Excitation of Spin Waves in Ferromagnets: Eddy Current and Boundary Condition Effects*

P. PINCUS
Department of Physics, University of California, Berkeley, California
(Received November 16, 1959)

The boundary condition for the transverse magnetization is derived when there is a surface anisotropy field H_s. Writing the transverse magnetization as $\alpha \sin kz + \beta \cos kz$, we have

$$\alpha/\beta = -\tfrac{1}{2}ka + (H_s/H_e ak),$$

where H_e is the exchange field and a the lattice constant. A similar result is found when there is an antiferromagnetic surface layer. For $H_s \neq 0$ spin-wave modes can be excited by a uniform rf field in a ferromagnet. The power absorbed in each mode in an insulator is calculated as a function of the surface anisotropy field. The excitation of the exchange modes is calculated for a metal with eddy-current damping. The eddy currents are found to have a large effect only on long wavelength spin waves. The line shape in a thick metal plate is calculated for H_0 normal to the plate.

I. INTRODUCTION

IT has been shown by Kittel[1] that under appropriate boundary conditions spin waves may be excited in a ferromagnetic insulator by a uniform rf field. Jarrett and Waring[2] have observed multiple resonances in $NiMnO_3$ which have been tentatively identified as spin-wave resonances. Seavey and Tannenwald[3] and Kooi and Moss[4] have observed spin-wave excitations in thin permalloy films. We consider here the theory of these excitations in an insulator in more detail than was done in reference 1, and we also investigate the phenomenon in a metal where there is eddy-current damping.

It is convenient in dealing with special interactions on the surface atoms to work with the Hamiltonian in the Heisenberg form, rather than in the Landau form, as surface singularities are avoided. We assume nearest-neighbor exchange forces, for convenience. Then

$$\mathcal{H} = -2J \sum_{i>j} \mathbf{S}_i \cdot \mathbf{S}_j - g\mu_B \sum_i \mathbf{S}_i \cdot \mathbf{H}_i, \qquad (1)$$

where $\mathbf{H}_i$ is the effective magnetic field at the ith spin resulting from external fields, demagnetizing fields, and anisotropy fields. We know that S^2 commutes with the Hamiltonian if $\mathbf{H}_i$ is the same for all spins, and hence no spin waves can be coupled to the uniform mode. This holds only in the absence of dipolar interactions. We know that interactions having a dipolar character can couple the uniform mode to a degenerate spin-wave mode and at high rf powers lead to the Bloembergen-Wang-Suhl breakdown.[5] In an inhomogeneous rf exciting field one may also observe higher modes, for example, the magnetostatic modes observed by White and Solt.[6] It is further possible to excite exchange modes by a uniform rf field in a thin film if the direction of the surface magnetization is pinned by local anisotropy interactions; this is the effect with which we are concerned here. Such interactions may arise from the lower symmetry of atoms at the surface or with differences in chemical composition, such as the formation of antiferromagnetic oxide layers at the surface.

In Sec. II, we discuss the effect of surface boundary conditions on the transverse components of the magnetization. We show that surface anisotropies are likely to be adequate to pin the directions of the end spins. In Sec. III, we consider the modes of a film of a ferromagnetic insulator as a function of the degree of pinning. We derive an expression for the power absorbed in each mode. In Sec. IV, we treat the problem of a metal: here eddy-current damping of the rf fields broadens the resonance lines and can also give rise to spin-wave excitations. We calculate the surface impedance for a metal specimen in which the direction of the surface magnetization is completely pinned; we treat the eddy-current problem only for a static magnetic field normal to the plane of the specimen. Existing experimental results are also discussed.

II. BOUNDARY CONDITIONS

We consider a slab of thickness L in a static magnetic field normal to the surface and in a uniform rf magnetic field parallel to the surface. In the discussion of boundary conditions, because of the symmetry of the problem, we may usually restrict ourselves to a one-dimensional line of spins. We give a more general discussion than is found in the original paper by Kittel, and we incidentally correct an unimportant error in his analysis.

Ament and Rado[7] have suggested that at a boundary the normal derivative of the transverse part of the magnetization must vanish. However, their derivation of the boundary condition does not correctly treat the discontinuity at a surface when the Landau form of the Hamiltonian is used. In fact, near the surface, an

* Work partially supported by the National Science Foundation.
[1] C. Kittel, Phys. Rev. **110**, 1295 (1958).
[2] H. S. Jarrett and R. K. Waring, Phys. Rev. **111**, 1223 (1958).
[3] M. H. Seavey, Jr., and P. E. Tannenwald, Phys. Rev. Letters **1**, 168 (1958).
[4] C. F. Kooi and R. W. Moss, Bull. Am. Phys. Soc. **4**, 353 (1959).
[5] H. Suhl, J. Phys. Chem. Solids **1**, 209 (1957).
[6] R. L. White and J. H. Solt, Phys. Rev. **104**, 56 (1956).
[7] W. S. Ament and G. T. Rado, Phys. Rev. **97**, 1558 (1955).

additional term proportional to the gradient of the magnetization must be included in their analysis. The antinode boundary condition is only rigorously true for the uniform ($k=0$) mode when there is no surface anisotropy and is never true when there is surface anisotropy. We know that relatively large anisotropies can act on the surface spins since interactions which cancel in the interior as a result of cubic symmetry will not cancel at the surface as pointed out by Néel.[8] An antiferromagnetic oxide layer on the surface of a metallic film can also give rise to an anisotropy which may tend to pin the end spins. Such an exchange surface anisotropy was first observed by Meiklejohn and Bean[9] in Co with a CoO surface layer.

