better agreement with experimental spectra, and even better agreement might possibly be obtained with other values of r_0 and a. However, the cross sections for the emission of deuterons, tritons, He^3, and He^4 particles are far too high with this correction, while that for the neutrons is too low (Table VI). It should be remembered that by use of Eq. (2) only the emission of charged particles is corrected, while that of the neutrons remains unchanged. This is evidently unsatisfactory, and a more rigorous treatment should also attempt to describe the dependence of the neutron-capture cross section on the excitation energy of the target nucleus. The unfavorable effect of the correction on the ratio of protons to alpha particles emitted also indicates that its form is unsatisfactory.

The same correction was applied also to the calculation of the alpha-particle spectra from the bombardment of natural Ni with 162-Mev O^{16} ions (Fig. 10). Here it seems to be somewhat too powerful, indicating that the form chosen was not quite correct. However, no attempts were made to find better forms of this correction.

Clearly an excitation-energy-dependent Coulomb barrier, as suggested by Fulmer and Cohen[20] and others, will not in itself lead to a satisfactory agreement between calculated and experimental spectra and particle cross sections.

A more rigorous treatment of inverse-reaction cross sections, taking into account both the diffuse edge of the nucleus and its excitation, is highly desirable. Only then will it be possible to examine more quantitatively the validity of the statistical model.

ACKNOWLEDGMENTS

During the course of this work, one of the authors (L.W.) was a guest at the Weizmann Institute of Science. He is deeply grateful for the cordiality extended to him. We wish to thank the authors named in reference 16 for giving us the results of their calculations. The assistance of Miriam Berko in tabulating the data and preparing the figures is gratefully acknowledged.

PHYSICAL REVIEW VOLUME 118, NUMBER 3 MAY 1, 1960

Monte Carlo Calculations of Nuclear Evaporation Processes. V. Emission of Particles Heavier Than He^4

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(Received November 20, 1959)

Previous Monte Carlo calculations of nuclear evaporation reactions have been extended to include the emission of He^6, Li^6, Li^7, Li^8, and Be^7 from Cu, Ag, Au, and Bi targets bombarded with high-energy protons (340–2000 Mev). Comparison with available experimental results shows good agreement in most cases. A discrepancy has been observed between the calculated and observed variation of Be^7 formation cross section with the mass of the target nucleus, but even here the agreement is within a factor of three. It is shown that, for the usually chosen parameters of the calculation, a level density parameter of $a=A/10$ is necessary.

STUDIES of the evaporation of nucleons, deuterons, tritons, He^3, and He^4 from various nuclei excited to energies from a few Mev to a few hundreds of Mev were reported in previous papers of this series.[1–3] The increasing availability of experimental measurements of the formation cross sections and kinetic energy spectra of particles heavier than He^4 produced in high-energy interactions[4–7] make it useful to extend our calculations to include such particles. In particular, it is interesting to establish whether calculations based on the statistical model are still valid for particles heavier than He^4. Approximate calculations of the emission of Be^7 from various targets bombarded with high-energy protons have recently been reported by Hudis and Miller[8] to be in reasonable agreement with measured cross sections.[4]

The Monte Carlo computer program described by Dostrovsky, Bivins, and Rabinowitz[1] was modified so as to include the emission of He^6, Li^6, Li^7, Li^8, and Be^7 as competing process to the emission of the lighter particles and fission. In dealing with these heavier particles one slight complication arises in that most of these have bound excited states. In the calculation account has to be taken of the fact that these particles may be emitted either in their ground state or in excited states. This was done by considering a particle in its

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TABLE I. Mass excess and statistical weight of various particles and their excited states.

Particle	Spin	Mass excess (Mev)	Statistical weight
n	$\frac{1}{2}$	8.368	2
p	$\frac{1}{2}$	7.585	2
H^2	1	13.726	6
H^3	$\frac{1}{2}$	15.835	6
He^3	$\frac{1}{2}$	15.817	6
He^4	0	3.607	4
He^6	0	19.398	6
Li^6	1	15.862	18
Li^{6*}	0	19.422	6
Be^7	$\frac{3}{2}$	17.840	28
Be^{7*}	$\frac{1}{2}$	18.271	14
Li^7	$\frac{3}{2}$	16.977	28
Li^{7*}	$\frac{1}{2}$	17.455	14
Li^8	2	23.310	40
Li^{8*}	1	24.285	24

excited state as a discrete evaporating entity with its appropriate spin and binding energy. The list of particles, their excited states and statistical weights used are shown in Table I.

