following statements can be made about the effects of the potentials in quantum theory:

1. These effects change only the phase of the wave function, and the phase changes are independent of the kinetic energy and kinetic momentum of the particle. Thus they are intrinsically quantum-mechanical effects, with no analog in classical theory.
2. These effects do not affect the gauge invariance of the theory.
3. These effects can have objective meaning only when they act differently on different parts of the wave function of the same particle. A classical particle could show effects of an electrostatic potential difference or a magnetic flux only by following a path passing through a region of nonvanishing field. A quantum-mechanical particle need not enter such a region to detect its existence; the de Broglie waves can pass on either side of it and receive a relative phase shift that is in principle observable.
4. These effects are not accidental results of a particular way of formulating the theory, and they do not constitute any paradox or inconsistency in the theory. On the contrary, our discussions of conceptual experiments have shown that these effects prevent paradoxes and are essential for the consistency of the theory.

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Electric Field Distributions in an Ionized Gas. II*

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A method previously described is used to calculate the probability distribution of the low-frequency component of the electric field at a neutral point, the distribution of the low-frequency component at an ion, and that of the high-frequency component at an electron. The results are compared with those obtained by other authors.

1. INTRODUCTION

In a previous paper,[1] a method was described by which the Holtsmark[2] distribution can be systematically corrected for the correlations between the particles producing the field, provided that this correction is not too large. The result was to put the Fourier transform $F(\mathbf{k})$ of the field distribution in the form

$$F(\mathbf{k})=\exp\left[\sum_{P=1}^{\infty}(n^P/P!)h_P(\mathbf{k})\right],\tag{1}$$

where n is the density of particles and the functions $h_P(\mathbf{k})$ correspond to increasing orders in a cluster expansion. Those can in turn be expressed in terms of other functions g_P,

$$h_P(\mathbf{k})=\int\varphi_1\varphi_2\cdots\varphi_P \times g_P(\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_P)d^3x_1d^3x_2\cdots d^3x_P,\tag{2}$$

where

$$\varphi_i=\exp(i\mathbf{k}\cdot\mathbf{E}_i)-1,\tag{3}$$

and $\mathbf{E}_i$ is the electric field produced by the particle of coordinates $\mathbf{x}_i$. The present calculations include only the first two terms of the series in (1). The corresponding g functions will be rewritten here explicitly,

$$g_1(\mathbf{x})=\mathcal{V}P_1(\mathbf{x}),\tag{4}$$

$$g_2(\mathbf{x}_1,\mathbf{x}_2)=\mathcal{V}^2[P_2(\mathbf{x}_1,\mathbf{x}_2)-P_1(\mathbf{x}_1)P_1(\mathbf{x}_2)],\tag{5}$$

P_1 and P_2 being the single-particle and pair distribution functions, respectively, and $\mathcal{V}$ being the volume of the container. The field distribution itself, $W(\mathbf{E})$, is obtained from

$$W(\mathbf{E})=(2\pi)^{-3}\int\exp(-i\mathbf{k}\cdot\mathbf{E})F(\mathbf{k})d^3k.\tag{6}$$

The distinction was made in I between the *low-frequency component* and the *high-frequency component* of the electric field in an ionized gas or plasma. The former is that part of the field whose time variation is governed by the motion of the ions. It is obtained by averaging the total field over a time long compared to typical electronic relaxation times, but short compared to ionic times. Therefore, it consists of the sum of the fields from the ions, each field being shielded by a cloud of electrons. Only the case of singly charged ions will be considered. Other cases would be equally easy to treat. The electron and ion densities are then equal and both denoted by n. Thermal equilibrium will also be assumed. One can then use the shielded field given by

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[1] M. Baranger and B. Mozer, Phys. Rev. 115, 521 (1959), referred to in the following as I.