We now derive the boundary conditions when an anisotropy field $\mathbf{H}_s$ acts on the end spins by a semiclassical treatment of the equations of motion. In Appendix A we derive precisely the same condition by the quantum-mechanical method of Bloch.[10] We show also how an antiferromagnetic surface layer may pin the directions of the end spins of a ferromagnetic chain.

The equation of motion for a surface spin $\mathbf{S}_1$ is

$$\partial\mathbf{S}_1/\partial t=(2J/\hbar)\mathbf{S}_1\times\mathbf{S}_2+\gamma\mathbf{S}_1\times(\mathbf{H}_0+\mathbf{H}_s), \quad (2)$$

where J is the exchange integral, $\mathbf{H}_0$ is the z-directed static field, $\mathbf{H}_s$ is the surface anisotropy field, and $\gamma=ge/2mc$. We take $\mathbf{H}_s$ in the z direction, normal to the surface. If we expand $\mathbf{S}_2$ as

$$\mathbf{S}_2\approx\mathbf{S}_1+a(\partial\mathbf{S}_1/\partial z)+\tfrac{1}{2}a^2(\partial^2\mathbf{S}_1/\partial z^2)+\cdots, \quad (3)$$

where a is the lattice constant, and let $S^+=S^x+iS^y$, Eq. (2) becomes

$$-i(\partial S_1^+/\partial t)=\omega_e[a(\partial S_1^+/\partial z)+\tfrac{1}{2}a^2(\partial^2 S_1^+/\partial z^2)]-(\omega_0+\omega_s)S_1^+, \quad (4)$$

where $\omega_e=2JS/\hbar$, $\omega_0=\gamma H_0$, and $\omega_s=\gamma H_s$. For an interior spin, the linear term in the lattice constant and the term in ω_s vanish by symmetry, giving in the interior

$$-i(\partial S^+/\partial t)=\omega_e a^2(\partial^2 S^+/\partial z^2)-\omega_0 S^+. \quad (5)$$

We now look for a solution of (5) of the form

$$S^+=e^{-i\omega t}(\alpha\sin kz+\beta\cos kz), \quad (6)$$

where α and β are constants whose magnitudes are determined by the strength of the exciting field. Substitution of (6) into (5) gives the usual ferromagnetic dispersion law:

$$\omega=\omega_0+\omega_e a^2k^2, \quad (7)$$

where ω_0 is the Zeeman frequency corrected for demagnetization effects. The ratio α/β is determined by substitution of (6) into (4), using (7) and setting $z=0$ for the surface spin. We find

$$\alpha/\beta=-\tfrac{1}{2}ka+(\omega_s/\omega_e ak). \quad (8)$$

For the spin waves of interest at microwave frequencies $ka\sim10^{-2}$, so we may neglect the first term on the right. If $\omega_s\sim0.1\omega_e$, we find $\alpha/\beta\sim10$, which implies that the end spins are effectively pinned in direction. The solution (6) is then dominated by the term in $\alpha\sin kz$ which has a node at the surface. When ω_s tends to zero, there is very little pinning because ka is usually quite small, but α is identically zero, i.e., antinode at surface, only when $k=0$. That is, the modes of the pure exchange problem are not simply of the form $\cos kz$.

We can estimate the magnitude of the surface anisotropy by assuming that an anisotropic exchange interaction acts on the surface spins

$$\mathcal{H}_s=C\sum_{i>j}S_i^zS_j^z. \quad (9)$$

This form is used for the anisotropy rather than just a pseudodipolar interaction because, as Néel[8] has shown, for certain orientations of the surface in a cubic lattice [(100) and (111) planes in a bcc and (111) plane in sc] the pseudodipolar anisotropy still vanishes at the surface in the classical limit. This disappearance of the pseudodipolar anisotropy energy at a surface arises only because of a cancellation of the anisotropy energy with part of the exchange energy. However, the symmetry of a surface is really uniaxial and hence we expect a uniaxial type anisotropy, such as the anisotropic exchange, which will not vanish at the surface for any orientations of a cubic crystal. The magnitude of the constant C can be estimated. Suppose we assume all the bulk anisotropy in nickel, for example, arises from pseudodipolar interactions. We fit C to the extrapolated experimental values[11] of the anisotropy constant K_1 at 0°K using the theoretical expression obtained by Van Vleck[12] and given in the present form by Keffer and Oguchi,[13]

$$K_1(0)=-3NSC^2/64J, \quad (10)$$

for a face-centered cubic lattice; here N is the number of spins per unit volume. For $C=\hbar\omega/zS$, where z is the number of nearest neighbors to a surface spin, we obtain $\omega/\omega_e\sim0.1$. At finite temperatures, the surface anisotropy is expected to fall off slower with increasing temperature than the volume anisotropy because of the lower symmetry at the surface. We should emphasize that there is no firm evidence that the pseudodipolar anisotropy accounts for the observed bulk anisotropy of nickel. Our argument above is simply to show what would happen if this were so.

