High-Orbital S-State Capture of π^- Mesons by Protons*

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(Received November 30, 1959)

The consequences of the very short capture time for π^- mesons in liquid hydrogen, recently measured by Fields, Yodh, Derrick, and Fetkovich, are discussed. It is pointed out that collisional de-excitation mechanisms, even including the Stark effect enhancement of capture, seem inadequate to explain the experiment. Alternative possibilities are discussed.

THERE has been very little experimental evidence on the slowing-down and capture times for π^- mesons in liquid hydrogen, until quite recently.[1] Usually, in any situation which required estimates of these times, appeal was made to the calculation of Wightman and of Fermi and Teller.[2] [Panofsky *et al.*,[3] e.g., estimate total de-excitation times to the ground state for the (π^-,p) atom to be of the order $10^{-10}-10^{-9}$ sec.]

Fields *et al.*[1] recently measured the time which elapses between that point in the slowing down of a π^- meson in liquid H_2 when it has velocity $V\simeq 0.05c$ and the moment of capture by a proton. To within a factor of three, this time interval is found to be

$$t_{\text{exp}}\simeq 3.5\times 10^{-12}\text{ sec},$$

or

$$\gamma_{\text{exp}}\simeq 3.1\times 10^{11}\text{ sec}^{-1}. \tag{1}$$

This time is much shorter than average radiation times of excited states of the (π^-,p) atom, except for the $2P\rightarrow 1S$ transition, for which[4]

$$t_R(2P\rightarrow 1S)=6.8\times 10^{-12}\text{ sec}. \tag{2}$$

We would like to consider first whether the Stark effect and capture process previously proposed for the (K^-,p) atom can explain this new experimental result.[5] We briefly recapitulate this argument here in a slightly modified form. Consider a highly excited (π^-,p) atom moving through liquid H_2 with a velocity $V\simeq 10^6$ cm/sec.[6,7] When this neutral object penetrates within the electron cloud of a neighboring H atom, it feels a strong electric field ($\simeq e^2/a_0^2$, a_0= electron Bohr radius) which induces Stark oscillations between the degenerate angular momentum sub-levels of the given principal quantum number. The time for these oscillations is very short {e.g., for the π^- in the $n=6$ level, $t_{\text{Stark}}[6l\rightarrow 6\times(l-1)]\simeq 10^{-16}$ sec}. Hence, during the collision, the other states will be coupled with the S state, from which the pion can be captured.[8]

The capture rate for a pion in the S state may be estimated just as before for the K^- meson. Using the low-energy pion cross-section data,[9] we find

$$\gamma_{\text{capt}}{}^{1S}=1.1\times 10^{15}\text{ sec}^{-1}. \tag{3}$$

Thus, if the pion were in a state of the (π^-,p) atom with principal quantum number n, and magnetic quantum number $m_l=0$, there would be $n-1$ degenerate angular

* This research was supported in part by the U. S. Air Force Office of Scientific Research of the Air Research and Development Command and in part by the U. S. Atomic Energy Commission.

[1] T. Fields, G. B. Yodh, M. Derrick, and J. Fetkovich, Bull. Am. Phys. Soc. Ser. II, 4, 402 (1959); and private communication.

[2] A. S. Wightman, Phys. Rev. 77, 521 (1950); and Ph.D. thesis, Princeton University, 1949 (unpublished). E. Fermi and E. Teller, Phys. Rev. 72, 406 (1947).

[3] W. K. F. Panofsky, R. L. Aamodt, and J. Hadley, Phys. Rev. 81, 565 (1951).

[4] H. A. Bethe and E. E. Salpeter, *Quantum Mechanics of One- and Two-Electron Atoms* (Springer-Verlag, Berlin, 1957), p. 253.

[5] T. B. Day, G. A. Snow, and J. Sucher, Phys. Rev. Letters 3, 61 (1959).

[6] We wish to thank Professor S. B. Treiman and Professor R. Karplus for their many helpful comments on the original model of reference 5. We now believe that a velocity of 10^6 cm/sec is more likely than one of 10^5 cm/sec since the first few de-excitations of the atom release volts of energy to atomic motion, and thermalizing times are much longer than the time required for further collisions with energy release.

[7] R. K. Adair, Phys. Rev. Letters 3, 438 (1959) has questioned the estimates of reference 5. We agree with Adair's statement that the problem should be treated more accurately. However, although we have not been able to prove our results rigorously, we believe that Adair's criticism of reference 5, is incorrect, for the following reasons.

