DISCUSSION

The present calculations exemplify a practical problem that can occur in any attempt to calculate a physical property other than the energy of a system of more than a very small number of particles. In such cases variational methods must be used, with the goal of obtaining a wave function sufficiently close to an eigenfunction of the Hamiltonian that mean values of other operators, not necessarily commuting with the Hamiltonian, will be close to the values obtained for a true eigenfunction. If the wave function depends on a number of parameters, ordinarily one would proceed by a steepest descent calculation to approach a stationary value of the energy, expressed as a function of these parameters. By carrying such a calculation through to its limit, eventually any property of the system other than the energy would approach its correct value.

As shown in the present calculations, it can happen that certain changes in parameters have a large effect on the energy with only a small effect on the mean value of some operator other than the Hamiltonian, and conversely. Under these circumstances the error in the calculated value of such an operator is not necessarily small, even though the error in the energy is small.

This difficulty can be dealt with by a slight modification of the method of steepest descent. If parameters have been varied sufficiently that further energy variations are small, a final sequence of variations should be carried out along the path, in the parameter hyperspace, determined by the gradient of the mean value of the auxiliary operator under consideration. The energy should be made stationary along this path. This process ensures that any further variation of the energy will be along a path in the parameter hyperspace orthogonal to that of greatest change of the auxiliary operator. The auxiliary operator is itself stationary for such variations.

The calculations reported here followed this general procedure, simplified by the fact that different contributions to the total energy are additive, in second-order perturbation theory, so that a large class of variations that do not appreciably affect the auxiliary operator can be neglected.

ACKNOWLEDGMENTS

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Total Neutron Cross Sections of Helium, Neon, Argon, Krypton, and Xenon

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The total neutron cross sections of the noble gases helium, neon, argon, krypton, and xenon have been measured for neutron energies from 120 kev to 6.2 Mev and from 12.1 Mev to 19.8 Mev by a transmission experiment. The neutrons were produced using the $\mathrm{Li}^7(p,n)\mathrm{Be}^7$, the $\mathrm{T}(p,n)\mathrm{He}^3$, the $\mathrm{D}(d,n)\mathrm{He}^3$, and the $\mathrm{T}(d,n)\mathrm{He}^4$ reactions in the appropriate energy intervals. A Van de Graaff accelerator was the source of the protons or deuterons. In general, the results obtained agree with previous work where such work exists. A previously unobserved S-wave scattering resonance was found in neon at about 500 kev, indicating the presence of an excited state in Ne^{21} with $J=\frac{1}{2}$ and even parity. The results for argon, krypton, and xenon exhibit general agreement with the cross sections of neighboring elements, as would be expected from the previously observed smooth variation of the $\sigma(A,E)$ surface.

I. INTRODUCTION

In the present experiment, total neutron cross sections of helium, neon, argon, krypton, and xenon were obtained from 120 kev to 6.2 Mev, and from 12.1 to 19.8 Mev. Results have previously been published by other investigators for some of these elements at energies within these intervals. The total neutron cross section of helium has been reported up to about 6.1 Mev and from 12.5 Mev to 20.5 Mev.[1–6] Previous measurements of the neon cross section extend from 0.8 Mev to 3.5 Mev,[7–8] and results for argon have been

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[1] T. A. Hall and P. G. Koontz, Phys. Rev. **72**, 196 (1947).

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[5] J. H. Coon, quoted in J. D. Seagrave, Phys. Rev. **92**, 1222 (1953).

[6] Unpublished Los Alamos work, results given in *Neutron Cross Sections*, compiled by D. J. Hughes and R. Schwartz, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1958), 2nd ed.

[7] C. P. Sikkema, Nuclear Phys. **3**, 375 (1957).

[8] H. O. Cohn and J. L. Fowler, Phys. Rev. **114**, 194 (1959).

