Surface Transport in Semiconductors

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A transport theory is given for electrons and holes in space-charge layers at semiconductor surfaces. For diffuse surface scattering, the effective surface mobilities may differ significantly from the bulk mobility for any strength of space-charge layer. Agreement with Schrieffer's formulas is found only for strong space-charge layers, and the discrepancy is explained. The results are extended to cover an arbitrary degree of diffuseness of surface scattering and to cover samples of small thickness.

1. INTRODUCTION

IT is well known[1] that it is possible to induce in a semiconductor a surface space-charge layer containing enough additional carriers to alter strongly the conductance parallel to the surface. Schrieffer[2] has presented a transport theory for this situation, taking into account the decrease of mobility due to diffuse scattering from the surface. His treatment follows, in general, the analyses of Fuchs[3] and Sondheimer[4] of the conductance of thin metal films, but introduces the additional feature of a space-charge potential into the Boltzmann equation and into the equilibrium distribution function. In Schrieffer's theory,[2] however, the space-charge potential was approximated by a formula that suppresses the bulk current. The resulting formula for the mobility of the induced carriers is then satisfactory only when they are in deep potential wells at the surface.

Several attempts have been made to improve on this theory by taking into account the presence of the bulk current. Zemel[5] and Flietner[6] treated a truncated linear potential, and Frankl[7] treated the Poisson potential, all for infinitely thick slabs.

The present paper gives a surface transport theory retaining the exact Poisson potential, but otherwise following Schrieffer in the use of constant effective mass, constant relaxation time, and nondegenerate statistics. The case of a thick slab with completely diffuse surface scattering is treated in Sec. 2. The total current is shown to consist of a bulk current term plus a term containing the product of the surface excess of carriers and their effective surface mobility μ_S.

This μ_S agrees with Schrieffer's result for deep potential wells, as expected. However, contrary to what is usually expected, μ_S may be appreciably less than the bulk mobility, μ, even when the well is shallow or inverted. The formulas are evaluated for the particular case of electrons and holes in germanium and silicon at room temperature and presented graphically. It is pointed out that in some circumstances μ_S is not a true mobility and that, in fact, it may be negative.

In Sec. 3, the theory is extended to cover slabs whose thickness is comparable to the electrostatic screening distance, and whose surface scattering may not be entirely diffuse.

In Sec. 4, the formulas are discussed. A rough intuitive argument is presented to describe the main physical features of the theory. The reader not interested in the mathematical details of the Boltzmann theory may get most of the physical picture from this section.

2. BOLTZMANN THEORY FOR THICK SLABS

We consider a slab whose thickness $2d$ is much greater than the electrostatic screening distance[8] $L_D=(\kappa kT/8\pi n_i e^2)^{\frac{1}{2}}$. If an electric field E_z is induced normal to the surface $z=0$ the resulting equilibrium electrostatic potential u (in units of kT/e) goes from the surface value u_S at $z=0$ to the bulk value u_B for $z\gg L_D$. Measuring the potential relative to the intrinsic Fermi level, the variation in u is related to the variation of the carrier concentrations by

$$n=n_i e^u, \quad p=n_i e^{-u}. \tag{2.1}$$

Schrieffer[2] shows that the Boltzmann equation (in the ohmic approximation) for the electron distribution function f in the presence of a small uniform electric field E_z, parallel to the surface, can be readily solved

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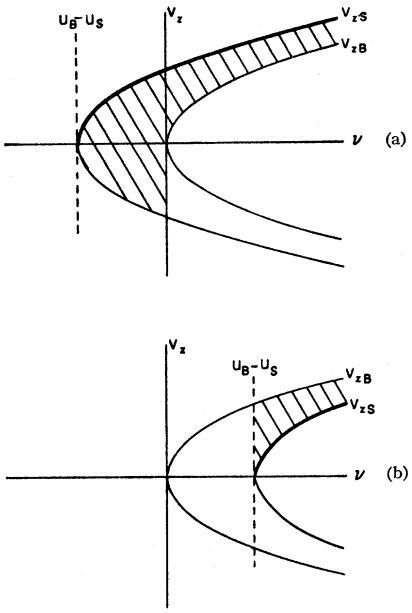

FIG. 1. Domains of solutions of Boltzmann equation; (a) for electrons attracted to surface, (b) for electrons repelled from surface.

by transforming from ordinary (z,v_z) space to a (ν,v_z) space where

$$\nu(z,v_z)=mv_z^2/kT-(u-u_B). \tag{2.2a}$$

The Boltzmann equation then takes the form

$$(\partial/\partial v_z)f_1e^K=f_{11}(\partial/\partial v_z)e^K, \quad (\nu=\text{const}), \tag{2.2b}$$

where

$$f_1=f-f_0, \tag{2.2c}$$

$$f_0=n_i(m/2\pi kT)^{\frac{3}{2}}\exp(u-mv^2/2kT), \tag{2.2d}$$

$$f_{11}=-e\tau E_xv_xf_0/kT, \tag{2.2e}$$

$$K=K(v_z,v_{z1})=-\int_{v_{z1}}^{v_z}mdv_z/e\tau E_z, \quad (\nu=\text{const}). \tag{2.2f}$$

We adopt, with Schrieffer,[2] the b.c. (boundary condition) of random surface scattering at $z=0$ (a more general condition is used in Sec. 3)

$$f_1(v_{zS})=0, \tag{2.3}$$

where v_{zS} is the z velocity of electrons leaving the surface $z=0$ with "energy" ν given by

$$\nu=mv_{zS}^2/2kT-(u_S-u_B), \quad v_{zS}\geq 0. \tag{2.4}$$

However, (2.3) does not determine f_1 completely. For electrons not bound in the surface potential well an additional b.c. is needed. The reason for this may be seen as follows: The domain of integration of (2.2b) is the strip in the (ν,v_z) plane bounded by the parabolas, Eq. (2.2a), for $z=0$ $(u=u_S)$ and $z=z_B$ $(u=u_B)$, shown in Fig. 1. Equation (2.3) fixes f_1 everywhere on the locus of v_{zS} (heavy lines in Fig. 1) and integration of (2.2b) along contours $\nu=$const (vertical lines in Fig. 1) generates f_1 at all *accessible* interior points of the domain, viz., the shaded regions of Fig. 1.