[2] J. Holtsmark, Ann. Physik 58, 577 (1919).

the Debye-Hückel[3] theory. Its range of validity is the same as that of the present cluster expansion method. The shielded field at the origin due to an ion located at point **x**, distance r, is then

$$\mathbf{E}_L = -er^{-3}\mathbf{x}(1+r/\lambda)\exp(-r/\lambda), \tag{7}$$

with

$$\lambda = (\kappa T/4\pi ne^2)^{\frac{1}{2}} \tag{8}$$

It is important to realize that only the electrons participate in this shielding of the field of each ion. Ion-ion correlations cannot be treated by the Debye-Hückel theory, as far as their influence on the field distribution is concerned, because the Debye-Hückel field is a long-time average, while one only wants to average over a time short compared to ion-ion collision times. Ion-ion correlations are taken into account by the cluster expansion. The high-frequency component is the difference between the total field and the low-frequency component. The surmise was made in I that its distribution was correctly given by considering an electron gas of density n with uniform positive background. It consists, therefore, of the sum of the unshielded Coulomb fields of each electron. For an electron at point **x**, distance r, this is

$$\mathbf{E}_H = er^{-3}\mathbf{x}. \tag{9}$$

The method has already been applied in I to the calculation of the distribution of the high-frequency component at a neutral point. In Sec. 2, the distribution of the low-frequency component at a neutral point is discussed. In Sec. 3, the method is applied to the distribution of the high-frequency component at an electron and that of the low-frequency component at an ion. In Sec. 4, the results are discussed and compared with the work of other authors.

2. DISTRIBUTION OF THE LOW-FREQUENCY COMPONENT AT A NEUTRAL POINT

Let the origin be the point at which the field distribution is calculated. The one-body function $h_1(\mathbf{k})$ is given by

$$h_1(\mathbf{k}) = \int [\exp(i\mathbf{k}\cdot\mathbf{E}_L) - 1]d^3x, \tag{10}$$

since the function $g_1(\mathbf{x})$ is unity for this case. As in I, all lengths will be expressed in units of r_0 given by

$$(4/15)(2\pi)^{\frac{3}{2}} r_0^3 n = 1, \tag{11}$$

and y, x, and β will be defined by

$$y = r_0/\lambda, \tag{12}$$

$$x = kE_0, \tag{13}$$

$$\beta = E/E_0, \tag{14}$$

where E_0 is the normal field strength

$$E_0 = e/r_0^2. \tag{15}$$

With these notations, the contribution of one-body clusters to the series in (1) can be written in the form

$$nh_1(k) = -x^{\frac{3}{2}}\chi'(x^{\frac{1}{2}}y) \tag{16}$$

The function χ' was found by numerical integration. It depends only on the variable $x^{\frac{1}{2}}y$ and is plotted in Fig. 1.

For calculating the second term of the series in (1), the pair correlation function is needed. For this, it is proper to use the result of the Debye-Hückel theory. The corresponding expression for g_2 was derived in I and is

$$g_2(\mathbf{x}_1,\mathbf{x}_2) = -e^2\Phi_{12}'/\kappa T, \tag{17}$$

with

$$\Phi_{12}' = |\mathbf{x}_1 - \mathbf{x}_2|^{-1}\exp(-|\mathbf{x}_1-\mathbf{x}_2|/\lambda'). \tag{18}$$

The Debye length which enters this equation must express the shielding of the ion-ion interaction by both the electrons and the other ions; hence, it must be calculated with $2n$ as particle density. It differs then by a factor $\sqrt{2}$ from the previously defined λ,

$$\lambda' = (\kappa T/8\pi ne^2)^{\frac{1}{2}} = \lambda/\sqrt{2}. \tag{19}$$

The two-body function $h_2(\mathbf{k})$ is given by

$$h_2(\mathbf{k}) = -(e^2/\kappa T)\iint \varphi_1\varphi_2\Phi_{12}'d^3x_1d^3x_2. \tag{20}$$

