In view of the large amount of interference required from the widely spaced levels of Pu^{239} the interference is much closer to that of a single fission channel (see Fig. 4 and Table I) than in the uranium isotopes. Bollinger[15] previously arrived at the same conclusion. The deviations from single channel seem necessary however to reduce the amount of destructive interference in the vicinity of 9 ev.

V. CONCLUSIONS

The neutron cross sections of the common fissionable isotopes exhibit anomalous shapes which may be reasonably ascribed to interference between levels. However, an anomalously large bound level does not appear to be required by each of the isotopes. The analysis of the U^{235} data show that a variety of evidence supports the existence of a fairly unusual negative energy level in that isotope. Because of the strong effect of this bound level on the cross sections of U^{235} the fit is fairly unique. For U^{233} and Pu^{239}, the cross section fit is much less unique. For each of the latter a simple fit is shown which does not involve an anomalous bound level. Better data and analyses for these elements may impose more uniqueness on the fits. At present there is little evidence for a basic anomaly in the neutron cross sections of the fissionable isotopes.

In each of the fissionable isotopes studied the magnitude of the interference between levels implies that only a few channels are involved in the fission process.

ACKNOWLEDGMENT

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Shell Model Assignments for the Energy Levels of C^{14} and N^{14}†

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Electromagnetic transition widths, reduced widths, and inelastic scattering cross sections are calculated for the following states of N^{14} and C^{14}: (1) The levels arising from the ground-state configuration, s^4p^{10}, (2) the odd-parity levels arising from excitation of a $1p$ nucleon into the degenerate $2s_{\frac{1}{2}}$ and $1d_{\frac{5}{2}}$ shells, (3) the even-parity group of levels formed by excitation of two $1p$ nucleons into the $2s$ and $1d$ shells. The calculations for the s^4p^{10} configuration are carried out using the wave functions of Elliott and of Visscher and Ferrell, and in jj coupling. The calculations for the odd-parity levels are done in the jj-coupling scheme. For the even-parity excited configuration an inert C^{12} core is assumed and $M1$ radiative widths are calculated for states arising from s^2+d^2+sd. The calculations are compared to the existing data. On the basis of this comparison shell-model assignments are proposed for 19 of the 27 known levels below 11-Mev excitation in N^{14} and for all the known levels in C^{14} below 9-Mev excitation.

I. INTRODUCTION

It is expected that the $T=0$ energy levels of N^{14} below, say, 8 Mev and the $T=1$ levels below, say, 11 Mev belong to three groups. One group consists of levels arising from the ground-state configuration,[1] s^4p^{10}. Another group consists of those levels which belong to the mixed s^4p^9s and s^4p^9d configurations. The third group of levels is formed by promoting two p-shell nucleons into the degenerate, or nearly degenerate, $2s$ and $1d$ shells. The latter group we shall refer[2] to as $s^4p^8(s,d)$. That the pairing energy is large enough so that $s^4p^8(s,d)$ should be lower in energy than the s^4p^92p and s^4p^91f configurations (which are expected above 8-Mev excitation in N^{14}) may not be obvious; however, it is implied by the work of the Pittsburgh group[3] on the $C^{14}(d,t)C^{13}$ reaction and is predicted by the binding energy calculations of Unna and Talmi.[4] One other configuration which might conceivably be expected to contribute to the energy region indicated is that formed

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[1] We shall often write the s^4p^{10} configuration in the hole notation, i.e., p^{-2}. Also, when no confusion should arise, we shall leave off principal quantum numbers and the closed $1s^4$ shell.

[2] This notation is intended to suggest that these levels belong to the configurations $s^4p^8s^2+s^4p^8d^2+s^4p^8sd$. We shall sometimes refer to these levels as belonging to the (s,d) configuration. This loose interpretation of configuration should not cause any misunderstanding.

[3] W. E. Moore, J. N. McGruer, and A. I. Hamburger, Phys. Rev. Letters 1, 29 (1958); E. Baranger and S. Meshkov, Phys. Rev. Letters 1, 30 (1958).

[4] I. Unna and I. Talmi, Phys. Rev. 112, 452 (1958).

by raising a $1s$ nucleon into the $1p$ shell; however, the results of Halbert and French[5] on the non-normal parity states of N^{15} indicate that this configuration will appear at a considerably higher excitation that s^4p^9s and s^4p^9d.

Extensive shell model calculations[6–8] have been made in recent years on the ground-state configuration of N^{14}, the main purpose of these being to explain the anomalously long lifetime of the C^{14} beta decay. However, no conventional shell model calculations have been made on the mixed s^4p^9s and s^4p^9d configurations or upon the $s^4p^8(s,d)$ group of levels, and in view of the difficulties involved it is not likely that they will be. Therefore, it is of interest to obtain as much theoretical understanding of the excited configurations of mass 14 as is possible by other means.

In this paper we will bypass the first and most difficult step in the usual shell model procedure—solving for the energies and wave functions resulting from an assumed nuclear Hamiltonian—and from the beginning will assume relatively simple wave functions for the states expected in the low energy spectrum of mass 14. These wave functions are then used to calculate electromagnetic transition widths, nucleon widths, and inelastic scattering cross sections. The comparison of these predictions with the available experimental data enables us to make shell-model configuration and spin-parity assignments which agree fairly well with all the relevant data.

Our calculations involving the levels of the p^{-2} configuration are based to a large extent on the calculations of Elliott[7] and of Visscher and Ferrell[8]—our numerical results being obtained by using their wave functions. Our primary aim is to identify the states of this configuration which lie between its three lowest members—which are already known—and 11-Mev excitation in N^{14}.

Unna and Talmi[4] have treated several of the levels arising from the (s,d) configuration. They assumed extreme jj coupling in their calculations so that the lowest levels are described by the configurations, $1s_{\frac{1}{2}}^4 1p_{\frac{3}{2}}^8 2s_{\frac{1}{2}}^2$ and $1s_{\frac{1}{2}}^4 1p_{\frac{3}{2}}^8 1d_{\frac{5}{2}}^2$. Calculations by Redlich[9] and by Elliott and Flowers[10] for the mass 18 nuclei indicate that the nearly degenerate configurations, $2s^2$, $1d^2$, and $2s1d$, strongly interact in contradiction to the assumption of extreme jj coupling. Furthermore, the assumption of a closed $1p_{\frac{3}{2}}$ core is in contradiction with the results of the intermediate coupling model.[11] One must, however, settle on a relatively simple scheme in order to be able to calculate interesting quantities such as transition rates. The jj model is the simplest scheme for this purpose. As an improvement on the jj scheme we use a model similar to that of Baranger and Meshkov,[3] i.e., we assume that the eight p particles from an inert spin-zero core (not necessarily a $p_{\frac{3}{2}}^8$ core) and that the $2s$ and $1d$ nucleons outside this core interact much the same as they do outside an O^{16} core. This approximation neglects the coupling of the core and the outside particles—probably not a bad approximation in view of the high excitation energy of the C^{12} first-excited state—and enables us to use the published mass 18 wave functions in calculating transition rates and reduced widths and also enables us to use the experimental mass 18 energy scheme in estimating the (s,d) level order in mass 14. In addition to identifying the lowest (s,d) levels in mass 14, we are interested in obtaining information supplementary to that already provided by the Pittsburgh group[3] on the interaction between the p^{-2} configuration and the (s,d) group of levels.

The simple model of Lane[12] for the non-normal parity states of nuclei in the $1p$-shell region provides a basis for the study of the p^9s and p^9d levels of mass 14. Lane suggested that such states may be describable as the weak coupling of a $2s_{\frac{1}{2}}$, $1d_{\frac{5}{2}}$, or $1d_{\frac{3}{2}}$ nucleon to a definite state of C^{13}. Since the first excited state of C^{13} is at 3.68 Mev[13] and the splitting of the $1d_{\frac{5}{2}}$ and $1d_{\frac{3}{2}}$ shells is about 5 Mev,[6] Lane's model would predict that the four lowest $T=0$ levels of mass 14 were formed by coupling a $2s_{\frac{1}{2}}$ or $1d_{\frac{5}{2}}$ nucleon to the $J^\pi=\frac{1}{2}^-$ C^{13} ground state and likewise for the four lowest $T=1$ states. These states would have single-particle reduced widths since they have the C^{13} ground state as their unique parent. A gap of about 3 Mev or more to the next odd-parity state of the same isotopic spin is expected. Because the C^{13} ground state is predominantly $(p_{\frac{3}{2}}^8 p_{\frac{1}{2}})$ these states should be predominately $(p_{\frac{1}{2}}s_{\frac{1}{2}})$ or $(p_{\frac{1}{2}}d_{\frac{5}{2}})$. Recently[14] these predictions have been compared to the available experimental information on the nucleon reduced widths of the mass 14 levels and an identification of these 8 non-normal parity states of mass 14 has been proposed mainly on the basis of this comparison. The proposed $T=0$ levels are at N^{14} excitations in the range 5–6 Mev, while the $T=1$ levels are at 8–10 Mev. The excitations of these p^9s and p^9d states are in good agreement with the predictions[4,14,15] based on a jj-coupling description of these states.

At the time of the work of WRH the N^{14} $T=1$ levels at 9.16 Mev and 10.43 Mev had been given[13,16] most

[5] E. C. Halbert and J. B. French, Phys. Rev. **105**, 1563 (1957).
[6] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).
[7] J. P. Elliott, Phil. Mag. **1**, 503 (1956).
[8] W. M. Visscher and R. A. Ferrell, Phys. Rev. **107**, 781 (1957).
[9] M. G. Redlich, Phys. Rev. **110**, 468 (1958), and references therein.
[10] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A229**, 536 (1955).
[11] D. Kurath, Phys. Rev. **101**, 216 (1956).
[12] A. M. Lane, Massachusetts Institute of Technology Report, 1955 (unpublished).
[13] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).
[14] E. K. Warburton, H. J. Rose, and E. N. Hatch, Phys. Rev. **114**, 214 (1959), hereafter referred to as WRH.
[15] J. P. Elliott (unpublished calculations); E. Baranger (unpublished calculations).
[16] H. B. Willard, J. K. Bair, H. O. Cohn, and J. O. Kington, Phys. Rev. **105**, 202 (1957).

probable assignments, $J^\pi=2^-$. Recently the 9.16-Mev level has been shown[17] to have even parity while preliminary results[18] indicate that the 10.43-Mev level most probably has $J^\pi=2^+$. Therefore all the known[19,20] $T=1$ levels in N^{14} below 11-Mev excitation, except for the four p^9s and p^9d states in question, seem to have even parity. There is a gap of 1.5 Mev between the C^{14}, 8.32-Mev level—which is probably the analog of the N^{14}, $T=1$, 10.43-Mev level[21]—and the next higher known C^{14} level at 9.80 Mev.[19] The spin-parity of the C^{14} 9.80-Mev level is not known. It would seem then that there is a gap of at least 3 Mev between the lowest four $T=1$ odd-parity states and the next higher odd-parity state with $T=1$, in agreement with the predictions of Lane's model and also with extreme jj coupling.

Definite parity assignments have not been made for any of the N^{14} levels in the region, 4–8 Mev and, in addition, the experimental information on the nucleon reduced widths of the N^{14} levels below 8 Mev is quite meager at the present time. A comparison with Lane's model, such as was made by WRH, is consequently much less definite for the four $T=0$ levels than it was for the $T=1$ levels.

The radiative widths and inelastic scattering cross sections calculated in the present paper provide a more stringent test of the identification of the $T=0$ non-normal parity levels proposed in WRH. For these calculations we assume extreme jj coupling, i.e., $(p_{\frac{1}{2}}s_{\frac{1}{2}})$ and $(p_{\frac{1}{2}}d_{\frac{5}{2}})$ states. For the electromagnetic transitions the effects of admixtures of $(p_{\frac{1}{2}}d_{\frac{3}{2}})$ in the $J=1$ and $J=2$ states are considered, but an inert $p_{\frac{3}{2}}$ core is assumed throughout since relaxation of this assumption would entail more complicated calculations than seem warranted. As discussed in WRH, justification for, and interest in, calculations based on extreme jj coupling for the non-normal parity states of mass 14 is provided by (1) the agreement of the spectrum and nucleon widths of the non-normal parity states with extreme jj coupling, (2) the qualitative agreement with extreme jj coupling of the experimental data on those electromagnetic transitions which involve the non-normal parity states, (3) the close resemblance of the p^9s and p^9d states of mass 14 to the $p^{-1}s$ and $p^{-1}d$ states of mass 16, since Elliott and Flowers[22] found the four lowest $T=1$ (but not $T=0$) non-normal parity states in mass 16 to be remarkedly well described by extreme jj coupling.

[17] A. A. Strassenburg, R. E. Hubert, R. W. Krone, and F. W. Prosser, Bull. Am. Phys. Soc. **3**, 372 (1958).
[18] E. Kashy, R. R. Perry, and J. R. Risser, Bull. Am. Phys. Soc. 4, 96 (1959), and private communications from E. Kashy. We are grateful to Dr. Kashy for informing us of these preliminary results.
[19] F. Ajzenberg-Selove and T. Lauritsen, Nuclear Phys. **11**, 1 (1959).
[20] See Sec. II of this paper.
[21] E. K. Warburton, Phys. Rev. **113**, 595 (1959).
[22] J. P. Elliott and B. H. Flowers, Proc. Roy. Soc. (London) **A242**, 57 (1957).

II. EXPERIMENTAL ENERGY SPECTRA OF N^{14} AND C^{14}

The experimentally known information on the energy spectra of N^{14} and C^{14} is shown in Fig. 1. Except for some changes and additions, which will be discussed here, Fig. 1 is taken from WRH, and is similar to the C^{14} and N^{14} spectra given by Ajzenberg-Selove and Lauritsen.[19]

The preference for odd parity for the N^{14} 5.69-Mev level has been discussed previously.[14,23] The N^{14} 6.23-Mev level had previously been assigned odd-parity[19] because the great strength of the N^{14} $8.62\rightarrow 6.23$ transition would seem to favor $E1$ radiation. No parity preference is given here for the 6.23-Mev level because the $C^{13}(d,n)N^{14}$ stripping results[24] are not in good agreement with an $l_p=0$ capture into this level. In addition, as will be discussed in Sec. IIIC, the strength of the N^{14} $8.62\rightarrow 6.23$ transition is compatible with $M1$ radiation.

The N^{14} 6.44- and 9.16-Mev levels have recently been established as $J=3$, $T=0$, and $J^\pi=2^+$, $T=1$.[25] The assignments for the N^{14} 10.24- and 10.43-Mev levels are from preliminary analysis of $C^{13}(p,p)C^{13}$ scattering results.[18]

Some isotopic-spin assignments of $T=0$ have been made in addition to those given by Ajzenberg-Selove and Lauritsen.[19] The $T=0$ assignments to the N^{14} 4.91-, 5.10-, 5.69-, and 5.83-Mev levels are supported by observation of deuteron groups corresponding to those levels in the $N^{14}(d,d')N^{14}$ reaction at $E_d=14.8$ Mev.[26] The N^{14} 4.91- and 5.10-Mev levels were also excited by $N^{14}(d,d')N^{14}$ with $E_d=9$ Mev.[27] The N^{14} 5.10-, 5.83-, 7.96-, 8.45-, and 10.05-Mev levels were observed[28] to have relatively large cross sections for the inelastic scattering of α particles by N^{14} indicating $T=0$ for these five levels. The $T=0$ assignments to these levels are supported by the absence of C^{14} levels which could be their isotopic-spin analogs.

Although there is no direct evidence, it seems most probable that the 8.98-, 9 39-, 9.72-, and 10.24-Mev levels have $T=0$ since these levels are all observed[19] to have sufficiently large proton widths so that the non-observation[19] of their counterparts in C^{14} by $C^{13}(d,p)C^{14}$ is most likely due to their having $T=0$ rather than to the C^{14} counterparts having small neutron widths. It will be assumed throughout this work that there are no $T=1$ levels in N^{14} below 8-Mev excitation, except for the 2.31-Mev level (i.e., no C^{14} levels below 6-Mev excitation).

[23] E. K. Warburton, W. T. Pinkston, H. J. Rose, and E. N. Hatch, Bull. Am. Phys. Soc. 4, 219 (1959).
[24] R. E. Benenson, Phys. Rev. **90**, 420 (1953).
[25] H. J. Rose, W. Trost, and F. Riess, Nuclear Phys. **12**, 510 (1959), and references therein.
[26] E. K. Warburton and J. N. McGruer (unpublished, 1955).
[27] T. S. Green and R. Middleton, Proc. Phys. Soc. (London) **A69**, 28 (1956).
[28] D. W. Miller, B. M. Carmichael, U. C. Gupta, V. K. Rasmussen, and M. B. Sampson, Phys. Rev. **101**, 740 (1956).

