are many open fission channels, and the "threshold for fission" for this spin state in U^{234} lies lower in excitation than the other.

At present, the understanding of slow neutron induced fission is far from complete. The analysis presented here is reasonably self-consistent, but it is certainly not unique. In the analysis of U^{235}, it has been found necessary to postulate an anomalously large negative energy resonance in order to explain the high thermal cross section.[2,4] In the present analysis, a negative energy resonance was not necessary. The presence of a noninterfering component, possibly arising from broad overlapping levels, was required and accounts for the large thermal cross section as well as for the high value of η observed at thermal energies. The consequences of the possible existence in U^{233} of two distinct types of resonances, belonging to different spin states, would seem to warrant further study.

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Low-Energy Neutron Cross Sections of Fissionable Nuclei

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The method of analysis developed in a previous paper is applied to the low-energy neutron cross sections of the common fissionable isotopes. Further evidence is presented to show that U^{235} possesses the unusual negative energy level required by the previous analysis. However, good fits are obtained for the cross sections of both U^{233} and Pu^{239} without such an unusual bound level, suggesting that the neutron resonance cross sections of the fissionable isotopes do not exhibit a basic anomaly. The size of the level interference effects in each of the isotopes implies that the fission process involves more than one but no more than a few fission channels.

I. INTRODUCTION

In a previous paper[1] the general Wigner-Eisenbud resonance theory was used to derive a method of fitting the low-energy neutron resonance cross sections of the fissionable nuclei. The method was capable of describing the interference between levels of the same spin and parity using only a few parameters to describe the interference and without any assumption concerning the number of fission channels. Rather, the number of fission channels is found from the average value of the interference parameters. The method was employed, in I, to describe the low-energy neutron cross sections of U^{235} measured by Shore and Sailor.[2]

The purpose of the present paper is to apply the methods derived, in I, to the analysis of the cross-section data of the other common fissionable isotopes, U^{233} and Pu^{239}. In doing so we shall attempt to answer the question which originally motivated this investigation. For many years it has been asserted that the low-energy neutron cross sections of the common fissionable isotopes involved some basic peculiarities: for example, that each isotope had an anomalously large resonance just below the neutron binding energy. The question is: are the peculiarities real or have the assertions rested on inadequate applications of the resonance theory to the data? We shall also try to determine what the cross-section analysis has to say about the number of channels involved in fission.

[1] E. Vogt, Phys. Rev. **112**, 203 (1958), hereafter referred to as I.
[2] F. J. Shore and V. L. Sailor, Phys. Rev. **112**, 191 (1958).

The next section gives a brief review of the results obtained in I for U^{235} together with some amplifications of those results and some minor improvements in the method of analysis. The review and the amplifications serve two purposes. First of all it displays the variety of evidence, in U^{235}, for a negative energy resonance with a large neutron reduced width. Secondly, since the data on U^{235} are the most detailed and the analysis least ambiguous of all the fissionable isotopes, the discussion of U^{235} is helpful in showing how the method of I is to be applied to the other fissionable isotopes.

The analysis of U^{233} and Pu^{239} is given in Secs. III and IV.

II. U^{235}

The fit obtained to the cross sections of U^{235} is shown in Fig. 1 and the constants employed in the fit are given in Table I. The data employed and the results found are those previously reported in I except that $1+\alpha$, the ratio of the absorption cross section to the fission cross section, has now also been calculated and compared to the experimental data of the Brookhaven neutron cross-section compilation.[3]

Two minor improvements have been added to the method of analysis outlined in I which account for the slight change of the fit to U^{235} and the corresponding parameters from those given in I. The first improvement

[3] D. J. Hughes and R. B. Schwartz, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958), second edition.

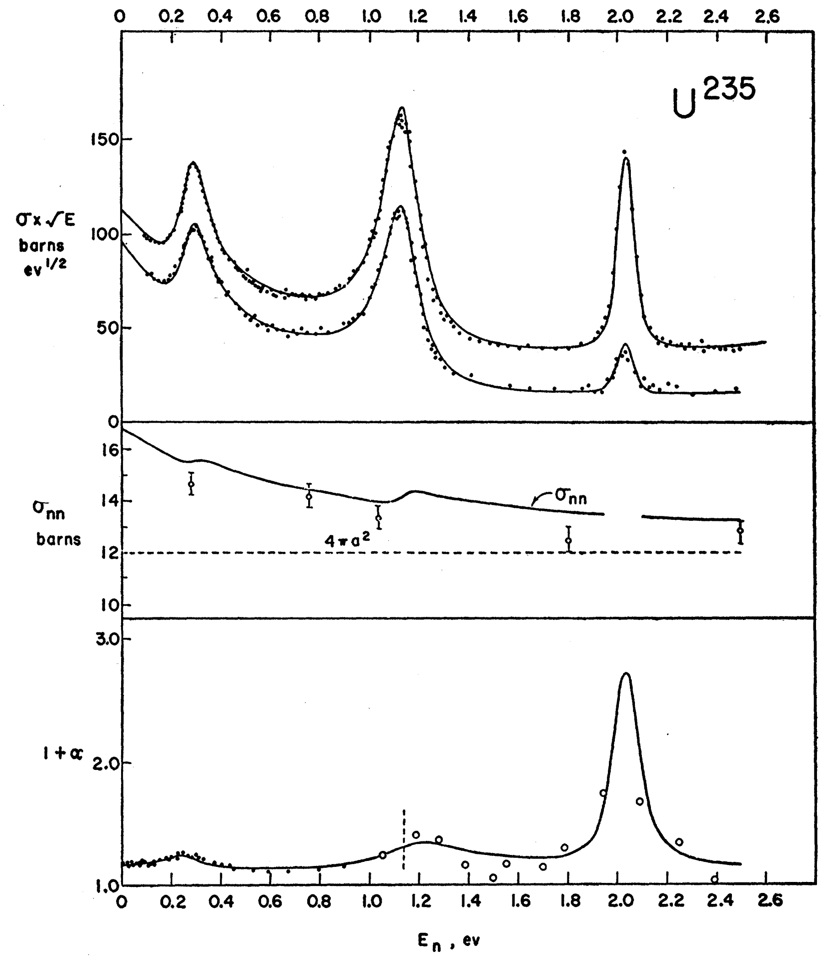

FIG. 1. The data and the theoretical fit for the low-energy neutron cross section of U^{235}. The abscissa for each part of the figure is the neutron energy, E_n, in ev. In each part of the figure the solid line is the multilevel fit and the points are the data described in the text. The top part of the figure gives the product of total cross section, σ_{nT}, with $E_n^{\frac{1}{2}}$, lying slightly above the similar product of σ_{nF} with $E_n^{\frac{1}{2}}$. The middle part of the figure gives the scattering cross section compared to the potential scattering cross section of $4\pi a^2$, where a is the nuclear radius (Table I). The bottom part of the figure is the ratio of the absorption cross section to the fission cross section. The broken vertical line indicates the resonance energy of the 1.14-ev state.