The starting point of each evaporation calculation was one of the excited nuclei resulting from the prompt nucleon cascade initiated in the target nucleus by an incident proton. Distributions in A, Z, and excitation energy of such excited nuclei from various targets and for various proton energies have been computed by Metropolis et al.,[9,10] and were made available to us through the courtesy of Dr. G. Friedlander. In order to improve the statistics of the calculations ten complete evaporation cascades were computed from each starting nucleus. In all 5000–10 000 evaporations were computed for each target and given proton energy.

Calculations were limited to cases for which such distributions were directly available or which could be approximated by small shifts of the origin of the distribution. For example, the interactions of Au were approximated using the available distribution for Bi after shifting each A by 12 units and each Z by 4 units. This shifting procedure, if not excessive, does not introduce appreciable errors in the medium and heavy nuclide region. It is possible that for lighter nuclides (e.g., Cu) where the sides of the stability valley are steeper and many shell edges are encountered in the course of an evaporation cascade, the shifting procedure is less trustworthy.

All calculations were made using a nuclear radius parameter of 1.5×10^{-13} cm, and with one exception with a level density parameter of $a=A/10$. The calculations for Ag and 1.84-Bev protons were repeated using a level density parameter of $a=A/20$ to demonstrate the effect of a change in this parameter on the yield of heavy particles.

In Table II are shown the calculated emission cross sections of He^6, Be^7, Li^6, Li^7, and Li^8 as well as experimental results for He^6, Be^7, and Li^8. Since no nucleon cascade results are available for the precise proton bombarding energy used in the experiments, calculations were carried out from data available for the nearest energy. Thus, experiments with 1-Bev protons are compared with calculations for 940-Mev protons and experiments with 2-Bev protons (He^6 production) and 2.2-Bev protons (Be^7 and Li^8) are compared with calculations for 1.84-Bev protons. In view of the low precision of both experimental and calculated results, these relatively small differences in proton bombarding energies were not considered important.

The agreement between calculated and experimental results for the formation cross section of He^6 is within the experimental and statistical error except for pb and 1.84-Bev protons. The calculations reproduce the rise of the cross section with both A and bombarding energy. The agreement of the Be^7 cross section is satisfactory for Cu and for 1.84 Bev protons on Ag. The calculated values for Au and for 940-Mev protons on Ag are too high by a factor of three. The rough Li^8 cross sections are again in agreement with the estimates of Katcoff[6]

TABLE II. Experimental and calculated cross sections[a] (in mb) for the emission of He^6, Be^7, Li^6, Li^7, and Li^8 from various targets.

Target[b]	Proton energy Mev	He^6 Exp[c]	He^6 Calc	Be^7 Exp[d]	Be^7 Calc	Li^8 Exp[e]	Li^8 Calc	Li^6 Calc	Li^7 Calc
Cu	940	2±1	2.2±0.5	4.4±1.1	5.3±0.7	···	1.7±0.4	13.7±1.1	9.8±1.0
Cu	1840	4±2	6.3±0.8	11.7±2.9	10.8±1.0	3	3.4±0.6	25.8±1.6	17.6±1.3
Ag	940	4±2	5.9±1.0	2.5±0.6	6.8±1.0	···	3.4±0.7	24.3±2.0	15.9±1.6
Ag	1840	7±4	11.3±1.6	11.3±2.8	13.6±1.9	4	8.5±1.5	43.8±3.4	38.9±3.3
Au	940	···	···	1.3±0.3	4.4±1.0	···	5.8±1.0	20.7±2.1	31.7±2.6
Au	1840	···	···	5.9±1.5	15.2±2.0	9	19.2±2.4	63.2±4.2	81.6±4.7
Pb	940	10±5	13.3±2.4						
Pb	1840	21±11	39.8±4.7						

[a] The calculated cross sections take into account the factor $(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})^2\times[(A_1+A_2-1)^{\frac{2}{3}}(1+A_2/A_1)]^{-1}$ (here A_1, A_2 are the mass numbers of the residual nucleus and the evaporating particle respectively) which was neglected in equation 3 of reference 1.
[b] The calculations were made for the natural isotopic mixture.
[c] Experimental results of Rowland and Wolfgang (reference 5) for 1.0- and 1.9-Bev proton energy.
[d] Experimental results of Baker, Friedlander, and Hudis (reference 4) for 1.0- and 2.2-Bev proton energy.
[e] Estimated cross sections of Katcoff (reference 6).

