Theory of the Photodisintegration of the Deuteron*

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By means of a covariant field-theoretic technique, a formally exact expression has been derived for the amplitude for photodisintegration of the deuteron. By expanding the result only in the number of mesons exchanged and by making a series of nonrelativistic approximations, the expression is reduced to one in which the corrections to the conventional dipole matrix element depend only on the amplitude for photomeson production, the renormalized meson-nucleon coupling constant, and the appropriate two nucleon wave functions. One finds that virtual meson effects play little role at energies below 100 Mev, in justification of recent calculations based on the conventional nonrelativistic theory. At higher energies good agreement with the total cross section was obtained by the inclusion of both hard core and tensor force effects in the wave functions. In addition the folded angular distribution could be fitted by using a reasonable extrapolation of the phase shifts in the $^1S_0$ and $^1D_2$ states.

I. INTRODUCTION

BOTH the existing experimental information and previous theoretical considerations suggest that it is convenient to divide the analysis of the photodisintegration of the deuteron into parts corresponding to two energy regions. In one, the region below 100 Mev, the experimental results can be completely understood within the framework of conventional quantum mechanics.[1] In the other region, however, above 100 Mev, there is clear evidence of the presence of virtual meson currents which become predominant with increasing photon energy. It is our purpose to justify the present method of explaining the lower energy phenomena and to exhibit a satisfactory theory for the higher energies.

For the former consideration it is necessary to show that any virtual mesonic contributions are negligible in this region. Moreover, we show that such contributions to the charge operator, responsible for electric multiple disintegration, constitute a relativistic correction to the standard operator which vanishes identically in the limit of zero photon energy. This last condition is commonly known as Siegert's theorem.[2]

To date, the only relatively complete and moderately successful account of the latter higher energy range has been given by Zachariasen.[3] However, his theory was limited by the fact that the treatment was a noncovariant one, and thus one may question whether the renormalization effects, which must be included, were correctly incorporated. In addition since this calculation was performed, advances which have been made in our theoretical understanding of the pion-nucleon interaction permit the inclusion of such effects in treating two nucleon problems in a more complete and systematic way.

We have therefore considered it worthwhile to renew the attack on this problem by means of a completely covariant formalism, designed not only to include the possibility of renormalization in a straightforward manner, but also to provide a means of approximation in which the appropriate physical effects are confined to the lowest orders.

In our final results all references to the abstract operators and state vectors of the theory will be eliminated in favor of wavefunction like amplitudes and operators in configuration space. In this form the details of the two nucleon system will be incorporated only in the appropriate wavefunctions; whereas it is the operators which will contain reference to the familiar meson-photon-single nucleon interactions. The detailed analysis of the matrix element will indicate first that there is a strong need for a hard core in the two nucleon potential, and second, to the extent that one admits the hard core *ab initio*, we arrive at a semi-empirical justification for our method of extrapolating pion-nucleon interaction operators off the energy shell, the method employed being that which is exact in the fixed source limit.[4]

To proceed we exhibit a formally exact expression for the S matrix for photodisintegration.[5] However, owing to our ignorance of the correct relativistic dynamics for strong interactions it will be necessary (in order to carry throug han explicit evaluation) to resort to a phenomenological procedure, which relates this expression to parameters available from more fundamental phenomena (such as meson-nucleon scattering, photomeson production, and nucleon-nucleon interactions). Our object, ultimately, is to obtain a version of the theory in which such information can be incorporated. To effect this conveniently will require certain

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[1] See for example: J. Iwadare, S. Osuki, M. Sano, S. Taketani, and M. Watasi, Progr. Theoret. Phys. (Kyoto) **16**, 455, 472, 604, 585 (1956); S. Hsieh, Progr. Theoret. Phys. (Kyoto) **18**, 637 (1957); J. J. DeSwart and R. E. Marshak, Phys. Rev. **111**, 272 (1958); A. F. Nicholson and G. E. Brown, Bull. Am. Phys. Soc. **3**, 172 (1958); S. H. Hsieh, Progr. Theoret. Phys. (Kyoto) **21**, 585 (1959).
[2] A. J. F. Siegert, Phys. Rev. **49**, 904 (1936).
[3] F. Zachariasen, Phys. Rev. **101**, 371 (1956).
[4] This is the method derived by C. F. Chew and F. E. Low, Phys. Rev. **101**, 1570, 1579 (1956).
[5] We use a formalism due to one of the authors A. Klein (A.K.) and C. Zemach, Phys. Rev. **108**, 126 (1958).

approximations: We neglect the meson-meson interaction; we assume that nucleons interact only via p-wave mesons; and we invoke the adiabatic limit.

