PHYSICAL REVIEW VOLUME 118, NUMBER 3 MAY 1, 1960

Muon Capture in He^3†

AKIHIKO FUJII
Department of Physics, Purdue University, Lafayette, Indiana
(Received November 20, 1959)

The hard-core wavefunction for a three-nucleon system is used to calculate the capture rate of the reaction $\mu^- + He^3 \rightarrow H^3$(ground state)$+\nu$. It is found to be 1.66×10^3 sec^{-1}.

THE capture rate of the reaction $\mu^- + He^3 \rightarrow H^3 + \nu$ was calculated by Fujii and Primakoff[1] in terms of the beta decay rate $H^3 \rightarrow He^3 + e^- + \bar{\nu}$. Use was made of Irving's wavefunction[2] for the three-nucleon system, but the calculated capture rate is supposed to depend on the adopted nuclear model fairly sensitively. Hence the same calculation has been repeated with a more realistic wavefunction.

The general expressions have been given already in the reference 1. For the reaction in question the problem is just reduced to find the numerical estimate of the parameter[3]

$$R = 1.04\times\left[\frac{1}{3}\int\left(\sum_{i=1}^{3}\exp(-i\mathbf{p}\cdot\mathbf{r}_i)\right)\psi^2(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3\right]^2,$$

where $\mathbf{p}$ is the neutrino momentum ($p=|\mathbf{p}|=5.26\times 10^{12}$ cm^{-1}), ψ is the wavefunction of the three-nucleon system and $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$ are the nucleon position vectors. The wave function proposed by Kikuta, Morita, and Yamada,[4,5] who took the effect of the hard core nuclear potential into account, reads

$$\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = N\prod_{i=1}^{3}\left[e^{-\mu(\rho_i-D)} - e^{-\nu(\rho_i-D)}\right] \quad \text{for all } \rho_i \geqq D,$$

$$\psi(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3) = 0 \qquad \text{otherwise},$$

where $\boldsymbol{\varrho}$'s are the internucleon distances

$$\rho_1 = |\boldsymbol{\varrho}_1| = |\mathbf{r}_2-\mathbf{r}_3|, \quad \rho_2 = |\boldsymbol{\varrho}_2| = |\mathbf{r}_3-\mathbf{r}_1|, \quad \rho_3 = |\boldsymbol{\varrho}_3| = |\mathbf{r}_1-\mathbf{r}_2|,$$

and N is a certain normalization constant. The parameters μ and ν are to be determined by variational calculation for a chosen core radius D. It is found that

$$R = 1.04\times[\eta(p)/\eta(0)]^2,$$

$$\eta(p) = \int \exp\left[-\frac{i}{3}\mathbf{p}\cdot(\boldsymbol{\varrho}_2-\boldsymbol{\varrho}_3)\right]\psi^2(\rho_1,\rho_2,\rho_3) \times\delta(\boldsymbol{\varrho}_1+\boldsymbol{\varrho}_2+\boldsymbol{\varrho}_3)d\boldsymbol{\varrho}_1 d\boldsymbol{\varrho}_2 d\boldsymbol{\varrho}_3$$

$$= 8\pi^2\sum_{l=0}^{\infty}(2l+1)(-1)^l\int\int\int j_l(p\rho_2/3)j_l(p\rho_3/3) \times P_l\left(\frac{\rho_2^2+\rho_3^2-\rho_1^2}{2\rho_2\rho_3}\right)\psi^2(\rho_1,\rho_2,\rho_3)\rho_1\rho_2\rho_3 d\rho_1 d\rho_2 d\rho_3,$$

in which j_l and P_l are the spherical Bessel and Legendre function of order l, respectively. The integration over ρ's is subject to the condition that ρ_1, ρ_2, ρ_3 form three sides of a triangle. One can see that the $l=0$ and 1 terms are dominant, since for higher l terms $|P_l|\leqq 1$, and the overlap between j_l and the wavefunction is very small hence hardly makes any appreciable contribution. The completely analytic formula is obtained for the $l=0$ term by the method of integration shown in the reference 4. For the $l=1$ term the approximation $D=0$ is good, and again the integral can be performed analytically.

The constants used for the numerical estimate are[5]: $D=0.40\times 10^{-13}$ cm, $\mu=0.500\times 10^{13}$ cm^{-1}, $\nu=4.27\times 10^{13}$ cm^{-1} for $l=0$, and $D=0$, $\mu=0.503\times 10^{13}$ cm^{-1}, $\nu=\infty$ for $l=1$. The ratio $\eta(p)/\eta(0)$ is found to be 0.932, thus $R=0.903$. The predicted capture rate is $w^{(\mu)}=1.66\times 10^3$ sec^{-1}, which is roughly 14% larger than the value $w^{(\mu)}=1.46\times 10^3$ sec^{-1} in the reference 1. The increase is expected if the hard core wavefunction gives smaller mean square radius ($\sim 1.5\times 10^{-13}$ cm) than Irving's wavefunction ($\sim 1.8\times 10^{-13}$ cm).[6]

The author expresses his thanks to Dr. M. E. Rose of Brookhaven National Laboratory for his interest and hospitality, to Dr. M. Morita of Columbia University for helpful discussions, and to Mr. R. Rose of Princeton University for assisting the computation.

† Work done at Brookhaven National Laboratory under the auspices of the U. S. Atomic Energy Commission in the summer, 1959.

[1] A. Fujii and H. Primakoff, Nuovo cimento **12**, 327 (1959).
[2] J. Irving, Phil. Mag. **42**, 338 (1951).
[3] Reference 1, Eqs. (9), (10), and (11).
[4] T. Kikuta, M. Morita, and M. Yamada, Progr. Theoret. Phys. (Kyoto) **15**, 222 (1956).
[5] T. Ohmura (to be published).
[6] Reference 1, Eq. (17).