Thus, an additional b.c. is needed for the unshaded regions,[9] and here we set

$$f_1(-v_{zB})=f_{11} \tag{2.5}$$

where v_{zB} is the z velocity of electrons in the bulk with "energy" given by

$$\nu=mv_{zB}^2/2kT, \quad v_{zB}\geq 0. \tag{2.6}$$

This condition, which is consistent with (2.3), requires f_1 to approach, as $z\to d$, the distribution function given by the theory of bulk mobility.

The solution of (2.2b) satisfying (2.5) and (2.3) is

$$f_1=f_{11}(1-h), \tag{2.7a}$$

where, for $u_S\geq u_B$,

$$h=\exp[-K(v_z,v_{zS})] \quad \text{for} \quad \nu\leq 0, \tag{2.7b}$$

$$h=\exp[-K(v_z,v_{zS})] \quad \text{for} \quad \nu\geq 0 \quad \text{and} \quad v_z\geq 0, \tag{2.7c}$$

$$h=0 \quad \text{for} \quad \nu\geq 0 \quad \text{and} \quad v_z<0, \tag{2.7d}$$

and, for $u_S\leq u_B$

$$h=\exp[-K(v_z,v_{zS})] \quad \text{for} \quad \nu\geq u_B-u_S \quad \text{and} \quad v_z\geq 0, \tag{2.7e}$$

$$h=0 \quad \text{for} \quad \nu\geq u_B-u_S \quad \text{and} \quad v_z<0, \tag{2.7f}$$

$$h=0 \quad \text{for} \quad \nu\leq u_B-u_S. \tag{2.7g}$$

The behavior of f_1 near the other surface $z=2d$ is easily gotten from (2.7) by symmetry arguments.

The electron current density and the electron current in a unit strip are

$$j_x(z)=\int_{-\infty}^{\infty}dv_x\int_{-\infty}^{\infty}dv_y\int_{-\infty}^{\infty}dv_z\,(-ev_xf_1), \tag{2.8}$$

$$I_n=2\int_0^d j_x(z)dz$$
$$=2eE_xn_B\mu_n(m/2\pi kT)^{\frac{3}{2}}\int_0^d dz$$
$$\times\int_{-\infty}^{\infty}dv_z(1-h)\exp(-\nu), \tag{2.9}$$

where $\mu_n=e\tau/m$ is the bulk mobility. The first term of (2.9) may be integrated once directly, and the second term may be integrated once after transforming the integration variables to (v_z,ν). Note that the transformation Jacobian is $(-kT\mu_n/e)$ $(\partial K/\partial v_z)$ and that $K(v_{zB},v_{zS})=\infty$. After some manipulation one finds

$$I_n=2eE_xn_B\mu_n(d-\lambda_n)-2eE_x\mu_{nS}\Delta N, \tag{2.10a}$$

[9] In Schrieffer's approximation, only the accumulation layer for electrons is treated, and the parabolic strip is of infinite width, so that no additional b.c. is needed.

where

$$\Delta N=\int_0^d dz\,(n-n_B)\quad(d\to\infty)\qquad(2.10b)$$

is the surface excess of electrons,[10,11]

$$\lambda_n=\tau(kT/2\pi m)^{\frac{1}{2}}\qquad(2.10c)$$

is a thermal mean free path, and μ_{nS} is the effective surface mobility, given by

$$\mu_{nS}=\mu_n[1-(\lambda_n n_B/\Delta N)(e^{u_S-u_B}-1)],\quad u_S\leq u_B,\qquad(2.10d)$$

$$\mu_{nS}=\mu_n\left(1-\frac{\lambda_n n_B}{\Delta N}\int_{u_B-u_S}^0 e^{-\nu}(1-e^{2K(v_{zS},0)})d\nu\right),$$

$$u_S\geq u_B.\qquad(2.10e)$$

Kingston and Neustadter[10] have given a convenient graphical solution of the Poisson equation for surface space-charge situations such as the present one (i.e., uniform donor and acceptor concentration, complete ionization, nondegenerate statistics):

$$(eL_D/kT)E_z=F(\Delta u,u_B)=-F(-\Delta u,-u_B),\qquad(2.11a)$$

where

$$F(\Delta u,u_B)=2\sinh\frac{\Delta u}{2}\times\left\{\cosh u_B\left[1+\tanh u_B\frac{\sinh\Delta u-\Delta u}{\cosh\Delta u-1}\right]\right\}^{\frac{1}{2}},\qquad(2.11b)$$

and $\Delta u=u-u_B$. The surface excesses of electrons and holes are then expressed as

$$\Delta N=n_i L_D G(-u_S,-u_B),\qquad(2.12a)$$

$$\Delta P=n_i L_D G(u_S,u_B),\qquad(2.12b)$$

where

$$G(-u_S,-u_B)=e^{+u_B}\int_0^{u_S-u_B}d(\Delta u)(e^{\Delta u}-1)/F(\Delta u,u_B).\qquad(2.12c)$$

Using these solutions of Poisson's equation we may re-express (2.10) as

$$\mu_{nS}/\mu_n=1-\frac{e^{u_B}(\lambda_n/L_D)}{G(-u_S,-u_B)}[e^{u_S-u_B}-1]\quad(u_S\leq u_B),\qquad(2.13a)$$

$$\mu_{nS}/\mu_n=1-\frac{e^{u_B}(\lambda_n/L_D)}{G(-u_S,-u_B)}\int_{u_B-u_S}^0 d\nu e^{-\nu}(1-e^{2K_n})\quad(u_S\geq u_B),\qquad(2.13b)$$

where

$$2K_n=-(L_D/\lambda_n)\pi^{-\frac{1}{2}}\int_{-\nu}^{u_S-u_B}d(\Delta u)(\Delta u+\nu)^{-\frac{1}{2}}/F\leq 0.\qquad(2.13c)$$

The corresponding formulas for μ_{pS}/μ_p for holes may be obtained from (2.13) by replacing u_S and u_B by $-u_S$ and $-u_B$, respectively, ν by $-\nu$ in (2.13c) and n by p, throughout.