The first assumption suggests a description of two nucleon interactions in terms of the numbers of mesons exchanged, since it allows a natural separation of the nucleon self field quanta from the exchange quanta. Consequently, we can develop a perturbation treatment in which successive orders represent larger numbers of mesons exchanged. In this work we limit ourselves to expression arising from the exchange of a single meson at most.

The above considerations will eventually lead to a representation of the S matrix of the form

$$S=S_0+S_1, \tag{1}$$

where S_0 will be the matrix element of the standard theory, and S_1 will arise from the one meson exchange effect. The latter will be characterized by f^2, the renormalized p-wave meson nucleon coupling constant and by the amplitude for photo-meson production off the energy shell.

In Sec. II we develop the formal solution for the S matrix. In order to arrive at the final working form of this matrix element, which will finally be derived and exhibited in Sec. IV, it is first necessary to relate the relativistic amplitudes (RA) which describe the respective two nucleon systems defined in Sec. II to the appropriate Schrödinger wavefunctions. To accomplish this we must here also develop a perturbation treatment in which the RA consists of a sum of terms, each characterized by the numbers of mesons exchanged. The effects of such a treatment are to remove, in the final results, all diagrams which (from a temporal point of view) contain no virtual mesons in intermediate states excepting, of course, the lowest order, standard matrix element.

Finally in Sec. V we report the results of the numerical analysis and describe and estimate the effects of the various approximations made throughout the analysis.

II. FORMAL EXPRESSION FOR THE S-MATRIX

In this section we exhibit the formal solution for the S matrix for the photodisintegration of a deuteron (4-momentum P') by an impinging photon (4-momentum k and polarization λ) resulting in two nucleons (4-momentum p_1 and p_2), which is defined by

$$S_{\alpha\beta}=\langle p_1p_2^{(-)}|P'k(\lambda)^{(+)}\rangle. \tag{2}$$

The superscripts distinguish in the usual manner the ingoing wave-state $(-)$ from the outgoing wave-state $(+)$, each being a member of a complete set of states. We have suppressed the polarization degrees of freedom necessary to specify the particle aspects of the initial and final states.

The starting point for developing a useful expression for the definition (2) is a formalism for bound state problems proposed by Klein and Zemach,[5] in which all quantities of interest are developed with the aid of the renormalized many body Green's functions.[6,7] For our case it will be sufficient to consider the two-nucleon Green's function[8]

$$\begin{aligned}G(12)&=G(x_1x_2,x_1'{}_2x')\\&=i^2\langle 0|T(\psi(x_1)\psi(x_2)\bar{\psi}(x_2')\bar{\psi}(x_1'))|0\rangle\end{aligned} \tag{3}$$

and the two nucleon single photon Green's function,

$$\begin{aligned}G_\lambda(12;\xi)&=G_\lambda(x_1x_2,x_1'x_2';\xi)\\&=i^2\langle 0|T(\psi(x_1)\psi(x_2)\bar{\psi}(x_2')\bar{\psi}(x_1')A_\lambda(\xi))|0\rangle,\end{aligned} \tag{4}$$

where T is the chronological ordering symbol of Wick[9] and $\psi(x)[A(\xi)]$ are the renormalized Heisenberg operator for the nucleon [photon] field with

$$\bar{\psi}(x)=\psi(x)^\dagger\gamma_0. \tag{5}$$

The two-nucleon Green's function satisfies a differentio-integral equation of the symbolic form[10]

$$(G_1^{-1}G_2^{-1}-I(12))G(12)=1(2), \tag{6}$$

or

$$G(12)^{-1}G(12)=1(2), \tag{7}$$

where G_i is the free nucleon Green's function, $I(12)$ is the interaction for two nucleons, and $1(2)$ is the antisymmetric δ function, i.e.,

$$\begin{aligned}\langle x_1x_2|1|x_1'x_2'\rangle=\delta(x_1-x_1')\delta(x_2-x_2')\\-\delta(x_1-x_2')\delta(x_2-x_1').\end{aligned} \tag{8}$$