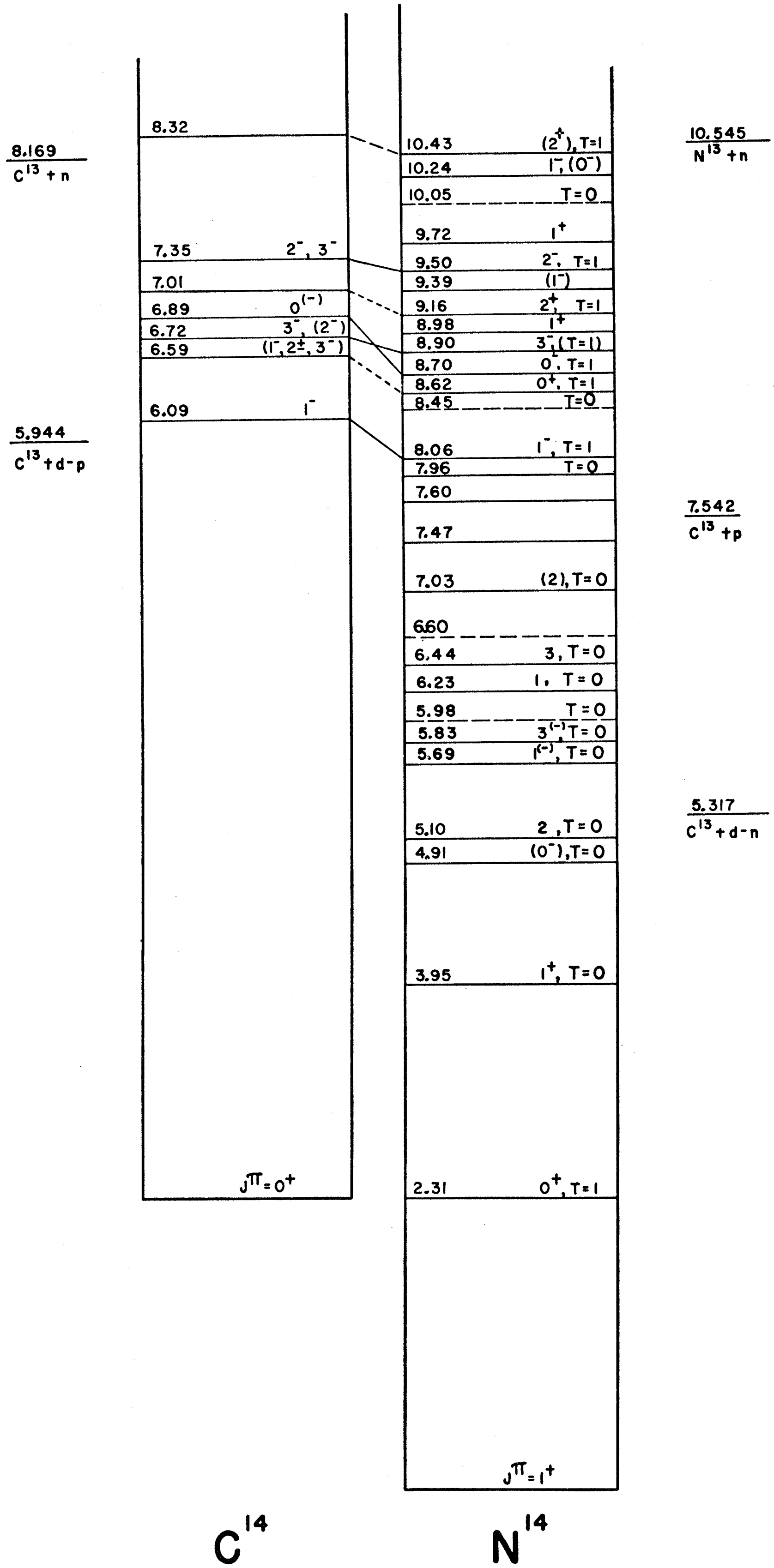

FIG. 1. Energy levels of C^{14} and N^{14}. The references for the information given in the energy level diagram are given in reference 14 except for later additions and corrections (see text). Uncertain or less likely assignments are enclosed in parenthesis. Uncertain levels are denoted by dashed lines. The C^{14} ground state is matched in energy with its isotopic-spin analog in N^{14} at 2.31 Mev in order to exhibit the correspondence of levels belonging to isotopic-spin triads. Levels for which the correspondence seems well established are connected by solid lines, while levels for which the correspondence is less certain are connected by dashed lines.

The tentative pairing of the C^{14} 6.59- and 7.01-Mev[29] levels with the N^{14} 8.62- and 9.16-Mev levels will be discussed in Sec. IV. The pairing of the other C^{14} levels with the $T=1$ N^{14} levels has been discussed.[14,21,29a]

III. ELECTROMAGNETIC TRANSITIONS AND REDUCED WIDTHS

A. Introduction

Expressions for the transition probabilities for multipole radiations in nuclei are given in Blatt and Weisskopf.[30] For an electric multipole transition of order L the width of the transition may be put in the form,

$$\Gamma(EL)=\frac{L+1}{2L(2L+1)[(2L-1)!!]^2}\frac{e^2E_\gamma^{2L+1}}{(\hbar c)^{2L+1}}\Lambda(EL), \quad (1)$$

where

$$\Lambda(EL)=\frac{|\langle J_f\|\mathbf{H}(EL)\|J_i\rangle|^2}{(2J_i+1)}. \quad (2)$$

The reduced matrix element is that of Racah,[31] E_γ is the energy of the gamma ray. The tensor, $\mathbf{H}_M(EL)$, is given by

$$\mathbf{H}_M(EL)=\sum_{i=1}^{A} r_i^L\mathbf{C}_M{}^L(\Omega_i)[1-\tau_3(i)].$$

The tensor $\mathbf{C}_M{}^L$ is an unnormalized surface harmonic tensor, i.e.,

$$\mathbf{C}_M{}^L(\Omega_i)=[4\pi/(2L+1)]^{\frac{1}{2}}\mathbf{Y}_L{}^M(\Omega_i).$$

The symbol, $\tau_3(i)$, represents the Pauli spin matrix for the ith nucleon. We use the convention that $\tau_3=+1$ for a neutron and $\tau_3=-1$ for a proton. The quantity Λ is the same as that of Lane and Radicati[32] except for $E1$ transitions. Following these authors Λ will be referred to as the transition strength. The present paper considers $E1$, $E2$, and $E3$ transitions. For γ-ray energies in Mev and nuclear distances in fermi, the radiative widths in ev are calculated from

$$\Gamma(E1)=6.25\times10^{-2}E_\gamma{}^3\Lambda(E1)\text{ ev}, \quad (3)$$

$$\Gamma(E2)=8.02\times10^{-8}E_\gamma{}^5\Lambda(E2)\text{ ev}, \quad (4)$$

$$\Gamma(E3)=5.24\times10^{-14}E_\gamma{}^7\Lambda(E3)\text{ ev}. \quad (5)$$

It has been observed,[22,33] that $\Delta T=0$ $E2$ and $E3$ transition rates in the $1p$-shell region are too fast to be accounted for by the independent particle model. Elliott and Flowers[22] have found that the theoretical calculations can be brought into line with the data by including the effect of quadrupole and octupole surface vibrations weakly coupled to the shell-model states. The contribution of a surface vibration of given order, say λ, may be shown equivalent, in the weak coupling approximation, to giving the nucleons effective charges, $\beta_\lambda e$ for the neutron and $(1+\beta_\lambda)e$ for the proton, in the shell model matrix element for the electric transition of order, λ. Elliott and Flowers have estimated β_2 and β_3 from the $E2$, O^{17}, $0.87\rightarrow0$ transition and the $E3$, O^{16}, $6.14\rightarrow0$ transition. They found $\beta_2=0.64$ and $\beta_3=1.1$. We shall assume that these values hold for the $\Delta T=0$ $E2$ and $E3$ transitions in mass 14, but that $\Delta T=1$ transitions in N^{14} have no collective enhancement.[34]

For magnetic transitions we have

$$\Gamma(ML)=\frac{L+1}{2L(2L+1)[(2L-1)!!]^2}\times\frac{E_\gamma{}^{2L+1}}{(\hbar c)^{2L+1}}\left(\frac{e\hbar}{Mc}\right)^2\Lambda(ML), \quad (6)$$

in which,

$$\Lambda(ML)=\frac{|\langle J_f\|\mathbf{H}(ML)\|J_i\rangle|^2}{(2J_i+1)}, \quad (7)$$

and

$$\mathbf{H}_M(ML)=\sum_{i=1}^{A}(\nabla r^L\mathbf{C}_M{}^L)_i{}^*\cdot\left[\frac{1}{L+1}\mathbf{l}+\mu_+\mathbf{s}+\tau_3\left(\frac{-1}{L+1}\mathbf{l}+\mu_-\mathbf{s}\right)\right]_i.$$

In this expression μ_+ and μ_- stand for $\mu_p+\mu_n$ and $\mu_p-\mu_n$, respectively, and $\mathbf{l}$ and $\mathbf{s}$ are the single particle orbital and spin angular momentum vectors, respectively. In the same units as before,

$$\Gamma(M1)=2.76\times10^{-3}E_\gamma{}^3\Lambda(M1)\text{ ev}, \quad (8)$$

$$\Gamma(M2)=3.55\times10^{-9}E_\gamma{}^5\Lambda(M2)\text{ ev}. \quad (9)$$

In calculating these widths with shell model wave functions a radial integral, $\langle r^L\rangle$, appears in the expression for $\Lambda(EL)$ and a similar integral, $\langle r^{L-1}\rangle$, appears in $\Lambda(ML)$. We have calculated these using harmonic oscillator radial wave functions. For such wave func-

[29] J. C. Armstrong, W. E. Moore, and A. G. Blair, Bull. Am. Phys. Soc. 4, 17 (1959).

[29a] *Note added in proof.*—Recent $C^{13}(p,p)C^{13}$ work [E. Kashy, R. R. Perry, and J. R. Risser, Bull. Am. Phys. Soc. 5, 108 (1960)] indicates assignments of $J^\pi=2^+$, 1^-, 2^+, and 1^- for N^{14} excited states at 10.084, 10.20, 10.415, and 10.51 Mev. The 10.084- and 10.20-Mev levels are almost certainly associated with the 10.05- and 10.24-Mev levels of Fig. 1. The establishment of the 10.415-Mev level (10.43-Mev in Fig. 1 and the text) as $J^\pi=2^+$ is essential to the conclusions regarding this level which are discussed in Secs. IIIB and C.

[30] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley & Sons, Inc., New York, 1952), pp. 595–599.

[31] G. Racah, Phys. Rev. **62**, 438 (1942). All quantities tabulated in this paper will follow the definitions and phase conventions of this reference. This is important, for example, in predicting the angular distributions of radiations in which electric and magnetic radiations compete.

[32] A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954).

[33] D. Kurath, Phys. Rev. **106**, 975 (1957).

[34] E. K. Warburton, Phys. Rev. **1**, Letters 68 (1958).

tions, which have radial fall-offs of the form, $\exp(-\frac{1}{2}\gamma r^2)$, we follow Visscher and Ferrell[8] and take $\gamma^{-\frac{1}{2}}=1.68$ fermi. Although this value was arrived at from a consideration of the Coulomb energy and, hence, only holds for the $1p$ radial wave function, we shall take it to hold for the $2s$ and $1d$ shells as well. Any method of evaluating these radial integrals is subject to uncertainties, conceivably as much as 10–20% according to Lane and Radicati.[32] $M1$ transitions are independent of the radial wave functions and so do not contain an uncertainty from this source. We shall sometimes give $M1$ transition strengths in Weisskopf units ($\Lambda=0.13$ Weisskopf unit) since experimental results are often given in these units.

The nucleon reduced widths of the non-normal parity states of mass 14 were discussed by WRH. We are also interested in the p-wave resonant reduced widths, $\theta^2(p)$, of several even-parity levels of N^{14}. For N^{14} levels below the neutron binding energy (10.545 Mev) $\theta^2(p)$ can be obtained directly from the level width since, for these levels, $\Gamma=\Gamma_\gamma+\Gamma_p$ and $\Gamma_\gamma\ll\Gamma_p$. We quote all reduced widths in units of $3\langle T_i\frac{1}{2}T_{zi}-\frac{1}{2}|T_fT_{zf}\rangle^2\hbar^2/2MR$, where $\langle T_i\frac{1}{2}T_{zi}-\frac{1}{2}|T_fT_{zf}\rangle$ is the isotopic-spin vector addition coefficient and the nuclear radius is taken to be 4.8×10^{-13} cm. For comparison with theory we need the relative reduced width $\mathcal{S}=\theta^2(p)/\theta_0^2(p)$ where $\theta_0^2(p)$ is the single-particle p-wave reduced width for resonant reactions. For $\theta_0^2(p)$ we adopt the value 0.75 which we estimate from the He^4+p and He^4+n resonant reactions.[19] The theoretical relative reduced width $\mathcal{S}$ is given in terms of shell-model wave functions by Elliott[7] and by Auerbach and French.[35]

B. $M1$ and $E2$ Transitions between the Different Levels of the s^4p^{10} Configuration

Intermediate coupling shell model calculations[6–8] of the levels arising from the p^{-2} configuration of mass 14 show that, barring strong interactions of these states with excited configurations, the order of the lowest p^{-2} states is $(J,T)=(1,0)$, $(0,1)$, $(1,0)$, $(2,0)$, $(2,1)$, and $(1,1)$. Of these six states the three lowest have been identified as the N^{14} ground state, the 2.31-Mev level, and the 3.95-Mev level. The (2,0), (2,1), and (1,1) states in N^{14}—which have not been identified—are predicted to be within, say, 50% of excitation energies of 6, 9, and 12 Mev, respectively. Assuming that these six p^{-2} states are in the order given above, we have calculated the strengths of the $M1$ and $E2$ transitions connecting them in an attempt to shed some light on their structure and/or their location by comparison of these calculations with experimental data. The following formulas were used in calculating the transition strengths:

$$\Lambda(M1)=0.144(2J'+1)\left|\sum_{LS}\sum_{L'S'}C_{LS}{}^{JT}C_{L'S'}{}^{J'T'}\times(-)^{1+L-S-J}\delta_{LL'}\delta_{SS'}[S(S+1)(2S+1)]^{\frac{1}{2}}\times W(SSJJ';1L)\right|^2, \quad (10)$$

for $\Delta T=0$,

$$\Lambda(M1)=(2J'+1)\left|\sum_{LS}\sum_{L'S'}C_{LS}{}^{JT}C_{L'S'}{}^{J'T'}\times[\delta_{LL'}(-)^{L-J+1}4.71(2L+1)^{-\frac{1}{2}}+6^{\frac{1}{2}}\delta_{SS'}(-)^{S-J'}(2L+1)^{\frac{1}{2}}(2L'+1)^{\frac{1}{2}}\times W(LJL'J';S1)W(11LL';11)]\right|^2, \quad (11)$$

for $\Delta T=1$, and

$$\Lambda(E2)=30(2J'+1)\left|\sum_{LS}\sum_{L'S'}C_{LS}{}^{JT}C_{L'S'}{}^{J'T'}\times\delta_{SS'}(-)^{1+S-J'}(2L+1)^{\frac{1}{2}}(2L'+1)^{\frac{1}{2}}\times W(1L1L';12)W(LJL'J';S2)\right|^2\gamma^{-2}, \quad (12)$$

for $\Delta T=0$ and $\Delta T=1$. $W(abcd;ef)$ is a Racah coefficient. The nuclear wave functions are assumed to have LS coupling amplitudes $C_{LS}{}^{JT}$ in the initial state and $C_{L'S'}{}^{J'T'}$ in the final state.

The results of calculations based on these formulas are presented in Table I. Three different coupling schemes were used in these calculations. They are (a) jj coupling, (b) wave functions derived[36] from the work of Elliott[7] and (c) the wave functions given[37] by Visscher and Ferrell.[8] In addition to the strengths $\Lambda(M1)$ and $\Lambda(E2)$ of a transition, the square root of the relative strengths of the $E2$ and $M1$ contributions to a γ transition is given in Table I since its sign and magnitude can be obtained from angular distribution or correlation measurements. Radiative widths can be calculated from these results if the energy differences are known or assumed.

Comparisons of the predictions of schemes (b) and (c) with experiment have already been made[7,8] for the transition from the N^{14} 2.31-Mev level to the ground state and for the branching ratio of the 3.95-Mev level. Experimental data available for comparison with the theory are the limits on the strength of the $2.31\rightarrow0$ transition, $0.1<\Lambda(M1)<3.0$,[19,38] and the branching ratio of the 3.95-Mev level, $\Gamma(3.95\rightarrow0)/\Gamma(3.95\rightarrow2.31)=0.037\pm0.006$.[39] All three predictions given in Table I are within the limits on $\Lambda(M1)$ for the $2.31\rightarrow0$ transition. Schemes (b) and (c) predict a considerably smaller width than scheme (a), jj coupling. This is a well known result of forcing the C^{14} and N^{14} wave functions to bring about the accidental cancellation of the

[35] T. Auerbach and J. B. French, Phys. Rev. 98, 1276 (1955).

[36] The LS expansion coefficients for the six p^{-2} states in question were calculated from Elliott's work by H. J. Rose (private communication) assuming that the excitation energy of the (2,0) level in N^{14} is 6 Mev and using the following values of Elliott's parameters: $L/K=6$, $\zeta/K=4$, $q=0.7$, $K=-1.5$, $t=\frac{1}{3}$, $x=0.4$, and $y=1.0$. We wish to thank H. J. Rose for permission to use his results.

[37] The wave function of the (2,1) level is not given by Visscher and Ferrell. The calculations involving this level were carried out assuming $\Psi(2,1)=0.90\psi(^1D_2)-0.43\psi(^3P_2)$, which results from one set of parameters listed by Visscher and Ferrell.

[38] F. Metzger, V. K. Rasmussen, and C. P. Swann (unpublished). We wish to thank Dr. Metzger, for communicating to us the limit $\Gamma(2.31\rightarrow0)<0.1$ ev which these authors obtained from a resonance fluorescence experiment. This limit corresponds to $\Lambda(M1)<3.0$.

[39] D. A. Bromley, E. Almqvist, H. E. Gove, A. E. Litherland, E. B. Paul, and A. J. Ferguson, Phys. Rev. 105, 957 (1957).

TABLE I. $M1$ and $E2$ transition strengths between the s^4p^{10} states of N^{14}.