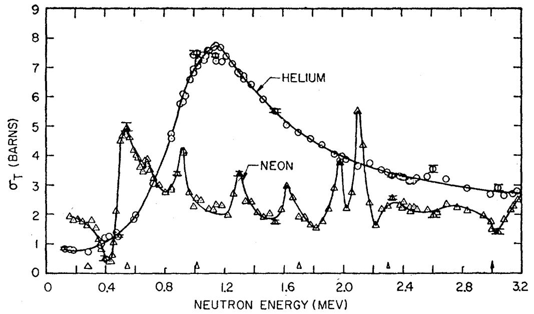

FIG. 1. Total neutron cross sections of helium and neon from 120 kev to 3.2 Mev.

reported between 0.45 Mev and 1.1 Mev.[9] No total neutron cross sections have been reported for krypton or xenon in the energy range covered by the present experiment.

II. EXPERIMENTAL PROCEDURES

A simple transmission experiment was performed to determine the total neutron cross sections of the five gases at neutron energies obtainable using a 3-Mev Van de Graaff accelerator and the usual neutron source reactions. Four source reactions were used in various energy intervals. They were the $\mathrm{Li}^7(p,n)\mathrm{Be}^7$, the $\mathrm{T}(p,n)\mathrm{He}^3$, the $\mathrm{D}(d,n)\mathrm{He}^3$, and the $\mathrm{T}(d,n)\mathrm{He}^4$ reactions.

The $\mathrm{Li}^7(p,n)\mathrm{Be}^7$ reaction was used to produce neutrons of energies from 122 kev to 1180 kev. A lithium layer was evaporated under vacuum on a tantalum backing. The target thickness was measured by observing the rise in neutron production near threshold. Targets of two thicknesses were used at different times, one being about 4-kev thick and the other about 42 kev. Observations were always made in the direction of the incident proton beam when using the lithium target.

The $\mathrm{T}(p,n)\mathrm{He}^3$ reaction was employed in the neutron energy range from 370 kev to 2.45 Mev. A solid zirconium-tritium target was used. Its thickness, measured by observing the rise in neutron production near the reaction threshold, was about 35 kev. With this neutron source reaction, observations were also made only in the direction of the incident proton beam. Over the energy interval from 370 kev to 1180 kev, in which neutrons from both the $\mathrm{Li}^7(p,n)\mathrm{Be}^7$ and the $\mathrm{T}(p,n)\mathrm{He}^3$ reactions were used, no systematic differences in the results were observed.

For energies from 2.39 to 6.23 Mev, the $\mathrm{D}(d,n)\mathrm{He}^3$ reaction served as the neutron source. Target nuclei in the form of deuterium gas were contained in a small gas cell separated from the beam pipe by a nickel foil 0.00038 cm thick. The deuterium gas pressure used ranged from 0.5 to 1.0 atmosphere. A cell 3.28 cm long was used for observations in the direction of the deuteron beam. In order to cover the energy range from 2.39 Mev to about 2.8 Mev, observations were made at angles of 70°, 90°, and 105° to the direction of the incident deuteron beam. At these angles a gas cell 1.64 cm long was employed.

Neutrons of energies from 12.1 to 19.8 Mev were produced by the $\mathrm{T}(d,n)\mathrm{He}^4$ reaction. The same solid zirconium-tritium target was used as for the $\mathrm{T}(p,n)\mathrm{He}^3$ reaction. Observations were made at angles of 0°, 60°, 75°, 105°, 115°, 125°, and 160° from the incident beam direction in order to cover this neutron energy range.

The samples of He, Ne, A, Kr, and Xe were contained in identical stainless steel cylinders, 2.54 cm in outside diameter, and 30.5 cm long. Both the cylinder walls and the caps soldered over the cylinder ends had a thickness of 0.16 cm. Hypodermic tubing about 60 cm long led from each cylinder to a high-pressure valve through which the cylinder was filled. Five cylinders contained gas samples at high pressure. The xenon sample was at a pressure of about 80 atmospheres, while the other gases were at pressures between 170 and 190 atmospheres. A sixth identical cylinder was evacuated; it was inserted between the neutron source and the detector when the counting rates with no gas scatterer in the neutron beam were measured.