The three factors of the integrand were expanded in spherical harmonics, in analogy with Eqs. (24), (25), (26) of I. As in the case of I, one can see that the series thus obtained is convergent and that the main contribution comes from the second term, i.e., $l=1$. Only the first three terms of the series were evaluated. The calculations are harder than those of I because one must use the shielded field $\mathbf{E}_L$ instead of the straight Coulomb field $\mathbf{E}_H$. The result will be written in the form

$$\tfrac{1}{2}n^2h_2(k) = x^{\frac{3}{2}}\psi'(x^{\frac{1}{2}}y). \tag{21}$$

The function ψ' is plotted in Fig. 1. The pair corrections for the low-frequency component are an order of magnitude smaller than those for the high-frequency com-

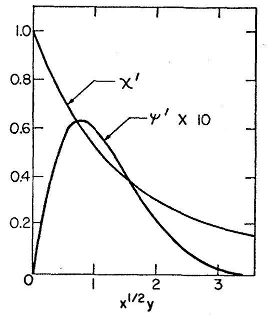

FIG. 1. The one-body function χ' and the two-body function ψ' for the low-frequency component at a neutral point versus $x^{\frac{1}{2}}y$. See text for the explanation.

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[3] P. Debye and E. Hückel, Physik. Z. 24, 185 (1923).

ponent. This is a consequence of the use of the shielded field and is due to the fact that the main contribution to the pair effects is made by pairs that are fairly distant from the field point, of the order of a Debye length away. Therefore, the use of the shielded field reduces their effect drastically.

Finally, the Fourier transform of the field distribution takes the form

$$F(x)=\exp\{-x^{\frac{3}{2}}[\chi'(x^{\frac{1}{2}}y)-\psi'(x^{\frac{1}{2}}y)]\}. \tag{22}$$

For all values of the parameter y that were used, ψ' is much smaller than χ', which indicates that the cluster expansion is good. The field distribution itself is given by (6) and is independent of angles. Therefore, the probability distribution $H(\beta)$ for the scalar quantity β is

$$\begin{aligned} H(\beta)&=4\pi\beta^2W(\beta)\\ &=2\pi^{-1}\beta\int_0^\infty \sin(\beta x)F(x)xdx. \end{aligned} \tag{23}$$

3. FIELD DISTRIBUTIONS AT ONE OF THE CHARGED PARTICLES

For the calculation of the distribution of the high-frequency component at an electron, and that of the low-frequency component at an ion, the function $g_1(\mathbf{x})$ is not unity. Rather, it is equal to the pair correlation function with one member of the pair located at the origin. For this it is again appropriate to use the Debye-Hückel result which is

$$\begin{aligned} g_1(\mathbf{x})&=\exp[-e^2(r\kappa T)^{-1}\exp(-r/\lambda)] \\ &\qquad\qquad\text{(high-freq. comp.)}, \\ &\quad\exp[-e^2(r\kappa T)^{-1}\exp(-r/\lambda')] \\ &\qquad\qquad\text{(low-freq. comp.)}. \end{aligned} \tag{24}$$

The first term of the series in (1) was then obtained by performing numerically the integral in Eq. (2) for $P=1$, the electric field used being $\mathbf{E}_H$ and $\mathbf{E}_L$ for the high- and low-frequency components, respectively. The result is a complicated function of both variables x and y.

The two-body function $g_2(\mathbf{x}_1,\mathbf{x}_2)$ may be considered as the sum of two parts. The first part is the same two-body function that was used in the calculation of the field distribution at a neutral point. The second part is the correction necessary to take into account the existence of a charged particle at the origin. This correction, however, is a three-body effect whose order of magnitude would be the same as that of the third term of the series in (1). Since all other three-body effects have been neglected, it is consistent to drop this one too and to use as the second-term of the cluster expansion (1) the same expression that was used for the field distributions at a neutral point, namely, (21) for the low-frequency component and (I, 31) for the high-frequency component. This was done, and further justification was obtained from a rough numerical estimate of the correction, which showed it to be small for all values of r_0/λ considered. The smallness of the correction may be understood on the basis of the remark, made in Sec. 2, that the pairs that contribute to $h_2(\mathbf{k})$ are rather distant from the origin; therefore, the presence of a charged particle at the origin does not matter much.