Transition $(J_iT_i)\rightarrow(J_fT_f)$	E_i (Mev)	E_f (Mev)	$\Lambda(M1)$ a	$\Lambda(M1)$ b	$\Lambda(M1)$ c	$\Lambda(E2)$[d] a	$\Lambda(E2)$[d] b	$\Lambda(E2)$[d] c	$[\Lambda(E2)/\Lambda(M1)]^{\frac{1}{2}}$[e] a	$[\Lambda(E2)/\Lambda(M1)]^{\frac{1}{2}}$[e] b	$[\Lambda(E2)/\Lambda(M1)]^{\frac{1}{2}}$[e] c
(01) → (10)	2.31	0	2.45	0.46	0.70	...	...	...	...	...	...
(10) → (10)	3.95	0	0.032	0.0029	0.0033	19.92	14.94	17.43	+24.98	−71.97	−71.97
(10) → (01)	3.95	2.31	7.88	13.1	11.3	...	...	...	...	...	...
(20) → (10)	?	0	0.096	0.125	0.11	11.95	7.47	4.78	+11.15	+7.83	+6.63
(20) → (01)	?	2.31	...	...	...	7.97	4.18	4.98	...	...	...
(20) → (10)	?	3.95	0.024	0.00032	0.0007	2.99	16.43	8.96	−11.15	+227.2	−113.6
(21) → (10)	?	0	11.8	12.5	12.6	11.95	3.48	8.07	+0.99	+0.53	+0.78
(21) → (01)	?	2.31	...	...	...	7.97	11.50	10.94	...	...	...
(21) → (10)	?	3.95	0.38	0.011	0.085	2.99	1.64	9.34	−2.82	+12.14	+10.51
(21) → (20)	?	?	16.4	20.7	19.3	6.97	2.59	3.94	−0.65	−0.35	−0.45
(11) → (10)	?	0	3.93	0.15	1.23	19.92	25.9	22.9	+2.26	+13.19	+4.30
(11) → (01)	?	2.31	0.064	0.033	0.04	...	...	...	...	...	...
(11) → (10)	?	3.95	9.26	3.29	11.8	4.98	0.065	0.18	−0.73	+1.43	−0.11
(11) → (20)	?	?	0.63	0.63	0.63	4.98	4.98	4.98	−2.82	−2.82	−2.82
(11) → (21)	?	?	0.040	0.015	0.023	4.98	1.84	2.81	−11.15	−11.15	−11.15

[a] Extreme jj coupling.
[b] Calculated from LS amplitudes extracted from the work of J. P. Elliott (reference 7) by H. J. Rose (private communication).
[c] Calculated from the LS amplitudes of Visscher and Ferrell (reference 8). For the (2,1) state, for which Visscher and Ferrell do not give the wave function, the wave function was assumed to be $\Psi=0.90\psi(^1D_2)-0.43\psi(^3P_2)$. The results are quite insensitive to this assumption.
[d] To take collective enhancement into account multiply $\Lambda(E2)$ by $\epsilon(\Delta T)$, where $\epsilon(1)=1$, $\epsilon(0)=(1+2\beta_2)^2=5.2$ (see Sec. IIIA).
[e] The amplitude ratio of the $M1$ and $E2$ contributions to a transition is given by $\delta=[\Gamma(E2)/\Gamma(M1)]^{\frac{1}{2}}=5.38\times10^{-3}E_\gamma[\Lambda(E2)/\Lambda(M1)]^{\frac{1}{2}}$.

$C^{14}\rightarrow N^{14}$ β-decay matrix element. This cancellation causes the spin part of the $M1$ transition operator which ordinarily dominates $M1$, $\Delta T=1$ transitions to give a vanishing contribution. Substantial admixture of $s^4p^8(s,d)$ states into the p^{-2} (1,0) and (0,1) levels could conceivably cause considerable departure from the values of $\Lambda(2.31\rightarrow0)$ of schemes (b) and (c), the departure being due to the interference between configurations in the orbital terms. A further narrowing of the experimental limits on the $2.31\rightarrow0$ lifetime could shed light on the wave functions of these two states.

The predicted ratios, $\Gamma(3.95\rightarrow0)/\Gamma(3.95\rightarrow2.31)$, corresponding to jj coupling, the wave functions of Elliott and the wave functions of Visscher and Ferrell are 0.073, 0.010, and 0.014, assuming no collective enhancement of the $E2$, $3.95\rightarrow0$ transition. The predicted ratios are 0.14, 0.041, and 0.055 if we assume the same magnitude of collective enhancement which Elliott and Flowers[22] found necessary to explain the O^{17} $0.87\rightarrow0$ $E2$ transition rate. It is seen that the wave functions of Elliott and the wave functions of Visscher and Ferrell give good agreement with the observed branching ratios assuming collective enhancement, but that agreement is also possible closer to jj coupling with no collective enhancement of the $E2$ rate. In order to decide whether collective enhancement is actually involved, the additional information obtainable from a measurement of the lifetime of the 3.95-Mev level and a determination from an angular correlation or distribution measurement of $[\Lambda(E2)/\Lambda(M1)]^{\frac{1}{2}}$ for the $3.95\rightarrow0$ transition would be useful.

The theoretical widths corresponding to the values of Λ given in Table I indicate that the N^{14}, $p^{-2}(2,0)$ level should decay to the N^{14} ground state with a mixture of $M1$ and $E2$ radiation, but that decays to the N^{14} 2.31- and 3.95-Mev levels are expected to be negligible for all three choices of coupling schemes. Any one of the N^{14} levels (see Fig. 1) at 7.96, 7.60, 7.47, 7.03, and 5.10 Mev and the uncertain levels at 10.05, 8.45, 6.60, and 5.98 Mev could conceivably have $J^\pi=2^+$, $T=0$. The decay modes[14,19] of the 7.96-Mev and the 5.10-Mev levels are inconsistent with the above remarks about the (2,0) decay. The 5.10-Mev level has a large branch to the 2.31-Mev level; the 7.96-Mev level branches strongly to the 3.95-Mev level. As will be discussed in Sec. IVA it seems improbable that the levels at 7.60, 7.47, 6.60, or 5.98 Mev in N^{14} could be the $p^{-2}(2,0)$ level, since none of these three levels has been observed[19] in the $N^{14}(\alpha,\alpha')N^{14*}$ reaction, as is expected for the $p^{-2}(2,0)$ level. If, therefore, the level in question is below 9 Mev in N^{14} as is theoretically expected, and if it has been observed, then it is most probably the 7.03- or 8.45-Mev level. The 7.03-Mev level has been observed to decay to the ground state; no other modes of decay have been observed for this level. No decays have been observed from the 8.45-Mev level.

Next we turn to a consideration of the $p^{-2}(2,1)$ level. Its decay to the $p^{-2}(2,0)$ level is the strongest listed in Table I; therefore, a study of its possible location and its γ-decay modes should also help us in our search for the (2,0) level. In the region around 9 Mev the only known levels for which the assignment 2^+, $T=1$, seems likely are the 9.16- and 10.43-Mev levels. The 9.16-Mev level has $T=1$, $J^\pi=2^+$, while the 10.43-Mev level has $T=1$, and a most probable assignment of $J^\pi=2^+$. In the remainder of this discussion it will be assumed that the N^{14}, 10.43-Mev level has $J^\pi=2^+$, $T=1$. From the values of Λ given in Table I we expect the $p^{-2}(2,1)$ level to decay strongly to the N^{14} ground state as well as to the $p^{-2}(2,0)$ level, but negligibly to the N^{14} 2.31-

and 3.95-Mev levels. The γ-decay modes of both the 9.16-Mev and the 10.43-Mev levels have been investigated by means of the $C^{13}(p,\gamma)N^{14}$ reactions.[16,40–44] The major decay modes of both levels are reported to be to the N^{14} ground state and to the 6.44- and 7.03-Mev levels. No decays have been observed from either level to the 2.31- or the 3.95-Mev level, in keeping with the above predictions for the $p^{-2}(2,1)$ level. The radiative width of the $9.16\rightarrow 0$ transition is $\Gamma_\gamma = 8.7\pm 1.5$ ev,[43] while the radiative width of the $10.43\rightarrow 0$ transition is reported to be 17 ev.[16] These two transitions are both too strong to contain appreciable contributions from quadrupole radiation for reasonable values of the quadrupole matrix element. Neither transition involves a parity change, so that the radiative widths of both transitions corresponds to $M1$ radiation. The experimental widths correspond to values of $\Lambda(M1)$ of 4.1 for the ground-state decay of the 9.16-Mev level and 5.5 for that of the 10.43-Mev level. These experimental strengths are to be compared to the theoretical values of $\Lambda(M1)$ for the three sets of wave functions used in the calculations of Table I. All three choices predict $\Lambda(M1)\simeq 12$ for the $p^{-2}(2,1)$ ground-state transition. The calculated strength of this transition is quite insensitive to reasonable changes in the wave functions for initial and final states which are pure p^{-2}.

Therefore, there are two $J^\pi = 2^+$, $T=1$ N^{14} levels in the energy region in which the $p^{-2}(2,1)$ level is expected, both of which decay strongly to the ground state by $M1$ radiation. Only one p^{-2} level can be in this region and since $M1$ radiation is forbidden between states belonging to different shell model configurations, we conclude that one of these levels arises from an excited even-parity configuration but contains a large admixture of the p^{-2} configuration. Presumably, this even-parity configuration is $s^4p^8(s,d)$ since a (2,1) level from this configuration is expected near 10-Mev excitation in N^{14} (see Sec. IIIC). Since neither decays to the ground state as strongly as a pure $p^{-2}(2,1)$ state should, we suggest that the two (2,1) states contaminate each other quite strongly—the strength of the interaction between the two levels being due to their proximity. The ground state is sufficiently far below any (s,d) levels (see Sec. IIIC) so that the contribution to the $(2,1)\rightarrow$ ground-state $M1$ transitions from its contamination is expected to be relatively small. The interaction of the (2,1) levels would explain the smallness of $\Lambda(M1)$ for both ground-state transitions, since in this picture $\Lambda(M1)$ is reduced by the fraction of p^{-2} in the total wave function of the excited state.

[40] J. D. Seagrave, Phys. Rev. **85**, 197 (1952).
[41] H. H. Woodbury, R. B. Day, and A. V. Tollestrup, Phys. Rev. **92**, 1199 (1953).
[42] Private communication from H. J. Rose (unpublished). We would like to thank Dr. Rose for communicating his results to us.
[43] S. S. Hanna and L. Meyer-Schützmeister, Phys. Rev. **115**, 986 (1959).
[44] R. W. Krone, J. J. Singh, and F. W. Prosser, Jr., Bull. Am. Phys. Soc. **4**, 219 (1959).

We would expect the $p^{-2}(2,0)$ level, like the ground state, to be relatively pure since it does not have a nearby $s^4p^8(s,d)$ level with which it can interact (see Sec. IIIC). Therefore, we expect that both (2,1) levels would decay to the $p^{-2}(2,0)$ level with a strength less than the value, $\Lambda(M1)\simeq 18$, predicted for the $(2,1)\rightarrow(2,0)$ transition in Table I. Preliminary results of Rose[42] suggest $\Lambda(M1)=12\pm 5$ for the $9.16\rightarrow 7.03$ transition and $\Lambda(M1)\simeq 10$ (with an uncertainty of about a factor of 2) for the $10.43\rightarrow 7.03$ transition if the 7.03-Mev level has even parity. These decays provide evidence that the N^{14} 7.03-Mev level is the (2,0) level of the p^{-2} configuration. The question of the interaction of the $p^{-2}(2,1)$ level with a second (2,1) level will be returned to in the next subsection.

There are only two known $J^\pi = 1^+$ levels in N^{14} above 8-Mev excitation. These are the 8.98-Mev and the 9.72-Mev levels. Neither of these levels have γ-decay modes[19] which fit the predictions of Table I for the $p^{-2}(1,1)$ level. In addition, analogs of these two levels have not been observed in C^{14} so that they are probably $T=0$. It seems most likely that the $p^{-2}(1,1)$ level is above 11 Mev as predicted.

C. The $s^4p^8(s,d)$ States of Mass 14

If our assumption of an inert spin zero core for the (s,d) levels of mass 14 is at all accurate the low-energy spectrum of these levels in N^{14} will bear a rather strong resemblance to that of F^{18}. The two lowest $T=0$ states in F^{18} are the $J^\pi=1^+$ ground state and the $J^\pi=3^+$, 0.94-Mev level. The two lowest $T=1$ levels are the 0^+, 1.08-Mev level and the 2^+, 3.07-Mev level.[19] We therefore expect to find the two lowest $(s,d)T=0$ levels and the two lowest $T=1$ levels in N^{14} in the order, (1,0), (3,0), and (0,1), (2,1), although they need not necessarily have the same energy separations as their counterparts in F^{18}.

We have calculated the strengths of the $M1$, $\Delta T=1$ transitions connecting these states in N^{14}. Electric quadrupole transitions and $\Delta T=0$, $M1$ transitions were not considered since they are expected to be negligibly small by comparison to $\Delta T=1$, $M1$ transitions.[45] The wave functions of these states are assumed to be expanded in a jj-coupling basis, i.e.,

$$\Psi(J,T)=C_{\frac{5}{2}\,\frac{5}{2}}{}^{J\;T}\psi(d_{\frac{5}{2}}{}^2)+C_{\frac{5}{2}\,\frac{1}{2}}{}^{J\;T}\psi(d_{\frac{5}{2}}s_{\frac{1}{2}})+C_{\frac{1}{2}\,\frac{1}{2}}{}^{J\;T}\psi(s_{\frac{1}{2}}{}^2)+C_{\frac{5}{2}\,\frac{3}{2}}{}^{J\;T}\psi(d_{\frac{5}{2}}d_{\frac{3}{2}})+C_{\frac{3}{2}\,\frac{1}{2}}{}^{J\;T}\psi(d_{\frac{3}{2}}s_{\frac{1}{2}})+C_{\frac{3}{2}\,\frac{3}{2}}{}^{J\;T}\psi(d_{\frac{3}{2}}{}^2). \quad (13)$$

With this notation, the $M1$ transition strengths are given by

$$\Lambda(M1)=3(4.58C_{\frac{5}{2}\,\frac{5}{2}}{}^{0\;1}C_{\frac{5}{2}\,\frac{5}{2}}{}^{1\;0}+4.71C_{\frac{1}{2}\,\frac{1}{2}}{}^{0\;1}C_{\frac{1}{2}\,\frac{1}{2}}{}^{1\;0}+2.17C_{\frac{5}{2}\,\frac{5}{2}}{}^{0\;1}C_{\frac{5}{2}\,\frac{3}{2}}{}^{1\;0}+2.66C_{\frac{3}{2}\,\frac{3}{2}}{}^{0\;1}C_{\frac{5}{2}\,\frac{3}{2}}{}^{1\;0}-0.76C_{\frac{3}{2}\,\frac{3}{2}}{}^{0\;1}C_{\frac{3}{2}\,\frac{3}{2}}{}^{1\;0})^2, \quad (14)$$

[45] G. Morpurgo, Phys. Rev. **110**, 721 (1958).

for $(0,1)\rightarrow(1,0)$,

$$\begin{aligned}\Lambda(M1)=(&4.80C_{\frac52\frac52}^{2\,1}C_{\frac52\frac52}^{1\,0}+1.99C_{\frac52\frac52}^{2\,1}C_{\frac52\frac32}^{1\,0}\\&-0.75C_{\frac32\frac32}^{2\,1}C_{\frac32\frac32}^{1\,0}+0.65C_{\frac32\frac32}^{2\,1}C_{\frac52\frac32}^{1\,0}\\&-2.53C_{\frac52\frac12}^{2\,1}C_{\frac32\frac12}^{1\,0}+2.68C_{\frac32\frac12}^{2\,1}C_{\frac32\frac12}^{1\,0}\\&-1.60C_{\frac52\frac32}^{2\,1}C_{\frac52\frac52}^{1\,0}-2.99C_{\frac52\frac32}^{2\,1}C_{\frac32\frac32}^{1\,0}\\&+1.26C_{\frac52\frac32}^{2\,1}C_{\frac52\frac32}^{1\,0})^2,\end{aligned}\tag{15}$$

for $(2,1)\rightarrow(1,0)$, and

$$\begin{aligned}\Lambda(M1)=(&-5.40C_{\frac52\frac52}^{2\,1}C_{\frac52\frac52}^{3\,0}+1.30C_{\frac52\frac52}^{2\,1}C_{\frac52\frac32}^{3\,0}\\&+0.70C_{\frac32\frac32}^{2\,1}C_{\frac32\frac32}^{3\,0}+3.99C_{\frac32\frac32}^{2\,1}C_{\frac52\frac32}^{3\,0}\\&-4.62C_{\frac52\frac12}^{2\,1}C_{\frac52\frac12}^{3\,0}+3.15C_{\frac32\frac12}^{2\,1}C_{\frac52\frac12}^{3\,0}\\&-3.20C_{\frac52\frac32}^{2\,1}C_{\frac52\frac52}^{3\,0}-0.53C_{\frac52\frac32}^{2\,1}C_{\frac32\frac32}^{3\,0}\\&-1.46C_{\frac52\frac32}^{2\,1}C_{\frac52\frac32}^{3\,0})^2,\end{aligned}\tag{16}$$

for $(2,1)\rightarrow(3,0)$.

If the wave functions given by Redlich[9] for the $s^4p^{12}(s,d)$ levels are assumed for the $s^4p^8(s,d)$ levels, the strengths of the $M1$ transitions given by Eqs. (14), (15), and (16) are $\Lambda(M1)=77$, 1.55, and 32.8, corresponding to $|M|^2=10$, 0.20, and 4.26 Weisskopf units, respectively. Equation (14) predicts an $M1$ strength relatively insensitive to reasonable changes in the Redlich wave functions. The wave functions of Elliott and Flowers,[10] for example, give $|M|^2=9.2$ Weisskopf units. With initial and final states arising from a pure $d_{\frac52}^2$ configuration, we have $|M|^2=8.2$. Pure $s_{\frac12}^2$ gives $|M|^2=8.6$ Weisskopf units. We expect, therefore, that the $(s,d)(0,1)\rightarrow(1,0)$ transition in N^{14} will be extremely strong, about 5–10 Weisskopf units. The strengths predicted by Eqs. (15) and (16) are, however, more sensitive to changes in the wave functions. This is particularly true for Eq. (15). The small value of the $(2,1)\rightarrow(1,0)$ strength for the Redlich wave function results from a partial cancellation in Eq. (15) of terms with different signs. With initial and final states which are pure $d_{\frac52}^2$, for example, the strength would be $|M|^2=3$ Weisskopf units, 15 times the values resulting from the use of Redlich's wave function. Because of this sensitivity we feel that the study of the $(2,1)\rightarrow(3,0)$ and $(2,1)\rightarrow(1,0)$ transitions will not be as useful in identifying the (s,d) levels in N^{14} as will the large, insensitive $(0,1)\rightarrow(1,0)$ transition strength.[46]

We propose the 0^+, N^{14} 8.62-Mev level and the $J=1$, 6.23-Mev level as the (s,d) states analogous to the 0^+, F^{18} 1.08-Mev level and the 1^+, F^{18} ground state.[47] The 8.62-Mev level and the p^{-2}, 2.31-Mev level are the only known $J^\pi=0^+$ levels in N^{14}. For any reasonable value of the spin-orbit coupling, the separation of the $T=1$, $J^\pi=0^+$ levels of p^{-2} is greater than 10 Mev.[8] Since it is most unlikely that it arises from p^{-2}, the configuration assignment (s,d) seems the most likely choice for the 8.62-Mev level.