We have evaluated μ_{nS}/μ_n and μ_{pS}/μ_p numerically for a range of values of u_S and u_B for germanium and silicon. Using the data compiled by Conwell[12] for the conductivity effective masses and drift mobilities of n-Ge and n-Si one finds (at 300°K)

$$L_D/\lambda_n=33.3\text{ (Ge)},\quad=2270\text{ (Si)}.$$

Also we use

$$L_D/\lambda_{ph}=54.8\text{ (Ge)},\quad=4610\text{ (Si)}\quad\text{(heavy holes)},$$
$$L_D/\lambda_{pl}=18.3\text{ (Ge)},\quad\cdots\quad\text{(light holes)}.$$

Here we obtained the mobilities of the light and heavy holes in germanium from the formula

$$\mu_p=(p_h\mu_{ph}+p_l\mu_{pl})/(p_l+p_h),\qquad(2.14)$$

using the ratio $p_l/p_h=0.0423$ given by Lax and Mavroides[13] and using the ratio $\mu_{pl}/\mu_{ph}=8$ estimated from the magnetic field dependence of the Hall coefficient by Willardson, Harman, and Beer[14] and from magneto-surface experiments by Zemel and Petritz.[15] The corresponding situation in silicon is somewhat obscure, but we provisionally represent p-type silicon in this calculation by neglecting the light holes.

Programs for evaluating μ_S/μ numerically from (2.13) have been written by Thomas[16] for an IBM 704 computer and A. Brown[17] for a Bendix G15D computer. These results are presented in Figs. 2–6 in a somewhat unconventional form, suggested by Brattain,[18] that minimizes the amount of interpolation needed to obtain the usual plot of μ_S/μ vs u_S for arbitrary values of u_B. The latter is illustrated in Fig. 7 for light holes in germanium at 300°K.

The values of L_D/λ in Figs. 2–6 cover a sufficiently broad range that most practical cases can be handled by interpolation with respect to this parameter, remembering to change the signs of u_S and u_B whenever the sign of the carrier charge is changed.

3. THIN SLABS

We now consider a semiconductor slab having surface charge density ω on both faces, counterbalanced

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[11] C. G. B. Garrett and W. H. Brattain, Phys. Rev. **99**, 376 (1955).
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[18] W. H. Brattain (private communication).

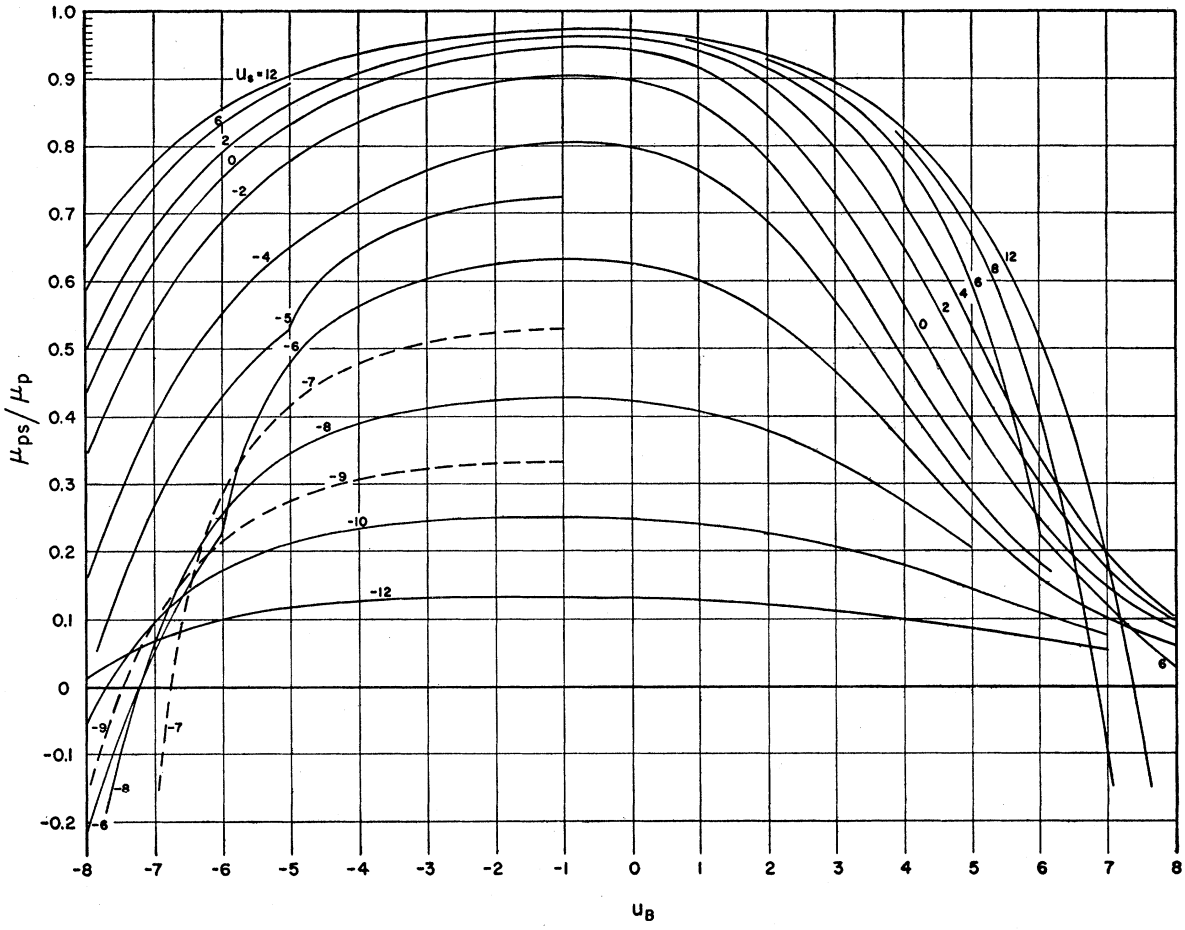

FIG. 2. Surface mobilities of holes with $L_D/\lambda=18.3$, e.g., light holes in germanium at 300°K.