TABLE I. Distribution of the low-frequency component, $H(\beta)$, at a neutral point for several values of r_0/λ. This distribution is to be used in line broadening calculations involving a neutral radiator when the perturbing ions are treated by the static approximation. The accuracy of the numerical work, for this and the following tables, is of a few units in the last figure quoted. The case $r_0/\lambda=0$, the Holtsmark distribution, was calculated entirely by numerical integration and therefore shows some very slight deviations from the distribution previously published[a]; the present calculation is to be preferred.

β	$r_0/\lambda=0$	$r_0/\lambda=0.2$	$r_0/\lambda=0.4$	$r_0/\lambda=0.6$	$r_0/\lambda=0.8$
0.1	0.00422	0.00729	0.01327	0.02333	0.03813
0.2	0.01667	0.02848	0.05102	0.08757	0.13867
0.3	0.03664	0.06168	0.10751	0.17774	0.26834
0.4	0.06308	0.10401	0.17482	0.27530	0.39237
0.5	0.09460	0.15201	0.24450	0.36386	0.48754
0.6	0.12959	0.20201	0.30911	0.43269	0.54520
0.7	0.16636	0.25054	0.36324	0.47746	0.56760
0.8	0.20323	0.29472	0.40392	0.49898	0.56241
0.9	0.23864	0.33234	0.43032	0.50101	0.53841
1.0	0.27122	0.36203	0.44324	0.48846	0.50329
1.1	0.29987	0.38320	0.44451	0.46611	0.46291
1.2	0.32378	0.39591	0.43645	0.43795	0.42121
1.3	0.34246	0.40078	0.42143	0.40703	0.38073
1.4	0.35570	0.39873	0.40159	0.37545	0.34287
1.5	0.36357	0.39091	0.37878	0.34457	0.30829
1.6	0.36633	0.37852	0.35444	0.31521	0.27721
1.8	0.35850	0.34459	0.30527	0.26253	0.22516
2.0	0.33694	0.30475	0.25951	0.21848	0.18476
2.2	0.30684	0.26444	0.21935	0.18245	0.15345
2.4	0.27275	0.22682	0.18523	0.15325	0.12896
2.6	0.23822	0.19338	0.15673	0.12960	0.10945
2.8	0.20557	0.16454	0.13314	0.11034	0.09361
3.0	0.17606	0.14012	0.11367	0.09457	0.08056
3.25	0.14437	0.11508	0.09401	0.07873	0.06726
3.50	0.11837	0.09514	0.07845	0.06623	0.05667
3.75	0.09741	0.07927	0.06606	0.05624	0.04827
4.00	0.08067	0.06660	0.05611	0.04814	0.04163
4.25	0.06733	0.05642	0.04804	0.04154	0.03629
4.50	0.05667	0.04818	0.04146	0.03613	0.03188
4.75	0.04811	0.04147	0.03603	0.03163	0.02810
5.00	0.04118	0.03596	0.03152	0.02785	0.02481
5.25	0.03553	0.03139	0.02774	0.02465	0.02198
5.50	0.03089	0.02757	0.02455	0.02196	0.01957
5.75	0.02704	0.02437	0.02185	0.01966	0.01758
6.00	0.02380	0.02165	0.01954	0.01766	0.01593
6.5	0.01879	0.01735	0.01583	0.01443	0.01325
7.0	0.01514	0.01414	0.01303	0.01197	0.01098
7.5	0.01241	0.01170	0.01087	0.01005	0.00919
8.0	0.01032	0.00981	0.00918	0.00854	0.00794
8.5	0.00870	0.00832	0.00783	0.00732	0.00688
9.0	0.00741	0.00713	0.00674	0.00634	0.00589
9.5	0.00638	0.00617	0.00585	0.00552	0.00514
10.0	0.00554	0.00537	0.00512	0.00485	0.00461

[a] See reference 1.