takes account of the Doppler broadening of the cross sections. Since the resonance peaks are not symmetrical the Doppler broadening cannot be taken into account by means of the tabulated functions for Doppler broadening. Instead we follow Bethe and Placzek[4] in assuming that the target atoms have a Maxwellian velocity distribution. If so, the Doppler broadened cross section, $\bar{\sigma}$, is related to the one, σ, without Doppler broadening by

$$\bar{\sigma}(E_n)=\int_0^\infty \omega(E)\sigma(E)dE\Big/\int_0^\infty \omega(E)dE, \qquad (1)$$

where

$$\omega(E)=(1/\pi^{\frac{1}{2}}\Delta)\exp[-(E-E_n)^2/\Delta^2], \qquad (2)$$

in which E_n is the energy of the incident neutron in the laboratory system, E is the energy of the neutron relative to the moving target and Δ is defined by:

$$\Delta=2(kTm/M)^{\frac{1}{2}}E_n^{\frac{1}{2}}. \qquad (3)$$

T is the temperature of the target, k is the Boltzmann constant and m/M is the ratio of neutron mass to target nucleus mass. For the fissionable isotopes at room temperature $\Delta\sim E_n^{\frac{1}{2}}/48.5$ with both E_n and Δ in electron volts. In all cases discussed below, the computed cross sections were Doppler broadened by means of (1) above. This correction is very important for the 2.04-ev resonance of U^{235}, and for some of the resonances of the other fissionable isotopes discussed below.

The second correction to the computed cross sections is made to take into account the levels which are ignored in the many-level formula. In the cross-section formulas, (9) to (12), of reference 1, it is assumed that only a few levels near the energy of interest contribute to the cross section.[5] The remaining levels are completely ignored and the level matrix $A_{\lambda\lambda'}$ as well as the sum over levels in the cross-section formulas, refer only to the few retained levels. The contribution of the retained levels to the cross section is treated in great detail, including

[4] H. A. Bethe and G. Placzek, Phys. Rev. 51, 450 (1937).

[5] J. A. Harvey and G. R. Satchler have kindly pointed out to the author that the formula for the total cross section, in I, contains an error in sign. Thus instead of (9), in I, σ_{nT} is given by

$$\sigma_{nT}E^{\frac{1}{2}}=6.52\times10^5\,\mathrm{Re}[(1-e^{-2ika})2gE^{-\frac{1}{2}} -ie^{-2ika}\textstyle\sum_{\lambda,\lambda'}(\Gamma_{\lambda n}{}^0)^{\frac{1}{2}}(\Gamma_{\lambda' n}{}^0)^{\frac{1}{2}}A_{\lambda\lambda'}]$$
$$+\text{similar term for the other spin.}$$

The correct formula for σ_{nT} was employed in all of the computations of I as well as in the present paper.

TABLE I. The parameters employed in the multilevel fit to the neutron cross sections of the fissionable isotopes. The first column lists the target nucleus. The next eight columns give the resonance energy, E_λ, the neutron reduced width [Eq. (8) of I], the radiation width, $\Gamma_{\lambda\gamma}$, and the fission width, $\Gamma_{\lambda F}$, for each resonance employed in the present multilevel analysis, as well as the corresponding quantities listed in the recent Brookhaven neutron cross-section compilation.[a] The four columns which follow describe the interference parameters. The first of these columns attaches a label to each resonance; the second gives the angle, θ_λ, in a plane (see Fig. 4) of the fission width vector, $\mathbf{g}_\lambda$, relative to the lowest lying level (of the same spin and parity) of those employed in the multilevel fit; the third gives the pairs of levels which interfere and the fourth, the interference parameter. The third last column gives the small contribution of the distant ignored levels to the total cross section, as discussed in Sec. II. The next column gives the corresponding result for the fission cross section. The final column gives the nuclear radius which determines the potential scattering.

	E_λ, ev		$2g\Gamma_{\lambda n}^0$, ev$\times10^{-3}$		$\Gamma_{\lambda\gamma}$, ev$\times10^{-3}$		$\Gamma_{\lambda F}$, ev$\times10^{-3}$		Interference parameters						
Element	Multi-level	Hughes and Schwartz[a]	Multi-level	Hughes and Schwartz[a,b]	Multi-level	Hughes and Schwartz[a]	Multi-level	Hughes and Schwartz[a,b]	λ	θ_λ	λ, λ'	$\cos(\theta_\lambda-\theta_{\lambda'})$	$E^{\frac{1}{2}}\times\sigma_{nT}$(distant) barns ev$^{\frac{1}{2}}$	$E^{\frac{1}{2}}\times\sigma_{nF}$(distant) barns ev$^{\frac{1}{2}}$	Radius 10^{-13} cm
U^{233}	0.10	0.20	0.0586	0.00029	56	40	994	60	1	0°	3, 4	−0.495			
	1.45	1.47	0.152	0.055	54		716	508	2	99.6°	3, 5	−0.860			
	1.76	1.75	0.235	0.105	49	44	231	200	3	0°	3, 8	−0.866			
	2.30	2.32	0.116	0.049	47	36	49	48	4	119.7°	4, 5	−0.012	3.25	2.90	10.00
	3.64	3.61	0.084	0.034	48	54	212	180	5	210.7°	4, 8	0.000			
	4.80	4.70	0.164	0.051	60		740	650	6	225.0°	5, 8	1.000			
		5.75		0.022		50		320	7		1, 2	−0.166			
	6.77	6.79	0.203	0.176	50	50	160	146	8	210.0°	1, 6	−0.707			
		9.05		0.018		56		280	9		2, 6	−0.590			
U^{235}	−0.95		1.488		27.6		169.4		1	0°	1, 2	0.148			
	0.273	0.282	0.00563	0.0052	29	32	99	82	2	277.5°	1, 3	−0.240			
	1.140	1.138	0.01613	0.014	44	42	124.6	106	3	103.9°	1, 6	−0.707			
	2.035	2.04	0.00537	0.0046	35	31	12	12	4		2, 3	−0.996	2.00	1.30	9.75
		2.82		0.0015					5		2, 6	0.594			
	3.16	3.14	0.01823	0.016	31.1		155	115	6	225.0°	3, 6	−0.516			
		3.60		0.024		37		45	7						
		4.84		0.025		25		4	8						
Pu^{239}	−1.200		1.156		39		201		1	0°	1, 3	−0.80			
	0.296	0.297	0.209	0.22	38.6	39	55.4	61	2		1, 4	−1.00			
	7.90	7.84	0.47	0.47	38	38	42	42	3	126.9°	1, 8	−1.00			
	11.00	10.93	0.83	0.83	32	32	147	147	4	180°	3, 4	0.80	2.20	1.30	9.50
		11.90		0.51		41		22	5		3, 8	0.80			
		14.3		0.21				60	6		4, 8	1.00			
		14.7		0.081				33	7						
	15.50	15.5	0.32	0.32	40		760	760	8	180°					