We exhibit a relationship between (3) and (4) by introducing an auxiliary external field characterized by a source function $J_\mu(\xi)$ coupled linearly to the photon field. This coupling is described by an addition to the Lagrange density of the form

$$\mathfrak{L}'(x)=\textstyle\sum_\mu J_\mu(x)A_\mu(x). \tag{9}$$

By means of the quantity

$$\langle Q[J]\rangle=\frac{\langle 0(+\infty)|Q(x)|0(-\infty)\rangle}{\langle 0(+\infty),0(-\infty)\rangle}, \tag{10}$$

where the vacuum states $0(+\infty)$, $0(-\infty)$ are defined if we assume $J_\mu(x)$ to vanish in the remote past and

[6] We shall be dealing throughout with a theory which can be renormalized via a unitary scale transformation. Hence, we assume that such a transformation has been applied.

[7] R. P. Feynman, Phys. Rev. **80**, 440 (1950); J. Schwinger, Proc. Natl. Acad. Sci. U. S. **37**, 452, 455 (1951) and unpublished lectures; E. Freese, Nuovo cimento **11**, 312 (1954); R. T. Matthews and A. Salam, Proc. Roy. Soc. (London) **221**, 128 (1953); F. Coester, Phys. Rev. **95**, 1318 (1954); H. Umezawa and A. Visconti, Nuovo cimento **1**, 1079 (1955); Y. Nambu, Phys. Rev. **100**, 294 (1955); **101**, 459 (1956); J. M. Yauch, Helv. Phys. Acta. **29**, 287 (1956).

[8] We use the notation $\gamma_i^+=-\gamma_i$, $\gamma_0^+=\gamma_0$, $x_\mu=(x_j it)$, $\gamma x=\gamma_\mu x_\mu =\boldsymbol{\gamma}\cdot\mathbf{x}-\gamma_0 t$, $dx=d^4x$, $d\mathbf{x}=d^3x$, $\hbar=c=1$, $\gamma_5^2=-1$, $\{\gamma_\mu,\gamma_\nu\}=-2\delta_{\mu\nu}$.

[9] G. C. Wick, Phys. Rev. **80**, 268 (1950).

[10] Equation (6) has been derived in unpublished lectures by J. Schwinger. See also J. Schwinger and Umezawa and Visconti, reference 7.

future, we obtain by a consideration of the variation of $\langle Q[J]\rangle$ with respect to $J_\mu(\xi)$, the formula[10]

$$\langle 0|T(Q(\xi)A_\mu(\xi))|0\rangle = (\langle A_\mu(\xi)\rangle - i\delta/\delta[J_\mu(\xi)])\langle Q[J]\rangle|_{J=0}. \quad (11)$$

Applied to the two nucleon propagator, Eq. (11) yields

$$G_\lambda(12;\xi) = (\langle A_\lambda(\xi)\rangle - i\delta/\delta[J_\lambda(\xi)])G(12)|_{J=0}. \quad (12)$$

Recalling that

$$D_{\mu\nu}(\xi,\xi') = i\langle 0|T(A_\mu(\xi)A_\nu(\xi'))|0\rangle = [\delta\langle A_\mu(\xi)\rangle/\delta J_\nu(\xi')]|_{J=0}. \quad (13)$$

Equation (12) yields, since $A_\mu(\xi)|_{J=0}$ vanishes,

$$G_\lambda(12;\xi) = -i\int D(\xi,\xi')[\delta G(12)/\delta\langle A_\lambda(\xi')\rangle]|_{J=0}d\xi', \quad (14)$$

where we have chosen the diagonal form of the photon propagator, $D_{\mu\nu}=\delta_{\mu\nu}D$.