TABLE II. Distribution of the low-frequency component, $H(\beta)$, at an ion for several values of r_0/λ. This distribution should be used in line broadening calculations involving a singly-charged ion as the radiator.

β	$r_0/\lambda=0.2$	$r_0/\lambda=0.4$	$r_0/\lambda=0.6$	$r_0/\lambda=0.8$
0.1	0.00755	0.01443	0.02697	0.04699
0.2	0.02948	0.05538	0.10087	0.16949
0.3	0.06382	0.11648	0.20368	0.32423
0.4	0.10754	0.18885	0.31327	0.46752
0.5	0.15700	0.26315	0.41060	0.57207
0.6	0.20838	0.33126	0.48377	0.62953
0.7	0.25810	0.38743	0.52861	0.64475
0.8	0.30313	0.42862	0.54684	0.62840
0.9	0.34124	0.45415	0.54339	0.59177
1.0	0.37105	0.46517	0.52425	0.54433
1.1	0.39198	0.46386	0.49502	0.49291
1.2	0.40417	0.45286	0.46025	0.44185
1.3	0.40828	0.43481	0.42329	0.39373
1.4	0.40533	0.41206	0.38641	0.34981
1.5	0.39652	0.38657	0.35105	0.31049
1.6	0.38312	0.35985	0.31799	0.27573
1.8	0.34726	0.30690	0.26005	0.21862
2.0	0.30581	0.25853	0.21291	0.17512
2.2	0.26428	0.21672	0.17521	0.14193
2.4	0.22580	0.18162	0.14518	0.11641
2.6	0.19181	0.15261	0.12124	0.09655
2.8	0.16265	0.12880	0.10205	0.08090
3.0	0.13807	0.10929	0.08657	0.06846
3.25	0.11299	0.08976	0.07122	0.05622
3.50	0.09310	0.07442	0.05923	0.04672
3.75	0.07735	0.06229	0.04975	0.03925
4.00	0.06481	0.05261	0.04218	0.03330
4.25	0.05477	0.04481	0.03606	0.02849
4.50	0.04668	0.03847	0.03107	0.02456
4.75	0.04010	0.03327	0.02696	0.02132
5.00	0.03470	0.02897	0.02355	0.01862
5.25	0.03024	0.02539	0.02069	0.01637
5.50	0.02653	0.02238	0.01828	0.01447
5.75	0.02341	0.01984	0.01623	0.01285
6.00	0.02077	0.01767	0.01448	0.01145
6.5	0.01660	0.01421	0.01167	0.00923
7.0	0.01350	0.01161	0.00955	0.00754
7.5	0.01115	0.00963	0.00792	0.00623
8.0	0.00933	0.00808	0.00664	0.00523
8.5	0.00790	0.00685	0.00563	0.00441
9.0	0.00676	0.00587	0.00482	0.00377
9.5	0.00584	0.00507	0.00415	0.00324
10.0	0.00508	0.00441	0.00361	0.00279

4. RESULTS AND DISCUSSION

Tables I to III contain the calculated values of the three field distributions for several values of r_0/λ up to 0.8. The distribution of the high-frequency component at a neutral point will be found in I. Other values of r_0/λ can easily be obtained by interpolation. The theory is not expected to be good when r_0/λ is larger than unity. In terms of the ion density n, in cm^{-3}, and the temperature T, in degrees Kelvin, r_0/λ is given by

$$r_0/\lambda = 0.0898 n^{1/6} T^{-\frac{1}{2}}. \tag{25}$$

In all cases the peak of the distribution shifts smoothly toward smaller fields as r_0/λ increases.