The measured width of the N^{14} $8.62\rightarrow6.23$ transition corresponds to $|M|^2=7.3$ Weisskopf units[48] if, in actual fact, it is $M1$. The great strength of this transition was previously taken to indicate $E1$ radiation and therefore odd-parity for the 6.23-Mev level. As was discussed above, however, the strength of this transition is consistent with the anomalously large $M1$ strength theoretically predicted for the $(s,d)(0,1)\rightarrow(1,0)$ transition. The agreement between the measured strength of the $8.62\rightarrow6.23$ transition and the theoretical prediction for the (s,d), $(0,1)\rightarrow(1,0)$ transition is, in fact, the main reason for proposing the 6.23- and 8.62-Mev levels as $s^4p^8(s,d)$ states.

The 8.62-Mev level has also been observed to decay to the 3.95-Mev level and to the ground state. The strengths of the transitions, assuming $M1$ radiation, are $|M|^2=0.57$ Weisskopf unit for the $8.62\rightarrow3.95$ transition and $|M|^2=0.09$ Weisskopf unit for the $8.62\rightarrow0$ transition.[48] Since electromagnetic transitions between (s,d) and p^{-2} levels are forbidden, these observed decays are sources of information on the mutual contamination of these configurations if the 8.62- and 6.23-Mev levels are the (0,1) and (1,0) levels of (s,d) as assumed. The 8.62-Mev level may be contaminated with p^{-2} states by interaction with the 2.31-Mev level and with the excited $p^{-2}(0,1)$ level which is expected to be well above 10-Mev excitation in N^{14}. The 6.23-Mev level may interact with the ground state and the 3.95-Mev level. The other $p^{-2}(1,0)$ level is probably too far above the 6.23-Mev level to have an appreciable interaction with it. The great strength of the $8.62\rightarrow6.23$ transition enables us to make rough estimates of the (s,d) contamination in the ground state configuration. No $M1$ transition between p^{-2} levels is expected to have a strength nearly as great as the $8.62\rightarrow6.23$ transition so we assume that the major contribution to the $8.62\rightarrow3.95$ and the $8.62\rightarrow0$ transitions comes from the (s,d) contamination of the p^{-2} levels. With this assumption, the contaminations in the p^{-2} levels are given by the ratios of their $M1$ strengths to that of the $8.62\rightarrow6.23$ decay, i.e., ~8% in the 3.95-Mev level and ~1% in the ground state.

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[46] The F^{18}, $(2,1)\rightarrow(1,0)$ and $(2,1)\rightarrow(3,0)$ transitions presumably correspond to the F^{18}, $3.07\rightarrow0$ and $3.07\rightarrow0.94$ transitions. The ratio of the radiative widths of these transitions has been measured to be $\Gamma(3.07\rightarrow0.94)/\Gamma(3.07\rightarrow0)=4.0$ (reference 19). The ratio calculated from Eqs. (15) and (16) with the wave functions of Redlich is 7.1. Considering the sensitivity of Eq. (15) the agreement must be considered as quite good.

[47] These assignments are in contradiction to those of Unna and Talmi (reference 4). These authors, assuming no configuration mixing between the $2s$ and $1d$ shells, predicted the $1s_{\frac12}^4 1p_{\frac32}^8 2s_{\frac12}^2$ (0,1) and (1,0) levels at 8.76 and 4.95 Mev in N^{14} and proposed an identification of the N^{14} 8.62- and 5.10-Mev levels with these configurations. They also identified the second mass 18 $T=1$, $J=0$ state, which has been predicted (reference 9) 3.92 Mev above the first $T=1$, $J=0^+$ level in F^{18}, as belonging to $1s_{\frac12}^4 1p_{\frac32}^8 1p_{\frac12}^4 2s_{\frac12}^2$. Presumably the first (0,1) level belongs to the $d_{\frac52}$ configuration in this scheme and lies in between the 5.10- and 8.62-Mev levels, say ~5–6 Mev in N^{14} and ~3–4 Mev in C^{14}. Since such a state has not been observed, we feel it more likely that the 8.62-Mev level is the lowest $(s,d)T=1$, $J^\pi=0^+$ level in mass 14 and is analogous to the 1.08-Mev level in F^{18} rather than the second $T=1$, $J^\pi=0^+$ level in F^{18}. The N^{14}, 5.10-Mev level has been shown by WRH to have $J=2$ since the work of Unna and Talmi was completed, so that it cannot be the (s,d), (1,0) level in question.

[48] D. H. Wilkinson and S. D. Bloom, Phil. Mag. **2**, 63 (1957).

If we also assume that the ground state and the 3.95-Mev level have roughly the same interaction matrix element with the 6.23-Mev level, first order perturbation theory predicts admixtures in the ratio $(6.23/2.28)^2 = 8.6$ for the 3.95-Mev level relative to the ground state, in good agreement with the values obtained above. If the 6.23-Mev level contaminates these (1,0) levels of p^{-2}, one can say roughly that they contaminate it by the same amount; hence, the 6.23-Mev level may be contaminated with $\sim$8–10% p^{-2}.

This analysis provides no information on the structure of the 8.62-Mev level. Information relative to its p^{-2} contamination can be obtained from a study of its proton reduced width. The proton reduced width of the 8.62-Mev level is $\theta_p^2 = 0.026$,[19] corresponding to $\mathcal{S} = 0.035$. If the relative reduced width $\mathcal{S}$ is taken to result entirely from the presence of p^{-2} contamination in the 8.62-Mev level, a lower limit on the amount of this contamination can be estimated.

The theoretical calculation of the relative reduced width, $\mathcal{S}$, for the $p^{-2}(0,1)$ state is straightforward.[35] The greatest value possible is $\mathcal{S} = 2.0$ and occurs in extreme jj coupling when the (0,1) state in question is $p_{\frac{1}{2}}^{-2}$. The width is of course zero in jj coupling for the other (0,1) state, since it arises from $p_{\frac{3}{2}}^{-2}$. For intermediate coupling wave functions near the jj limit a value slightly smaller, say 1.8 is obtained. If the C^{13} ground state is assumed to be pure p^9 and if the jj-coupling value of 2.0 is taken for the relative reduced width of the p^{-2} contamination in the 8.62-Mev level, then a lower limit of $\sim$2% is obtained for this contamination. Any p^{-2} contamination different from $p_{\frac{1}{2}}^{-2}$ would lead to a larger lower limit. This lower limit on the p^{-2} impurity in the 8.62-Mev level is compatible with the estimate of Baranger and Meshkov[3] of $\sim$11% d^2 contamination in the C^{14} ground state.

If the N^{14}, (s,d), (1,0) level is at 6.23 Mev then both the 9.16- and the 10.43-Mev levels are in the energy region where we would expect the (2,1) level of this configuration. The strengths of the $10.43 \rightarrow 6.23$ and the $9.16 \rightarrow 6.23$ transitions must both be rather weak to have escaped observation. That they have not been observed is consistent with the small value predicted for the strength of the (s,d), $(2,1) \rightarrow (1,0)$ transition using Redlich's wave functions. Both the 9.16- and the 10.43-Mev levels decay strongly to the $J=3$, N^{14} 6.44-Mev level. Assuming the transition proceeds by $M1$ radiation, the strength of the $9.16 \rightarrow 6.44$ transition obtained from several measurements[40–44] is $\Lambda(M1) = 16 \pm 5$, while the strength of the $10.43 \rightarrow 6.44$ transition is $\Lambda(M1) \simeq 10$.[16,42] Either of these values is in reasonable agreement with our expectations for the strength of the (s,d), $(2,1) \rightarrow (3,0)$ transition, so that the N^{14}, 6.44-Mev level is a possible candidate for this (3,0) level. In the last subsection we proposed that the 9.16- and 10.43-Mev levels were two $J^{\pi} = 2^+$, $T=1$ levels resulting from a strong interaction between the lowest (2,1) levels of the p^{-2} and (s,d) configurations. The fact that both levels decay strongly to the $J=3$, 6.44-Mev level in N^{14} is in agreement with this suggestion if the 6.44-Mev level has even parity—in which case it is likely that it is a $s^4p^8(s,d)$ level. Further information on the structure of the 9.16- and 10.43-Mev levels can be obtained from a study of their proton reduced widths.

The total width, Γ, of the N^{14} 9.16-Mev level is 77 ± 12 ev[43] while that of the 10.43-Mev level is 30 ± 3 kev.[16] The corresponding proton p-wave reduced widths θ_p^2 are calculated to be 0.85×10^{-4} and 1.2×10^{-2}, with $\mathcal{S}$ equal to 1.2×10^{-4} and 1.6×10^{-2}, respectively.

We have calculated $\mathcal{S}$ for the $p^{-2}(2,1)$ level in intermediate coupling. Using an intermediate coupling wave function corresponding to $L/K=6$ and $a/K=6.75$, which was given by Bennett[49] for the C^{13} ground state, and Elliott's wave function for the N^{14}, $p^{-2}(2,1)$ level, we find $\mathcal{S} = 2.7 \times 10^{-2}$. For larger values of a/K, $\mathcal{S}$ is even smaller, vanishing in the limit of jj coupling. The theoretical value quoted above is in fairly good agreement with the experimentally determined value of $\mathcal{S}$ for the 10.43-Mev level. If N^{14} were described by an intermediate coupling scheme much closer to jj coupling, then the theoretical prediction could be brought into agreement with the width of the 9.16-Mev level. A coupling so close to the jj limit would, however, be at variance with the schemes used so successfully in previous investigations of the p shell. On the other hand, a comparison of the predicted width of the $p^{-2}(2,1)$ level in intermediate coupling with the experimental reduced width of the 9.16-Mev level would seem to indicate that the 9.16-Mev level contains at most a few percent in intensity of the configuration p^{-2}. The predicted p^{-2} width is, however, such a small fraction of a single-particle reduced width that small admixtures of higher configurations in the C^{13} ground state might diminish the width by an interference of terms arising from different configurations. The following very crude calculations serve to illustrate this possibility.

We have previously conjectured that the 9.16-Mev level is described by a wave function,

$$\Psi_{14}(2,1) = a_1\psi[p^{10}; 2,1] + a_2\psi[p^8(s,d); 2,1].$$

Assume for present purposes that the main features of the C^{13} ground state are incorporated in the wave function,

$$\Psi_{13}(\tfrac{1}{2},\tfrac{1}{2}) = b_1\psi[p^9; \tfrac{1}{2},\tfrac{1}{2}] + b_2\psi[p^7(s,d); \tfrac{1}{2}\tfrac{1}{2}].$$

Further assume a jj-coupling $p_{\frac{3}{2}}^{8}$ core for the (s,d), (2,1) configuration in Ψ_{14}. For the $p^7(s,d)$ state in Ψ_{13} take the (s,d), (2,1) level antisymmetrically coupled to a $p_{\frac{3}{2}}^{7}$ parent. With these crude assumptions and with the previously used intermediate coupling wave functions—$a/K=6.75$ for p^9 and the Elliott wave function for p^{10}—the reduced width is readily shown to be

$$\mathcal{S} = 15(0.042a_1b_1 + 0.13a_2b_2)^2. \quad (17)$$

[49] E. F. Bennett, Atomic Energy Commission Report NYO-8082, 1958 (unpublished).

TABLE II. $\Delta T=1$, $E1$ transition strengths in N^{14}.

Transition (jj coupling)	E_i (Mev)	E_f (Mev)	$\Lambda(E1)$[a]	$\Lambda(E1)$[b]	$\Gamma(E1)$[b] (ev)	Γ(exp)[c] (ev)
$(p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=1} \rightarrow (p_{\frac{1}{2}}^2)_{J=1}$	8.06	0	1.24	1.06	35	9.4(±~20%)
$(p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=1} \rightarrow (p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-1})_{J=1}$	8.06	3.95	0	0.17	0.7	1.2(±~25%)
$(p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=1} \rightarrow (s_{\frac{1}{2}}^2)_{J=1}$	8.06	6.23	1.24	0.26	0.10	not observed(<0.5)
$(p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=0} \rightarrow (p_{\frac{1}{2}}^2)_{J=1}$	8.70	0	1.92	1.69	70	43(±~20%)
$(p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=0} \rightarrow (p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-1})_{J=1}$	8.70	3.95	0	0.26	1.7	0.9(±~25%)
$(p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=0} \rightarrow (s_{\frac{1}{2}}^2)_{J=1}$	8.70	6.23	1.92	0.41	0.38	not observed(<0.5)
$(s_{\frac{1}{2}}^2)_{J=0} \rightarrow (p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=1}$	8.62	5.69	1.92	0.30	0.5	0.7(±~30%)
$(p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=1} \rightarrow (p_{\frac{1}{2}}^2)_{J=0}$	5.69	2.31	0.62	0.58	1.4	unknown
$(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=2} \rightarrow (d_{\frac{5}{2}}d_{\frac{3}{2}})_{J=1}$	9.50	6.23	1.41	0.45	1.0	<0.1
$(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=2} \rightarrow (d_{\frac{5}{2}}d_{\frac{3}{2}})_{J=3}$	9.50	6.44(?)	1.47	0.071	0.13	<0.05
$(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=3} \rightarrow (d_{\frac{5}{2}}d_{\frac{3}{2}})_{J=3}$	8.90	6.44(?)	1.30	0.062	0.06	0.01(±~33%)

[a] Extreme jj coupling.
[b] Extreme jj coupling for the odd-parity states, the wave functions of Visscher and Ferrell (reference 8) for the s^4p^{10} states, and the wave functions of Redlich (reference 9) for the assumed $s^4p^8(s,d)$ states at 8.62, 6.44, and 6.23 Mev in N^{14}.
[c] Experimental widths (references 14, 40, 41, 48, 50).

Let us next take $a_1=(\frac{1}{3})^{\frac{1}{2}}$, $a_2=\pm(\frac{2}{3})^{\frac{1}{2}}$ for the 9.16-Mev level. This assumption enables us to estimate the relative values of b_1 and b_2 needed for any desired degree at cancellation in the reduced width. In order to reduce $\mathcal{S}$ by a factor of 10—from 2.7×10^{-2} to 2.7×10^{-3}—one needs only a 1% contamination, or an amplitude b_2 of 0.1 in the C^{13} wave function. For a reduction by another factor of 10, which is needed for close agreement with the observed width of the 9.16-Mev level, an amplitude of 0.2, i.e., a 4% contamination in the C^{13} ground state is needed. The cancellation is even more easily achieved if the ratio, a_1/a_2, is decreased beyond our arbitrary estimate. These considerations are consistent with the observed reduced width of the 10.43-Mev level since it will have amplitudes of the wrong phase to provide cancellation if the phases of the 9.16-Mev level amplitudes favor cancellation. Unless a cancellation of the sort which is outlined here is involved the reduced width of the 9.16-Mev level is in disagreement with our assumption that the 9.16- and 10.43-Mev levels are mixtures of $p^{-2}(2,1)$ and (s,d), (2,1) states.

D. $\Delta T=1$, $E1$ Transitions in N^{14}

The calculated strengths of the isotopic-spin allowed $E1$ transitions connecting the singly-excited p^9s and p^9d odd-parity levels of N^{14} with the p^{-2} and (s,d) states are given in Table II. The identification of the p^9s and p^9d states is that of WRH. The strengths listed in column 4 were calculated assuming the extreme jj-coupling configurations listed in Table II for all states involved. The strengths listed in column 5 were calculated assuming extreme jj coupling for the odd-parity states, the wave functions of Visscher and Ferrell for the p^{-2} states, and the wave functions of Redlich for the (s,d) states. Column 6 lists the radiative widths corresponding to the strengths of column 5, and the available experimental information[14,40,41,48,50] on the radiative widths is listed in column 7. For all measured widths the contributions of higher multipole orders are expected to be negligible. In view of the assumptions made in the calculations, the overall agreement between the calculated and measured $E1$ widths is surprisingly good.

The factor of four disagreement of the measured $8.06\rightarrow0$ $E1$ transition strength with that calculated for $(p_{\frac{1}{2}}s_{\frac{1}{2}})\rightarrow(p_{\frac{1}{2}}^2)$ is presumably due to departures from extreme jj coupling for the 8.06-Mev level. To check the sensitivity of this calculation to an admixture of $(p_{\frac{1}{2}}d_{\frac{3}{2}})$, the radiative width of the $8.06\rightarrow$ground-state transition was calculated assuming a 8.06-Mev level wave function of the form

$$\Psi(1,1)=a_1\psi(p_{\frac{1}{2}}s_{\frac{1}{2}})+a_2\psi(p_{\frac{1}{2}}d_{\frac{3}{2}}),$$

with the result

$$\Gamma(8.06\rightarrow \text{g.s.})=35\,|a_1-(\tfrac{5}{2})^{\frac{1}{2}}a_2|^2. \tag{18}$$

From the $C^{13}(d,p)C^{14}$ stripping results the weight of s^4p^9d in the C^{14} 6.09-Mev level—which is the analog of the N^{14} 8.06-Mev level—was found to be <13%.[51] Taking $a_2^2=0.13$ with the same phase for a_1 and a_2, Eq. (18) gives $\Gamma(8.06\rightarrow0)=9.8$ ev as compared to the experimental width of 9.4 ev. Therefore, a $(p_{\frac{1}{2}}d_{\frac{3}{2}})$ admixture compatible with the upper limit set by the stripping results could cause a factor of ~4 reduction in the $E1$ rate. This consistency argument cannot be used, however, to fix the amount of $(p_{\frac{1}{2}}d_{\frac{3}{2}})$ in the 8.06-Mev level wave function, since the actual $E1$ transition strength is expected to be affected by admixtures of $(p_{\frac{3}{2}}^7p_{\frac{1}{2}}^2)$, etc., in the predominantly $(p_{\frac{3}{2}}^8p_{\frac{1}{2}})$ core as well as by admixtures of p^9d.

