Hence of the two channels $|1\rangle$ and $|2\rangle$, one will have the phase $\delta_1+\delta_2$, and the other phase will be $\pi/2$.

The application of this to the pion nucleon problem comes when we consider channel $|1\rangle$ to be the two-body, pion-plus-nucleon channel with the appropriate total quantum numbers, while $|2'\rangle$ as shown above can represent the inelastic scattering. The third channel represents the photon-plus-nucleon channel. If we have a resonance of the sort discussed in the last section, the elastic phase must increase through $\pi/2$ on passing through the resonance region, while the parameter λ starts at 1, decreases to 0, and then increases to 1 again. Hence the photoproduction phase starts off together with the scattering phase shift and finally rejoins it, but there is an intermediate region in which the phase is given by one of the two forms given above. The choice of $\delta_1+\delta_2$ seems to agree somewhat better with the experiments unless the phase shift δ_2 is small. The model as outlined in the previous section would predict a phase shift δ_2 of $\pi/2$ at all energies through the second resonance, and a phase shift increasing through π for the third. Perhaps further experiments on the scattering and photoproduction will enable a comparison to be made.

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Quantum Limitations on the Measurement of Gravitational Fields*

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By means of the analogy that exists between the gravitational field, in the weak, quasi-static case, and the electromagnetic field, uncertainty relations are obtained for the average values of some of the Christoffel symbols measured in two domains, similar to those for the components of the quantized electromagnetic field. Furthermore, it is shown that there exists a limitation on the accuracy to which the average value of a single one of these Christoffel symbols can be measured. The existence of uncertainty relations provides an argument in support of the standpoint that the gravitational field must be quantized.

THERE has recently been some controversy about the necessity of quantizing the gravitational field.[1] It is therefore of interest to show that it must be subject to some uncertainty relations. This can be done on the basis of an analogy between the gravitational and the electromagnetic fields.

In the case of gravitation, the motion of a test particle is described by the equation of the geodesic[2]:

$$du^k/ds=-\Gamma^k{}_{00}(u^0)^2-2\Gamma^k{}_{0n}u^0u^n-\Gamma^k{}_{mn}u^mu^n.$$

For slow motion, $u^0\approx 1$, $u^k\approx 0$, so that the last term can be neglected. This equation then has a form analogous to the equation of motion of a particle of charge e and mass m acted upon by the Lorentz force in a given electromagnetic field

$$du^k/ds=-(e/m)(F^k{}_0u^0+F^k{}_nu^n).$$

Here $F^k{}_0=-E_k$ and $F^k{}_n=-B_m$, where $\mathbf{E}$ and $\mathbf{B}$ are the electric and magnetic field vectors and (k, n, m) is a cyclic permutation of (1, 2, 3).

We see then, that there is a correspondence between the Christoffel symbols and the electromagnetic field components given by

$$F^k{}_0\rightarrow\Gamma^k{}_{00},\quad F^k{}_n\rightarrow 2\Gamma^k{}_{0n},$$

provided we also let $e\rightarrow m$.

In what follows, it will be assumed that the gravitational field is weak and quasi-static, and that we are using quasi-Galilean coordinates. The analogy can then be pursued further, because $-\Gamma^k{}_{00}$ and $-2\Gamma^k{}_{0n}$ ($\approx 2\Gamma^n{}_{0k}$) are produced by masses (multiplied by G,

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[1] P. G. Bergmann, *Summary of the Colloque International de Royaumont* (to be published).
[2] u^μ is the velocity four-vector of the particle. The velocity of light is taken as unity. Latin indices run from 1 to 3.

the gravitational constant) in the same way as **E** and **B** are produced by charges.[3]

Now, Bohr and Rosenfeld[4] have made a detailed analysis of the measurement of the electromagnetic field. On the basis of the above correspondence it is possible to carry over many of their results to the case of the gravitational field.

For instance, let us assume that the measurements of the average values of the fields are carried out in two space regions I_1 and I_2, each having dimensions of the order of L, during the corresponding time intervals T_1 and T_2, each of the order of T, and let the mean distance between I_1 and I_2 be of the order of r. If we further assume that light signals emitted during the measurement performed in I_1 during T_1 will reach a substantial part of I_2 during T_2, then one gets, on the basis of the work of Bohr and Rosenfeld, a number of uncertainty relations, e.g.,[5]

$$(\Delta\Gamma^k{}_{00})_1(\Delta\Gamma^k{}_{0n})_2 \gtrsim Gh/r^2LT, \quad (T \ll L),$$
$$\gtrsim Gh/r^2T^2, \quad (T \gg L),$$

where there is no summation on the index k.

On the other hand, in some cases modifications or new limitations are necessary. As an example, let us consider the result obtained by Bohr and Rosenfeld, that one can measure the average value of the electromagnetic field, in a given region of space and during a given interval of time, to any desired accuracy. The procedure which they considered involved essentially the use of a body of large charge and mass uniformly distributed, occupying the region in question, and the determination of the momentum acquired by it during the given time interval. The corresponding result in the gravitational case would then be: by means of suitable measurements involving a test body of large mass one can determine the average values of the Christoffel symbols $\Gamma^k{}_{00}$ ("electric-type" field) and $\Gamma^k{}_{0n}$ ("magnetic-type" field), in a given space region and time interval, to any desired accuracy.

While this is probably adequate for most cases, it should be pointed out that, in principle, there exists nevertheless a limitation in the gravitational case. This arises from the fact that, according to the interior Schwarzchild solution of the Einstein gravitational field equations, a particle of mass m has a minimum radius of the order of magnitude Gm. Thus one can write for the case of an "electric-type" gravitational field, described by $\Gamma^k{}_{00}$,

$$\Delta\Gamma^k{}_{00} = \Delta p^k/mT,$$

where $\Delta\Gamma^k{}_{00}$ is the error in the measurement of the field component, m is the particle mass, T the time interval of the measurement, and Δp^k the uncertainty in the measurement of the momentum component. The latter is of the order of $h/\Delta x^k$, so that

$$\Delta\Gamma^k{}_{00} \gtrsim h/mT\Delta x^k.$$

Now if the region being measured has dimensions of the order of L, then we must have

$$\Delta x^k \lesssim L, \quad Gm \lesssim L,$$

so that we get

$$\Delta\Gamma^k{}_{00} \gtrsim Gh/L^2T.$$

For the case of a "magnetic-type" gravitational field, described by $\Gamma^k{}_{0n}$ $(n \neq k)$, one readily obtains relations of the form

$$u^n\Delta\Gamma^k{}_{0n} \gtrsim h/mT\Delta x^k.$$

Proceeding as before and taking into account further that, in order for the measuring body to remain in the given space region, we must have

$$u^n \lesssim L/T,$$

we see that

$$\Delta\Gamma^k{}_{0n} \gtrsim Gh/L^3.$$

The existence of these quantum uncertainties in the gravitational field is a strong argument for the necessity of quantizing it. It is very likely that a quantum theory of gravitation would then generalize these uncertainty relations to all other Christoffel symbols.

[3] A. Einstein, *The Meaing of Relativity* (Princeton University Press, Princeton, 1953), p. 102.
[4] N. Bohr and L. Rosenfeld, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **12**, 8 (1933).
[5] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954), p. 80.