[8] L. Néel, Compt. rend. **237**, 1468 (1953); J. phys. radium **15**, 225 (1954).
[9] W. H. Meiklejohn and C. P. Bean, Phys. Rev. **102**, 1413 (1956). A similar effect has also been observed in Fe with an FeO surface layer. W. H. Meiklejohn and C. P. Bean, J. Appl. Phys. **29**, 454 (1958).
[10] F. Bloch, Z. Physik **61**, 206 (1930).
[11] R. Bozorth, *Ferromagnetism* (D. Van Nostrand Company, Inc., Princeton, 1951).
[12] J. H. Van Vleck, Phys. Rev. **52**, 1178 (1937).
[13] F. Keffer and T. Oguchi, Phys. Rev. **117**, 718 (1960).

We now show how an antiferromagnetic surface layer may pin the direction of the end spins of a ferromagnetic chain. Qualitatively, because it usually takes more energy to excite an antiferromagnet than a ferromagnet, the end spins of the ferromagnetic region will be effectively frozen into the antiferromagnetic lattice, in which excitations at the ferromagnetic resonant frequency fall off exponentially. In order to derive the boundary condition, we consider the equations of motion of a layer of N antiferromagnetically coupled spins followed by the ferromagnetic region, i.e., the Nth spin is coupled antiferromagnetically to the $N+1$st, and the $N+1$st spin is coupled ferromagnetically to the $N+2$nd spin. If $\mathbf{H}_A$ is the anisotropy field in the antiferromagnetic region, and J and J' (both written as positive) are the exchange integrals in the ferromagnetic and antiferromagnetic regions, respectively, the equations of motion are

$$\begin{aligned}
d\mathbf{S}_1/dt &= -(2J'/\hbar)\mathbf{S}_1\times\mathbf{S}_2+\gamma\mathbf{S}_1\times(\mathbf{H}_0+\mathbf{H}_A),\\
d\mathbf{S}_2/dt &= -(2J'/\hbar)\mathbf{S}_2\times(\mathbf{S}_1+\mathbf{S}_3)+\gamma\mathbf{S}_2\times(\mathbf{H}_0-\mathbf{H}_A),\\
&\cdots\\
d\mathbf{S}_N/dt &= -(2J'/\hbar)\mathbf{S}_N\times(\mathbf{S}_{N+1}+\mathbf{S}_{N-1})\\
&\qquad +\gamma\mathbf{S}_N\times(\mathbf{H}_0-\mathbf{H}_A),\\
d\mathbf{S}_{N+1}/dt &= -(2J'/\hbar)\mathbf{S}_{N+1}\times\mathbf{S}_N\\
&\quad +(2J/\hbar)\mathbf{S}_{N+1}\times\mathbf{S}_{N+2}+\gamma\mathbf{S}_{N+1}\times\mathbf{H}_0,
\end{aligned}\tag{11}$$

where N is taken to be even. If the transverse components of the spins vary as $e^{-i\omega t}$ (11) can be rewritten, when we are driving the spins at the frequency $\omega_0+\omega_e a^2k^2$, as

$$\begin{aligned}
&S_1^+(\omega_e a^2k^2-\omega_A-\omega_e')=\omega_e'S_2^+,\\
&S_2^+(\omega_e a^2k^2+\omega_A+2\omega_e')=-\omega_e'(S_1^++S_3^+),\\
&\cdots\\
&S_N^+(\omega_e a^2k^2+\omega_A+2\omega_e')=-\omega_e'(S_{N-1}^++S_{N+1}^+),\\
&S_{N+1}^+(\omega_e a^2k^2-\omega_e-\omega_e')=\omega_e'S_N^+-\omega_e S_{N+2}^+,
\end{aligned}\tag{12}$$

where $\omega_e'=2J'S/\hbar$. Now (12) is a set of difference equations having the solution

$$S_N^+=R_NS_{N+1}^+,\tag{13}$$

where

$$R_N=F^+[1-F^+F^-/(1-F^-R_{N-2})]^{-1},\tag{14}$$

and

$$R_2=F^+/(1-F^+F_0).\tag{15}$$

Here F^+, F^-, and F_0 are given by

$$\begin{aligned}
F^+&=-\omega_e'/(\omega_A+2\omega_e'),\\
F^-&=\omega_e'/(2\omega_e'-\omega_A),\\
F_0&=-\omega_e'/(\omega_e'+\omega_A).
\end{aligned}\tag{16}$$

If we try a solution of the form of (6) in the ferromagnetic region and use (13) and the last of (12), we find the pinning condition for the end spin of the ferromagnetic chain:

$$\alpha/\beta=-\tfrac{1}{2}ka+\omega_e'(1+R_N)/\omega_e ak.\tag{17}$$

For $\omega_e'\gg\omega_A$, it is easy to show from (14), (15), and (16) that

$$1+R_N\approx N\omega_A/\omega_e'.\tag{18}$$

The boundary condition becomes, for $ka\ll1$,

$$\alpha/\beta=N\omega_A/\omega_e ak.\tag{19}$$

If $\omega_A\sim0.1\omega_e$ and $N\sim10$, then we find $\alpha/\beta\sim10^2$. Hence even a thin antiferromagnetic surface oxide layer can be quite effective in causing the end spins of a ferromagnetic chain to be pinned.