[50] C. Broude, L. L. Green, J. J. Singh, and J. C. Wilmott, Phil. Mag. **2**, 499 (1957); Phil. Mag. **2**, 1006 (1957); P. Lehmann, A. Lévêque, and R. Pick, Compt. rend. **243**, 743 (1956).
[51] See Table IX of WRH.

The calculated $9.50 \rightarrow 6.23$ transition arises from the $(d_{\frac{5}{2}}d_{\frac{3}{2}})$ component of the $(s,d)(1,0)$ wave function of Redlich, this being the only component of the $(s,d)(1,0)$ state which can contribute to the transition if the 9.50-Mev level is assumed to be pure $(p_{\frac{1}{2}}d_{\frac{5}{2}})$. The calculated strengths of the $(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=2} \rightarrow (s_{\frac{1}{2}}d_{\frac{3}{2}})_{J=1}$ and $(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=2} \rightarrow (d_{\frac{3}{2}}^2)_{J=1}$ $\Delta T=1$, $E1$ transitions are quite weak, so that the most obvious explanation for the discrepancy between the calculated and observed radiative width of the $9.50 \rightarrow 6.23$ transition is either that the $(d_{\frac{5}{2}}d_{\frac{3}{2}})$ coefficient of the 6.23-Mev level is much smaller than that obtained from Redlich's mass 18 wave functions, or that the assumption of an inert $p_{\frac{3}{2}}^8$ core breaks down for this transition.

The observed strengths of the transitions from the N^{14} 9.50- and 8.90-Mev levels to the N^{14} 6.44-Mev level are consistent with the 6.44-Mev level being the (3,0) level of $s^4p^8(s,d)$, with some indication of $C_{\frac{5}{2}\frac{5}{2}}^{30}$ being smaller here than its value in Redlich's mass 18 wave functions if such is the case.

Dipole radiative widths are proportional to E_γ^3 so that, for a given transition strength, lower energy transitions are difficult to detect. For this reason transitions from the 9.50- and 8.90-Mev levels and from the $J^\pi=2^+$ levels at 9.16 and 10.43 Mev (see Sec. IIIC) to the (3,0) level of $s^9p^8(s,d)$ could easily have been overlooked if the (3,0) level were at an high enough excitation. Therefore, we conclude that the experimental evidence presented in Table II and in the last subsection is consistent with either the 6.44-Mev level or a level above $\sim$7 Mev being the (3,0) level of $s^4p^8(s,d)$.

Other $\Delta T=1$, $E1$ transitions in N^{14} for which some information exists are the $9.50 \rightarrow 0$, $9.50 \rightarrow 3.95$, and $8.90 \rightarrow 7.03$ transitions (assuming the 7.03-Mev level is the $J^\pi=2^+$, $T=0$ level of p^{-2}). These three transitions —which are forbidden in extreme jj coupling—have been observed to be extremely weak. The $9.50 \rightarrow 0$ and $9.50 \rightarrow 3.95$ transitions were discussed in WRH. The weakness of these two transitions is among the best evidence that the $T=1$ odd-parity levels of mass 14 are fairly well described by jj coupling. The weakness of the $8.90 \rightarrow 7.03$ transition is consistent with an assignment of $p^{-2}(2,0)$ for the N^{14} 7.03-Mev level.

E. $M1$ and $E2$ Transition Strengths Connecting the Singly-Excited Odd-Parity States of Mass 14

The $M1$ and $E2$ transition strengths connecting the p^9s and p^9d states proposed by WRH were calculated assuming extreme jj coupling. The results are presented in Table III. The agreement between the experimental and calculated radiative widths is illustrated in the last two columns of Table III. Except for the N^{14} $9.50 \rightarrow 5.83$ and $5.83 \rightarrow 5.10$ transitions, the agreement is satisfactory. As discussed by WRH, the absence of transitions connecting the p^9s states with the p^9d states is consistent with the Δl selection rule for $M1$ transitions.

The $9.50 \rightarrow 5.83$ transition is predicted to be extremely weak. This is a case, then, in which we expect the effects of departures from extreme jj coupling to dominate the transition rate. This remark also holds for the $8.90 \rightarrow 5.10$ transition, so that we regard the agreement with experiment for the latter transition to be fortuitous. Departures from extreme jj coupling for the p^9d, $J^\pi=3^-$ levels can only occur through breakup of the $p_{\frac{3}{2}}^8$ core so that it would be difficult to estimate the effect of departures from $(p_{\frac{1}{2}}d_{\frac{5}{2}})$ for the 5.83-Mev level. For the p^9d, $J^\pi=2^-$ levels, however, admixtures of $(p_{\frac{1}{2}}d_{\frac{3}{2}})$ are possible. Let us calculate the possible effect of such an admixture by assuming that the p^9d, $J^\pi=2^-$ wave functions are given by

$$\Psi(2,1)=a_1\psi(p_{\frac{1}{2}}d_{\frac{5}{2}})+a_2\psi(p_{\frac{1}{2}}d_{\frac{3}{2}}),$$
$$\Psi(2,0)=b_1\psi(p_{\frac{1}{2}}d_{\frac{5}{2}})+b_2\psi(p_{\frac{1}{2}}d_{\frac{3}{2}}),$$

and that the p^9d, $J^\pi=3^-$ state is pure $(p_{\frac{1}{2}}d_{\frac{5}{2}})$. Then the $9.50 \rightarrow 5.10$ and $9.50 \rightarrow 5.83$ transition strengths are calculated to be $\Lambda(M1)=12[a_1b_1-0.02a_2b_2-0.23(a_1b_2+a_2b_1)]^2$ and $0.11(a_1+9.4a_2)^2$, respectively. It is apparent that the $9.50 \rightarrow 5.10$ transition rate is quite insensitive to admixtures of $(p_{\frac{1}{2}}d_{\frac{3}{2}})$ in either $J^\pi=2^-$ state, but that even if the N^{14} 9.50-Mev level contained only 1% in intensity of $(p_{\frac{1}{2}}d_{\frac{3}{2}})$ the calculated value of the $9.50 \rightarrow 5.83$ transition given in Table III would be meaningless—and we certainly expect a larger admixture of $(p_{\frac{1}{2}}d_{\frac{3}{2}})$ than 1%. For a reasonable admixture of $(p_{\frac{1}{2}}d_{\frac{3}{2}})$ in the 9.50-Mev level, however, the $9.50 \rightarrow 5.83$ transition is still about a factor of 4 stronger than can be explained with our assumption of an inert $p_{\frac{3}{2}}^8$ core.

The N^{14} $5.83 \rightarrow 5.10$ transition was studied by WRH who obtained the limits on the radiative width of this transition by means of a Doppler-shift method. We shall see that the disagreement of at least a factor of 50 between the experimental limits and the theoretical prediction for extreme jj coupling is quite serious. The weakness of the calculated $5.83 \rightarrow 5.10$ $M1$ transition is due primarily to the inhibition of $\Delta T=0$, $M1$ transitions in self-conjugate nuclei. As shown by Morpurgo,[45] the $\Delta T=0$, $M1$ rate in a self-conjugate nucleus is proportional to the matrix element of the intrinsic spin $\mathbf{S}$ taken between initial and final states. If we drop the extreme jj-coupling assumption and assume only $s^4p^9(2s,1d)$ for both the 5.83- and 5.10-Mev levels, the largest possible value of $\mathbf{S}$ in either state is $\mathbf{S}=2$. It can easily be shown that the largest possible value of $\Lambda(M1)$ for the $5.83 \rightarrow 5.10$ transition, assuming $\mathbf{S}=2$ and $T=0$ for both levels, is $\Lambda(M1)=0.9$ corresponding to $\Gamma(M1)=1.2\times10^{-3}$ ev. This width can certainly be considered as a model-independent upper limit to $\Gamma(5.83 \rightarrow 5.10)$ with $T=0$ and odd-parity for both states. Actually, we expect the effective value of $\mathbf{S}$ in the transition to be considerably less than unity for any reasonable assumptions about the wave functions.

TABLE III. Extreme jj-coupling $M1$ and $E2$ transition strengths between the singly-excited odd-parity states of mass 14.

Transition $(J_iT_i) \rightarrow (J_fT_f)$	Nucleus	E_i (Mev)	E_f (Mev)	$\Lambda(M1)$	$\Lambda(E2)$[a]	$[\Lambda(E2)/\Lambda(M1)]^{\frac{1}{2}}$ [b]	$\Gamma(M1)$[c] (ev)	$\Gamma(\text{exp})$[d] (ev)
$p_{\frac{1}{2}}s_{\frac{1}{2}} \rightarrow p_{\frac{1}{2}}s_{\frac{1}{2}}$								
$(01) \rightarrow (10)$	N^{14}	8.70	5.69	10.8	...	...	0.81	<4.8[e]
$(11) \rightarrow (10)$	N^{14}	8.06	5.69	15.8	0.0	0.0	0.58	$0.56(\pm\sim25\%)$
$(11) \rightarrow (00)$	N^{14}	8.06	4.91	3.62	...	...	0.31	$0.19(\pm\sim30\%)$
$(10) \rightarrow (00)$	N^{14}	5.69	4.91	0.065	...	...	8.5×10^{-5}	not observed
$(01) \rightarrow (11)$	C^{14}	6.89	6.09	19.5	...	...	2.75×10^{-2}	$>2.2\times10^{-3}$
$p_{\frac{1}{2}}d_{\frac{5}{2}} \rightarrow p_{\frac{1}{2}}d_{\frac{5}{2}}$								
$(21) \rightarrow (20)$	N^{14}	9.50	5.10	12.0	17.86	-1.22	2.82	$3.85(\pm16\%)$
$(21) \rightarrow (30)$	N^{14}	9.50	5.83	0.11	4.49	-6.30	1.55×10^{-2}	$0.80(\pm20\%)$
$(31) \rightarrow (20)$	N^{14}	8.90	5.10	0.081	3.22	-6.30	1.23×10^{-2}	$2\times10^{-2}(\pm50\%)$
$(31) \rightarrow (30)$	N^{14}	8.90	5.83	19.3	19.13	-1.00	1.55	$0.37(\pm16\%)$
$(30) \rightarrow (20)$	N^{14}	5.83	5.10	0.017	3.22	-13.63	1.83×10^{-5}	$1.3\times10^{-2}>\Gamma>10^{-3}$
$(21) \rightarrow (31)$	C^{14}	7.35	6.72	2.43	17.96	$+2.79$	1.68×10^{-3}	unknown

[a] To take collective enhancements into account multiply $\Lambda(E2)$ by $\epsilon(T_z,\Delta T)$ where $\epsilon(0,1)=1$, $\epsilon(0,0)=(1+2\beta_2)^2=5.2$, $\epsilon(1,0)=\beta_2^2=0.41$ (see Sec. IIIA).
[b] The amplitude ratio of $M1$ and $E2$ contributions to a transition is given by $\delta=[\Gamma(E2)/\Gamma(M1)]^{\frac{1}{2}}=5.38\times10^{-3}E_\gamma[\Lambda(E2)/\Lambda(M1)]^{\frac{1}{2}}$.
[c] The calculated $M1$ radiative width corresponding to $\Lambda(M1)$. $\Gamma(E2)$ is negligible compared to $\Gamma(M1)$ so that $\Gamma(M1)$ is also the total calculated radiative width.
[d] Experimental widths (references 14, 19, 40, 41, 48, 50).
[e] The measured width is an upper limit because of the unknown background to the N^{14} 8.70-Mev level (reference 23).

For $\mathbf{S}=1$ for both states, the limit on $\Gamma(M1)$ for $\Delta T=0$ would be $\Gamma(M1)<0.3\times10^{-3}$ ev. It is apparent, then, that the measured limits on the radiative width of the $5.83\rightarrow5.10$ transition are in disagreement with the identification of the 5.83- and 5.10-Mev levels as $T=0$, singly-excited, odd-parity states. We suggest three possible explanations for this disagreement. (1) Either the 5.83- or 5.10-Mev level has even parity, (2) the 5.83- and 5.10-Mev levels have appreciable admixtures of $T=1$, (3) the Doppler-shift measurement made by WRH of the $5.83\rightarrow5.10$ transition lifetime is in error. In view of the other evidence presented in this paper we do not believe the first alternative to be likely. However, the need of an experimental determination of the parity of both levels in order to check this point is obvious.

Let us see how large an isotopic spin impurity would be necessary in order to obtain a width $\Gamma(M1)>10^{-3}$ ev for the $5.83\rightarrow5.10$ transition. We follow the notation of Radicati[52] and write the wave function of the initial or final state of the transition in the form

$$\Psi=\psi(T)+\alpha_T(T')\psi(T'),$$

where the states are assumed to contain contributions from $T=0$ and $T=1$ only and $\alpha_T(T')$ is the amplitude of the T' impurity in a state with isotopic spin T.

To obtain a crude estimate, we assume $\alpha_0(1)$ is the same for both states and neglect the $\Delta T=0$ part of the transition. We also assume that the $\psi(T')\rightarrow\psi(T)$ and $\psi(T)\rightarrow\psi(T')$ contributions to the $M1$ matrix element are equal in magnitude and phase. With these assumptions we obtain $\Lambda(M1)=4\alpha_0^2(1)\Lambda(\Delta T=1)$. The limit $\Gamma(M1)>10^{-3}$ ev for the $5.83\rightarrow5.10$ transition corresponds to $\Lambda(M1)>1$, so that our crude estimate gives $\alpha_0^2(1)\gtrsim[4\Lambda(\Delta T=1)]^{-1}$. For $\Lambda(\Delta T=1)$ we arbitrarily take 5.8, which is the $\Lambda(M1)$ corresponding to the measured $\Delta T=1$, $9.50\rightarrow5.83$ transition (see Table III). There is some justification for this choice in that we expect the $T=1$ part of the $T=0$, 2^- and 3^- levels to arise chiefly from interactions with the $T=1$, 2^- and 3^- levels at 9.50 and 8.90 Mev; however, the main reason for taking $\Lambda(\Delta T=1)=5.8$ is that a strength of approximately this value or larger is needed in order to obtain reasonable values for the $\alpha_0^2(1)$. Putting the strength $\Lambda(\Delta T=1)=5.8$ into the relationship $\alpha_0^2(1)\gtrsim[4\Lambda(\Delta T=1)]^{-1}$ gives $\alpha_0^2(1)\gtrsim0.05$. From the nature of the assumptions made in obtaining this crude estimate it is clear that, in as far as the N^{14} 5.83- and 5.10-Mev levels are $J^\pi=3^-$ and 2^- and the lifetime measurement of WRH is correct, the estimate can be regarded as a rather firm lower limit to the isotopic-spin impurity of either state. It is hard to envisage a mechanism which could cause such a large contamination; however, there is some empirical evidence[53] that states with large $2s$- or $1d$-reduced widths tend to contain large isotopic-spin impurities.

We have shown that the limits set by WRH on the radiative width of the $5.83\rightarrow5.10$ transition demand a surprisingly large isotopic-spin contamination of the N^{14} 5.10- and 5.83-Mev levels if these states are $J^\pi=2^-$ and 3^- as we propose. In view of this consequence and the difficulty of Doppler-shift lifetime determinations, a re-measurement of the N^{14} 5.83-Mev lifetime would appear to be quite worthwhile, as would a parity measurement of the N^{14} 5.10- and 5.83-Mev levels. The question of the isotopic-spin impurities of the p^9s and p^9d levels will be discussed further in Sec. IIIG.

[52] L. A. Radicati, Proc. Phys. Soc. (London) **A66**, 139 (1953); **A67**, 39 (1954), and references therein.

[53] D. H. Wilkinson, *Proceedings of the Rehovoth Conference on Nuclear Structure*, edited by H. J. Lipkin (North-Holland Publishing Company, Amsterdam, 1958), Session IV, p. 175.

F. $\Delta T=0$, $E1$, $M2$, and $E3$ Transitions in Mass 14

The $E1$, $M2$, and $E3$ ground-state transitions listed in Table IV and the $M1$ $7.35\to 6.72$ and $6.89\to 6.09$ transitions which were listed in Table III are the only known C^{14} γ-ray transitions. The strengths of the three ground-state transitions in C^{14} were calculated assuming extreme jj coupling for the odd-parity states and for the C^{14} ground state. A strict shell-model calculation would yield zero for the $E3$ $(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=3}\to(p_{\frac{1}{2}}^2)_{J=0}$ transition strength since C^{14} consists of two neutrons outside a C^{12} core. Following Elliott and Flowers,[22] the neutron was endowed with a charge of $1.1e$ in calculating the $E3$ radiative width of the C^{14} $6.72\to 0$ transition.

In Table IV the calculated radiative widths are compared to measured[54] limits on the radiative widths of the C^{14} 6.09- and 6.72-Mev levels. These limits were obtained from Doppler shift measurements. It is apparent that the experimental limits on the radiative widths are consistent with, but give no real test of, the predictions of Table IV. It should be pointed out that a measurement of the lifetime of the C^{14} $6.72\to 0$ transition would provide additional information on the role of electric octupole oscillations in light nuclei.

There is no experimental information on the lifetime of the C^{14} 7.35-Mev level; however, the ratio $\Gamma(7.35\to 0)/\Gamma(7.35\to 6.72)$ has been determined[54] to be within a factor of five of unity. This ratio is consistent with the extreme jj-coupling prediction (see Tables III and IV) of $\Gamma(7.35\to 0)/\Gamma(7.35\to 6.72)=4.4$.