by an internally distributed space-charge. (Such a situation might result, for example, from exposure of the slab to some gaseous ambient.) When the slab thickness $2d$ is *not* large compared to the screening length L_D, the space-charge may extend entirely across the slab. Then the potential $u=\ln(n/n_i)$ will differ from the bulk value u_B even at the slab center, and u will everywhere depend upon the surface value u_S. We now treat the electron transport in this situation, introducing, furthermore, the possibility that the surface scattering may not be entirely diffuse.

If a small uniform electric field E_x is applied parallel to the surface, the distribution function will be symmetric about the midplane $z=d$. We can then confine our attention to the region $0\leq z\leq d$, noting that the boundary condition (2.5) has to be replaced here by

$$f_1(v_{zd})=f_1(-v_{zd}), \tag{3.1}$$

where $mv_{zd}^2/2kT=\nu+(u_d-u_B)$. For the boundary condition at $z=0$ we use Fuchs'[3,4] simple scheme of interpolating between the model of completely diffuse surface scattering, viz.:

$$f_1(v_{zS})=0, \tag{2.5}$$

and the model of specular reflection, viz.:

$$f_1(v_{zS})=f_1(-v_{zS}). \tag{3.2}$$

Fuchs[3,4] introduces an interpolation parameter w_n such that

$$f_1(v_{zS})=w_nf_1(-v_{zS}),\quad 0\leq w_n\leq 1. \tag{3.3}$$

Fuchs interprets w_n as the probability that an electron striking the surface will be specularly reflected.

The solution of the Boltzmann equation (2.2b) satisfying (3.1) and (3.3) is

$$f_1=f_{11}(1-h_w), \tag{3.4a}$$

where for $u_S\geq u_B$,

$$h_w=\frac{(1-w_n)\exp[-K(v_z,v_{zS})]}{1-w_n\exp[2K(v_{zS},0)]}$$

$$\text{for}\quad \nu\leq u_B-u_d, \tag{3.4b}$$

$$h_w=\frac{(1-w_n)\exp[-K(v_z,v_{zS})]}{1-w_n\exp[2K(v_{zS},v_{zd})]}$$

$$\text{for}\quad \nu\geq u_B-u_d,\ v_z\geq 0, \tag{3.4c}$$

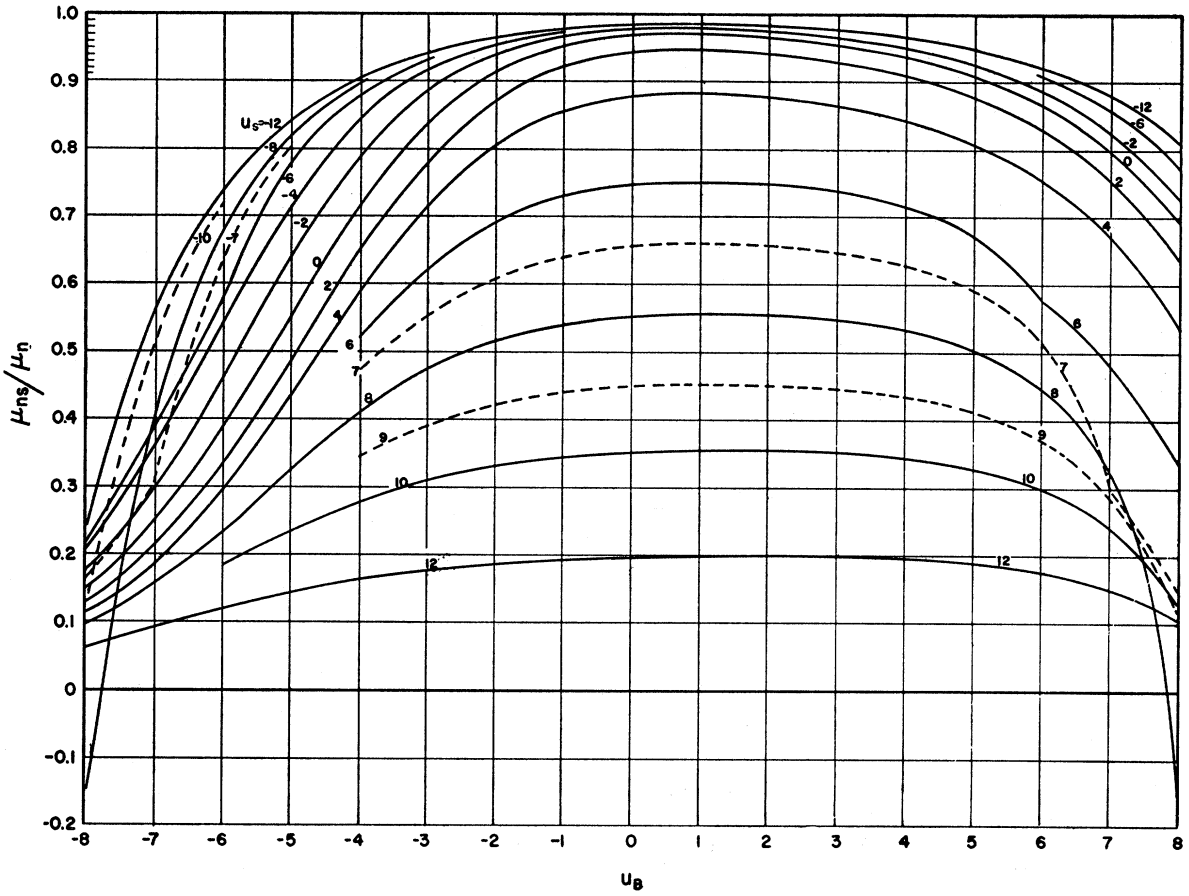

FIG. 3. Surface mobilities of electrons with $L_D/\lambda=33.3$, e.g., electrons in germanium at 300°K.