[a] See reference 3.
[b] The reduced neutron widths of U^{233}, as listed in reference 3, correspond to definite spin assignments. The author is grateful to M. S. Moore for supplying the values of $2g\,\Gamma_{\lambda n}^0$ for U^{233} corresponding to those of reference 3. As suggested by Moore, some typographical errors in reference 3 are also corrected in the table.

their mutal interference. The effect of the discarded levels on the cross section can easily be calculated in an approximate way. We observe first of all, as in I, that the interference terms of the discarded levels with themselves and with the retained levels have random sign fluctuations. More explicitly, at any energy the interference contribution to the cross section arising from a discarded level and any other level has a sign (i.e., constructive or destructive interference) which is random.[6] This can be seen from (17) of I which applies to the discarded level because it is fairly distant from the energy under consideration. According to (17) the sign of the interference term of the distant level is proportional to the sign of the reduced neutron width amplitude, $\gamma_{\lambda n}$, of that level. Presumably $\gamma_{\lambda n}$ has a random sign. Because of the random sign and small value of the interference terms arising from the distant levels, we can ignore the interference terms and include the distant levels only through their direct contribution to the cross section. The resulting modifications of the cross-section formulas (9) to (12) of I is only to add to those formulae the contribution from each distant level calculated with the Breit-Wigner single-level formula. For example, the contribution, $\sigma_{nT}{}^{(\text{distant})}$, of the distant nonretained levels to the total cross section, σ_{nT}, is

$$\sigma_{nT}{}^{(\text{distant})}=\frac{\pi}{k^2}\sum_{\lambda}{}' g_J\frac{\Gamma_{\lambda n}\Gamma_\lambda}{(E_\lambda-E)^2+1/4\Gamma_\lambda{}^2}, \tag{4}$$

where the prime on the sum means that the nearby levels which are retained in the many-level formulas are to be excluded from the sum. In (4) $\Gamma_{\lambda n}$ is the neutron width, Γ_λ the total width, E the resonance energy of each distant level λ, E is the neutron energy, k the neutron wave number, and g_J the usual statistical spin factor. We must remember that all the distant negative as well as positive energy levels are to be taken into account in the sum of (4). Since the discarded levels are distant, $(E_\lambda-E)\ll\Gamma_\lambda$, and since only the levels formed by s-wave neutrons are considered, we have

$$\begin{aligned}
&E^{\frac{1}{2}}\times\sigma_{nT}{}^{(\text{distant})}\\
&\quad\approx(6.52\times10^5)\sum_{\lambda}{}'\frac{\Gamma_{\lambda n}{}^0\Gamma_\lambda}{(E_\lambda-E)^2}\\
&\quad\approx6.52\times10^5\frac{\langle\Gamma_{\lambda n}{}^0\rangle}{D}\langle\Gamma_\lambda\rangle\left[\int_0^{E(1)}+\int_{E(2)}^{\infty}\right]\frac{dE_\lambda}{(E_\lambda-E)^2}\\
&\quad\approx6.52\times10^5\frac{\langle\Gamma_{\lambda n}{}^0\rangle}{D}\langle\Gamma_\lambda\rangle\left(\frac{1}{E-E^{(1)}}+\frac{1}{E^{(2)}-E}\right),
\end{aligned}\tag{5}$$

where $E^{(2)}$ and $E^{(1)}$ are, respectively, the upper and lower end of the energy interval in which all the levels are retained in the many-level formulas. D is the average level spacing for levels of a given species. (E is in ev and the reduced neutron width $\Gamma_{\lambda n}{}^0$ is defined to be $2g\Gamma_{\lambda n}E^{-\frac{1}{2}}$). The similar contributions of the distant levels to the partial cross sections are obtained by replacing $\langle\Gamma_\lambda\rangle$ in (5) by the appropriate width[6]: that is, by $\langle\Gamma_{\lambda F}\rangle$ for $\sigma_{nF}{}^{(\text{distant})}$, etc. The strength function, $\langle\Gamma_{\lambda n}{}^0\rangle/D$, and $\langle\Gamma_\lambda\rangle$ can be estimated from the observed positive energy levels. We can improve (5) by explicitly putting the observed positive energy levels into the sum on the right-hand side of (5) and approximating the sum by an integral only for the nonmeasured positive energy levels and for the negative energy levels. If the number of levels included in the many-level formula is sufficiently large so that $E^{(2)}$ and $E^{(1)}$ are far from the energy E of interest then the square bracket of (5) will be practically constant. Over the energy range in which we have computed cross sections below we have approximated each $\sigma^{(\text{distant})}$ by a constant value. The constant is given in Table I. As can be seen from a comparison of the values in the table with the figures, the direct contribution of the distant levels to the cross sections is usually a very small part of the cross sections.

It should be noted that the effect of the distant levels is taken into account only in a very crude manner by the above approximation. Although the interference terms of the distant levels with each other and with the nearby included levels have a zero expectation value, the variance of these interference terms may well be large—perhaps larger than the square of $\sigma^{(\text{distant})}$ as calculated above. The above treatment has included only the positive definite contribution from the distant levels.

In U^{235} the ratio, α, of capture to fission possesses an unusual property; α is found to be larger at resonance than in between cross-section peaks not only for the resonances shown on Fig. 1 but even for the first few resonances lying above the energy of Fig. 1. The value of α calculated from the parameters of the multilevel cross-section fit (without any attempt to fit α) has the same behavior. The explanation appears to lie in the rather large negative energy level of U^{235}. As can be seen from Table I, the negative energy level has not only a large neutron reduced width but also a rather large fission width. Therefore, α is small for the negative energy level. Since the negative energy state dominates the cross section in between the first few resonances at positive energy, α is unusually small there. Because this regular behavior of α extends even beyond the energy of detailed cross-section fitting (Fig. 1) it is additional, though somewhat minor, evidence for an unusual negative energy state in U^{235}.