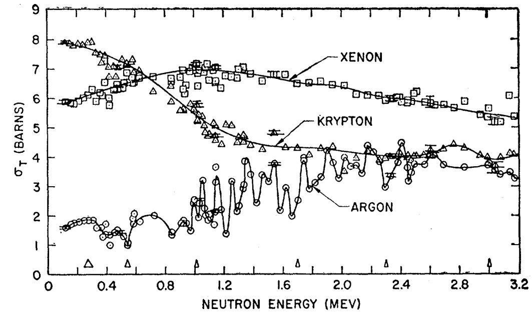

FIG. 2. Total neutron cross sections of argon, krypton, and xenon from 120 kev to 3.2 Mev.

For neutron energies up to about 800 kev, an $\mathrm{Li}^6\mathrm{I}$ crystal, 3.81 cm in diameter and 1.27 cm thick, was used as a neutron detector. The crystal was enriched to about 95% in Li^6. It was sandwiched between two pieces of Lucite, each about 2.5 cm thick, which served as a neutron moderator and as a light pipe. The sandwich was mounted on the face of an RCA-6655 photomultiplier tube. The crystal, moderator, and light pipe were surrounded by a layer of boron carbide, which in turn was contained within a 0.050-cm-thick cadmium shell. The outside diameter of the cadmium container was 5.40 cm. The cadmium and boron served to absorb thermal neutrons, while the moderator slowed the incident neutrons in order to increase the counting efficiency.

At neutron energies above 800 kev, the detector was a plastic phosphor, 2.54 cm in diameter and 2.54 cm thick, mounted on a photomultiplier tube.

Most of the observations were made with the detector at a distance of about 1 meter from the neutron source.

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[9] J. B. Guernsey and C. Goodman, Phys. Rev. 92, 323 (1953).

The scatterer was midway between the source and detector. Measurements were made with a source-detector separation of about 55 cm when using the $T(p,n)He^3$ source reaction to produce neutrons of energies from about 900 kev to about 2 Mev.

The procedure followed in measuring the total cross sections at a particular neutron energy was as follows. Counting rates in the detector were measured with each gas scatterer and with the evacuated cylinder between the source and detector. The background counting rate with a copper or polyethylene absorber 30 cm long placed between the source and detector was also observed. When the $D(d,n)He^3$ reaction was used, additional background measurements were made by repeating the above sequence of observations with helium rather than deuterium in the target cell. During all observations, the neutron flux was monitored by either a "long counter" or another plastic phosphor detector, placed so that the scattering samples would not block the direct path from the neutron source to the monitor.

The areal densities of the noble gas samples were determined by carefully weighing the cylinders before and after the addition of the gas and by accurately measuring the cylinder dimensions. The cylinders were reweighed several times during the course of the experiment to check on possible gas leaks.

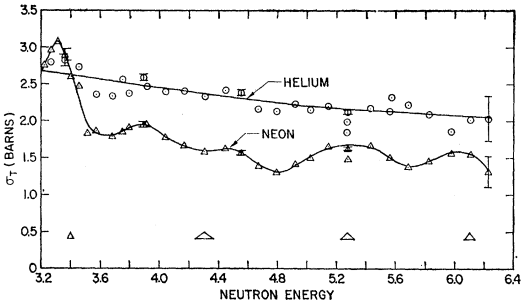

FIG. 3. Total neutron cross sections of helium and neon from 3.2 Mev to 6.2 Mev.