The dipole field giving rise to the Stark effect can be attractive. As a result, even for velocities as small as 10^5 cm/sec, the $(K^- - p)$ atom speeds up to energies of the order of the Stark shift energies ($\sim$1 ev). This means that the relative orbital angular momenta that are present at the time of absorption are $\sim$10, instead of $\sim$1 as Adair claims.

If one insists on calculating a cross section for the complete process, then one should include the absorption interaction (both for the transition rates and for level shifts) in which case the final reaction particles have a spectrum of final momenta, of their center of mass motion relative to the proton, extending to energies of the order of the energy release. This spectrum is heavily weighted about the value for elastic scattering with no absorption interaction as considered by Adair, but still with a width of the order of the Stark energies, corresponding to the speed-up discussed above.

Moreover, a simple Born approximation approach to the complete cross section calculation, such as done by Adair, is open to question. For example, such a calculation of the inelastic scattering process for $6P\rightarrow 6S$, with no level shift in the S state, and no absorption, gives a cross section of the order $7\times 10^5\pi a_0^2$ which would violate the optical theorem for such low-energy collisions. This indicates that any simple angular momentum barrier argument which uses the first Born approximation is very unreliable.

[8] The nP-state capture is smaller than the nS-state capture by several orders of magnitude; see K. Brueckner, R. Serber, and K. Watson, Phys. Rev. 81, 575 (1951).

[9] *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, New York, 1957), p. II-15.

momentum states with $m_l=0$ coupled with the nS state, and the Stark capture rate would be

$$\gamma_{\mathrm{capt}}{}^n(\mathrm{Stark})\simeq\frac{1}{n}\frac{1.1\times10^{15}}{n^3}\ \mathrm{sec}^{-1}. \tag{4}$$

Since the inverse of the transit time is

$$\gamma_{\mathrm{trans}}\simeq(\tfrac{2}{3}2a_0/V)^{-1}\simeq1.5\times10^{14}\ \mathrm{sec}^{-1}, \tag{5}$$

there is very little capture during a collision of the (π^-,p) atom. The amount may be estimated by saying that there is a cross section for capture of the pion in the n level with $m_l=0$ of

$$\sigma_{\mathrm{capt}}(m_l=0)\simeq[1-\exp(-\gamma_{\mathrm{capt}}{}^n(\mathrm{Stark})/\gamma_{\mathrm{trans}})]\pi a_0{}^2, \tag{6}$$

and by further saying that the number of $(n, m_l=0)$ states is only $1/n$ of the total number possible.[10] Thus, the average capture rate for the pion in an n state is

$$\begin{aligned}\Gamma_{\mathrm{coll}}{}^n(\mathrm{Stark})&\simeq NV(1/n)\sigma_{\mathrm{capt}}(m_l=0)\\ &=3\times10^{12}(1/n)[1-\exp(-7.3/n^4)]\ \mathrm{sec}^{-1},\end{aligned} \tag{7}$$

where N is the number density of H atoms in an H_2 bubble chamber. This is much smaller than the experimental rate, Eq. (1), for any of the initially high values of n. Thus, it is clear that the collisional Stark mechanism alone cannot explain the experimental result.[11]

Moreover, while it is true that the Stark collision rates, Eq. (7), are $\sim10\times$radiation rates for $n\sim3$–4, and thus could be expected to dominate for these n's, the time estimated by Wightman[2] for a pion to go from initial capture in a molecule to $n\sim4$ in an atom is $\sim5\times10^{-11}$ sec. Thus the experimental result seems to rule out also the possibility that the Stark mechanism takes over after the usual collisional de-excitation brings the pion to low n values. Let us, then, adopt Wightman's picture that the π^- slows down and is captured by replacing one of the electrons in the H_2 molecule more or less adiabatically. We now consider whether nuclear capture of the π^- for an isolated $(\pi^-,H_2{}^+)$ molecule can occur in times of the order of Eq. (1).