The $M2$ and $E3$ transitions rates of the lowest $T=0$, $J^\pi=2^-$ and 3^-p^9d levels of N^{14}—assumed to be the 5.10-Mev and 5.83-Mev levels—to the $p^{-2}(1,0)$ and $(0,1)$ ground state and first excited state were calculated assuming extreme jj coupling. For the $E3$ transitions, the particle making the transition was endowed with an extra charge of $1.1e$. For N^{14} this leads to an enhancement of a factor of 10. The results are presented in Table V.

A limit on the total radiative width of the N^{14} 5.10-Mev level of $\Gamma_\gamma<2.2\times10^{-3}$ ev was determined by WRH using a Doppler-shift method. The 5.10-Mev level branches 66% to the N^{14} ground state and 34% to the N^{14} 2.31-Mev level so that the radiative width given in Table V for the $M2$, $5.10\to 2.31$ transition is compatible with this limit. WRH also analyzed the angular distribution of the 5.10-Mev γ ray emitted following the capture of protons into the N^{14} 9.50- and 8.90-Mev levels. Assuming $E1$ and $M2$ radiation only for the $5.10\to 0$ transition, they found $\Gamma(M2)/\Gamma(E1)$ to be 0.03 ± 0.01. This ratio, and the branching ratio $\Gamma(5.10\to 0)/\Gamma(5.10\to 2.31)=2$, gives 17 ± 6 for the experimental ratio of the $5.10\to 2.31$ to $5.10\to 0$ $M2$ transitions. This ratio is to be compared to the calculated ratio (see Table V) of 4.6. The agreement is not unsatisfactory, however, it should be remarked that the analysis made by WRH on the angular distribution of the $5.10\to 0$ transition is not justified since the contribution of $E3$ radiation to the $5.10\to 0$ transition is not expected to be negligible (see Table V). The N^{14} $5.10\to 0$ transition, then, is a rather unusual case in which we expect an appreciable effect on the distribution of the radiation from three multipole orders. This situation arises because of the inhibition of electric dipole and magnetic radiation for $\Delta T=0$ transitions in self-conjugate nuclei and the possible enhancement of electric octupole radiation.

From the angular distribution studies of WRH, the $5.83\to 0$ transition was found to have an intensity ratio in the range $0.16<\Gamma(\text{Octupole})/\Gamma(\text{Quadrupole})<16$. Such a ratio would be quite surprising if the 5.83-Mev level had even parity so that the $5.83\to 0$ transition were a mixture of $E2$ and $M3$ radiation, especially in view of the inhibition of $\Delta T=0$ magnetic radiation in self conjugate nuclei.[34,45] However, with $J^\pi=3^-$ for the 5.83-Mev level, the $5.83\to 0$ transition is a mixture of $M2$ and $E3$ radiation, for which we predict $\Gamma(E3)/\Gamma(M2)=0.16$ (see Table V). If there were no collective enhancement of the $E3$, $5.83\to 0$ transition the calculated ratio would be 0.016 so that there is some evidence here for enhancement. The limits on the radiative width of the $5.83\to 0$ transition obtained by WRH from measurements of the branching ratio and lifetime of the 5.83-Mev level are $1.3\times10^{-4}<\Gamma<2.6\times10^{-3}$ ev. The calculated radiative width of the $5.83\to 0$ transition is $\Gamma=\Gamma(M2)+\Gamma(E3)=0.95\times10^{-4}$ ev. If the lifetime determination of WRH is correct the evidence for enhancement is considerably strengthened since an $E3$ width smaller than that given in Table V would be inconsistent with the measurements of Γ and $\Gamma(E3)/\Gamma(M2)$. As discussed in the last sub-

TABLE IV. Ground-state transitions of the singly-excited odd-parity levels of C^{14}.

Transition	E_i (Mev)	Γ_γ(calc)[a] (ev)	Γ_γ(exp)[b] (ev)
$(p_{\frac{1}{2}}s_{\frac{1}{2}})_{J=1}\to(p_{\frac{1}{2}}^2)_{J=0}$	6.09	8.9	$>2.2\times10^{-3}$
$(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=2}\to(p_{\frac{1}{2}}^2)_{J=0}$	7.35	7.5×10^{-3}	unknown
$(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=3}\to(p_{\frac{1}{2}}^2)_{J=0}$	6.72	1.3×10^{-5}	$<2.2\times10^{-3}$

[a] Calculated in extreme jj coupling using the configurations of column 1.
[b] Experimental radiative widths (reference 54).

TABLE V. $\Delta T=0$, $(p_{\frac{1}{2}}d_{\frac{5}{2}})\to(p_{\frac{1}{2}}^2)$, $M2$ and $E3$ radiative widths in N^{14}.

J_i	J_f	E_i (Mev)	E_f (Mev)	Multi-polarity	Γ_γ(calc) (ev)
3	1	5.83	0	$M2$	8.2×10^{-5}
3	1	5.83	0	$E3$	1.3×10^{-5}
2	1	5.10	0	$M2$	1.7×10^{-5}
2	0	5.10	2.31	$M2$	7.8×10^{-5}
2	1	5.10	0	$E3$	0.8×10^{-5}

[54] E. K. Warburton and H. J. Rose, Phys. Rev. **109**, 1199 (1958).

section, one consequence of the 5.83-Mev level lifetime determination is that the 5.83- and 5.10-Mev levels contain $\sim 5\%$ in intensity of $T=1$. In this case, if the 5.83-Mev level and the $(p_{\frac{1}{2}}d_{\frac{5}{2}})$, (3,1) level at 8.90-Mev in N^{14} were each other's chief contaminators, the calculated $\Delta T=1$ part of the $M2$, $5.83\rightarrow 0$ width would be larger than the $\Delta T=0$ part by a factor $\sim 0.05(\mu_p-\mu_n-\frac{1}{3})^2/(\mu_p+\mu_n-\frac{1}{3})^2=3.2$.[34] The corresponding increase in the calculated $M2$, $5.83\rightarrow 0$ width would be in better agreement with the experimental range for the $5.83\rightarrow 0$ width but would imply a collective enhancement of the $E3$ transition even larger than the factor of 10 assumed.

Finally, we compare the experimental and calculated ratio $\Gamma(5.83\rightarrow 0)/\Gamma(5.83\rightarrow 5.10)$. From Tables III and V we find $\Gamma(5.83\rightarrow 0)/\Gamma(5.83\rightarrow 5.10)=5.2$, while the experimental ratio was determined by WRH to be 0.18. The discrepancy of a factor of 30 is approximately what we would expect if the lifetime measurement of WRH were correct. This comparison, then, gives some support to the proposal of large isotopic-spin impurities in the N^{14} 5.83- and 5.10-Mev levels.

G. Isotopic-Spin Impurities in N^{14}

The isotopic-spin forbidden $E1$ transitions of the lowest N^{14} p^9s $T=1$ and $T=0$ levels—which we take to be the ones at 8.06 and 5.69 Mev—were discussed by Wilkinson and Bloom[48] who compared the strengths of these transitions with the allowed transitions to the N^{14} ground and first excited states, respectively. If the p^{-2} ground state and first excited state are assumed to have negligible isotopic-spin impurities, this comparison leads to estimates of $\alpha_1^2(0)_{8.06}\simeq 0.061$ and $\alpha_0^2(1)_{5.69}\sim 0.15$.[55] From the closeness of these admixtures, Wilkinson and Bloom suggested that the $J^\pi=1^-$, $T=0$ and 1 levels were each other's chief contaminators. In the spirit of this suggestion we can take the T' part of both wave functions to be $(p_{\frac{1}{2}}s_{\frac{1}{2}})$ and see if the $(p_{\frac{1}{2}}s_{\frac{1}{2}})\rightarrow(p_{\frac{1}{2}}^2)$ $E1$ strengths given in Table II lead to appreciably different $\alpha_T^2(T')$ for the $(p_{\frac{1}{2}}s_{\frac{1}{2}})$ states in question. As a matter of fact, they do not if the estimate is made by modifying the estimate of Wilkinson and Bloom to take into account the factor of two difference in the calculated $(p_{\frac{1}{2}}s_{\frac{1}{2}})\rightarrow(p_{\frac{1}{2}}^{-2})$, $(1,1)\rightarrow(1,0)$ and $(1,0)\rightarrow(0,1)$ rates (see Table II). In this case, both of the $\alpha_T^2(T')$ are changed by a factor of two, so that $\alpha_1^2(0)_{8.06}\simeq 0.12$ and $\alpha_0^2(1)_{5.69}\simeq 0.075$.

In principle, the same sort of analysis can be made to estimate the isotopic-spin impurity of the N^{14} 5.10-Mev level—assuming it has odd-parity and is predominantly $(p_{\frac{1}{2}}d_{\frac{5}{2}})$. As is the case for the $(p_{\frac{1}{2}}s_{\frac{1}{2}})$, $T=0$ and 1 levels, we can assume that the $J^\pi=2^-$, $T=1$, $(p_{\frac{1}{2}}d_{\frac{5}{2}})$ level at 9.50 Mev and the 5.10-Mev level are each other's chief contaminators. This assumption is justified to some extent because the 9.50-Mev level is probably the only $J^\pi=2^-$, $T=1$ level below 11 Mev in N^{14} (see Fig. 1). The $T=1$ part of the N^{14} 5.10-Mev level would then be predominantly $(p_{\frac{1}{2}}d_{\frac{5}{2}})$, and we have no meaningful estimate for the strength of the $\Delta T=1$ part of the $E1$, $5.10\rightarrow 0$ transition since an $E1$ $(p_{\frac{1}{2}}d_{\frac{5}{2}})\rightarrow(p^{-2})$ transition is forbidden. One method of obtaining an estimate of $\alpha_0^2(1)_{5.10}$ is to estimate the $5.10\rightarrow 0$ transition rate from the calculated $M2$ $(p_{\frac{1}{2}}d_{\frac{5}{2}})_{J=2}\rightarrow(p_{\frac{1}{2}}^2)_{J=0}$ transition and the known ratio $\Gamma(5.10\rightarrow 0)/\Gamma(5.10\rightarrow 2.31)$, and to assume for the strength of the $\Delta T=1$ part of the $5.10\rightarrow 0$ $E1$ transition a limit corresponding to the measured limit $\Gamma(9.50\rightarrow 0)<7.6\times 10^{-3}$ ev. These assumptions give $\alpha_0^2(1)_{5.10}>0.13$. This estimate, although hardly reliable, does show that the decay modes of the N^{14} 5.10-Mev level are not inconsistent with a large isotopic-spin impurity for this level.

There is another way to estimate the isotopic-spin impurities of the p^9s and p^9d levels of N^{14}. Barker and Mann[56] have shown that the ratio of the proton and neutron reduced widths is given by

$$\frac{\theta_p^2}{\theta_n^2}=\left|\frac{1+\alpha_T(T')}{1-\alpha_T(T')}\right|^2 \qquad (19)$$

if the $T=0$ and $T=1$ parts of the wave function are assumed to differ only in their T values. In principle, then, the isotopic-spin impurity of the 8.06-Mev level, for instance, could be obtained from a comparison of the $C^{13}(p,p)C^{13}$ and $C^{13}(d,n)N^{14}$ reactions. Alternatively, we can obtain an estimate of the isotopic-spin impurity of a p^9s or p^9d level by assuming the deviation of the reduced width from the single-particle value is due to an isotopic-spin impurity. In this case the relative reduced width $\mathcal{S}$ is given from Eq. (19) by

$$\mathcal{S}=|1+\alpha_T(T')|^2. \qquad (20)$$

The proton reduced widths of the N^{14} 8.06-, 8.70-, 8.90-, and 9.50-Mev levels correspond[14] to $\mathcal{S}=0.6$, 0.9, 0.9, and 0.7, which, with the above assumptions, give $\alpha_1^2(0)=0.05$, 0.0025, 0.0025, and 0.027, respectively. These estimates are not very meaningful since we expect the breakup of the C^{13} core, and the interaction of the $2s$ and $1d$ shells to give $\mathcal{S}<1$ regardless of the isotopic-spin contamination. In addition, the values of $\mathcal{S}$ have an uncertainty of $\simeq 30\%$ and Eq. (20) is quite sensitive to $\mathcal{S}$. However, the expected effect of isotopic-spin impurities does offer a qualitative explanation for the discrepancy between the relative reduced widths of N^{14} $T=1$ states and their analogs in C^{14}. As discussed by WRH, the two s-wave reduced widths in C^{14} are equal while the s-wave reduced widths of N^{14} are not and likewise for the d-wave reduced widths. For the s states at least, the discrepancy is outside the experimental

[55] Actually Wilkinson and Bloom proposed $J^\pi=1^-$ for the N^{14} 6.23-Mev level. Their analysis, then, was for the $6.23\rightarrow 0$ transition, rather than the $5.69\rightarrow 0$ transition. The decay of the 6.23-Mev level is similar to that of the 5.69-Mev level so there is no real difference in the analysis. They obtained $\alpha_0^2(1)_{6.23}\simeq 0.078$.

[56] F. C. Barker and A. K. Mann, Phil. Mag. 2, 5 (1957).

error. This difference is just what we would expect if the $\alpha_0(1)$ of the two $T=1$ s states in N^{14} had different magnitudes or phase and likewise for the two d states.

IV. INELASTIC SCATTERING

In previous sections of the present paper we have studied several excited states of N^{14} using the time-honored procedures of comparing observed radiative and nucleon widths with the predictions of the shell model. Any reaction data is in principle capable of yielding information on shell structure. In particular, the direct interaction theory of inelastic scattering has recently been used[57,58] to study the structure of nuclear states in the p shell. In our attempt to compare our conjectured mass 14 configuration and spin assignments with as much experimental data as possible, we have calculated the theoretical differential cross sections for the inelastic scattering of protons, deuterons and α particles and have compared these with the available data. The results do not add as much as had hoped to the results of the preceding sections, mostly because of the sparcity of the data, but also because in the cases of the (p,p') and (d,d') reactions, the simpler theories are of doubtful applicability.

The expressions for the inelastic scattering cross sections for α particles and protons are derived in the appendix, the final formulas for the differential cross section for $N^{14}(\alpha,\alpha')N^{14*}$ and $N^{14}(p,p')N^{14*}$ being given by Eqs. (5a) and (14a), respectively. Also given in the appendix is an expression [Eq. (15a)] for the $C^{14}(p,p')C^{14*}$ cross section. There is no experimental data available at the present time with which to compare this cross section; it is given in the event that such experimental data becomes available.

For both the (α,α') and (p,p') reactions, a single-particle transition operator is assumed. A direct result of this assumption is that the cross section for excitation of double-excited configurations is predicted to be zero. This assumption is also made in a completely qualitative discussion of some experimental (d,d') results. It is found that, in general, this assumption gives results consistent with the results presented in the preceding sections. A brief report of the work presented in this section has been made previously.[59]

A. $N^{14}(\alpha,\alpha')N^{14*}$

Miller *et al.*[28] have measured the relative cross sections for exciting various N^{14} levels by the inelastic scattering of alphas. They used a gas target and took data at lab angles between 75° and 105°. Their results are given in Table VI, the numbers being our estimates of the relative heights of the peaks on a graph for $\theta_L=90°$ in their paper. There is no evidence for excitation of the 2.31-Mev level. This is to be expected, of course, since this level has $T=1$. Neither is there evidence for the excitation of the 6.23-Mev level, or the doubtful 5.98-Mev level. On the basis of other evidence we have suggested that the 6.23-Mev level is a $J=1$, $T=0$ level arising from the excitation of two $1p$-nucleons to the nearby $2s$ and $1d$ shells. Its absence in the (α,α') spectrum is consistent with this conjecture, since the single-particle operator which causes the transition is incapable of causing more than one nucleon to "jump". The other peaks are all roughly the same size, with the exception of the one corresponding to the 5.10-Mev level, which is two or three times larger than the others. There are no peaks corresponding to the 4.91-Mev and 5.69-Mev levels, but these may be hidden by the broad peaks of the 5.10- and 5.83-Mev levels which, according to the theory, should be somewhat larger than those of the 4.91- and 5.69-Mev levels.

Miller *et al.* also found that the N^{14} 6.44-Mev level was excited quite strongly in (α,α'). This level is also excited[60] by $C^{13}(d,n)N^{14}$, for which the excitation of doubly-excited configurations is also forbidden if the stripping mechanism is involved. These results indicate that the $J=3$, 6.44-Mev level is not the (3,0) level of the doubly-excited (s,d) configuration—unless the (3,0) level contains a large contamination from the $p^{-2}(3,0)$ level expected[7,8] at a much higher excitation. Such a large contamination seems unlikely, and it is hard to imagine any other configurational assignment for a $J^\pi=3^+$ level at 6.44 Mev in N^{14}. One suggestion that arises is that the 6.44-Mev level has $J^\pi=3^-$ and is a p^9d level, which in Lane's scheme, would be predominantly a $d_{\frac{5}{2}}$ nucleon coupled to the first excited p^9 state of C^{13}. In this case, a strong interaction with the $(p_{\frac{1}{2}}d_{\frac{5}{2}})$, 5.83-Mev level would be expected—thus providing an explanation for the excitation of the 6.44-Mev level by (α,α') and $C^{13}(d,n)N^{14}$. Another possibility is that the 6.44-Mev level is the $J^\pi=3^+$, $T=0$ level of $(p_{\frac{1}{2}}f_{7/2})$, although this level is expected at ~9 Mev in N^{14} since the $f_{7/2}$ level of O^{17} is thought to be 3.8-Mev above the $d_{\frac{5}{2}}$ ground state.[19] If either description of the 6.44-Mev level were correct it would be hard to understand the strong preferential γ decays to this level from the (2,1) levels at 9.16 and 10.43 Mev. It would seem that the various experimental information on the 6.44-Mev level cannot be understood without further information. A striking result of the (α,α') work of Miller *et al.* was the absence of peaks corresponding to the N^{14} 7.47- and 7.60-Mev levels. The (α,α') data, then, is consistent with one or both of these levels being $s^4p^8(s,d)$; and it is possible that one of them is the (3,0) level in question.