$$h_w=\frac{(1-w_n)\exp[-K(v_z,-v_{zS})+2K(v_{zS},v_{zd})]}{1-w_n\exp[2K(v_{zS},v_{zd})]}$$

$$\text{for}\quad \nu\geq u_B-u_d,\ v_z\leq 0, \tag{3.4d}$$

and, for $u_S\leq u_B$

$$h_w=0 \qquad \text{for}\quad \nu\leq u_B-u_S, \tag{3.4e}$$

$$h_w=\frac{(1-w_n)\exp[-K(v_z,v_{zS})]}{1-w_n\exp[2K(v_{zS},v_{zd})]}$$

$$\text{for}\quad \nu\geq u_B-u_S,\ v_z\geq 0, \tag{3.4f}$$

$$h_w=\frac{(1-w_n)\exp[-K(v_z,-v_{zS})+2K(v_{zS},v_{zd})]}{1-w_n\exp[2K(v_{zS},v_{zd})]}$$

$$\text{for}\quad \nu\geq u_B-u_S,\ v_z\leq 0. \tag{3.4g}$$

The everpresent factor $(1-w_n)$ in h_w serves as a reminder that h_w describes the momentum dissipation by the surface scattering mechanism. The modulation of this dissipation by the surface electric field is less apparent but can be seen to occur via the functions K.

The current (per unit strip of y direction) is computed as in Sec. 2, resulting in an expression similar to (2.10a)[19]:

$$I_n=eE_x n_B\mu_n[2d-2\lambda_n(1-w_n)]+2eE_x\mu_{nS}\Delta_d N, \tag{3.5a}$$

where

$$\Delta_d N=\int_0^d dz(n-n_B), \tag{3.5b}$$

$$\mu_{nS}/\mu_n=1-\frac{(1-w_n)\lambda_n n_B}{\Delta_d N}\Bigg(e^{(u_S-u_B)}-1-(1-w_n)\int_{u_B-u_S}^{u_B-u_d}\frac{d\nu\exp[-\nu+2K_n(v_{zS},0)]}{1-w_n\exp[2K_n(v_{zS},0])}-(1-w_n)\int_{u_B-u_d}^{\infty}\frac{d\nu\exp[-\nu+2K_n(v_{zS},v_{zd})]}{1-w_n\exp[2K_n(v_{nS},v_{zd})]}\Bigg),$$

$$u_S\geq u_B, \tag{3.5c}$$

[19] *Note added in proof:* When $\lambda_n\gtrsim d$ and $\Delta_d N\to 0$ one can show that $\mu_{ns}\Delta_d N$ remains finite and I_n remains positive.

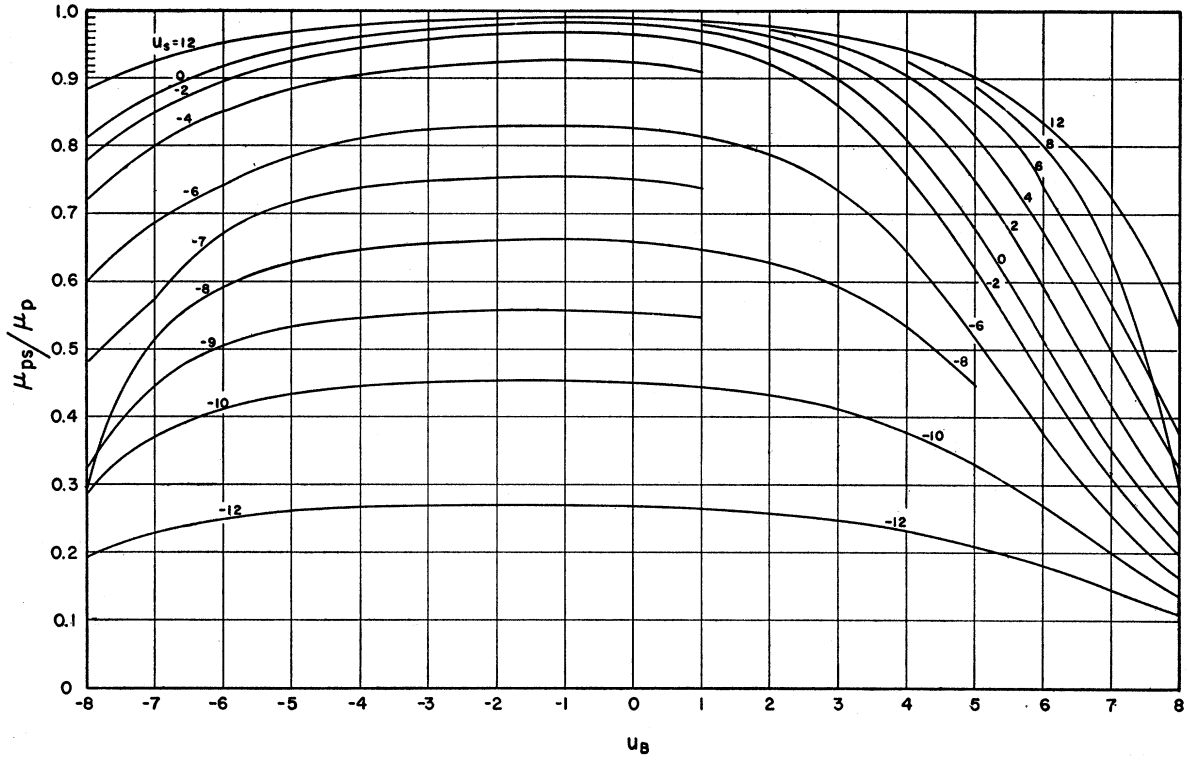

FIG. 4. Surface mobilities of holes with $L_D/\lambda=54.8$, e.g., heavy holes in germanium at 300°K.