III. EXCITATION IN A FERROMAGNETIC INSULATOR

In this section we discuss the excitation of spin-wave modes in an insulator, and we calculate the power absorbed in each mode as a function of the boundary conditions. We omit all relaxation mechanisms; consequently all the calculated peaks will be infinitely sharp. We discuss here the case of H_0 normal to the film, and defer until Appendix B the case of H_0 parallel to the film. If

$$H^+=h^+e^{-i\omega t}\tag{20}$$

is a small transverse rf field, (5) becomes

$$-i(\partial S^+/\partial t)=\omega_e a^2(\partial^2S^+/\partial z^2)-\omega_0S^++\gamma Sh^+.\tag{21}$$

We shall assume eigenmodes of the form of (6) and write

$$g(k)=\alpha/\beta.\tag{22}$$

Because the Hamiltonian is symmetric with respect to reflection in the center of the film we must have

$$g(k)\sin kL+\cos kL=\pm1,\tag{23}$$

which determines the eigenvalues k. Equation (23) reduces to the following transcendental equations for the wave vectors, according to the choice of sign above:

$$g(k)=\tan(kL/2),\tag{24a}$$

$$g(k)=-\cot(kL/2),\tag{24b}$$

with $g(k)$ given by (8). Here (24a) corresponds to modes which are even with respect to reflection in the xy plane at $z=L/2$, and (24b) gives the odd modes. With a uniform field, only the even modes can be excited. However, if h^+ is not homogeneous and not invariant under the reflection, the odd modes can also be excited. It is easily seen from (8) that for large surface anisotropies the solutions of (24a) are very close to

$$k=p\pi/L,\tag{25}$$

where p is an odd integer. For very small ω_S the solutions are approximately given by (25) with even integers p.

If we let

$$\Psi_p=\cos k_p z+g(k_p)\sin k_p z, \tag{26}$$

we can write the solution to (21) as

$$S^+=e^{-i\omega t}\sum_p \beta_p\Psi_p, \tag{27}$$

where the sum is over all solutions of (24). Substitution of (27) into (21) gives

$$\gamma S h^+=\sum_p \beta_p\Psi_p(\omega_p-\omega), \tag{28}$$

where

$$\omega_p=\omega_0+\omega_e a^2k_p^2. \tag{29}$$

We now calculate the amplitude coefficients β_p by multiplying both sides of (28) by Ψ_n and integrating. The two integrals in question are

$$I_1(k_p)=\int_0^L \Psi_p\Psi_n dz=L\delta_{0p}\delta_{np} +I_2(k_p)\delta_{n0}+Q(k_p)\delta_{pn}, \tag{30}$$

where

$$Q(k_p)=\tfrac{1}{2}L+g_pk_p^{-1}+\tfrac{1}{2}g_p^2L, \tag{31}$$

and

$$\begin{aligned} I_2(k_p)=\int_0^L \Psi_p dz&=L \quad \text{if} \quad p=0\\ &=2gk_p^{-1} \quad \text{if} \quad g=\tan kL/2\\ &=0 \quad \text{if} \quad g=-\cot kL/2. \end{aligned} \tag{32}$$

We see from (32) that only the modes even with respect to reflection are excited. From (30) and (32), it is readily seen that when $\omega_s=0$,

$$\beta_0=\gamma S h^+/(\omega_0-\omega), \quad \beta_{p\neq 0}=0. \tag{33}$$

Thus, when there is no surface anisotropy or field inhomogeneity, there is no coupling between a uniform rf field and nonuniform spin wave modes. However, for nonvanishing ω_s, we obtain

$$\beta_p=2g(k_p)\gamma S H^+/k_pQ(k_p)(\omega_p-\omega), \tag{34}$$

where the k_p's are the solutions of (24a). The power absorbed in the spin-wave modes is then

$$P=4\hbar\gamma^2Sh^{+2}\sum_p[\delta(\omega_p-\omega)g^2(k_p)/k_p^2LQ(k_p)]. \tag{35}$$

For $\omega_s\rightarrow\infty$ the pinning is complete and the power absorbed per mode is inversely proportional to k_p^2.