The N^{14} 7.96-Mev level and the N^{14} levels, or groups of levels, at 8.45 and 10.05 Mev were excited quite strongly in the work of Miller *et al.*, implying that these

[57] C. A. Levinson and M. K. Banerjee, Ann. Phys. **2**, 471 (1957); **3**, 67 (1958).
[58] W. T. Pinkston, Phys. Rev. **115**, 963 (1959).
[59] W. T. Pinkston and E. K. Warburton, Bull. Am. Phys. Soc. **4**, 254 (1959).
[60] W. A. Ranken, T. W. Bonner, J. M. McCrary, and T. A. Rabson, Phys. Rev. **109**, 917 (1958).

levels belong to singly-excited configurations. These three levels, as well as the 6.44-Mev level, are possible candidates for the $T=0$, $J^\pi=3^+$ and 4^+ levels of $(p_{\frac{1}{2}}f_{7/2})$.

The differential cross section for the excitation of the 3.95-Mev level has been measured by Ploughe *et al.*[61] The Bessel function shape characteristic of nuclear surface interactions was observed, the data corresponding rather well to $[j_2(qR_0)]^2$ with the radius of interaction, $R_0=5.2\times10^{-13}$ cm. Unfortunately, this is the only inelastic α-particle differential cross section which has been measured for mass 14.

In order to compare theoretical and experimental predictions one must evaluate the radial integrals $(n_fl_f|j_\lambda(rq)|1p)$, which appear in the (α,α') cross section. The surface theory of direct interactions[62] simply replaces this radial integral over a Bessel function by the Bessel function itself, evaluated at a "nuclear radius". Another method performs the volume integration using harmonic oscillator radial functions for the target nucleons. For comparison with the 90° data both methods have serious shortcomings.

The difficulty with the surface interaction approximation is that it predicts a differential cross section whose diffraction oscillations go to zero. At 90° for the radius used by Ploughe *et al.*[61] and for q values corresponding to energies of interest, the squares of Bessel functions of different orders may be quite different and their ratios extremely sensitive functions of angle. The observed differential cross sections never quite dip to zero and their ratios are unlikely to depend so sensitively on angle. The drawback of the volume integration method is that it has a Gaussian fall-off at large angles if harmonic oscillator radial functions are used. If the $\hbar\omega$ of the oscillator well is chosen so that the radial integral of $j_2(qr)$ reproduces the experimental first maximum of the 3.95-Mev level distribution, then by 90° all the differential cross sections are drastically attenuated. This is obviously in disagreement with the data. The aforementioned difficulties and the poor agreement of the theory with the data at angles beyond the region of the principal maximum, seem to be typical of the simple theories of direct interactions.

In order to compare our formulas with the 90° data we have devised two very rough approximations in an attempt to circumvent these shortcomings. First, we have replaced the value of the radial integral at 90° by an average over angles of a volume integration using oscillator radial functions. In order to see how sensitively the results depend on this procedure we have introduced a second approximation which is to assume that all the radial integrals at large angles are roughly constant and equal. The results for the cross sections are tabulated in Table VI for both these approximations. One can see that they give very similar results,

TABLE VI. Comparison of the experimental and theoretical $N^{14}(\alpha,\alpha')N^{14*}$ cross sections.

Excitation energy (Mev)	Shell model assignment	$\sigma_{\text{exp}}(90°)$[a] (arbitrary units)	Theoretical cross sections (arbitrary units) ⟨Radial integral⟩$_{\text{av}}$[b]	$\langle\sigma\rangle_{\text{av}}$[b]	σ[c]
3.95	$p_{\frac{1}{2}}^{-1}p_{\frac{3}{2}}^{-1}$, $J=1$	1.0	1/2	1.0	1.0
4.91	$p_{\frac{1}{2}}s_{\frac{1}{2}}$, $J=0$	3.1[d]	7/24	0.19	0.33
5.10	$p_{\frac{1}{2}}d_{\frac{5}{2}}$, $J=2$		1/4	0.83	1.67
5.69	$p_{\frac{1}{2}}s_{\frac{1}{2}}$, $J=1$	1.68[d]	7/24	0.39	0.67
5.83	$p_{\frac{1}{2}}d_{\frac{5}{2}}$, $J=3$		1/4	0.67	1.33
6.44	(?), $J=3$	1.18	...	...	...
7.03	$p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-1}$, $J=2$	0.64	1/2	1.0	1.0

[a] Estimated from Fig. 2 of reference 28.
[b] Calculated with harmonic oscillator radial wave functions and averaged over angles.
[c] All radial integrals assumed constant and equal.
[d] Unresolved.

indicating that the differences in the cross sections are caused mainly by differences in the angular momentum factors multiplying the radial integrals.

Comparing these results to the Indiana results in Table VI we conclude that the (α,α') data is in agreement with the shell assignments already made, although the general validity of such comparisons is questionable until differential cross sections are obtained for all the levels.

The results in Table VI were obtained using pure jj-coupling wave functions. When the excited states belong to the p^{-2} configuration it is not necessary to use jj-coupling wave functions; intermediate coupling wave functions can be used. The reduced matrix elements in Eq. (3a) of the appendix may be evaluated by means of Eq. (12) of Sec. IIIB since both equations contain the matrix element of a tensor of rank 2. We have done this for the wave functions which cause the $C^{14}\to N^{14}$ β-decay matrix elements to vanish. With Elliott's wave function the cross section for the excitation of the ${}^3D_2$, $(p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-1})$, (2,0) state, relative to that for the excitation of the 3.95-Mev level, is increased from 1.0 to 1.4. The wave functions of Visscher and Ferrell cause this ratio to be increased to 2.0. We do not regard these increases to be significant in the light of the aforementioned difficulties. In Table VI the N^{14} 7.03-Mev level is listed as the p^{-2}, (2,0) level for the reasons given in Secs. IIIB and C. The (α,α') data supports this assignment and indicates that the 6.60-, 7.47-, and 7.60-Mev levels could not be p^{-2}, (2,0) since they are not observed in the (α,α') work of Miller *et al.* However, it should be pointed out, that the data presented here and in Sec. III cannot rule out the possibility that the N^{14} 8.45-Mev level is the (2,0) state in question.

B. $N^{14}(p,p')N^{14*}$

The situation here is even worse than in the case of the alpha-particle bombardments. Very few angular distributions have been measured. Freemantle *et al.*,[63]

[61] W. D. Ploughe, E. Bleuler, and D. J. Tendam, Bull. Am. Phys. Soc. 4, 17 (1959).
[62] N. Austern, S. T. Butler, and H. McManus, Phys. Rev. **92**, 350 (1953).
[63] R. G. Freemantle, D. J. Prowse, and J. Rotblat, Phys. Rev. **96**, 1268 (1954).

have measured the angular distribution of 9.5-Mev, protons inelastically scattered from the 3.95-Mev level. They found the distribution to be approximately symmetrical about 90° in the center-of-mass system. This could be due to compound nucleus formation; however, it seems unlikely that the compound nucleus cross section would dominate the direct reaction cross section at a proton energy of 9.5 Mev. A much more likely explanation is that the rise in cross section at backward angles is due to the stripping-type exchange term considered by Banerjee.[64]

In their detailed treatment of $C^{12}(p,p')C^{12*}$, Levinson and Banerjee[57] considered two direct reaction terms. These are the direct term corresponding to the incoming proton with momentum K_i interacting with a nucleon and leaving with momentum K_f and the knockout term corresponding to the interacting nucleon—in this case a proton—exchanging with the incoming proton and leaving with momentum K_f. Banerjee has shown that the exchange term corresponding to the emission of this same proton due to the interaction of the incoming proton with the *remaining* nucleons of the nucleus is, in general, not negligible as was previously assumed. This term, in analogy to the similar heavy particle stripping in deuteron-induced reactions, is called the stripping term. The stripping term is expected to supply ~60% of the direct reaction cross section at low proton energies and to become less important as the proton energy increases with negligible contribution to the cross section for proton energies above approximately 14 Mev. Since the relative contribution of the stripping-type exchange term is difficult to calculate it is not included in the (p,p') cross-section formula derived in the appendix and we consider the $N^{14}(p,p')N^{14*}$ reaction for $E_p>14$ Mev. The only $N^{14}(p,p')N^{14*}$ work in this proton energy range that we are aware of is a cursory investigation of the angular distributions at $E_p=17$ Mev obtained with scintillation crystal resolution.[65]

Detenbeck[65] has observed that for 17-Mev protons the angular distribution of protons leading to the N^{14} 3.95-Mev level is very much like that for the excitation of the 4.43-Mev level in C^{12}, which has been analyzed so successfully by Levinson and Banerjee[57] in terms of the distorted-wave direct interaction model without the stripping-type exchange term. This means that this term can most probably be neglected at this high a proton energy, but that, unfortunately, the use of the Born approximation radial integrals in (p,p') is even less valid than it was for (α,α'). For this reason and also because of the sparcity of experimental data, only the most qualitative comparison between theory and experiment will be made here. In order to get a rough estimate of the cross sections we resort to the same approximations for the radial integrals in Eq. (15a) which were made in the (α,α') case. These approximations result in relative cross section predictions practically identical to the last two columns of Table VI, the reason for this being that the angular momentum factors for (p,p') are almost identical to those for (α,α') for the N^{14} wave functions we assumed. As in the case of (α,α'), the similarity between the cross sections obtained from the two very different procedures for estimating the radial integrals presumably indicate that it is the angular momentum factors which are most important. For the spin dependence in Eq. (15a) we took $a=\frac{5}{6}$, $b=\frac{1}{6}$ (see the appendix). The calculations were repeated for $a=1$, $b=0$ with no significant change in the results.

The major difference between the (p,p') and (α,α') predictions is that the N^{14} 2.31-Mev level can be excited in (p,p'). The predicted cross section for excitation of the 2.31-Mev level in (p,p') is $\frac{1}{6}$ of that for the 3.95-Mev level aside from the dependence on the radial integrals. There is no significant difference in the predicted cross section of the 2.31-Mev level for (a) jj coupling, (b) the wave functions of Visscher and Ferrell, and (c) the wave functions of Elliott. Detenbeck observed that the cross section for the 2.31-Mev level is about 10% or less than that of the 3.95-Mev level at $E_p=17$ Mev in agreement with theory. He also observed that the 3.95- and 7.03-Mev levels and the unresolved 5.69- and 5.83-Mev levels were excited with roughly equal cross sections, while the 6.44-Mev level cross section was about a factor of four smaller.[66] Proton groups corresponding to the 4.91- and 5.10-Mev levels were obscured by $C^{12}(p,p')C^{12*}$. The N^{14} 6.23-, 7.47-, and 7.60-Mev levels were excited weakly if at all.

We conclude that the (p,p') data, such as it is, is consistent with the theoretical predictions based on the assignments already made. For stronger confirmation it would be necessary to obtain differential cross sections for all the levels involved using high resolution magnetic spectroscopy. If this data were available it might be worthwhile to improve the theoretical calculations by means of the distorted wave method used by Levinson and Banerjee or by the simpler procedure of Glendenning,[67] and to include the stripping-type exchange term.

C. $C^{14}(d,d')C^{14*}$

The (d,d') reaction does not lend itself to simple theoretical interpretation. The binding energy of the deuteron is not great enough to consider it a simple projectile as we did for the α-particle inelastic scattering. In addition, if the 2 nucleons in the deuteron are each allowed to interact with the target the exchange terms due to the exclusion principle render the theory even

[64] M. K. Banerjee, International Congress of Nuclear Physics, July, 1958; M. K. Banerjee and D. Mitra (unpublished).

[65] R. W. Detenbeck (unpublished).

[66] Of course, the resolution of this experiment is not sufficient to positively identify the observed proton groups with these levels. Strictly speaking we should say the results are consistent with these relative cross sections.

[67] N. K. Glendenning, Phys. Rev. **114**, 1297 (1959).

more complicated than the (p,p') theory. Huby and Newns[68] have derived expressions for the (d,d') differential cross section when exchange terms are neglected. The theory of Huby and Newns is not sufficient for present purposes since in C^{14} the observed excitation[29] of the $J^\pi=2^-$, 7.35-Mev level can only take place through a spin-flip, that is, the transition from the $J^\pi=0^+$ ground state to this level must involve the matrix element of an odd rank tensor in $\mathbf{r}$ space in order to conserve parity, so that the $\Delta J=2$ transition must involve a spin-flip. In view of these difficulties we shall make only the most qualitative comparison of the $C^{14}(d,d')C^{14*}$ data with the general features of the direct interaction mechanism.

Armstrong *et al.*[29] have studied the inelastic scattering of 14.9-Mev deuterons from C^{14}. They obtained angular distributions for the known 6.72-, 7.35-, and 8.32-Mev levels and the previously unreported level at 7.01-Mev excitation. The angular distribution of these levels all peaked in the forward direction and have approximate cross sections at $\theta_L=25°$ of 1.04, 0.45, 0.58, and 0.83 mb per steradian, respectively. They also observed the 6.09- and 6.59-Mev levels with cross sections at $\theta_L=25°$ of 0.25 and 0.14 mb per steradian, respectively. The C^{14} 6.89-Mev level was not observed.

The identification of the C^{14} 6.89-, 6.09-, 7.35-, and 6.72-Mev levels with the 0^- and 1^-, $(p_{\frac{1}{2}}s_{\frac{1}{2}})$ and 2^- and 3^-, $(p_{\frac{1}{2}}d_{\frac{5}{2}})$ states, respectively, was discussed by WRH. This identification is illustrated in Fig. 1 by the solid lines connecting these C^{14} levels with the non-normal parity, $T=1$, N^{14} states discussed previously. It should be pointed out that the very different cross sections for the excitation of the 2^-, 7.35-Mev level and the 0^-, 6.89-Mev level are hard to understand since, on the direct interaction model, the excitation of both levels must arise from spin-flip terms. It is possible that the low cross section for the 6.89-Mev level arises from some accidental cancellation.

The C^{14} 6.59-Mev level is at approximately the right excitation to be the analog of the N^{14} 0^+, 8.62-Mev level. If it were not it would be difficult to understand the nonobservation of the N^{14}, $T=1$ level and the C^{14} level which would be the analogs at the C^{14} 6.59-Mev level and the N^{14} 8.62-Mev level, respectively. The uncertain spin-parity assignments shown in Fig. 1 for the C^{14} 6.59-Mev level are due to analysis of the $C^{13}(d,p)C^{14}$ reaction. As discussed by WRH, this analysis is quite uncertain and the 6.59-Mev level could well be $J^\pi=0^+$. In this case the cross section obtained[69] for this level in $C^{13}(d,p)C^{14}$ would yield a neutron reduced width[70] compatible with the proton reduced width of the N^{14} 8.62-Mev level. Assuming the direct interaction mechanism, the (d,d') cross section for the C^{14} analog of the $N^{14}(0,1)$ (s,d) level is expected to arise from the p^{-2} impurity in this state. Therefore, the small cross section for excitation of the C^{14} 6.59-Mev level in the (d,d') reaction is consistent with the 6.59-Mev level being the analog of the N^{14} 8.62-Mev level, which we proposed as the (0,1) (s,d) state (see Sec. IIIC). The conclusion that the C^{14} 6.59-Mev level is most likely the analog of the N^{14} 8.62-Mev level is indicated in Fig. 1 by the dashed line connecting these levels.

In Secs. IIIB and C it was argued that the N^{14} 9.16- and 10.43-Mev levels were the two $J^\pi=2^+$, $T=1$ levels resulting from a strong interaction between the lowest (2,1) levels of the p^{-2} and (s,d) configurations. On this picture, we would expect the C^{14} analogs of these two levels to have comparable (d,d') cross sections—the excitation of each arising from the p^{-2} admixture in the state. If such an interaction between the levels is assumed, the $C^{14}(d,d')C^{14*}$ results of Armstrong et al., are consistent with an identification of the C^{14} 7.01- and 8.32-Mev levels as the analogs of the N^{14} 9.16- and 10.43-Mev levels, respectively. This identification, which is indicated by dashed lines connecting these states in Fig. 1, is consistent with the relative energy positions of these levels. It is also consistent with the experimental information on the neutron reduced widths of these C^{14} levels. The nonobservation[69] of the C^{14} 7.01-Mev level via the $C^{13}(d,p)C^{14}$ reaction is consistent with the small neutron reduced width expected for the C^{14} analog of the N^{14} 9.16-Mev level—which has an extremely small proton reduced width (see Sec. IIIC)—while the neutron reduced width of the C^{14} 8.32-Mev level is compatible[69,70] with that expected for the analog of the N^{14} 10.43-Mev level.

V. CONCLUSIONS

On the basis of this and previous investigations we propose the N^{14} shell-model assignments which are given in Table VII. Our proposed assignments for C^{14} are those indicated by the lines connecting analog states of C^{14} and N^{14} in Fig. 1.

We have mentioned only briefly the $T=0$ states above the N^{14} 7.03-Mev level. On Lane's model of non-normal parity states, eight odd-parity levels with $T=0$ are expected at 7–11 Mev excitation in N^{14}. Two of these are the $T=0$, $J^\pi=1^-$ and 2^- levels arising from coupling a $d_{\frac{5}{2}}$ nucleon to the C^{13} ground state. Six odd-parity $T=0$ states are expected in this region from coupling an $s_{\frac{1}{2}}$ or $d_{\frac{5}{2}}$ nucleon to the first-excited p^9 state of C^{13} at 3.68-Mev. It is quite probable that the N^{14} 9.39- and 10.24-Mev levels are 2 of these expected 8 odd-parity levels. The $C^{13}(p,p)C^{13}$ results of Zipoy *et al.*,[71] indicate that the 9.39-Mev level is formed by capture of d-wave protons with a reduced width close

[68] R. Huby and H. C. Newns, Phil. Mag. **42**, 1442 (1951).
[69] J. N. McGruer, E. K. Warburton, and R. S. Bender, Phys. Rev. **100**, 235 (1955).
[70] The reduced widths of the C^{14} levels were calculated from the $C^{13}(d,p)C^{14}$ results (reference 69) by E. U. Baranger (unpublished).
[71] D. Zipoy, G. Freier, and K. Famularo, Phys. Rev. **106**, 93 (1957); D. M. Zipoy, Phys. Rev. **110**, 995 (1958).