[6] This rule applies to any reaction cross section. An exception to this rule occurs for the scattering cross section. As can be seen in the detailed analysis below, the principal effect of the resonances on the scattering cross section is an interference of the resonances with the potential scattering. The interference of a distant resonance with the potential scattering does not have an arbitrary sign—it is always destructive if the resonance lies above the energy of interest and constructive if below. The small effect of the distant resonance on σ_{nn} can easily be estimated by the methods discussed in the text.

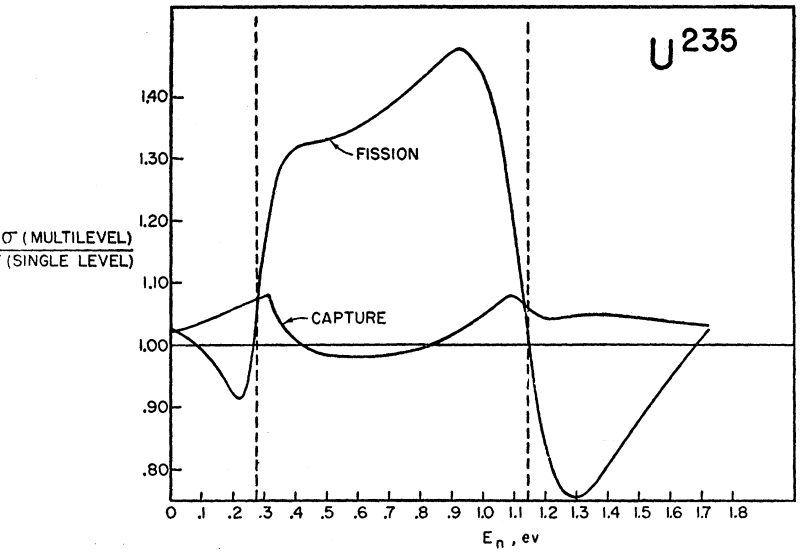

FIG. 2. The ratio of the computed multilevel cross sections of U^{235} to the computed cross sections obtained using the parameters of the multilevel fit with the Breit-Wigner single-level formula for each level. The abscissa is the neutron energy, E_n, in ev. The result for both the fission cross section and the capture cross section are shown. The vertical dashed lines identify the resonance energies, E_λ, of the two states which occur in the energy interval of the figure.

The measured ratio, α, of capture to fission, can be used to initiate a cross-section analysis. For a resonance which does not interfere strongly with any of its neighbors the maxima in $\sigma_{n\gamma}$ and σ_{nF} occur at the same energy so that α also has its maximum (or minimum) value at the same energy. However, interference displaces the peak in the fission cross section but not the peak in capture cross section. The peak displacement due to interference may be most apparent in α. Thus, for the 1.14-ev resonance of U^{235} the peak in α (or equivalently, $1+\alpha$) is shown, by Fig. 1, to occur at about 1.25 ev illustrating the fact that this state exhibits considerable interference.

To illustrate the asymmetry of the resonances of U^{235} we can compare the computed cross sections of Fig. 1 with the computed cross sections which we obtain using the parameters of Table I with the Breit-Wigner single-level formula for each resonance in place of the more general cross-section expressions derived in I. The comparison is made on Fig. 2. The result for $\sigma_{\text{multilevel}}/\sigma_{\text{Breit-Wigner}}$ vs neutron energy, is shown not only for fission but also for capture. We note first that the interference between levels makes the fission cross section deviate as much as 50% from its Breit-Wigner value. Moreover the striking asymmetry of the fission cross-section resonances is shown by the variation of the ratio of Fig. 2 about the resonance energies (broken vertical lines on Fig. 2).

For capture, the ratio of $\sigma_{\text{multilevel}}/\sigma_{\text{Breit-Wigner}}$ deviates by less than 10% from unity and even this deviation has only a small part which is asymmetrical about the resonance energy. The deviations of the capture cross section from its Breit-Wigner value has a rather different origin than the corresponding deviation of the fission cross section. Thus, to compute the cross section for incoming neutrons and an outgoing channel c we can begin by assigning to each resonance an amplitude, $\Gamma_{\lambda n}^{\frac{1}{2}}\Gamma_{\lambda c}^{\frac{1}{2}}/(E_\lambda-E-\frac{1}{2}i\Gamma_\lambda)$ whose absolute square is proportional to the estimate of the Breit-Wigner formula for the contribution of the level λ to the cross section. When interference between resonance levels takes place two changes are to be made in the computation. First of all, according to the results derived in I, for each outgoing channel we must take the square of the sum of the amplitudes from the various levels rather than the sum of the squares. Secondly, the simple energy denominators, $(E_\lambda-E-\frac{1}{2}i\Gamma_\lambda)^{-1}$, of the Breit-Wigner amplitudes, are replaced by the much more complicated ones given in terms of the level matrix A, in I. For levels which do not overlap very strongly the change in the energy denominators is a small effect. The large deviations of the fission cross section arise because fission involves only a few channels so that the cross terms between states, in the square of the sum of amplitudes, are not washed out by a sum over channels.

In the capture cross section we *assume* that the cross terms are zero because of the large number of radiation channels. The small deviations, Fig. 2, of the capture cross section from the Breit-Wigner value are caused solely by the change in the energy denominator, that is, by the replacement of $(E_\lambda-E-\frac{1}{2}i\Gamma_\lambda)^{-1}$ by $A_{\lambda\lambda}$. We remember that either of these energy "denominators" depends only on the levels and is the same for all reaction channels. For the nuclear reactions which we are considering the changes in the energy denominators from those of the Breit-Wigner formula are brought about by the fission width. However, even for capture channels we must use $A_{\lambda\lambda}$ in place of $(E_\lambda-E-\frac{1}{2}i\Gamma_\lambda)^{-1}$, in spite of the fact that we neglect the cross terms between levels for capture channels. The new energy denominators contain a few terms which are not symmetrical about the resonance energy. Their effect on the capture cross section is only one or two percent, an order of magnitude smaller than the effects of the new denominators which are symmetrical about the resonance energies.

The neglected cross terms between levels, i.e., terms proportional to $\sum_c \Gamma_{\lambda c}^{\frac{1}{2}}\Gamma_{\lambda' c}^{\frac{1}{2}}$ where the sum is over all radiation channels, would also give rise to asymmetries of a few percent if they had been included. According to (16) of I, for a large number of channels m, the interference between the roughly equal amplitudes of a pair of levels is, typically, about $(2/\pi m)^{\frac{1}{2}}$ times as large as the sum of the squares of the amplitudes. Thus the neglected cross terms, for say a hundred radiation channels could easily give rise to an asymmetry (about resonance) of a few percent in the capture cross section. It is reasonable, therefore, to assume that the capture cross section is symmetrical about the resonance energy. It is not quite so reasonable, however, to neglect those changes in the energy denominators whose effect on the cross section is symmetrical about the resonance energies and to use only the Breit-Wigner formula for the capture cross section. Figure 2 gives a quantitative illustration of the kind of effects that are brought about by interference.