In Fig. 2 the four different types of distributions, for a common value of r_0/λ, are compared together and with the Holtsmark distribution. One notes that each of the curves for the low-frequency component is appreciably displaced toward the left compared to the corresponding curve for the high-frequency component. This displacement is due to the use of the shielded field for the low-frequency component and emphasizes the importance of the ion-electron correlations. One can also compare a distribution at a neutral point with the corresponding distribution at a charged point. In both cases, the latter distribution has its maximum raised and its tail lowered compared to the neutral distribu-

TABLE III. Distribution of the high-frequency component, $H(\beta)$, at an electron for several values of r_0/λ. For the distribution of the high-frequency component at a neutral point.[a]

β	$r_0/\lambda=0.2$	$r_0/\lambda=0.4$	$r_0/\lambda=0.6$	$r_0/\lambda=0.8$
0.1	0.00569	0.00787	0.01089	0.01469
0.2	0.02235	0.03076	0.04229	0.05667
0.3	0.04881	0.06658	0.09062	0.12019
0.4	0.08325	0.11219	0.15056	0.19693
0.5	0.12338	0.16377	0.21600	0.27766
0.6	0.16667	0.21731	0.28085	0.35377
0.7	0.21055	0.26903	0.33985	0.41848
0.8	0.25266	0.31576	0.38912	0.46749
0.9	0.29098	0.35511	0.42632	0.49908
1.0	0.32394	0.38563	0.45063	0.51365
1.1	0.35049	0.40672	0.46247	0.51318
1.2	0.37009	0.41854	0.46319	0.50051
1.3	0.38266	0.42181	0.45456	0.47873
1.4	0.38853	0.41764	0.43873	0.45085
1.5	0.38827	0.40736	0.41769	0.41941
1.6	0.38271	0.39232	0.39325	0.38647
1.8	0.35927	0.35309	0.34000	0.32179
2.0	0.32540	0.30859	0.28755	0.26396
2.2	0.28718	0.26462	0.24028	0.21534
2.4	0.24897	0.22439	0.19978	0.17576
2.6	0.21341	0.18926	0.16607	0.14406
2.8	0.18178	0.15942	0.13844	0.11881
3.0	0.15447	0.13451	0.11596	0.09871
3.25	0.12616	0.10934	0.09370	0.07916
3.50	0.10350	0.08957	0.07649	0.06425
3.75	0.08549	0.07403	0.06308	0.05275
4.00	0.07119	0.06175	0.05256	0.04379
4.25	0.05979	0.05198	0.04421	0.03671
4.50	0.05065	0.04414	0.03752	0.03107
4.75	0.04326	0.03780	0.03211	0.02652
5.00	0.03725	0.03261	0.02769	0.02281
5.25	0.03231	0.02834	0.02405	0.01976
5.50	0.02822	0.02480	0.02103	0.01723
5.75	0.02480	0.02183	0.01850	0.01511
6.00	0.02193	0.01932	0.01636	0.01333
6.5	0.01743	0.01538	0.01299	0.01052
7.0	0.01411	0.01246	0.01050	0.00846
7.5	0.01161	0.01025	0.00861	0.00690
8.0	0.00968	0.00855	0.00716	0.00570
8.5	0.00818	0.00721	0.00602	0.00477
9.0	0.00698	0.00615	0.00511	0.00402
9.5	0.00601	0.00529	0.00438	0.00343
10.0	0.00523	0.00459	0.00379	0.00295

[a] See reference 1.

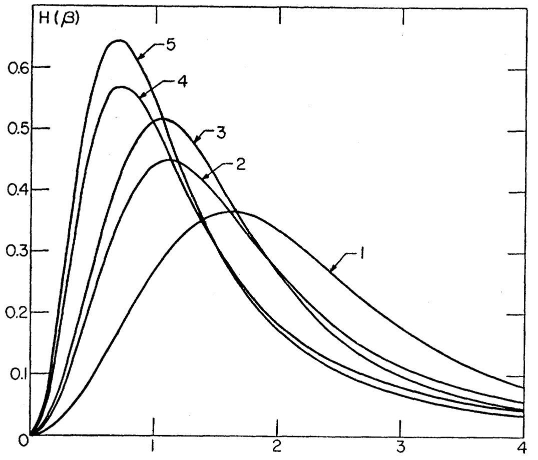

FIG. 2. A comparison of the field distributions for $r_0/\lambda=0.8$. Curve 1 is the Holtsmark distribution, curve 2—the high-frequency component at a neutral point, curve 3—the high-frequency component at an electron, curve 4—the low-frequency component at a neutral point, and curve 5—the low-frequency component at an ion.

