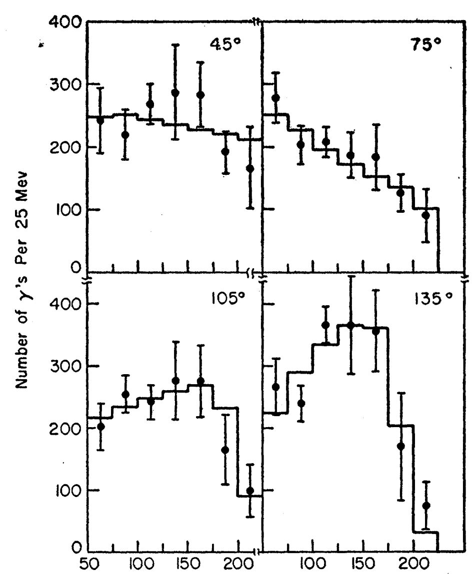

FIG. 4. Energy distributions of the charge-exchange gamma rays at each of the four angles of observation. The histograms indicate the numbers predicted by the parameters obtained in the fitting procedure. The points are the corresponding observed numbers, with the assigned errors attached. The errors shown are the combination of the statistical errors and the error arising from uncertainties in the calibration.

[6] R. L. Walker, J. G. Teasdale, V. Z. Peterson, and J. I. Vette, Phys. Rev. **99**, 210 (1955).
[7] R. H. Land, Phys. Rev. **113**, 1141 (1959).

TABLE III. Comparison of the results of this paper with those of reference 3 (Ashkin et al.). The coefficients are for the expansion $d\sigma/d\Omega = a + b\cos\theta + c\cos^2\theta$.

Coefficient	a	b	c
Present paper	1.61 ± 0.26	-1.54 ± 0.30	5.35 ± 0.88
Reference 3 normalized to present total cross sec.[a]	1.89 ± 0.15	-1.65 ± 0.13	4.46 ± 0.31
Reference 3 without renormalization	1.54 ± 0.09	-1.34 ± 0.09	3.63 ± 0.21

[a] The quoted errors on these include a contribution from the uncertainty in the normalization factor.

V. DISCUSSION

A direct comparison of the measurement reported in this paper can be made with the earlier results of Ashkin et al.,[3] which is $\sigma_{\text{tot}}(\pi^- \rightarrow \pi^0) = 34.6\pm1.2$ mb. The difference is mainly due to the method of calculating $\sigma_{\text{tot}}(\pi^- \rightarrow \pi^0)$. Ashkin et al., as was stated previously, normalized their value of the coefficients describing the angular distribution for charge exchange to the difference between their total transmission measurement and the integrated value of their elastic scattering measurement. Kruse and Arnold[8] have recently remeasured both of these cross sections and find a higher value for the total transmission measurement. There is good agreement between the present results and the results of Ashkin et al., if the angular distribution they report is renormalized to agree with the total cross section reported here. This amounts to an increase of 23% for their total cross section and all of their coefficients. The results of this comparison are contained in Table III. The present result agrees well with the result of Kruse and Arnold at 152 Mev. Their result is $\sigma_{\text{tot}}(\pi^-,\pi^0) = 40.4$ mb.

VI. ACKNOWLEDGMENTS

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The author is also indebted to Professor V. L. Telegdi and Professer S. C. Wright for their interest and advice.

[8] U. E. Kruse and R. C. Arnold, Phys. Rev. **116**, 1008 (1959). The value quoted for their result has been calculated by the present author from the angular distribution for elastically scattered pions and the total transmission measurement reported in their paper. No attempt has been made to calculate the error on their values.

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Charge-Exchange Scattering of Negative Pions at 61 Mev and 95 Mev*

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The charge-exchange scattering of negative pions by liquid hydrogen has been measured at 61 ± 1 Mev and 95 ± 2 Mev bombarding energy. The measurements were made with a gamma-ray spectrometer which employs a lead glass Čerenkov counter. If the charge exchange scattering cross section is expanded as a series of Legendre polynomials in the center-of-mass system of the collision, we find that at 61 Mev,

$$d\sigma/d\Omega = (1.00\pm0.05)[0.613\pm0.030 - (0.830\pm0.068)P_1(\cos\theta') + (0.183\pm0.150)P_2(\cos\theta')],$$

and at 95 Mev,

$$d\sigma/d\Omega = (1.00\pm0.03)[1.05\pm0.05 - (1.15\pm0.12)P_1(\cos\theta') + (0.33\pm0.25)P_2(\cos\theta')].$$

The total cross section for charge exchange, obtained by integration, is: $\sigma_{\text{tot}}(\pi^- \rightarrow \pi^0) = 7.7\pm0.6$ mb at 61 Mev and $\sigma_{\text{tot}}(\pi^- \rightarrow \pi^0) = 13.2\pm0.8$ mb at 95 Mev.

A table summarizing the measurements performed by this group at 61 Mev, 95 Mev, 128 Mev, and 150 Mev is given.

I. INTRODUCTION

THE study of the charge exchange reaction,

$$\pi^- + p \rightarrow \pi^0 + n \rightarrow 2\gamma + n, \qquad (1)$$

which has been carried out for incident pions of 128 Mev with the aid of a lead glass, gamma-ray spectrometer,[1] is extended in the present work to pions of 95 Mev and 61 Mev incident energy. Although some modifications of the apparatus and its mode of operation were made in this work, the technique is essentially the same as that used earlier. A similar measure-

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