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[10] N. Metropolis, R. Bivins, M. Storm, A. Turkevich, J. M. Miller, and G. Friedlander, Phys. Rev. **110**, 185 (1958).

TABLE III. Be^7 formation cross sections (mb).

	460-Mev protons			940-Mev protons			1.84-Bev protons		
		Calculated			Calculated			Calculated	
Target	Exp[a]	H and M[b]	Authors	Exp[c]	H and M[b]	Authors	Exp[c]	H and M[b]	Authors
Cu	0.6	0.75	1.42	4.4	2.1	5.3	11.7	4.5	10.8
Ag	0.1	0.7	1.07	2.5	3.2	6.8	11.3	6.5	13.6
Au	0.01	0.18	0.40	1.3	1.6	4.4	5.9	6.0	15.2

[a] Results for 330-Mev protons, Marquez and Perlman, reference 11.
[b] Hudis and Miller, reference 8.
[c] Baker, Friedlander, and Hudis, reference 4.

and show a rise with A and with proton energy. Thus there seems to be a discrepancy in the A dependence of the Be^7 cross section between the calculated and the experimental results. The approximate calculations of Hudis and Miller[8] while often closer to the experimental values than ours (see Table III) also show the same increase in Be^7 formation cross section with the mass number of the target.

It appears that at low proton bombarding energy (330 Mev) the experimental cross sections decreases with increasing A both for Be^7 [11] and for Li^8.[7] Unfortunately, no results are available for He^6 production at 340 Mev. At higher bombarding proton energies the decrease in the formation cross section of Be^7 with increasing A is less marked. For Li^8 the situation seems actually to reverse at higher proton energy and the formation cross section increases with A. We have attempted to calculate from Cu to Ag targets. The calculated cross sections for the emission of Be^7 from Cu, Ag, and Au targets bombarded with 460 Mev protons are shown in Table III. It is seen that there is no *sharp* decrease in the cross section in going from Cu to Ag targets. The large discrepancy between the calculated values for a bombarding energy of 460 Mev and the experimental cross sections for 330 Mev are probably due to the proximity of the threshold for this reaction.

Tables II and III show that the calculated cross sections are larger than the experimental cross sections and that the discrepancy increases with increasing mass of the target. It is believed that this overestimate is at least partially due to an overestimate of the interaction radius $R=r_0(A_1^{\frac{1}{3}}+A_2^{\frac{1}{3}})$ (here r_0 is the nuclear radius parameter and A_1, A_2 are the mass numbers of the residual nucleus and evaporated particle respectively) in the calculation of the inverse cross section and the Coulomb barrier (see equation 9, ref. 1).

A comparison of the results for Ag with two different values of the level density parameter (Table IV) shows clearly that for the particular choice of the other parameters used in the calculation a value of $a=A/20$ is utterly unsuitable, while $A/10$ gives reasonable agreement. This result is in agreement with other calculations in the high-energy region, but is in disagreement with low-energy calculations which require a smaller a. In fact, it appears that calculations in the range of initial excitation of 20–50 Mev fit experimental results best with $a=A/20$ or $A/30$, whereas there is no doubt that calculation for initial excitation in the hundreds of Mev region agree best with $a=A/10$.

TABLE IV. Calculated heavy particle formation cross sections from Ag bombarded with 1.84-Bev protons for two different values of level density parameters.

Particle emitted	Experimental cross section	Calculated cross section $a=A/10$	$a=A/20$
He^6	7 ±4	11.3±1.6	58.6±5.6
Be^7	11.3±2.8	13.6±1.9	109.3±7.8
Li^8	4	8.5±1.5	68.8±6.2
Li^6	···	43.8±3.4	200.7±10.5
Li^7	···	38.9±3.3	215.0±10.8

In common with all thermodynamical calculations the results while giving quantitative results do not throw light on the mechanism of the process. While for the evaporation of nucleons and perhaps alpha particles no difficulty is encountered in accepting the evaporation analogy of classical physics and chemistry, for heavy particles such as Li^8 such naive picture of the mechanism is clearly untenable. The apparent success of the statistical model of nuclear reaction in calculating cross sections for the evaporation of particles as heavy as Li^8 raises the interesting speculation as to how far such agreement will hold with increasing mass of emitted particle.

ACKNOWLEDGMENT

The authors wish to thank M. Berko for help with the data processing.

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