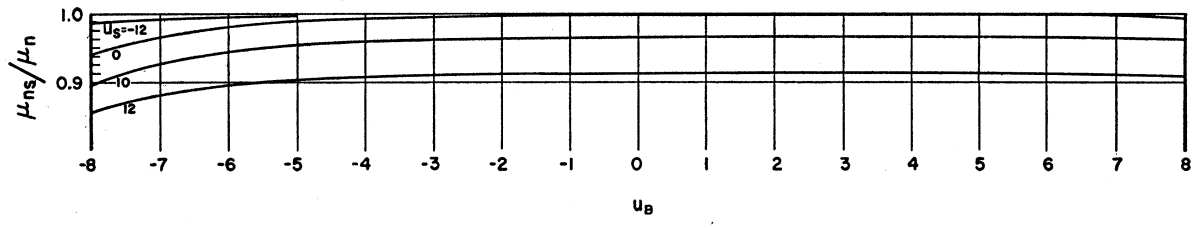

FIG. 5. Surface mobilities of electrons with $L_D/\lambda=2270$, e.g., electrons in silicon at 300°K.

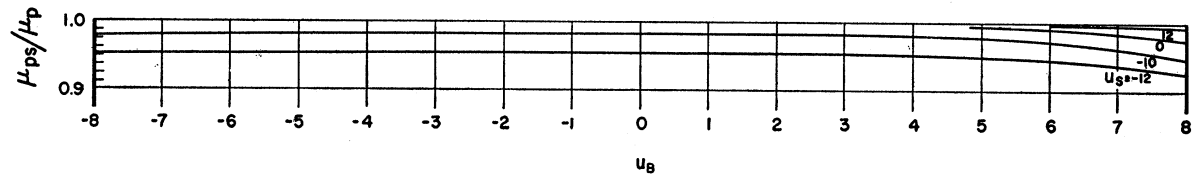

FIG. 6. Surface mobilities of holes with $L_D/\lambda=4610$, e.g., holes in silicon at 300°K.

$$\mu_{nS}/\mu_n=1-\frac{(1-w_n)\lambda_n n_B}{\Delta_d N}\left(e^{(u_S-u_B)}-1-(1-w_n)\int_{u_S-u_B}^{\infty}\frac{d\nu\,\exp[-\nu+2K_n(v_{zS},v_{zd})]}{1-w_n\exp[2K_n(v_{zS},v_{zd})]}\right),\quad u_S\leq u_B. \tag{3.5d}$$

When

$$d\gg\lambda_n, \tag{3.6a}$$

these formulas simplify somewhat because then

$$-2K(v_{zS},v_{zd})\gg 1, \tag{3.6b}$$

except for the few electrons having $\nu\gg 1$. Equation

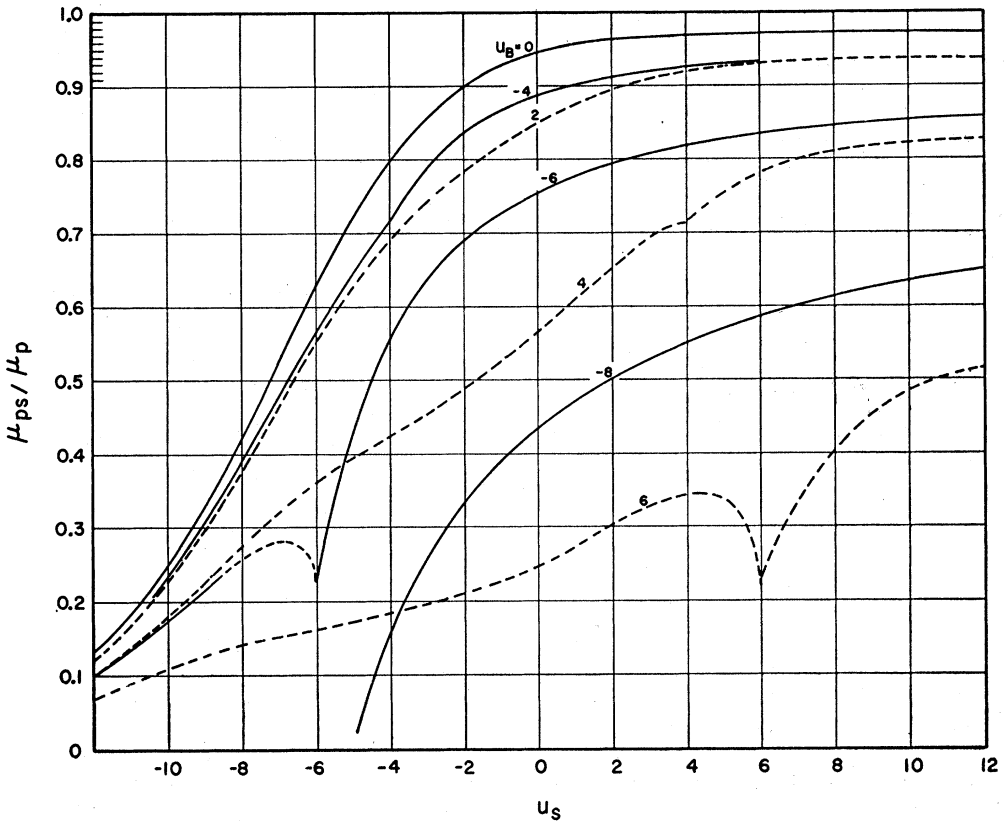

FIG. 7. Surface mobilities of holes with $L_D=18.3$, data of Fig. 2 plotted in conventional manner.