tion, but the two maxima occur for practically the same β. In other words, most of the effect of the repelling charge at the origin consists in cutting down the probability of large fields; then the whole curve has to be raised in order to preserve the normalization. That the presence of the charge at the origin affects mostly large fields has been mentioned previously by Edmonds[4] and Ecker and Müller.[5]

For very large values of β, all the neutral distributions should tend toward the Holtsmark distribution, which itself tends toward $1.496\beta^{-\frac{5}{2}}$. When there is a repelling charge at the origin, on the other hand, Lewis and Margenau[6] found that the leading term for large β is

$$H(\beta)\approx 1.496\beta^{-\frac{5}{2}}\exp[-0.334(r_0/\lambda)^2\beta^{\frac{1}{2}}], \tag{26}$$

in other words that, in spite of the repulsion, large fields are still preponderantly due to a single particle getting very close. As in the neutral case, unfortunately, expression (26) becomes good only for very large β, and for $\beta=10$ it is still too small by about 15%. Lewis and Margenau[6] also gave the second term in the asymptotic expansion, but it turns out that higher terms have much larger coefficients than it, and this term becomes significant only for extremely large values of β (of order 10 000). An accurate expression for moderately large values of β is therefore not available.

It is fairly easy, however, to calculate the average β^2 for the distributions at a charged point (at a neutral point, it is divergent). This is simply related to the behavior of $F(\mathbf{k})$ for small $\mathbf{k}$. It turns out to be almost the same for the high-frequency and the low-frequency components. The explanation of this phenomenon resides in the fact that most of the contribution to the average comes from quite large values of β, and that for large β the distributions of the two components of the field actually cross each other, the probability of the low-frequency component becoming larger than the other because its pair corrections are smaller. The root mean square β was found to be well represented by the formula

$$\beta_{\text{rms}}=2.99\lambda/r_0. \tag{27}$$

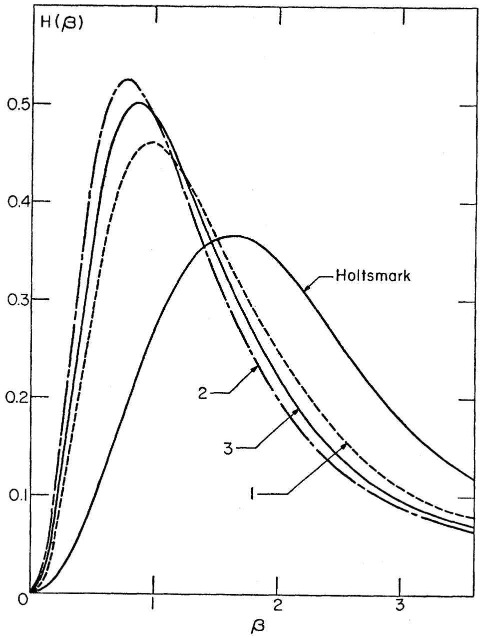

FIG. 3. A comparison of the distribution of the low-frequency component at a neutral point with the work of Ecker and Müller for $r_0/\lambda=0.6$. Curve 1 is the distribution neglecting pair corrections ($\psi'=0$), curve 2—Ecker and Müller's result ($\lambda\rightarrow\lambda'$), and curve 3—this paper.