To utilize the constructs so far introduced for the evaluation of (2) we introduce the related Relativistic Amplitudes (hereafter referred to as RA)

$$\bar{\chi}_{p_1p_2}{}^{(-)}(x_1x_2) = \langle p_1p_2{}^{(-)}|T(\bar{\psi}(x_2)\bar{\psi}(x_1))|0\rangle. \quad (15)$$

and

$$\chi_{P'k(\lambda)}{}^{(+)}(x_1x_2;\xi) = \chi_{p'}{}^{D}(x_1x_2)A_{k(\lambda)}(\xi) = \langle 0|T(\psi(x_1)\psi(x_2)A_\lambda(\xi))|P'k(\lambda)^{(+)}\rangle, \quad (16)$$

which will serve as relativistic descriptions of the n–p scattering system and deuteron, respectively.

By means of the limiting procedure of Gell-Mann and Low[11] one can demonstrate that the relativistic amplitudes satisfy the symbolic homogeneous equations

$$\bar{\chi}_{p_1p_2}{}^{(-)}(12)G(12)^{-1}=0 \quad (17)$$

and

$$G(12)^{-1}\chi_{P'}{}^{D}(12)=0 \quad (18)$$

or when converted to the integral form they obey

$$\bar{\chi}_{p_1p_2}{}^{(-)}(12) = \bar{\chi}_{p_1p_2}{}^{(0)}(12) + \bar{\chi}_{p_1p_2}{}^{(-)}(12)I(12)G_1G_2, \quad (19)$$

and

$$\chi_{P'}{}^{D}(12) = G_1G_2I(12)\chi_{P'}{}^{D}(12). \quad (20)$$

In Eq. (19) $\bar{\chi}_{p_1p_2}{}^{(0)}(12)$ is the solution of the free equation given by

$$\bar{\chi}_{p_1p_2}{}^{(0)}(12)G_1^{-1}G_2^{-1}=0, \quad (21)$$

and has been added to satisfy the boundary conditions.

Finally it will be necessary to understand what is meant by the T symbol in the limiting instance of equal times, namely

$$\lim_{x_{10}=x_{20}=t} T(\psi(x_1)\psi(x_2)) = \tfrac{1}{2}(\psi(x_1)\psi(x_2)-\psi(x_2)\psi(x_1)) \equiv [\psi(x_1)\psi(x_2)]. \quad (22)$$

For the case $t\rightarrow\pm\infty$ we define the special limits

$$\lim_{t\rightarrow\infty(-\infty)} [\psi(x_1)\psi(x_2)] = [\psi(x_1)\psi(x_2)]^{\text{out(in)}}. \quad (23)$$

The tools developed will now be used to construct the S matrix. This is done by noting that from a knowledge of the Green's function, $G_\lambda(12;\xi)$, the S matrix can be identified as follows: We note first that

$$\lim_{\substack{t\rightarrow\infty\\ t'\rightarrow-\infty\\ \xi_0\rightarrow-\infty}} G_\lambda(12;\xi) = i^2\langle 0|[\psi(x_1)\psi(x_2)]^{\text{out}} \times[\bar{\psi}(x_2')\bar{\psi}(x_1')A_\lambda(\xi)]^{\text{in}}|0\rangle. \quad (24)$$

Introducing the appropriate complete set of states and utilizing Eq. (2) results in

$$\lim_{\substack{t\rightarrow\infty\\ t'\rightarrow-\infty\\ \xi_0\rightarrow-\infty}} G_\lambda(12;\xi) = i^2\textstyle\sum_{\alpha\beta}\langle 0|[\psi(x_1)\psi(x_2)]^{\text{out}}|\alpha^{(-)}\rangle \times S_{\alpha\beta}\langle\beta^{(+)}|[\bar{\psi}(x_2')\bar{\psi}(x_1')A_\lambda(\xi)]^{\text{in}}|{}^0\rangle. \quad (25)$$