(3.5) then yields

$$\frac{\mu_{nS}}{\mu_n}=1-\frac{(1-w_n)\lambda_n n_B}{\Delta_d N}\left(e^{(u_S-u_B)}-1 - (1-w_n)\int_{u_B-u_S}^{u_B-u_d}\frac{d\nu\,\exp[-\nu+2K_n(v_{zS},0)]}{1-w_n\exp[2K_n(v_{zS},0)]}\right) \quad \text{for } u_S\geq u_B, \tag{3.7a}$$

$$\frac{\mu_{nS}}{\mu_n}=1-\frac{(1-w_n)\lambda_n n_B}{\Delta_d N}(e^{(u_S-u_B)}-1) \quad \text{for } u_S\leq u_B. \tag{3.7b}$$

These formulas may be written in a form analogous to (2.13) if we extend the formalism of Kingston and Neustadter[10] to cover the electrostatistics of slabs of finite thickness. The solution of Poisson's equation now has to satisfy the symmetry condition

$$E_x=0 \quad \text{at} \quad z=d. \tag{3.8}$$

The electric field E_z is then given by

$$F(\Delta u,u_d,u_B)=kTE_z/eL_D = -F(-\Delta u,-u_d,-u_B), \tag{3.9a}$$

where

$$F(\Delta u,u_d,u_B)=2\sinh\frac{\Delta u}{2}\left\{\cosh u_d\left[1+\tanh u_d \times\left(\frac{\sinh\Delta u-\Delta u(\sinh u_B/\sinh u_d)}{\cosh\Delta u-1}\right)\right]\right\}^{\frac{1}{2}}. \tag{3.9b}$$

This is a somewhat more cumbersome calculation than that for a thick slab inasmuch as some preliminary numerical calculation is necessary here even to get u_S and u_d. These are determined from the auxiliary formulas

$$d=\int_0^d dz=-L_D\int_{u_S}^{u_d}du/F(\Delta u,u_d,u_B), \tag{3.10}$$

and (Gauss' law)

$$F(\Delta_S u,u_d,u_B)=\omega/2n_i eL_D \quad (\Delta_S u=u_S-u_B), \tag{3.11}$$

where ω is the surface charge density on each face. The analog of (2.12) for the electron and hole excesses is

$$\Delta_d N=\int_0^d dz\,(n-n_B) = n_i L_D G(-u_S,-u_d,-u_B), \tag{3.12a}$$

$$\Delta_d P=\int_0^d dz\,(p-p_B)=n_i L_D G(u_S,u_d,u_B), \tag{3.12b}$$

where

$$G(-u_S, -u_d, -u_B) = e^{u_B}\int_{u_d-u_B}^{u_S-u_B} \frac{d(\Delta u)(\exp\Delta u - 1)}{F(\Delta u, u_d, u_B)}. \quad (3.12c)$$

Using (3.12), we find analogs of the thick slab formulas (2.13):

$$u_{nS}/\mu_n = 1 - \frac{(1-w_n)e^{u_B}(\lambda_n/L_D)}{G(-u_S, -u_d, -u_B)}\left\{e^{u_S-u_B} - 1 - (1-w_n) \times \int_{u_B-u_S}^{u_B-u_d} \frac{dv\, \exp[-v+2K_n(v_{zS},0)]}{1-w_n \exp[2K_n(v_{zS},0)]}\right\} \quad (d \gg \lambda_n,\ u_S \geq u_B), \quad (3.13a)$$

$$\mu_{nS}/\mu_n = 1 - \frac{(1-w_n)e^{u_B}(\lambda_n/L_D)}{G(-u_S, -u_d, -u_B)}[e^{u_S-u_B} - 1] \quad (d \gg \lambda_n,\ u_S \leq u_B). \quad (3.13b)$$

Here K_n is as defined in (2.13) except that $F(\Delta u, u_B)$ is replaced by $F(\Delta u, u_d, u_B)$. The corresponding formulas for holes are obtained by the same process as in the thick slab case, described after formulas (2.13).

4. DISCUSSION

The formulas derived in Sec. 2 can be made plausible by the following rough arguments. Consider a slab of thickness $2d$ with electron concentration n_B and, on each face, a square potential well of width L having additional concentration Δn. The effect of the surface scattering can be represented as producing a "stagnant layer" at each face. For the bulk carriers, not confined in the well, the thickness of this layer should be of the order of the bulk mean free path λ, whereas for the excess carriers, confined in the well, the layer will have some other thickness l, to be determined. The total electron current per unit strip will than be

$$I_n = 2eE_x n_B \mu_n (d-\lambda) + 2eE_x (L\Delta n)\mu_n (1 - l/L), \quad (4.1)$$

which essentially agrees in form with Eq. (2.10a). Now, as an estimate, we set $l = gL$, where g is the probability that the electrons in the well collide with the surface rather than in the bulk. For all the electrons starting at depth z, this probability is

$$\tfrac{1}{2}\exp(-z/\lambda) + \tfrac{1}{2}\exp[-(2L-z)/\lambda],$$

and, averaging over z, we find

$$g = (\lambda/2L)[1 - \exp(-2L/\lambda)].$$

Hence, the bracketed factor in (4.1) becomes

$$\mu_{nS} = \mu_n[1 - (\lambda/2L)(1 - e^{-2L/\lambda})]. \quad (4.2)$$

The Boltzmann result (2.13b) can be identified as a suitable energy average of (4.2) by rewriting (2.2f) in the form

$$K_n(v_{zS}, 0) = -\int_0^{L(v)} \frac{dz}{\tau v_z} \sim \frac{L(v)}{\lambda}. \quad (4.3)$$

The foregoing treatment, though instructive, suffers from the defect of relaying on an artificial separation of the carriers into two sets. Actually, the surface mobility formula includes not only the current contribution of the excess carriers, but also an alteration of the mobility of the bulk carriers brought about by the bending of their trajectories toward or away from the scattering surface. The competition between these two effects may be seen if we rewrite μ_{nS} in the form (Appendix A)

$$\frac{\mu_{nS}}{\mu_n} = \frac{\lambda_n n_B}{\Delta N}\left\{\int_{u_B-u_S}^{0} dv\, e^{-v}[e^{2K_n} - 2K_n - 1] + 2\int_0^{\infty} dv\, e^{-v}\int_0^d \frac{dz}{\tau}\left(\frac{v_{zB}-v_z}{v_{zB}v_z}\right)\right\}, \quad (4.4)$$

for electrons when $u_S \geq u_B$. The first term is always positive, while the second is negative because $v_z \geq v_{zB}$ when $u_S \geq u_B$. It is the latter term that is responsible for the sharp decrease in μ_{nS} when $u_S \cong u_B$, apparent in some of the curves of Fig. 7. If L_D is sufficiently small, it is even possible for μ_{nS} to become negative when the bands are nearly flat.