TABLE VII. Proposed shell model assignments for the energy levels of N^{14}

N^{14} excitation energy (Mev)	J^π; T[a]	Shell model assignment[b]
0	1^+; 0	(1,0); $p_{\frac{1}{2}}^2$
2.31	0^+; 1	(0,1); $p_{\frac{1}{2}}^2$
3.95	1^+; 0	(1,0); $p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-1}$
4.91	(0^-); 0	(0,0); $p_{\frac{1}{2}}s_{\frac{1}{2}}$
5.10	2; 0	(2,0); $p_{\frac{1}{2}}d_{\frac{5}{2}}$
5.69	$1^{(-)}$; 0	(1,0); $p_{\frac{1}{2}}s_{\frac{1}{2}}$
5.83	$3^{(-)}$; 0	(3,0); $p_{\frac{1}{2}}d_{\frac{5}{2}}$
(5.98)	...	(?)
6.23	1; 0	(1,0); (s,d)
6.44	3; 0	(s,d) or p^9d(?)
(6.60)	...	(?)
7.03	(2); 0	(2,0); $p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-1}$
7.47	...	(?)
7.60	...	(?)
7.96	(?); 0	(?)
8.06	1^-; 1	(1,1); $p_{\frac{1}{2}}s_{\frac{1}{2}}$
(8.45)	(?); 0	(?)
8.62	0^+; 1	(0,1); (s,d)
8.70	0^-; 1	(0,1); $p_{\frac{1}{2}}s_{\frac{1}{2}}$
8.90	3^-; (1)	(3,1); $p_{\frac{1}{2}}d_{\frac{5}{2}}$
8.98	1^+; (0)	(?)
9.16	2^+; 1	(2,1); $(s,d)+p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-1}$
9.39	(1^-); (0)	$p_{\frac{1}{2}}d_{\frac{3}{2}}$(?)
9.50	2^-; 1	(2,1); $p_{\frac{1}{2}}d_{\frac{5}{2}}$
9.72	1^+; (0)	(?)
(10.05)	(?); 0	(?)
10.24	(1^-); (0)	p^9s and/or p^9d
10.43	(2^+); 1	(2,1); $p_{\frac{3}{2}}^{-1}p_{\frac{1}{2}}^{-1}+(s,d)$

[a] Taken from Fig. 1 and the text.
[b] The predominant jj configuration is given except in the case of those configurations arising from $s^4p^8(s,d)$ which are denoted by (s,d).

to the single-particle value. Furthermore, the (α,α') cross section predicted by the method of Sec. IV is quite small, consistent with the nonobservation of this level by Miller *et al.*[28] Therefore, we tentatively assign this level $(p_{\frac{1}{2}}d_{\frac{3}{2}})$. However, if this assignment is correct it is surprising that a $9.39\to 2.31$ transition has not been observed, since the $\Delta T=1$, $E1(p_{\frac{1}{2}}d_{\frac{3}{2}})_{J=1}\to (p_{\frac{1}{2}}^2)_{J=0}$ transition is calculated to be quite strong.

No attempt has been made to assign configurations to the even-parity $T=0$ levels above the N^{14} 7.03-Mev level. As mentioned in the last section, some of these may belong to $p^8(s,d)$ or p^9f. Others may arise from p^92p or the single or double excitation of an $1s$ nucleon.

An important question which arose in making the shell-model assignments of Table VII is just how close must be the agreement between the theoretical predictions of our over-simplified models and the experimental data to constitute support for particular level assignments. Unfortunately, this question cannot be settled in advance; individual cases must be studied as they arise. In some instances the theoretical result could be shown to be insensitive to small changes in the wave functions. Such cases could be used as strong arguments for a particular assignment. An example is the great strength of the N^{14} $8.62\to 6.23$ transition which, if $M1$ as we assume, practically guarantees that the 8.62- and 6.23-Mev levels are (s,d). In other instances strong arguments for an assignment arose in cases in which there is rough agreement with a large group of data, even though agreement with any one of these data might be inconclusive. An example here is the assignment of $(p_{\frac{1}{2}}d_{\frac{5}{2}})$ to the N^{14} 5.10- and 5.83-Mev levels.

The question of what agreement to expect is most important in the case of electromagnetic transitions which are quite sensitive to the assumed wave functions. Even if the "correct" shell-model wave functions are used, electromagnetic transition calculations are not always completely successful. Kurath,[33] in his calculations of $M1$ and $E2$ radiative widths near the middle of the $1p$ shell, obtained results ranging from agreement to disagreement by an order of magnitude. We have taken the point of view that the stronger a transition is calculated to be in a realistic model the better should be the agreement with experiment. For a transition of average strength we expect our over-simplified models to reproduce the nuclear matrix element amplitude to about a factor of two or better; so that we take as a rough guide agreement to something like a factor of four as consistent with our shell-model assignments.

One means of judging the degree of agreement for the electromagnetic transitions is to see whether the calculated widths give any better over-all agreement with experiment than that obtained taking for each multipolarity the average rate for light nuclei[53] modified to include the expected inhibition[34,45] of $\Delta T=1$ magnetic transitions in N^{14}. Given that the spin-parity assignments we assume are correct, the calculated widths give appreciably better agreement. In fact, a major success of our calculations is that they give a generally consistent explanation for the dipole transition strengths which vary from the extremely weak (such as the $9.50\to 0$ transition) to the extremely strong (such as the $8.62\to 6.23$ transition).

In this work we have attempted not only to determine the predominant shell-model configuration of many mass 14 levels, but also to estimate the type and magnitude of admixtures of other configurations in these levels. We have done this, on the main, by offering admittedly *ad hoc* explanations for some of the discrepancies between experiment and predictions based on the single configuration models. We have shown that some discrepancies are semi-independent of the assumed model and have rather surprising results; namely, the large interaction of the $J^\pi=2^+$, $T=1$, p^{-2} and (s,d) levels and the large isotopic-spin impurities of the N^{14} 5.10- and 5.83-Mev levels. There are a few unexplained contradictions. An example is the data relating to the N^{14} 6.44-Mev level.

Our approach is frankly heuristic and optimistic. We have tried to indicate, implicitly or explicitly, experiments which will test our predictions and assist in an understanding of the mass 14 spectrum. It is our hope that this work will stimulate such experiments.

APPENDIX

In this appendix we derive the differential cross sections for the inelastic scattering of alpha particles from N^{14} and the inelastic scattering of protons from N^{14} and C^{14}. These cross sections were used to obtain the results presented in Sec. IV.

The inelastic scattering of alpha particles is treated on a very simple model. The alpha particle is spinless and tightly bound. For these reasons, but primarily for reasons of simplicity, we treat the inelastic scattering as the collision of a simple point particle with the target, the effective interaction of the alpha with the target being represented by an ordinary central potential between the alpha and each target nucleon. The cross section is calculated in Born approximation, i.e.,

$$\sigma_\alpha(\theta)=\frac{K_f}{K_i}\left[\frac{\mu}{2\pi\hbar^2}\right]^2\frac{1}{[J_i]}\sum_{M_i}\sum_{M_f}\bigg|\int\exp(-i\mathbf{K}_f\cdot\mathbf{r}_\alpha)\times\Psi^*(J_fM_f)\sum_{n=1}^{A}V(r_{\alpha n})\exp(i\mathbf{K}_i\cdot\mathbf{r}_\alpha)\Psi(J_iM_i)\bigg|^2. \quad (1a)$$

The K's are the incident and final wave vectors of the alpha and μ is its reduced mass. The symbol $[j]$ represents $(2j+1)$. The sum over n ranges over the A nucleons of the target. The calculation may be further simplified by the assumption, admittedly a crude one, that V may be approximated by a zero range potential,

$$V(r_{\alpha n})=V_0\delta(\mathbf{r}_n-\mathbf{r}_\alpha). \quad (2a)$$

Then using the Rayleigh expansion of a plane wave and standard angular momentum methods, Eq. (1a) may be reduced to

$$\sigma_\alpha(\theta)=\frac{K_f}{K_i}\left[\frac{\mu}{2\pi\hbar^2}\right]^2V_0^2A^2\sum_{\lambda=0}^{\infty}[\lambda](n_fl_f|j_\lambda(qr)|n_il_i)^2\times\frac{|\langle J_f\|\mathbf{C}^\lambda(\Omega_A)\|J_i\rangle|^2}{[J_i]}. \quad (3a)$$

In this formula $\mathbf{C}^\lambda$ is an unnormalized spherical harmonic, $j_\lambda(qr)$ is a spherical Bessel function of order λ, nl represents the one-particle quantum numbers of a bound nucleon, and $\mathbf{q}$ is the momentum transfer $\mathbf{q}=\mathbf{K}_f-\mathbf{K}_i$. The symbol Ω_A represents the angular variables of the position of the Ath nucleon.

In order to further reduce Eq. (3a), wave functions must be chosen for the initial and final states of the target. As before the states belonging to excited configurations will be treated in jj coupling. For excitation of such configurations the ground state is represented by the jj state, $p_{\frac{1}{2}}{}^2J=1$, $T=0$, and the excited states of non-normal parity by $p_{\frac{1}{2}}(nlj)J_fT_f$. With these restrictive assumptions the sum over λ in the differential cross section reduces to a single term, that for which $\lambda=2j-l$. If for the sake of brevity we set

$$\Delta=\frac{K_f}{K_i}\left[\frac{\mu}{2\pi\hbar^2}\right]^2V_0^2, \quad (4a)$$

then the differential cross section reduces to

$$\Delta^{-1}\sigma_\alpha(\theta)=2[j][J_f]W^2(jJ_f\tfrac{1}{2}1;\tfrac{1}{2}\lambda)(n_fl_f|j_\lambda|1p)^2. \quad (5a)$$

This formula has been used to calculate the (α,α') cross sections which are tabulated in Table VI. The data tabulated there and the method used to evaluate the radial integrals is discussed in Sec. IVA.

The direct interaction theory of (p,p') reactions has been treated in detail by Levinson and Banerjee. The formula for the differential cross section is made much more complicated than for (α,α') by the exclusion principle and the nonzero spin of the protons. In this section we use the Levinson-Banerjee transition amplitude calculated, as before, in Born approximation with a zero range potential between the incident proton and the target nucleons. In addition we permit the potential to have a spin dependence. This turns out to be unimportant for N^{14}, but may be important in weakening some selection rules in C^{14} where there is no exclusion principle for the incident proton relative to the two neutrons outside the C^{12} core.

According to Levinson and Banerjee the differential cross section is given by

$$\Delta_p^{-1}\sigma_p(\theta)=2[J_i]^{-1}\sum_{M_i}\sum_{M_f}\sum_{m_s}\sum_{m_s'}\times\bigg|\int\Psi_{12}^*(J_fM_fT_fM_T)e^{-i\mathbf{K}_f\cdot\mathbf{r}_3}\delta_3(s_z,m_s)\delta_3(t_z,m_t)\times V(2,3)(1-P_{23})e^{i\mathbf{K}_i\cdot\mathbf{r}_3}\delta_3(s_z,m_z')\delta(t_z,m_t')\times\Psi_{12}(J_iM_iT_iM_T)\bigg|^2. \quad (6a)$$

Here the μ in Δ should be that for a proton. The operator P_{ij} exchanges particles i and j. A spin-sum is implied by the integration, the free proton is particle 3 in the incident and final wave functions, and the δ's are the usual Pauli spinors for both ordinary and isotopic spin. We take for V_{23} the zero range charge independent potential,

$$V(2,3)=V_0\delta(\mathbf{r}_2-\mathbf{r}_3)(a+bP_{23}{}^\sigma). \quad (7a)$$

If we introduce the channel spin in the initial and final states, i.e., $\mathbf{I}_i=\mathbf{J}_i+\mathbf{s}_i$, $\mathbf{I}_f=\mathbf{J}_f+\mathbf{s}_f$, and again use the Rayleigh plane wave expansion, then Eq. (6a) reduced to

$$\Delta^{-1}\sigma_p(\theta)=2[J_i]^{-1}\sum_\lambda[\lambda]\sum_{I_i}\times\sum_{I_f}|\langle J_fs_3I_f\|\mathbf{X}^\lambda(2,3)\|J_is_3I_i\rangle|^2, \quad (8a)$$

in which

$$X_\nu^\lambda(2,3)=j_\lambda(qr_2)C_\nu^\lambda(\Omega_2)\sum_{t_z t_z'}\langle T_f M_T|_{12}\delta_3(t_z,m_t)$$
$$\times[(a+bP_{23}{}^\sigma)-(b+aP_{23}{}^\sigma)P_{23}{}^\tau]\delta_3(t_z,m_t')|T_1M_T\rangle_{12}$$
$$=j_\lambda(qr_2)C_\nu^\lambda(\Omega_2)(\alpha+\beta P_{23}{}^\sigma). \quad (9a)$$

By standard angular momentum recoupling techniques α and β may be shown to be

$$\alpha=a-Gb,\quad \beta=b-Ga, \quad (10a)$$

with

$$G=[T_i][T_f]\sum_{T_0}\langle T_f\tfrac{1}{2}M_Tm_t|T_0M_T+m_t\rangle$$
$$\times\langle T_i\tfrac{1}{2}M_Tm_t|T_0M_t+m_t\rangle W(T_i\tfrac{1}{2}\tfrac{1}{2}T_f;T_0\tfrac{1}{2}). \quad (11a)$$

In paritular $G=0$ for C^{14}, $G=\frac{1}{2}$ for N^{14} and $G=1$ for O^{14}. With the use of the jj-coupling wave functions assumed, Eq. (8a) can be reduced to

$$\Delta^{-1}\sigma_p(\theta)=2[J_i]^{-1}\sum_\lambda[\lambda]\sum_{I_i}\sum_{I_f}|S_\lambda(I_i,I_f)|^2, \quad (12a)$$

where

$$S_\lambda(I_i,I_f)=3[I_i][I_f][J_f][j](n_fl_f|j_\lambda(qr)|1p)^2$$
$$\times\Big\{\alpha W(J_fI_fJ_iI_i;\tfrac{1}{2}\lambda)W(jJ_f\tfrac{1}{2}J_i;\tfrac{1}{2}\lambda)\langle j\lambda\tfrac{1}{2}0|\tfrac{1}{2}\tfrac{1}{2}\rangle$$
$$+\beta\sqrt{2}\sum_{j'J'}[J'][j']^{\frac{1}{2}}W(J_fI_fJ'I_i;\tfrac{1}{2}\lambda)W(jJ_fj'J';\tfrac{1}{2}\lambda)$$
$$\times\langle j\lambda\tfrac{1}{2}0|j'\tfrac{1}{2}\rangle\begin{Bmatrix}\frac{1}{2} & 1 & \frac{1}{2}\\ J' & I_i & \frac{1}{2}\\ j' & \frac{1}{2} & 1\end{Bmatrix}\Big\}. \quad (13a)$$

The last term in brackets on the right of Eq. (13a) is a $9j$ coefficient. For our particular case of spins and configurations the sum over channel spins may be carried out to give, for N^{14},

$$\Delta^{-1}\sigma_p(\theta)=N[j'][J_f]\sum_\lambda(n_fl_f|j_\lambda(qr)|1p)^2$$
$$\times\Big\{2\alpha^2W^2(jJ_f\tfrac{1}{2}1;\tfrac{1}{2}\lambda)+2(6)^{\frac{1}{2}}[l]\langle l100|\lambda0\rangle$$
$$\times W^2(jJ_f\tfrac{1}{2}1;\tfrac{1}{2}\lambda)W(jl\tfrac{1}{2}1;\tfrac{1}{2}\lambda)\alpha\beta+6[l]\beta^2\langle l100|\lambda0\rangle^2$$
$$\times\Big[\frac{1}{18[j]}W^2(j\tfrac{1}{2}\lambda1;l\tfrac{1}{2})+\frac{4}{9[j]}W^2(j\tfrac{1}{2}\lambda1;l\tfrac{3}{2})$$
$$+\tfrac{1}{3}\delta(l,J_f)\frac{1}{[J_f]}W^2(jJ_f\tfrac{1}{2}1;\tfrac{1}{2}\lambda)$$
$$-\frac{1}{18[J_f]^2}\delta(l,J_f)\Big]\Big\}. \quad (14a)$$

The normalization factor, N, is 1 except when the final state is the state $p_{\frac{1}{2}}{}^2$, $J=0$, $T=1$, in which case it is 2. For the case of C^{14}, since $J_i=0$, the resulting expression is simpler. It is

$$\Delta^{-1}\sigma_p(\theta)=[j]\sum_\lambda(n_fl_f|j_\lambda(qr)|1p)^2\{(a^2+ab)\delta(J_f,\lambda)$$
$$+3b^2[J_f][l]\langle l100|\lambda0\rangle^2[W^2(jl\tfrac{1}{2}1;\tfrac{1}{2}\lambda)$$
$$\times([j]^{-1}-2[l]^{-1}\delta(J_f,l))+\tfrac{1}{9}\delta(J_f,l)[l]^2]\}, \quad (15a)$$

In deriving Eqs. (14a) and (15a) we have neglected the stripping-type exchange term which Banerjee has shown to be non-negligible for proton energies less than approximately 14 Mev. Therefore, Eq. (14a) is applicable for proton energies larger than about 14 Mev. In as far as C^{14} can be described as two neutrons outside a C^{12} core, however, there can be no knock-out (i.e., $G=0$ for C^{14}) or stripping exchange term in $C^{14}(p,p')C^{14*}$ so that Eq. (15a) gives the jj-coupling cross section in regions where compound nucleus formation can be neglected.