To conclude the discussion of U^{235} we sum up the evidence for a large negative energy level in that element. First of all, the detailed shape analysis of I for the fission cross section and the total cross section appears to require a bound level at about -1.0 ev with a neutron reduced width almost ten times average.[7] Secondly, the deviation of the scattering cross section of U^{235} from $4\pi a^2$ (see Fig. 1) is explained by the large bound state, as discussed in I. The theoretical value of σ_{nn} shown on Fig. 1 was obtained with the multilevel cross section formula and the level parameters and nuclear radius required to fit σ_{nT} and σ_{nF}. No attempt was made to fit σ_{nn}. Thirdly the regular behavior of $1+\alpha$, discussed above, is accounted for by the properties of the large negative energy level. A fourth feature, which has not been discussed so far, is the explanation of the famous "hole" in the fission cross section of U^{235} in the vicinity of 4 ev. As can be seen from Figs. 1 and 2, below 3 ev the interference between resonances is, on the whole, quite strongly *constructive*. Both the 1.14-ev resonance and the 3.16-ev state interfere constructively with the large bound state below their respective resonance energies. Eventually one must pay for this assist to the cross section by having strong *destructive* interference—that is by having the cross section much smaller than would be computed with the Breit-Wigner formula. In our fit this should happen beyond the 3.16-ev level since the 1.14-ev and the 3.16-ev level both interfere destructively with the negative energy level there. It is in that vicinity where the data show the fission cross section to be abnormally low. No detailed fit to the data for U^{235} has been carried out above 3 ev to illustrate this point more quantitatively.

III. U^{233}

Figure 3 shows the Brookhaven[8] data for σ_{nT} of U^{233}; the MTR[9] data for σ_{nF}; the data of Oleksa[10] for the scattering cross section, and the collected data[3] for $1+\alpha$. For the moment we shall ignore the theoretical fit to the data which is superimposed on the figure. The first feature which we note is that the 2.3-ev level is very asymmetric (see the data for $1+\alpha$ on Fig. 3). Thus the resonances of U^{233} have the asymmetries observed in the resonances of U^{235}. However, we observe that neither the scattering cross-section data nor $1+\alpha$ for U^{233} show the regularities found in U^{235} which indicated the existence of an unusual bound level. We notice, next, the very broad resonance in the vicinity of 4.8 ev, the broad shoulder, near 1.5 ev, of the 1.75-ev resonance as well as the broad plateau at lower energies. The simplest possible explanation for the broad shoulder and the broad plateau would appear, at first sight, to be a single broad level, at low energy, which produces the plateau directly and which, through its interference with the 1.75-ev level, produces the broad shoulder at 1.5 ev. This possibility was explored at length in a shape analysis of σ_{nT} and σ_{nF}, and found to be unsuccessful. So much constructive interference is required at 1.5 ev that the subsequent destructive interference above 1.75 ev is incompatible with the data. Consequently two levels are required below 1.75 ev. Several possibilities for the two exist. For example a broad level at $\sim$1.5 ev together with a negative energy level of moderate size could produce the broad plateau at low energies. Alternatively, a broad level at both 1.5 ev and at low positive energies should also suffice. Of these two alternatives, which are equally likely and

[7] Although the probability of finding such a large neutron reduced width is, according to the Porter-Thomas distribution, only a fraction of a percent, the probability would be considerably enhanced if the average reduced width turns out to be even slightly spin-dependent and if the bound level belongs to the spin having the larger average reduced width. Furthermore, an appreciable reduction of the reduced neutron width of the negative energy resonance could probably be made if the fluctuating interference terms of the distant levels were properly taken into account, or if any direct interaction occurs, augmenting the fission cross section in between resonances. Thus the scattering cross section of U^{235}, as shown on Fig. 1, might be interpreted to suggest that the negative energy resonance of U^{235} is large, but not quite as large as we have assumed it to be.

[8] Brookhaven National Laboratory crystal spectrometer group (unpublished data). The author is indebted to Dr. Vance Sailor for sending the data and for permission to use it in this article.

[9] M. S. Moore and C. W. Reich, preceding paper [Phys. Rev. **118**, 718 (1960)]; M. S. Moore, L. G. Miller, and O. D. Simpson, this issue [Phys. Rev. **118**, 714 (1960)]. See also R. G. Fluharty, M. S. Moore, and J. E. Evans, *Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), Paper P645. Because the data for U^{233} in the present paper is only the early work of Moore, Miller, and Simpson their data may differ slightly from that of Fig. 3. Many of the conclusions about U^{233} which are derived below and even some of the details of the fit were arrived at independently by the MTR group. The author is indebted to Charles Reich for communicating the MTR data and for useful discussions about U^{233}. M. S. Moore, L. G. Miller, and O. D. Simpson very kindly consented to the use of their data before publication.

[10] S. Oleksa, Phys. Rev. **109**, 1645 (1958).