III. RESULTS

The results of the total neutron cross-section measurements are given in Figs. 1–6. The neutron energy spread is indicated by triangles near the base line for a few representative points. This neutron energy spread was produced by the finite target thickness and also, when the $D(d,n)He^3$ reaction was used, by straggling in the nickel entrance foil. The effect of the finite lateral dimensions of the scatterer and detector on the energy resolution was negligible. The total energy spread varied from a few kilovolts up to about 200 kev, depending on the particular target used, the angle at which observations were made, and the neutron energy. It was usually between 20 and 60 kev.

At several energies, probable errors are indicated on the figures. These calculated errors include statistical counting uncertainties and also the possible errors in the areal densities of the gas samples caused by uncertainties in the weights of the gases and in the gas cylinder dimensions. No in-scattering corrections have been applied to the cross sections, since calculations indicate that this correction would in no case exceed 1.5%. It would, therefore, be small compared with other uncertainties.

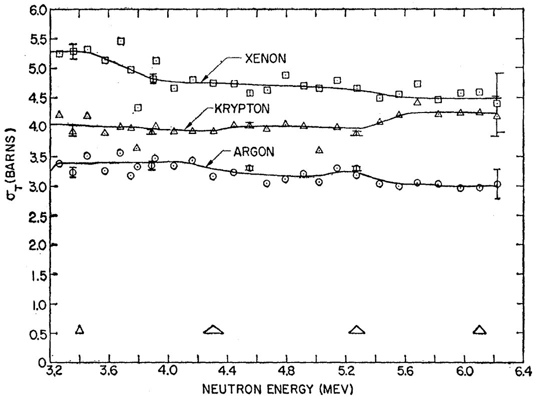

FIG. 4. Total neutron cross sections of argon, krypton, and xenon from 3.2 Mev to 6.2 Mev.

The total cross section of helium has previously been measured over the energy region covered in the present experiment.[1–6] The results obtained here indicate a somewhat higher cross section near the resonance at 1.15 Mev. Earlier observations in this energy region were made using the $Li^7(p,n)Be^7$ reaction, and no correction was made for the effect of the second group of lower-energy neutrons produced in this source reaction.

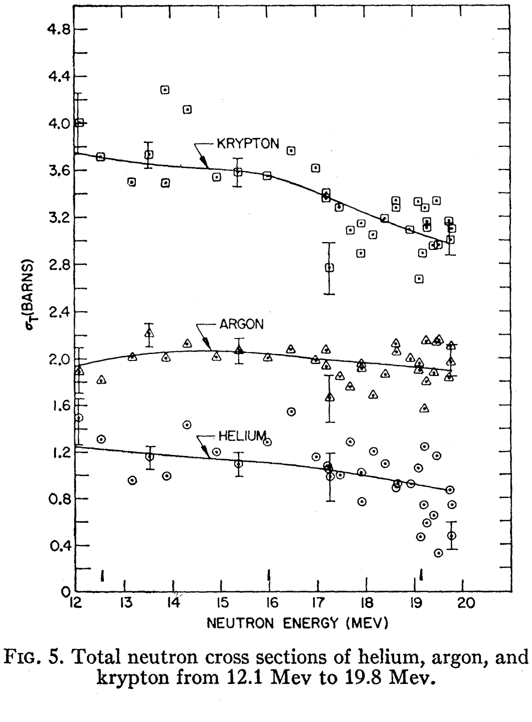

FIG. 5. Total neutron cross sections of helium, argon, and krypton from 12.1 Mev to 19.8 Mev.