Inserting for $G_\lambda(12;\xi)$ the results of Eq. (14) and the identity

$$\frac{\delta G(12)}{\delta\langle A_\lambda(\xi)\rangle} = -G(12)\left(\frac{\delta G(12)^{-1}}{\delta\langle A_\lambda(\xi)\rangle}\right)G(12), \quad (26)$$

we obtain on the other hand for the same limit as in (24)

$$-i^4\int\sum_{\alpha\beta\mu}\langle 0|[\psi(x_1)\psi(x_2)]^{\text{out}}|\alpha^{(-)}\rangle\bar{\chi}_\alpha{}^{(-)}(x_1''x_2'') \times\left(\frac{\delta G(x_1''x_2'',x_1'''x_2''')^{-1}}{\delta\langle A_u(\xi')\rangle}\right)\bigg|_{A=0}\chi_{\beta\mu}{}^{(+)}(x_1'''x_2'''\xi') \times\langle\beta^{(+)}|[\bar{\psi}(x_2')\bar{\psi}(x_1')A_\lambda(\xi)]^{\text{in}}|0\rangle dx_1''\cdots dx_2'''d\xi'$$
$$= i^2\sum_{\alpha\beta}\langle 0|[\psi(x_1)\psi(x_2)]^{\text{out}}|\alpha^{(-)}\rangle \times S_{\alpha\beta}\langle\beta^{(+)}|[\bar{\psi}(x_2')\bar{\psi}(x_1')A_\lambda(\xi)]^{\text{in}}|0\rangle. \quad (27)$$

From Eq. (27), we conclude that[12]

$$S_{\alpha\beta} = \int\textstyle\sum_\lambda\bar{\chi}_\alpha{}^{(-)}(x_1x_2)\left(\dfrac{\delta G(x_1x_2,x_1'x_2')^{-1}}{\delta\langle A_\lambda(\xi)\rangle}\right)\bigg|_{A=0} \times\chi_{\beta\lambda}{}^{(+)}(x_1'x_2'\xi)dx_1\cdots dx_2'd\xi, \quad (28)$$

or for our case

$$S_{\alpha\beta} = \int\textstyle\sum_\lambda\bar{\chi}_{p_1p_2}{}^{(-)}(x_1x_2)(\delta G(12)^{-1}/\delta\langle A_\lambda(\xi)\rangle)|_{A=0} \times\chi_{p'}{}^{D}(x_1'x_2')A_{k(\lambda)}(\xi),dx_1'\cdots dx_2'd\xi. \quad (29)$$

The expression for the S matrix given by (29) can also

[11] M. Gell-Mann and F. E. Low, Phys. Rev. 84, 350 (1951).

[12] For further justification, see A. Klein and C. Zemach, reference 5. Also note Gell-Mann and Low, reference 11.

be written in the form

$$S_{\alpha\beta}=-\int \sum_\lambda \bar{\chi}_{p_1p_2}{}^{(-)}(x_1x_2)\langle x_1x_2|j_\lambda(\xi)|x_1'x_2'\rangle \times\chi_{P'}{}^D(x_1'x_2')A_{k(\lambda)}(\xi),dx_1\cdots dx_2'd\xi, \quad (30)$$

where

$$\langle x_1x_2|j_\lambda(\xi)|x_1'x_2'\rangle=-\frac{\delta G(x_1x_2,x_1'x_2')}{\delta\langle A_\lambda(\xi)\rangle}\bigg|_{A=0}. \quad (31)$$

This form should not be unexpected since the variational derivative of the Hamiltonian with respect to the photon field $(-\delta H/\delta\langle A_\lambda\xi\rangle)|_{A=0}$ is the current density operator in the noncovariant formalism and as seen from Eqs. (17) and (18), the inverse 2-nucleon Green's function, in the covariant theory plays the role analogous to the Hamiltonian.