Although the formulas for depletion differ from those for accumulation, μ_{nS} is a continuous function of u_S at $u_S = u_B$. The limiting value is

$$\mu_{nS}/\mu_n = 1 - \frac{\lambda_n}{L_D/(\cosh u_B)^{\frac{1}{2}}} \quad (u_B = u_S). \quad (4.5)$$

Noting that the true bulk screening distance is $L_D(\cosh u_B)^{-\frac{1}{2}}$, it is clear that the flat-band surface mobility becomes negative when the bulk screening distance becomes less than λ_n, so that the space-charge region is narrower than the "stagnation layer." In such cases μ_{nS} is not a true mobility. In fact a strict local proportionality between current density and field does not exist when the field varies appreciably within λ: instead the current density at a point is influenced by the field strength at all points within a few mean free paths.

APPENDIX A. ALTERNATE FORM OF μ_{nS}

Integrating both terms of (2.9) in the (v_z, v) coordinate system one finds, for $u_S \geq u_B$

$$I_n=2eE_x n_B\mu_n\lambda_n \times\left(\int_{u_B-u_S}^{0} d\nu\, e^{-\nu}[e^{2K_n(v_{zS},0)}-2K_n(v_{zS},0)-1] + \int_0^{\infty} d\nu\, e^{-\nu}[e^{-K_n(v_{zB},v_{zS})}+2K_n(v_{zB},v_{zS})-1]\right). \quad (A.1)$$

But

$$K_n(v_{zB},v_{zS})=-\int_{v_{zB}}^{v_{zS}}\frac{m dv_z}{e\tau E_z}=\int_0^d dz/\tau v_z \quad (A.2)$$

$$=\frac{d}{\tau v_{zB}}-\int_0^d\frac{dz}{\tau}\left(\frac{v_z-v_{zB}}{v_z v_{zB}}\right). \quad (A.3)$$

Thus, for $d\gg\lambda_n$, the first term of the second integral in (A.1) vanishes, and, noting that $\nu=mv_{zB}^2/2kT$, (A.1) becomes

$$I_n=2eE_x n_B\mu_n(d-\lambda_n)+2eE_x\Delta N\mu_n\frac{n_B\lambda_n}{\Delta N} \times\left\{\int_{u_B-u_S}^{0} d\nu\, e^{-\nu}[e^{2K_n}-2K_n-1] +2\int_0^{\infty} d\nu\, e^{-\nu}\int_0^d\frac{dz}{\tau}\left(\frac{v_{zB}-v_z}{v_z v_{zB}}\right)\right\}, \quad (A.4)$$

from which (4.4) follows.

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Zeeman Effect of Impurity Levels in Silicon*

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Completely resolved Zeeman spectra for the bismuth donor in silicon including optical transitions from the $1s$ donor ground state to the excited states $2p_0$, $2p_\pm$, $3p_0$, $4p_0$, $3p_\pm$, $5p_0$, $4p_\pm$, and $5p_\pm$ are presented. The transitions were observed at liquid helium temperature, using linearly polarized radiation alternately parallel and perpendicular to the magnetic field, and field intensities up to 38.9 kilogauss oriented along each of the three principal crystallographic axes. Both linear splitting of the $p_\pm$ states and a quadratic dependence on field were observed. The use of impurity Zeeman spectra is demonstrated for evaluating effective mass parameters, determining the nature of energy bands and finding and identifying impurity excited states. The transverse effective mass for the electron in silicon was found to be $(0.186\pm0.006)\ m_0$ in agreement with recent cyclotron resonance results. From Zeeman splitting, electron effective masses up to $0.5\ m_0$ can be measured to within $\pm\frac{1}{2}\%$ at infrared frequencies in a field of 40 kilogauss. The behavior in a magnetic field of the first two donor excited states could be explained by treating the magnetic terms of the Hamiltonian as a perturbation to first order. Interactions among the higher closely-spaced Zeeman levels were observed above 20 kilogauss and were evaluated with a second-order treatment. The Zeeman structure for the aluminum acceptor reflected the complexity of the valence bands and the acceptor ground state and was in qualitative agreement with the theoretical results of Kohn and Schechter. Transitions were observed to eight excited states converging to the series limit. Evidence is given for the degeneracy of each state.

I. INTRODUCTION

IT has been realized that measurement of the Zeeman effect of the excited states of the monovalent donor and acceptor impurities in semiconductors offers a means of studying the energy band structure and the effective masses of carriers. Theoretical evaluation[1–3] of the linear Zeeman effect for donors in Ge and Si had indicated that the Zeeman splitting should be resolvable by spectroscopic techniques. For germanium, a linear splitting for the transition $1s\rightarrow 2p_\pm(m=\pm1)$ for both As and P impurities was reported by Fan and Fisher,[4] and similar measurements in the far infrared region were also reported by Boyle[5] for As impurity. Transitions from the ground state to quantized magnetic levels in the conduction band were also observed and the effective mass of the electron was determined. For silicon, both linear and quadratic Zeeman effects were observed for the bismuth donor[3] and a preliminary Zeeman spectrum for Si(Bi) was presented by Zwerd-

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