While the π^- is in a molecule it is in a strong, noncentral electric field at all times, rather than just during the time of collision with another atom, as before. For orientation, consider the pion in an orbit which has roughly the same root mean square radius and energy as the electron it replaces. As an approximate wave function, we could use the standard Heitler-London wave functions for the H_2 molecule.[12] This would correspond roughly to an excited (π^-,p) atom with $n\simeq12$–14 in the electric field of another H atom. Thus, we would have a situation similar to that considered before, only now the capture rate $\gamma_{\mathrm{capt}}{}^n(\mathrm{mol})$ would be given by

$$\frac{1}{n}\gamma_{\mathrm{capt}}{}^n(\mathrm{Stark})\lesssim\gamma_{\mathrm{capt}}{}^n(\mathrm{mol})\lesssim\gamma_{\mathrm{capt}}{}^n(\mathrm{Stark}). \tag{8}$$

[See Eq. (4) and reference 10.] In a molecule, the quantum number which corresponds to the m_l of the atom is Λ, and this is a good quantum number only if the coupling between pionic motion and nuclear vibration and rotation can be neglected. A measure of the validity of the approximations in a Heitler-London type wave function is the smallness of the expansion parameter $(m_\pi/M_p)^{\frac{1}{4}}$ (see reference 12, p. 263). Since the pion mass is so much larger than the electron mass, the simple hydrogenic wave functions used in a Heitler-London wave function are very poor approximations to the actual picture. Thus, the value of $\gamma_{\mathrm{capt}}{}^n(\mathrm{mol})$ is unknown, and Eq. (8) can only be considered as a very rough indication of the limits on $\gamma_{\mathrm{capt}}{}^n(\mathrm{mol})$. If $\gamma_{\mathrm{capt}}{}^n(\mathrm{mol})$ were approximately $\gamma_{\mathrm{capt}}{}^n(\mathrm{Stark})$ with $n\simeq12$, then this would explain the experimental result, Eq. (1).

Thus the results of this paper can be summarized as follows. The experimental number, Eq. (1) is so restrictive that the Stark effect process of reference 5 cannot explain the capture rates of pions in hydrogen. This is true either using the Stark mechanism alone at higher n's, or, if Wightman's results are correct, using the Stark mechanism after the usual collisional de-excitation processes have brought the π^- to low values of n. (Of course, the experiment says nothing about K^- mesons stopping in hydrogen, for which, presumably, the process of reference 5 could still be operative.[10])

Radiation is completely ineffective unless some process selectively populates only the $2P$ state. It appears from the μ-mesonic x-ray experiments of the Stearns[13] that there is some nonradiative process which competes very favorably with the K_α and L_α radiation in low Z materials (excluding H and He) and is roughly Z independent, with transition rate for the K_α case $\simeq6\times10^{13}$ sec^{-1}. No explanation of this anomaly exists, to our knowledge. If this process is still operative in hydrogen (which is not *a priori* clear), it is possible

[10] It should be pointed out that this additional factor of n represents the effect of neglecting Stark transitions between levels with different m_l, and corresponds to adiabatic motion of the atom through the electric field. Nonadiabatic motion will correspond to a smaller reduction factor, until the opposite limit is reached, where m_l is no longer a good quantum number at all, and all n^2 degenerate states are strongly Stark-coupled to the nS state. In this limit, the Stark capture rate of Eq. (5) becomes

$$\gamma_{\mathrm{capt}}{}^n(\mathrm{Stark})=(1/n^2)(\gamma_{\mathrm{capt}}{}^{1S}/n^3).$$

[11] From this point of view, the corresponding rates for the $(K^-,-p)$ atom are (with $V=10^6$ cm/sec)

$$n\Gamma_{\mathrm{coll}}{}^n(\mathrm{Stark})\simeq3\times10^{12}[1-\exp(-3.1\times10^3/n^4)]\ \mathrm{sec}^{-1}$$

$(\gamma_{\mathrm{capt}}{}^{1S}=4.7\times10^{17}$ sec^{-1} for the $K-$, p interaction.) Thus $\Gamma_{\mathrm{coll}}{}^n(\mathrm{Stark})$ is still $\sim100\times$radiation rates for $n\sim10$, as stated in reference 5, and the conclusions enumerated there are still valid.

[12] L. Pauling and E. B. Wilson, *Introduction to Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1935), p. 353.

[13] M. Stearns and M. B. Stearns, Phys. Rev. **105**, 1573 (1957).

that it is the explanation of the experiment of Fields *et al.*

As a final possibility, this paper proposes that the basic Stark effect model of reference 5 be applied to a (π^-,H_2^+) molecule. Thus, if it is assumed that such a molecule is formed, the result of Fields et al. could be accounted for with a molecular Stark effect. This would require a pion density at a proton of the same order as would be expected using a Heitler-London wave function for the pion in the Born-Oppenheimer approximation treatment of the molecule if the pion were in an $n\simeq 12$ state with $\Lambda=0$ only. While it is expected that this approximation is quite poor for the π-mesonic molecule, it is not unreasonable to expect that the true pion wave function has a comparable density at the proton.