The distribution of the low-frequency component at a neutral point will now be compared with the work of Ecker and Müller.[5] These authors do not make a separate calculation of the effects of ion-ion correlations. Instead, they argue that those can be taken into account by using an electric field shielded by both electrons and ions, i.e., by using λ' in (7) instead of λ. It was seen in the introduction to the present paper that this cannot be correct because the instantaneous field is desired, not a time average. Indeed, it would seem that the method of Ecker and Müller overestimates the correction due to ion-ion correlations by a factor 2 since the correlation of a given ion pair is taken into account twice: once when they use the λ'-shielded field for one of the ions, and once when they use it for the other. Figure 3 shows this to be the case: curve 1 is obtained by using the field (7), but no pair correlations, i.e., by omitting the function ψ' in (22). Curve 2 is obtained by substituting λ' for λ in (7), again omitting pair corre-

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lations; this is Ecker and Müller's result. Curve 3 is the result of the present theory, which is seen to lie almost halfway in between the other two. Indeed, were one to use

$$\lambda'' = (\kappa T/6\pi n e^2)^{\frac{1}{2}} \tag{28}$$

as a shielding length, and no pair correlations, one would obtain a very accurate result. It must be pointed out that the error in Ecker and Müller's distributions is actually small; all three curves in Fig. 3 are much closer together than they are to the Holtsmark distribution. Work similar to that of Ecker and Müller has also been done by Hoffman and Theimer,[7] but their numerical accuracy is inferior to that of the former authors.

Work on field distributions at a charged point has also been done by Broyles[8] and Lewis and Margenau.[6] The latter authors used an unshielded field, hence their work should be compared with results on the high-frequency component. This is the case where the pair term was found here to be quite important. This pair term is not included by Lewis and Margenau, so that their results differ appreciably from those of the present work, their distribution being shifted toward larger fields. The work of Broyles is hard to compare with others as it does not yield the Holtsmark distribution in the limit of small r_0/λ; this work would be useful for large values of r_0/λ, when the cluster expansion fails completely.

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Nuclear Quadrupole Spin-Lattice Relaxation in Alkali Halides*

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Nuclear quadrupole spin-lattice relaxation times have been measured in alkali halide crystals by the pulsed magnetic resonance technique. Measurements were made on Na^{23} in NaCl, NaBr, and NaI; Cl^{35} in NaCl and KCl; $Br^{79,81}$ in NaBr, KBr, RbBr, and CsBr; Rb^{87} in RbCl and RbBr; and I^{127} in NaI, KI, and CsI. Over a temperature range of 298°K to 195°K the relaxation times are inversely proportional to the square of the absolute temperature. The data are compared to relaxation times calculated from an ionic crystal model of Van Kranendonk and a covalent model of Yosida and Moriya. The ionic model is modified to include the interaction between the nuclear quadrupole moment and the electric field gradient due to electric dipole moments associated with optical modes of vibration. Neither of these models alone predicts the experimental relaxation times for all cases, but a combination of the two effects is required. The modified ionic model applies reasonably well to crystals which contain the lighter ions.

I. INTRODUCTION

TWO theories have been proposed to explain nuclear quadrupole spin-lattice relaxation times T_1 in crystalline solids. The relaxation due to fluctuations of the electric field gradient originating from ionic point charges is considered in the theory of Van Kranendonk,[1] referred to as the ionic model. The theory of Yosida and Moriya,[2] which applies a covalent model, attributes relaxation to the asymmetry of the electron charge cloud distribution when two ions are in a state of covalent bonding. These theories were applied to the alkali halides and our interest will be confined to these crystals. A recent attempt[3] to interpret relaxation-time data in terms of these models was inconclusive because of the lack of sufficient data. With the enlarged data presented in this paper, we attempt to confirm, in various cases, the proper combination of mechanisms that couple the nuclear quadrupole moment to the lattice-phonon distribution in the temperature region above the Debye temperature.

An important modification of the ionic model, calculated by one of the authors (W.E.B.), introduces the effect of induced electric dipole moments associated with optical modes of the lattice vibration. This

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