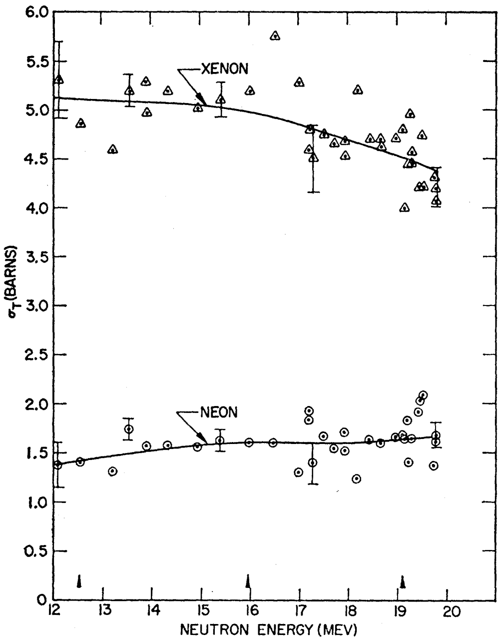

FIG. 6. Total neutron cross sections of neon and xenon from 12.1 Mev to 19.8 Mev.

The present measurements were made using both the $Li^7(p,n)Be^7$ reaction and the $T(p,n)He^3$ reaction in the energy range from 370 kev to 1180 kev. The results obtained using the former reaction were corrected for the second group of neutrons, and the corrected points agree well with those obtained using the $T(p,n)He^3$ reaction.

The neon cross section has previously been measured from 0.8 Mev to 3.5 Mev with about the same energy resolution as that employed in the present experiment.[7–8] The total cross-section results obtained here agree well with this earlier work.

The prominent S-wave scattering resonance observed in the present work at about 500 kev is due to scattering from an excited state in Ne^{21} at about 7.21 Mev. This level is probably the one previously reported at an excitation of 7.30±0.06 Mev; it was observed by examining protons emitted in the $Ne^{20}(d,p)Ne^{21}$ reaction.[10]

The experimental results obtained here have been analyzed to determine the resonance parameters of this level in Ne^{21}. The general shape of the cross-section curve, particularly the presence of the minimum at about 425 kev, and the magnitude of the peak at about 540 kev indicate an S-wave resonance. Since the target nucleus has zero spin and even parity, the excited state has a J value of $\frac{1}{2}$ and even parity.

The assumptions were made that only this resonance contributes to the scattering cross section between 200 kev and the peak at 540 kev, and that a constant background of 0.4 barn is contributed by Ne^{22} (about 9% abundant) and possibly by potential scattering of $l \neq 0$ neutrons. The experimental results were analyzed using the Breit-Wigner one-level formula to obtain the resonance energy, the width at resonance, and the S-wave potential scattering phase shift at resonance. The $l=0$ potential scattering phase shifts reported by Cohn and Fowler[8] at higher energies and the fact that this phase shift approaches kR at low energies were used to estimate the value of the potential phase shift over the region of the resonance. A process of variation of parameters was employed to find the magnitude and the estimated uncertainties of the resonance parameters. The results indicate a resonance energy of 473±3 kev, a width of 107±6 kev at resonance, and an S-wave potential scattering phase shift of $-52° \pm 4°$ at the resonance energy.

There is also some indication of a resonance in neon at about 675 kev. Because of its small magnitude, the resonance probably indicates scattering from Ne^{22}, and therefore the presence of a state in Ne^{23} at an excitation of about 6.0 Mev.

For argon, krypton, and xenon, the energy resolution used made it impossible to resolve individual levels. The results for these gases represent an average cross section over a region containing at least several resonances. Some indication of the level structure previously observed in argon at low energies[9] was seen, but at higher energies for argon and at all energies for krypton and xenon, the results gave no indication of level structure.

It has been pointed out that a three-dimensional plot of neutron total cross sections as a function of mass number and neutron energy exhibits a smooth variation of the cross section with these parameters.[11] The cross sections must be averaged over energy intervals large compared with the level spacing, so only gross structure is indicated in the plot. An examination of the total cross sections averaged over resonances of elements with mass numbers close to those of the gases observed in this experiment indicates agreement of the present results with the expected smooth cross-section variation.

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[10] R. Middleton and C. T. Tai, Proc. Phys. Soc. (London) **A64**, 801 (1951).

[11] H. H. Barschall, Phys. Rev. **86**, 431 (1952).