ACKNOWLEDGMENTS

We would like to thank Professor L. S. Rodberg for many stimulating discussions of the π- and μ-mesonic problems. We would also like to thank Dr. T. Fields for sending us the experimental results prior to publication.

PHYSICAL REVIEW VOLUME 118, NUMBER 3 MAY 1, 1960

Interference Phenomena in Nuclear Scattering of Neutral K Mesons*

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(Received November 16, 1959)

The scattering of neutral K mesons has been treated phenomenologically. The scattered beam, in general, contains both K_0 and $\bar{K}_0$ components having different amplitudes. These amplitudes interfere with each other in the generation of K_1 and K_2 components in the scattered beam. The relative sign of the two amplitudes may then be determined from the analysis of K_1, K_2 decays. The leptonic decay rates of the scattered beam show a dependence on ΔM, the mass difference between K_1, K_2 in such a way that the sign of ΔM can, in principle, be determined experimentally.

I. INTRODUCTION

Among the elementary particles, the neutral K mesons present a unique situation. They occur as two distinct kinds of particles according to their strong and weak interactions. The weakly interacting particles—the short-lived K_1 and the long-lived K_2—have been described as linear combinations of the strongly interacting K_0 and $\bar{K}_0$ particles, and vice versa.[1] Recent experiments support this description.[2] The encounter of such a mixture of particles and antiparticles shows some interesting phenomena, such as characteristic interaction in dense matter[3] and the interference between K_1 and K_2 components in the leptonic decay modes.[4]

The scattering of neutral K mesons may be explored to obtain some interesting results. Whereas a neutral K beam in dense material (Pais-Piccioni experiment) loses almost all of its $\bar{K}_0$ component,[5] such a beam being scattered by protons would contain both K_0 and $\bar{K}_0$ components having different amplitudes. In the subsequent decays, these amplitudes would interfere, and the decay ratios may be helpful in determining the relative sign of the K_0 and $\bar{K}_0$ nuclear potentials. The interference in the leptonic decay modes would also be expected to be different from that of an unscattered beam.

II. NUCLEAR SCATTERING ON PROTONS

For simplicity, we start with a K_0 beam, allowing the K_1 component to decay almost completely, and consider the scattering of K_2 mesons on protons. The attenuation of the beam due to its decay may then be neglected, because the K_2 mean life is rather large.

The wave functions for the different components are

$$\begin{aligned}\psi(K_1)&=(1/\sqrt{2})[\psi(K_0)+\psi(\bar{K}_0)],\\ \psi(K_0)&=(1/\sqrt{2})[\psi(K_1)+i\psi(K_2)],\\ \psi(K_2)&=(1/\sqrt{2}i)[\psi(K_0)-\psi(\bar{K}_0)],\\ \psi(\bar{K}_0)&=(1/\sqrt{2})[\psi(K_1)-i\psi(K_2)].\end{aligned}\tag{1}$$

The wave function $\psi(K_2)$ is modified after scattering as

$$\psi_{\text{scat.}}=(1/\sqrt{2}i)\{[\tfrac{1}{2}(1-\eta_1)+\tfrac{1}{2}(1-\eta_0)]\psi(K_0)-[1-\bar{\eta}_1]\psi(\bar{K}_0)\},\tag{2}$$

where $\eta_{1,0}$ is $\exp(2i\delta_{1,0})$, $\delta_{1,0}$ corresponding to the real

* This work was done under the auspices of the U. S. Atomic Energy Commission.

[1] M. Gell-Mann and A. Pais, Phys. Rev. **97**, 1387 (1955).

[2] M. Bardon, K. Lande, L. M. Lederman, and W. Chinowsky, Ann. Phys. **5**, 156 (1958). Reference to earlier works can be found in this article.

[3] A. Pais and O. Piccioni, Phys. Rev. **100**, 1487 (1955).

[4] R. G. Sachs and S. B. Treiman, Phys. Rev. **103**, 1545 (1956).

[5] A more general analysis of K_0 mesons traversing an absorber in the regeneration of K_1 and K_2 components has been made by Good in terms of forward-scattering amplitudes. [M. Good, Phys. Rev. **106**, 591 (1957).]