IV. FERROMAGNETIC METAL

In a metal one source of damping of the spin-wave modes is the interaction with the conduction electrons through eddy currents. The effect of eddy-current damping on the spin-wave resonances decreases with increasing k. The problem is to solve the spin-wave equation simultaneously with the Maxwell equation:

$$\begin{aligned} &c\,\mathrm{curl}\mathbf{H}=4\pi\sigma\mathbf{E},\\ &c\,\mathrm{curl}\mathbf{E}=-(\partial/\partial t)(\mathbf{H}+4\pi\mathbf{M}). \end{aligned} \tag{36}$$

The dc field H_0 interacting with the spins is given by

$$H_0=H-4\pi M, \tag{37}$$

where $4\pi M$ is the demagnetizing field. Eliminating the electric field from (36) gives

$$\left(\frac{c}{4\pi}\right)^2\frac{\partial^2H^+}{\partial z^2}=\frac{\sigma}{4\pi}\frac{\partial H^+}{\partial t}-\frac{\sigma M}{S}\frac{\partial S^+}{\partial t}. \tag{38}$$

If we assume solutions of H^+ and S^+ of the form of (6), and solve the secular determinant arising from (21) and (38), we obtain the dispersion law

$$\omega=\omega_0+\omega_e a^2k^2+4\pi\gamma M/(1+\tfrac{1}{2}i\delta^2k^2). \tag{39}$$

This is a well-known result. The skin depth δ is defined by

$$\delta^2=c^2/2\pi\sigma\omega. \tag{40}$$

We see from (39) that for short wavelength spin waves, i.e., $k\delta\gg 1$, the frequency ω is largely real and there is very little damping by the eddy currents. We now use the usual boundary condition that H^+ is continuous across the surface, and we assume that there is complete pinning of the spin directions at the surfaces. The four boundary conditions are

$$h^+(0)=h^+(L)=h_0; \quad S^+(0)=S^+(L)=0. \tag{41}$$

To calculate the absorption curve, we need the surface impedance Z defined by

$$Z=4\pi\left(\frac{\mathbf{n}\cdot(\mathbf{E}\times\mathbf{H}^*)}{|H|^2}\right)_{Z=0}=-\left(\frac{1}{h^+}\frac{\partial h^+}{\partial Z}\right)_{Z=0}, \tag{42}$$

where $\mathbf{n}$ is a unit vector normal to the surface. The surface impedance is related to the absorbed power per unit area by

$$P=(2/\sigma)(c/4\pi)^2|H|^2\,\mathrm{Re}Z. \tag{43}$$

On combining (6), (39), and (42) we obtain the following

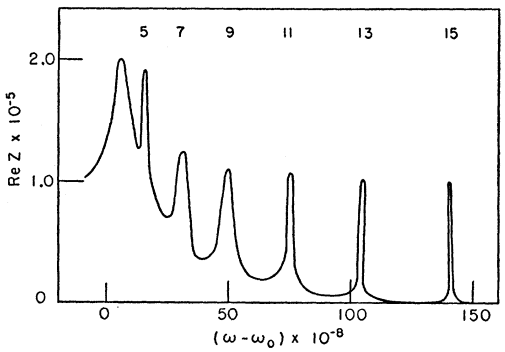

FIG. 1. Real part of the surface impedance vs $(\omega-\omega_0)$ for $L=4\times10^{-5}$ cm, $\omega_e a^2=10^{-2}$ rad cm²/sec, $\delta^2=10^{-8}$ cm², $4\pi M=10^4$ oersteds.

expression for the surface impedance:

$$Z=\frac{-2i[k_1\tan\tfrac{1}{2}k_1L-k_2\tan\tfrac{1}{2}k_2L+\tfrac{1}{2}i\delta^2k_1k_2(k_2\tan\tfrac{1}{2}k_1L-k_1\tan\tfrac{1}{2}k_2L)]}{\delta^2(k_1^2-k_2^2)}. \tag{44}$$

Here k_1^2 and k_2^2 are the two complex solutions to (39). One of them resembles an electromagnetic eddy-current solution mixed with a small amount of spin-wave excitation, while the other corresponds to a spin wave with a small admixture of electromagnetic field. In Fig. 1, we have plotted the first few modes in a thin film for typical values of the parameters. This absorption curve looks more like (although our calculated high modes are too narrow) the experimental curves of Seavey and Tannenwald[3] than the absorption calculated for antinode boundary conditions.[14] The latter boundary condition allows excitation of the spin-wave modes by a sort of White-Solt effect, and the inhomogeneity in the rf fields caused by the eddy currents is not large enough to give rise to the experimental resonances of Seavey and Tannenwald. In the high mode number (short wavelength) limit, $k\delta\gg1$,

$$k_1\approx k_0+4\pi\gamma Mi/k_0\delta^2(\omega-\omega_0), \tag{45}$$

where

$$k_0^2=(\omega-\omega_0)/\omega_e a^2; \tag{46}$$

here k_2 is quite small and approaches $2(i)^{\frac{1}{2}}/\delta$. In this limit, the surface impedance can approximately be written as

$$Z\approx-8\pi\gamma Mi\tan\tfrac{1}{2}k_1L/\delta^2k_1(\omega-\omega_0), \tag{47}$$

which gives resonances at $k_1=p\pi/L$, where p is an odd integer. The power absorption on resonance is given by

$$P_{\rm res}\approx c^2h_0^2/4\pi^2L\sigma. \tag{48}$$

We see then that for the short wavelength modes the amplitudes of the resonances tend to a constant value; however, the line widths can be seen to decrease with increasing k_1. The fact that the eddy-current damping decreases with increasing k was realized by Ament and Rado[7] and Kittel.[15]

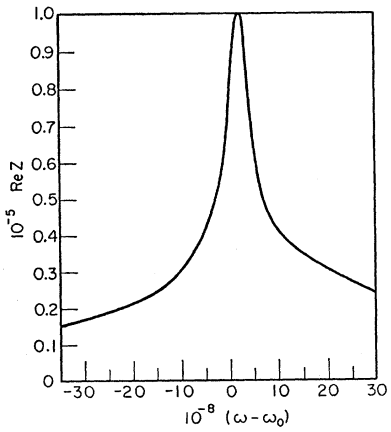

FIG. 2. Real part of the surface impedance for an infinite slab. The parameters are the same as in Fig. 1.