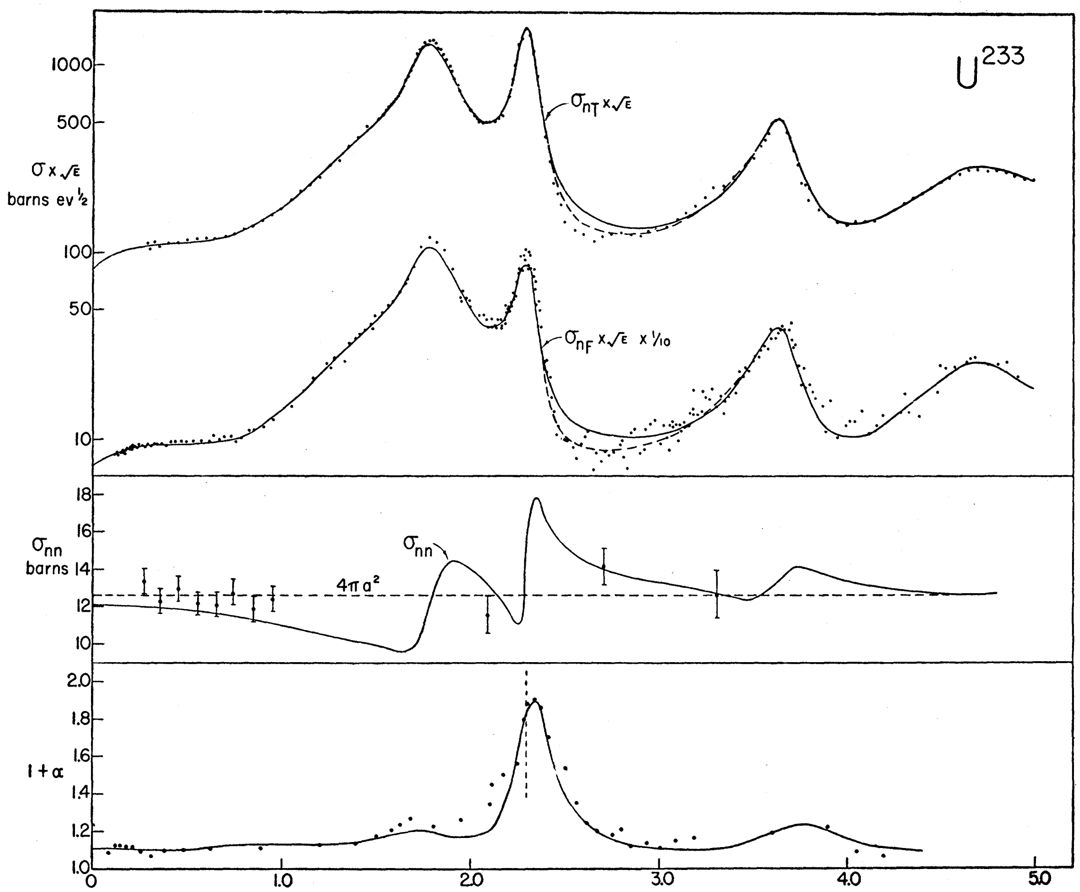

FIG. 3. The data and the theoretical fit for the low-energy neutron cross sections of U^{233}. The abscissa is the neutron energy, E_n, in ev. In each part of the figure the solid lines correspond to multilevel fits and the points to the experimental data described in the text. The top part of the figure gives the product of the total cross section, σ_{nT}, with $E_n^{\frac{1}{2}}$. Below this lies the product of σ_{nF} with $E^{\frac{1}{2}}$, multiplied by 1/10 to separate it from $\sigma_{nT}\times E^{\frac{1}{2}}$. The broken line gives a slight modification of the multilevel fit (see Sec. III). The middle part of the figure gives the scattering cross section compared to the potential scattering cross section of $4\pi a^2$, where a is the nuclear radius (Table I). The bottom part of the figure is the ratio of the absorption cross section to the fission cross section. The broken vertical line indicates the resonance energy of the 2.30-ev state.

equally simple, only the latter has been explored in detail. As will be shown, a good fit to the data above 0.1 ev is obtained in this way, without using any bound level.

To begin the analysis we now have six levels below 5 ev, three broad ones (the 4.7-ev level as well as the two below 1.75 ev) and three narrower ones. We remember that only levels of the same spin interfere with each other. Therefore we must decide which of the six levels belong to each of the two possible spin states. A suggestion toward the spin grouping comes from the shape of the cross section about the 2.3-ev level. The strong asymmetry of the 2.3-ev state suggests that it has the same spin as one of its nearest neighbors, e.g., the 1.75-ev level. We assume that these two levels do have a common spin. Then a fairly simple and believable grouping of the levels below 5 ev assigns the three broad levels to one spin state and the remaining levels to the other. No other grouping of the levels has been investigated.

The spin grouping chosen here agrees with the recent measurements of Regier, Burgus, and Tromp[11] of the variation, from resonance to resonance, of the ratio of asymmetric to symmetric fission. They found the value of this ratio at 4.7 ev to be the same as at thermal energies. However, at both the 1.75-ev and the 2.3-ev resonances the ratio was found to be significantly different from the value at thermal energies. According to Wheeler[12] the ratio of asymmetric to symmetric fission should depend on the spin of the compound state.

With the above basis the theoretical fit shown on Fig. 3 was obtained. The fit is good everywhere except in the region just above the 2.3-ev level and below 0.1 ev. The experimental cross sections show even more destructive interference above 2.3 ev than is predicted by the fit. Although an extra distant level ($E_\lambda = 6.8$ ev) has been added to the "narrow" levels no great effort

[11] R. B. Regier, W. H. Burgus, and R. L. Tromp, Phys. Rev. **113**, 1589 (1959).

[12] J. A. Wheeler, Physica **22**, 1103 (1956).

has been made to fit the cross section in that energy interval. The difficulty lies in having the "narrow" levels interfere strongly and destructively at ~2.6 ev and, at the same time, to interfere strongly and constructively near 3.2 ev, just below the 3.6-ev level. That is, the interference must reverse its sign in a region where no resonance state is assumed to lie. No serious attempt to resolve this dilemma was made—say by regrouping the levels, partly because the cross section data appear to show some structure in between the 2.3-ev and the 3.6-ev levels. If better data show an additional resonance there it would at once be simple to fit the cross sections over the whole interval. If no such level is found we remark here that the present fit can be much improved by adding another distant level to the fit. For example, the dotted curve on Fig. 3 shows the fit that is obtained if the known 10.33-ev level of U^{233} is assumed to interfere with the "narrow" levels in much the same way that the 6.8-ev level (Table I) does.

Although the multilevel cross-section fit of Fig. 3 employs no negative energy resonance, the fit differs from the data below 0.1 ev. Figure 3 does not show any of the large amount of data[3] for U^{233} cross sections below 0.1 ev. The theoretical fit is lower than the data there. The low-energy region has been disregarded in U^{233} because the present investigation wanted to show that the U^{233} data required no resonance with unusual properties. Any bound level of unusual size would affect the cross section much beyond 0.1 ev. Our fit shows that an unusual bound level is not necessary. On the other hand, the difference between the data and our multilevel fit at energies below 0.1 ev can be accounted for easily, in a variety of ways, by a bound level of normal size. Because the properties of such a level are far from unique, they are not discussed here. The fit near thermal energies for U^{233} is to be contrasted with that for U^{235}. In the latter, a very specific and large negative energy resonance was required and the multilevel fit, below 0.1 ev, does not differ from the data by more than a few percent.[13] The uniqueness of our entire fit to U^{233} is also to be contrasted with that to U^{235}. The large negative energy resonance of the latter limited the choice of parameters of the other levels quite strictly—for example, it appeared to be necessary to have the first two resonances at positive energy in U^{235} have the same spin as the large bound level so that they could interfere with it. As discussed in I, the unusual bound state makes the cross section fit to U^{235} fairly unique. In contradistinction, the cross sections of U^{233} require no unusual bound level and a variety of equally simple fits appear possible, only one of which has been investigated in the present paper.