Recalling the expression for the inverse Green's function given by (6) and (7), we write

$$S_{\alpha\beta}=S^{(1)}+S^{(2)}, \quad (32)$$

where

$$S^{(1)}=e\int \sum_\lambda \bar{\chi}_{p_1p_2}{}^{(-)}(12) \times\langle 12|\Gamma_\lambda{}^{(1)}(\xi)G_2{}^{-1}+\Gamma_\lambda{}^{(-)}(\xi)G_1{}^{-1}|1'2'\rangle \times\chi^D(1'2')A_{k(\lambda)}(\xi)dx_1\cdots dx_2'd\xi, \quad (33)$$

and

$$S^{(2)}=e\int \sum_\lambda \bar{\chi}_{p_1p_2}{}^{(-)}(12)\langle 12|\delta I/\delta_e\langle A_\lambda(\xi)\rangle|_{A=0}|1'2'\rangle \times\chi_{P'}{}^D(1'2')A_{k(\lambda)}(\xi)dx_1\cdots dx_2'd\xi. \quad (34)$$

In Eq. (33) the electromagnetic vertex operator for the ith particle, $\Gamma_\lambda{}^{(i)}(\xi)$ is defined by the equation

$$\Gamma_\lambda{}^{(i)}(\xi)=-\delta G_i{}^{-1}/\delta_e\langle A_\lambda(\xi)\rangle|_{A=0}. \quad (35)$$

The division of $S_{\alpha\beta}$ into $S^{(1)}$ and $S^{(2)}$, not to be confused with the one described in the beginning of this section [see Eq. (1)], is a natural one from a covariant point of view, since it appears to be a clean separation into one and two body effects. From a practical point of view this is an illusion, for in order to evaluate Eq. (32) it will be necessary to carry out a noncovariant expansion of the RA. It will then be seen that $S^{(1)}$ contains in addition to the standard interaction some mesonic contributions; of course $S^{(2)}$ will consist exclusively of meson interaction terms.

The expansion of the RA alluded to above is designed to relate it to the analogous Schrödinger two nucleon wave function. To effect this we use an iteration procedure[13] in which the RA depending on unequal times is related to an equal time amplitude, recognized to be essentially the appropriate Schrödinger wave function. This technique leads to a natural division into two contributions, one the lowest order amplitude (depending on no meson coordinates), and the other, the one meson exchange effects. The details of this procedure will be exhibited in Sec. III. It is after it has been carried out that one has the decomposition referred to in Eq. (1).

[13] This result has been derived by both E. Saltpeter, Phys. Rev. **87**, 328 (1952) and A. Klein, Phys. Rev. **90**, 1101 (1953).

III. REDUCTION OF THE RELATIVISTIC AMPLITUDE

To reduce the RA we will be guided by the fact that we wish to separate the amplitude into two terms

$$\chi=\chi_0+\chi_1, \quad (36)$$

where χ_0 is the lowest order forms and χ_1 contains one meson exchange corrections. We will concern ourselves only with the continuum amplitude, $\bar{\chi}_{p_1p_2}{}^{(-)}(x_1x_2)$ since for the deuteron amplitude, $\chi_{P'}{}^D(x_1x_2)$, the reduction will proceed in an analogous manner. We find it desirable to separate the former into its dependence on the total and relative coordinates

$$x=x_1-x_2,\quad p=\tfrac{1}{2}(p_1-p_2), \qquad R=\tfrac{1}{2}(x_1+x_2),\quad P=p_1+p_2, \quad (37)$$

which yields

$$\bar{\chi}_{p_1p_2}{}^{(-)}(x_1x_2)=(2\pi)^{-\frac{3}{2}}e^{-iPR}\bar{\chi}_p{}^{(-)}(x), \quad (38)$$

wherein we have chosen the standard normalization of one particle in a box of volume $(2\pi)^3$. It is our purpose to relate $\bar{\chi}_p{}^{(-)}(x)$, depending an unequal time amplitude $\bar{\varphi}_p{}^{(-)}(x)$, which in momentum space is equivalent to relating $\bar{\chi}_p{}^{(-)}(q)$ to $\bar{\varphi}_p{}^{(-)}(\mathbf{q})$ where,

$$\bar{\varphi}_p{}^{(-)}(\mathbf{q})=(2\pi)^{-\frac{1}{2}}\int\bar{\chi}_p{}^{(-)}(q)dq_0. \quad (39)$$