Another rather interesting limiting case of (44) is that of a thick slab, i.e., $L\rightarrow\infty$. H_0 is still taken to be normal to the surface of the slab. MacDonald[16] and Ament and Rado[7] have treated the case with H_0 parallel to the surface. A similar problem has been solved by Gurevich[17] using the Ament-Rado antinode boundary condition. For the antinode boundary conditions, the line width and exchange shift of the resonance are found to be approximately one half of the corresponding values for pinned boundaries. In this limiting case, for pinned boundaries, (44) becomes

$$Z=2(1-\tfrac{1}{2}i\delta^2k_1k_2)/\delta^2(k_1+k_2). \tag{49}$$

In Fig. 2 we plot the real part of (49) for the same values of the parameters as in Fig. 1. We see from (49) that all the subsidiary spin-wave resonances have disappeared and the resonant frequency is given by

$$\omega\approx\omega_0+(a/\delta)(32\pi\gamma M\omega_e)^{\frac{1}{3}}, \tag{50}$$

or in terms of the Landau exchange stiffness constant A,

$$\omega\approx\omega_0+(8\gamma/\delta)(\pi A)^{\frac{1}{3}}. \tag{51}$$

The exchange shift $\omega-\omega_0$ is quite small and, for example, in Permalloy at about 10^4 megacycles/second, is only a few percent of ω_0. The shift is equivalent to a very small increase in the spectroscopic splitting factor, g. The eddy current line width is approximately the same as the exchange shift. In materials of lower resistivity than Permalloy the exchange shift will of course be larger.

Rado and Weertman[18] have estimated the exchange stiffness constant A for a Fe-Ni alloy (66% Ni) as 3.3×10^{-6} erg/cm by fitting their experimental resonance curves to a theoretical curve for the antinode boundary condition. This value is in serious disagreement with values ($A=0.8\times10^{-6}-1\times10^{-1}$ erg/cm) obtained from experiments by Kondorsky and Fedotov[19] and Bean[20] on similar materials. However, the Rado-Weertman estimate may have been too high by as much as a factor of four if the surface spins were pinned. Then, the Rado-Weertman value would be changed to

[14] M. H. Seavey, Jr., and P. E. Tannenwald, J. phys. radium **20**, 323 (1959).

[15] C. Kittel, Phys. Rev. **110**, 836 (1958).

[16] J. R. MacDonald, thesis, oxford, 1950 (unpublished).

[17] V. L. Gurevich, J. Exptl. Theoret. Phys. U. S. S. R. **33**, 1497 (1957) [translation: Soviet Phys.-JETP **6**(**33**), 1155 (1958)].

[18] G. T. Rado and J. R. Weertman, J. Phys. Chem. Solids (to be published).

[19] E. Kondorsky and L. M. Fedotov, Bull. Acad. Sci. U. S. S. R. Phys. Ser. **16**, 432 (1952).

[20] C. P. Bean, Proceedings of the Pittsburgh Conference on Magnetism and Magnetic Materials, 1955 (unpublished), p. 365.

$A\sim 0.8\times 10^{-6}$ erg/cm, bringing their exchange constant into agreement with the other experiments.

Young and Uehling[21] have carried out ferromagnetic resonance experiments on Supermalloy with H_0 perpendicular to the surface of the sample. Their results show relatively large line widths of about 400 oersteds. Eddy current line widths of these materials are only about 50 oersteds. Hence, this is some indication that there is a line broadening mechanism in addition to the eddy current damping. The experimental curves fit quite well with a Bloch type damping with T_2 of the order of 10^{-9} sec. Young and Uehling state that their experiments were quite sensitive to surface conditions, and hence there may be some type of surface broadening mechanism, possibly from mechanical distortion of the surface.

Hoskins[22] has also carried out ferromagnetic resonance experiments in Fe-Ni alloys (30–50% Ni) for the perpendicular field geometry at 25 kMc/sec. He finds line widths of the order of 200 oersteds which are also too large to be completely explained by eddy current damping.

V. ACKNOWLEDGMENTS

The author would like to express his most sincere appreciation to Professor C. Kittel who closely followed the calculations. He would also like to thank Dr. P. G. de Gennes for his frequent invaluable discussions.