[13] Although the data to support this point are not shown on Fig. 1, a thorough comparison of the data and the fit at these energies has been made by W. W. Havens, Jr., and G. J. Safford (unpublished). The author is indebted to them for discussions about their work.

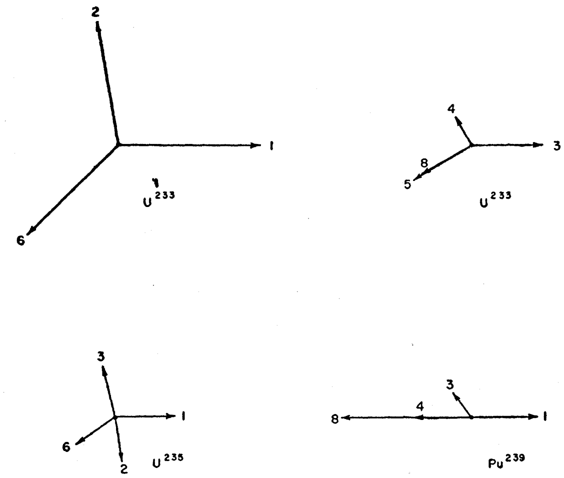

FIG. 4. The fission width vectors, $\mathbf{g}_\lambda$, employed in the multilevel fit. The numbers labelling each vector are given by the tenth column of Table I. The length of each of the vectors is drawn, on an arbitrary scale, to be proportional to $(\Gamma_{\lambda F}/2)^{\frac{1}{2}}$. The figure shows that all the multilevel fits were chosen to be consistent with two fission channels inasmuch as the vectors $\mathbf{g}_\lambda$ all lie in the same plane. For each interfering set of levels, the $\mathbf{g}_\lambda$ of the lowest lying state is chosen to lie at 0° in the plane. As described in the text and in Table I, the angle of each other vector $\mathbf{g}_\lambda$, in the plane, then determines the interference of the state λ.

The fit to the cross sections of U^{233} which is shown on Fig. 3 contains the interesting but very tentative suggestion that the levels of one spin state, in U^{233}, have very large fission widths ($\Gamma_{\lambda F}\sim 1$ ev), whereas the levels of the other spin state have fission widths which are smaller, more comparable to those of U^{235}. It is the very broad levels which, in our fit, provide the broad "base line" for the cross sections—a base line which has in the past sometimes been assumed to arise from a huge negative energy level lying quite far below zero neutron energy. The width of the broad levels is almost equal to their spacing.

Although the fit to the data of U^{233} is far from unique it nonetheless gives quite clear evidence that the number of fission channels is not large but more than one. According to I, the number of fission channels is related to the average value of the interference parameters, $\cos\theta_{\lambda\lambda'}$. In our fit, Table I, the amount of interference is moderate and corresponds to a few fission channels. In the actual fit the number of free variables was reduced, as for U^{235} in I, by requiring that the fit be compatible with two fission channels. This requirement introduces a few linear relationships among the interference parameters, $\cos(\theta_{\lambda\lambda'})$. If we define the fission width vector $\mathbf{g}_\lambda$ to have a component $\frac{1}{2}\Gamma_{\lambda F_c}{}^{\frac{1}{2}}$ in each fission channel, c, then the requirement of compatibility with two channels states that the vectors $\mathbf{g}_\lambda$ are all coplanar and each $\mathbf{g}_\lambda$ has associated with it its angle, θ_λ, in the plane. The interference parameters, $\theta_{\lambda\lambda'}$, are then simply the angles $\theta_\lambda-\theta_{\lambda'}$, between the vectors $\mathbf{g}_\lambda$ and $\mathbf{g}_{\lambda'}$. To illustrate this point and to make the meaning of the interference parameters more clear,

Figure 4 shows the vector diagrams which correspond to the parameters of Table I. The constraint on the interference parameters which makes them compatible with two channels does not imply that there are only two channels—it merely means that the remaining degrees of freedom of the interference parameters have no important effect on the cross sections so that the present data do not distinguish between a two-channel fit and a many-channel fit. According to the average value of the interference parameter of Table I and Fig. 1 of I, both the narrow and the broad levels of U^{233} have roughly three channels.

One does not need to rely on the detailed analysis for U^{233} to obtain the above rough information about the number of fission channels. Any reasonable analysis of the cross sections appears to require some very broad levels and some narrower ones. The broad levels must decay through more than one fission channel because otherwise their massive interference would distort the cross section much more than is observed. On the other hand the observed asymmetry of some of the narrow levels such as the 2.3-ev level shows that at least moderate interference occurs so that not very many fission channels can be important in this reaction. Thus any fit will employ the moderate interference corresponding to a few fission channels.

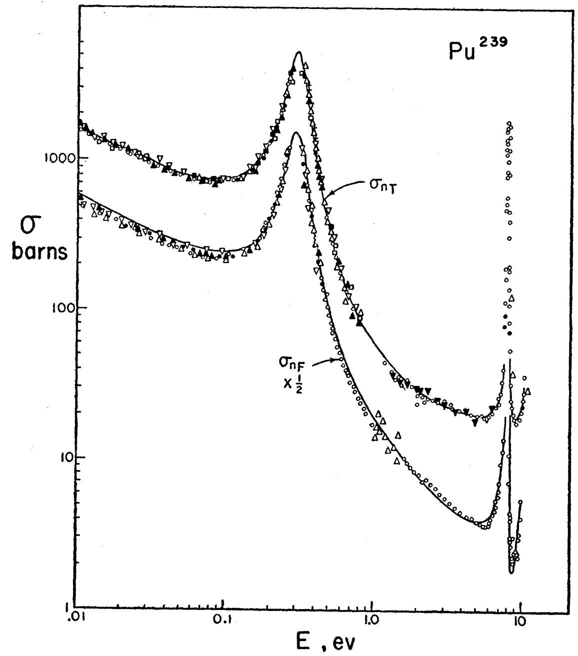

FIG. 5. The data and the multilevel fit for the total cross section, σ_{nT}, and the fission cross section, σ_{nF}, of Pu^{239}. The abscissa is the neutron energy in ev. The fission cross section has been multiplied by $\frac{1}{2}$ to separate it from σ_{nT}.

IV. Pu^{239}

Figure 5 gives the data of the Brookhaven neutron cross section compilation[3,14] for the total and fission cross section of Pu^{239}. The scattering cross section for this element has not been measured and α, which is not shown, has been measured only below 0.6 ev and above 7.0 ev.