APPENDIX A. QUANTUM-MECHANICAL DERIVATION OF BOUNDARY CONDITIONS

We now rederive Eq. (8), using the Bloch spin-wave states. We consider a finite line of $N+1$ spins ($S=\frac{1}{2}$), with the ends experiencing a surface anisotropy field $\mathbf{H}_S$. The Hamiltonian can be written as

$$\mathcal{H}=-\frac{J}{2}\sum_{i>j}[\sigma_i^z\sigma_j^z+\tfrac{1}{2}(\sigma_i^+\sigma_j^-+\sigma_i^-\sigma_j^+)]-\frac{g\mu_B}{2}H_S(\sigma_0^z+\sigma_N^z), \quad \text{(A1)}$$

where we consider only nearest-neighbor interactions; the σ's are the Pauli spin matrices. For one reversed spin, we look for an eigenfunction of the Bloch form:

$$\Psi_k=\textstyle\sum_j C_j^k\varphi_j, \quad \text{(A2)}$$

where

$$\varphi_j=\alpha_0\alpha_1\cdots\alpha_{j-1}\beta_j\alpha_{j+1}\cdots\alpha_N. \quad \text{(A3)}$$

Here α and β are the eigenvectors of σ^z with eigenvalues $+1$ and -1, respectively. Then

$$\begin{aligned}\langle\varphi_j|\mathcal{H}|\Psi_k\rangle&=-\frac{J}{2}\left[2C_{j-1}^k+2C_{j+1}^k+(N-4)C_j^k+\frac{2g\mu_BH_S}{J}C_j^k\right] && \text{for } j\neq 0, N,\\ &=-(J/2)[2C_1^1+(N-2)C_0^k] && \text{for } j=0,\\ &=-(J/2)[2C_{N-1}^k+(N-2)C_N^k] && \text{for } j=N. \quad \text{(A4)}\end{aligned}$$

If we write the eigenvalue equation as

$$\mathcal{H}\Psi_k=E_k\Psi_k, \quad \text{(A5)}$$

then

$$\langle\varphi_j|\mathcal{H}|\Psi_k\rangle=E_kC_j^k. \quad \text{(A6)}$$

If we try an expression for C_j^k of the form

$$C_j^k=\alpha\sin jak+\beta\cos jak, \quad \text{(A7)}$$

where a is the lattice constant, and combine Eq. (A6) with Eq. (A4), we obtain the usual energy expression:

$$E_k=-(J/2)(N-4)-2J\cos ak-g\mu_BH_S. \quad \text{(A8)}$$

Now, combining Eq. (A8) with Eq. (A4), and using the approximation $ka\ll 1$, we arrive at precisely the same boundary condition as that given by Eq. (8).

APPENDIX B

In a general geometry, there is an additional magnetic field $\mathbf{H}_k$, parallel to $\mathbf{k}$, arising from the dipolar interaction of the spins themselves. In the perpendicular field problem treated in Sec. III, $\mathbf{H}_k$ vanishes, but it is nonzero for other geometries. Kittel and Herring[23] have derived the ferromagnetic dispersion relation as a function of the angle, θ_k between $\mathbf{k}$ and the magnetization. We calculate here the power absorption per mode in the parallel case, i.e., $\theta_k=\pi/2$. The constant magnetic field is taken along the z axis, parallel to the surface of the film. The spin waves are directed along the x axis perpendicular to the surface.

The field $\mathbf{H}_k$ is determined by the Maxwell equations:

$$\operatorname{div}(\mathbf{H}_k+4\pi\mathbf{M})=0, \quad \operatorname{curl}\mathbf{H}_k=0. \quad \text{(B1)}$$

We look for solutions

$$\begin{aligned}S^x&=R(k)\cos\omega t(\alpha\sin kx+\beta\cos kx),\\ S^y&=\sin\omega t(\alpha\sin kx+\beta\cos kx),\end{aligned} \quad \text{(B2)}$$

where $R(k)$ is to be determined. From (B1) and (B2), we obtain

$$H_k^x=-4\pi MS^x/S, \quad H_k^y=H_k^z=0. \quad \text{(B3)}$$

The equations of motion become

$$\begin{aligned}\partial S^x/\partial t&=-\omega_e a^2(\partial^2\delta^y/\partial x^2)+\omega_0S^y,\\ \partial S^y/\partial t&=\omega_e a^2(\partial^2S^x/\partial x^2)-\omega_0S^x+\gamma SH_k^x.\end{aligned} \quad \text{(B4)}$$

[21] J. A. Young, Jr., and E. A. Uehling, Phys. Rev. **94**, 544 (1954).
[22] R. Hoskins, thesis, University of California, 1955 (unpublished).
[23] C. Herring and C. Kittel, Phys. Rev. **81**, 869 (1951).