We note immediately that the levels of Pu^{239} are much more widely spaced than in U^{233} or U^{235}: the average fission width $\langle\Gamma_{\lambda F}\rangle$ is almost equal to the average level spacing in U^{233}; in U^{235} $\langle\Gamma_{\lambda F}\rangle$ is perhaps five times smaller than D; in Pu^{239} $\langle\Gamma_{\lambda F}\rangle$ is about fifteen or twenty times smaller than D. Thus the levels overlap much less in Pu^{239} than in the U isotopes and we expect interference to be less important for Pu^{239}. This is indeed so. The resonance peaks of Pu^{239} are very large. We look for interference effects only in-between the resonances—many half-widths away from the resonance energy—where the cross section is very small compared to its peak values.

If one calculates the cross sections of Pu^{239} with the Breit-Wigner formula, using the accepted[3] resonance parameters one finds that in the vicinity of 3.0 and 4.0 ev the calculated fission cross section, for example, is too small by a factor of about two. Thus constructive interference is required there. The nearby levels at positive energies are not sufficiently large to provide this interference so we ask what the properties of a single bound level must be to fit the cross section near 4 ev. Since the 0.29-ev level has a relatively small value of $\Gamma_{\lambda n}\Gamma_{\lambda F}$, its contribution, even with interference, is small near 4 ev. Moreover any appreciable interference will distort this resonance more than is observed. Consequently we assume that the 0.29-ev level does not have the same spin as the 7.9-ev level or the single bound level, and the 0.29-ev state can therefore not interfere with the latter two. Of the known levels below 16 ev we choose the rather large level at 15.5 ev (whose asymmetry has been noted before[15]) and the 11.00-ev level to interfere with the 7.9-ev level and the bound one. The parameters of the former two are taken from reference 3 and not varied. The fit that is obtained to the Pu^{239} data is shown on Fig. 5. The parameters of the fit are listed in Table I.

The fit obtained to the Pu^{239} data is, again, far from unique. One can obtain as good a fit by using either a larger negative energy resonance or by using the interference of more of the known resonances above the energy of interest. Such fits are either less probable or less simple. The bound level of the fit has a reduced neutron width about three times average and, therefore, with about a 30% probability. Thus the fit of Fig. 5 shows that at least one fit without unusual properties is possible.

[14] Much of the data for Pu^{239} is that of L. M. Bollinger, R. E. Coté and G. E. Thomas, *Proceedings of the Second United Nations Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958* (United Nations, Geneva, 1958), Vol. 15, p. 127.

[15] L. M. Bollinger, Columbia University Conference on Neutron Physics, September, 1957 [Atomic Energy Commission Report TID-7547 (unpublished)].

In view of the large amount of interference required from the widely spaced levels of Pu^{239} the interference is much closer to that of a single fission channel (see Fig. 4 and Table I) than in the uranium isotopes. Bollinger[15] previously arrived at the same conclusion. The deviations from single channel seem necessary however to reduce the amount of destructive interference in the vicinity of 9 ev.

V. CONCLUSIONS

The neutron cross sections of the common fissionable isotopes exhibit anomalous shapes which may be reasonably ascribed to interference between levels. However, an anomalously large bound level does not appear to be required by each of the isotopes. The analysis of the U^{235} data show that a variety of evidence supports the existence of a fairly unusual negative energy level in that isotope. Because of the strong effect of this bound level on the cross sections of U^{235} the fit is fairly unique. For U^{233} and Pu^{239}, the cross section fit is much less unique. For each of the latter a simple fit is shown which does not involve an anomalous bound level. Better data and analyses for these elements may impose more uniqueness on the fits. At present there is little evidence for a basic anomaly in the neutron cross sections of the fissionable isotopes.

In each of the fissionable isotopes studied the magnitude of the interference between levels implies that only a few channels are involved in the fission process.

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Shell Model Assignments for the Energy Levels of C^{14} and N^{14}†

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Electromagnetic transition widths, reduced widths, and inelastic scattering cross sections are calculated for the following states of N^{14} and C^{14}: (1) The levels arising from the ground-state configuration, s^4p^{10}, (2) the odd-parity levels arising from excitation of a $1p$ nucleon into the degenerate $2s_{\frac{1}{2}}$ and $1d_{\frac{5}{2}}$ shells, (3) the even-parity group of levels formed by excitation of two $1p$ nucleons into the $2s$ and $1d$ shells. The calculations for the s^4p^{10} configuration are carried out using the wave functions of Elliott and of Visscher and Ferrell, and in jj coupling. The calculations for the odd-parity levels are done in the jj-coupling scheme. For the even-parity excited configuration an inert C^{12} core is assumed and $M1$ radiative widths are calculated for states arising from s^2+d^2+sd. The calculations are compared to the existing data. On the basis of this comparison shell-model assignments are proposed for 19 of the 27 known levels below 11-Mev excitation in N^{14} and for all the known levels in C^{14} below 9-Mev excitation.

I. INTRODUCTION

It is expected that the $T=0$ energy levels of N^{14} below, say, 8 Mev and the $T=1$ levels below, say, 11 Mev belong to three groups. One group consists of levels arising from the ground-state configuration,[1] s^4p^{10}. Another group consists of those levels which belong to the mixed s^4p^9s and s^4p^9d configurations. The third group of levels is formed by promoting two p-shell nucleons into the degenerate, or nearly degenerate, $2s$ and $1d$ shells. The latter group we shall refer[2] to as $s^4p^8(s,d)$. That the pairing energy is large enough so that $s^4p^8(s,d)$ should be lower in energy than the s^4p^92p and s^4p^91f configurations (which are expected above 8-Mev excitation in N^{14}) may not be obvious; however, it is implied by the work of the Pittsburgh group[3] on the $C^{14}(d,t)C^{13}$ reaction and is predicted by the binding energy calculations of Unna and Talmi.[4] One other configuration which might conceivably be expected to contribute to the energy region indicated is that formed

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[1] We shall often write the s^4p^{10} configuration in the hole notation, i.e., p^{-2}. Also, when no confusion should arise, we shall leave off principal quantum numbers and the closed $1s^4$ shell.

[2] This notation is intended to suggest that these levels belong to the configurations $s^4p^8s^2+s^4p^8d^2+s^4p^8sd$. We shall sometimes refer to these levels as belonging to the (s,d) configuration. This loose interpretation of configuration should not cause any misunderstanding.

[3] W. E. Moore, J. N. McGruer, and A. I. Hamburger, Phys. Rev. Letters 1, 29 (1958); E. Baranger and S. Meshkov, Phys. Rev. Letters 1, 30 (1958).

[4] I. Unna and I. Talmi, Phys. Rev. 112, 452 (1958).