Substituting (B2) into (B4), we obtain the dispersion law

$$\omega^2=(\omega_0+\omega_e a^2k^2)(\omega_0+\omega_e a^2k^2+4\pi\gamma M). \quad \text{(B5)}$$

When $k=0$, this is the usual $(BH)^{\frac{1}{2}}$ resonance condition; combining (B2), (B4), (B5), we find that $R(k)$ is given by

$$R(k)=-[(\omega_0+\omega_e a^2k^2)/(\omega_0+\omega_e a^2k^2+4\pi\gamma M)]. \quad \text{(B6)}$$

Following Sec. III, we apply a uniform transverse rf field ($h_0\cos\omega t$, $h_0\sin\omega t$, 0) and calculate the amplitude coefficients. With the additional rf driving field, the equations of motion become

$$\begin{aligned}\partial S^x/\partial t&=-\omega_e a^2(\partial^2 S^y/\partial s^2)+\omega_0 S^y-\gamma S h_0\cos\omega t,\\ \partial S^y/\partial t&=\omega_e a^2(\partial^2 S^x/\partial x^2)-\omega_0 S^x\\ &\quad+\gamma S h_0\sin\omega t+\gamma S H_k{}^x.\end{aligned} \quad \text{(B7)}$$

The boundary conditions are the same as those in Sec. III and, consequently, the eigenmodes are the same. We now define $\Psi_p{}^x$ and $\Psi_p{}^y$ by

$$\Psi_p{}^y=\alpha\sin k_p x+\beta\cos k_p x, \quad \Psi_p{}^x=R(k_p)\Psi_p{}^y. \quad \text{(B8)}$$

Then, we try solutions of the form

$$S^x=\cos\omega t\sum_p\beta_p{}^x\Psi_p{}^x, \quad S^y=\sin\omega t\sum_p\beta_p{}^y\Psi_p{}^y. \quad \text{(B9)}$$

From (B7) and (B9) we obtain the following coupled equations for the amplitude coefficients $\beta_p{}^x$ and $\beta_p{}^y$:

$$\begin{aligned}\gamma S h_0&=\sum_p[(\omega_0+\omega_e a^2k_p{}^2)\beta_p{}^y+\omega R(k_p)\beta_p{}^x]\Psi_p{}^y,\\ \gamma S h_0&=\sum_p[\omega\beta_p{}^y+(\omega_0+\omega_e a^2k_p{}^2\\ &\quad+4\pi\gamma M)R(k_p)\beta_p{}^x]\Psi_p{}^y.\end{aligned} \quad \text{(B10)}$$

The ratio $\beta_p{}^x/\beta_p{}^y$ is then found to be

$$\frac{\beta_p{}^x}{\beta_p{}^y}=\frac{\omega-\omega_0-\omega_e a^2k_p{}^2}{R(k_p)(\omega-\omega_0-\omega_e a^2k_p{}^2-4\pi\gamma M)}. \quad \text{(B11)}$$

The power absorbed per mode is now easily seen to be

$$P=(2\hbar\gamma^2 S h^{+2})\sum_p\frac{[1+R^2(k_p)]g^2(k_p)S(\omega-\omega_p)}{k_p{}^2LQ(k_p)}, \quad \text{(B12)}$$

where ω_p is given by (B5). We see that the absorption per mode in the parallel use is only $\frac{1}{2}[1+R^2(k_p)]$ of that in the perpendicular case.

PHYSICAL REVIEW VOLUME 118, NUMBER 3 MAY 1, 1960

Spin Relaxation and Line Width in Alkali Metal Vapors

ARNOLD L. BLOOM
Varian Associates, Palo Alto, California
(Received November 16, 1959)

Relaxation constants T_1 and T_2 have been computed for experiments involving optical pumping and optical detection in alkali metal vapor. The calculations have been performed for several possible spin relaxation mechanisms; namely, magnetic dipole, electric quadrupole, spin exchange, and the optical pumping process itself. For all of these mechanisms a reorientation experiment will approximately predict a spin resonance line width (equivalent to the statement $T_1\approx T_2$ for spin-$\frac{1}{2}$ particles.) However, a spin reorientation experiment of the type originally performed by Dehmelt, employing circularly polarized light, gives approximate results because of the nonexponential character of the reorientation. A more suitable experiment is one employing hyperfine population differences and unpolarized light.

INTRODUCTION

THE recent observation of long spin reorientation times in alkali metal vapors in buffer gases,[1] or with buffering wall coatings[2,3] has prompted considerable speculation as to the existence of correspondingly narrow line widths and application to magnetic field and frequency measurements. Implicit in this is the assumption that conditions in the alkali vapor are analogous to those of nuclear magnetic resonance of spin-$\frac{1}{2}$ nuclei in a nonviscous fluid, for which one has equal times ($T_1=T_2$). However, the alkali vapor differs from an ensemble of spin-$\frac{1}{2}$ particles in two important respects; first, because of the strong hyperfine coupling, and secondly, because of the special nature of the observables measured by optical detection. Thus, for example, the quantity measured by the optical detection of hyperfine population differences using unpolarized light[4] is not simply related to classical dynamical variables such as magnetic dipole moment, electric quadrupole moment, etc.

The observed values of reorientation times in alkali vapor, in the limit of vanishing light intensity, are of the

[1] H. G. Dehmelt, Phys. Rev. **105**, 1487 (1957).
[2] H. G. Robinson, E. S. Ensberg, and H. G. Dehmelt, Bull. Am. Phys. Soc. Ser. II, **3**, 9 (1958).
[3] W. Franzen, Phys. Rev. **115**, 850 (1959).
[4] W. E. Bell and A. L. Bloom, Phys. Rev. **109**, 219 (1958); M. Arditi and T. R. Carver, Phys. Rev. **109**, 1012 (1958).