Since by definition

$$\bar{\varphi}_p{}^{(-)}(\mathbf{q})=(2\pi)^{-\frac{1}{2}}\int\bar{\chi}_{0p}{}^{(-)}(q)dq_0, \quad (40)$$

we shall require as a condition for the determination of $\bar{\chi}_{1p}{}^{(-)}(q)$ that

$$\bar{\chi}_{1p}{}^{(-)}(q)dq_0=0. \quad (41)$$

The reduction of the relativistic amplitude is accomplished from a consideration of the integral equation (19) which in momentum space reads

$$\bar{\chi}_p{}^{(-)}(q)\delta^4(P-Q) =\int\bar{\chi}_p{}^{(-)}(q')\delta^4(P-Q')\langle q_1'q_2'|I|q_1q_2\rangle dq_1'\cdots dq_2 +\bar{\chi}_p{}^{(0)}(q)\delta^4(P-Q). \quad (42)$$

At this point we break away from the covariant form of the theory by taking only positive energy projections of the RA, denoted by the relation

$$\chi_{++p}^{(-)}(q)^\dagger=\chi_p^{(-)}(q)^\dagger\Lambda_+^{(1)}(q_1)\Lambda_+^{(2)}(q_2), \qquad (43)$$

where $\Lambda_+^{(1)}(q_1)$ is the Casimir positive energy projection operator for the first particle as defined by

$$\Lambda_+^{(1)}(\mathbf{q}_1)=(\boldsymbol{\alpha}^{(1)}\cdot\mathbf{q}_1+\beta_m^{(1)}+E_{q1})/2E_{q1}, \qquad (44)$$

with

$$E_{q1}=(\mathbf{q}_1^2+m^2)^{\frac{1}{2}}. \qquad (45)$$

In lowest order the $(+-)$ and $(-+)$ projection give a vanishing contribution whereas $(--)$ contains corrections negligible for our purposes.[14] These approximations are all consistent with retaining only the p-wave pion-nucleon interaction. In the considerations which follow we suppress the dependence on the projection operator.

Owing to the assumption that we keep only the one meson exchange term, the matrix element of the interaction, I, is given by[15]

$$\langle q_1q_2|I|q_1'q_2'\rangle=-[i/(2\pi)^4]\boldsymbol{\Gamma}_5^{(1)}(q_1,q-q')\cdot\boldsymbol{\Gamma}_5^{(2)}(q_2,q'-q)\delta^4(Q-Q'), \qquad (46)$$

with the meson-nucleon vertex functions, $\boldsymbol{\Gamma}_5(q,p)$, defined by the equation

$$\langle p|[\delta G^{-1}/\delta\langle\boldsymbol{\phi}(q)\rangle]|_{\phi=0}|p'\rangle=\boldsymbol{\Gamma}_5(p,q)\delta^4(p-p'-q), \qquad (47)$$

where $\boldsymbol{\phi}(q)$ is the renormalized meson field operator. The matrix element given by (46) is represented by the Feynman diagram in Fig. 1.

In evaluating (46) and (47), we shall make what is essentially a low-energy approximation, introducing forms for the Green's functions of the theory that have been utilized in the proof of the so-called low-energy theorems.[16] These forms depend on parameters all of which can be taken from experiment, such as nucleon charge, magnetic moment, mass, etc. We thus write[17]

$$\begin{aligned}G^{-1}(p,[\phi],[A])&=\gamma(p-cA)\tfrac{1}{2}(1+\tau_3)+m\\&\quad-(\mu_p'\tfrac{1}{2}(1+\tau_3)+\mu_N'\tfrac{1}{2}(1-\tau_3))\tfrac{1}{2}\sigma_{\mu\nu}F_{\mu\nu}\\&\quad-(f/\mu)\gamma_5\gamma_p\boldsymbol{\tau}\cdot\boldsymbol{\phi}+(ef/\mu)\gamma_5\gamma A\tfrac{1}{2}[\tau_3,\boldsymbol{\tau}\cdot\boldsymbol{\phi}], \qquad (48)\end{aligned}$$

and

$$\Delta^{-1}(q,[A])=(q-eA\mathcal{T}_3)^2+\mu^2. \qquad (49)$$

[14] See A. Klein, reference 13.

[15] See J. Schwinger, reference 7.

[16] See A. Klein, Phys. Rev. **99**, 998 (1955).

[17] This form is a reasonable approximation for threshold reactions. However for the higher energies one must make a more sophisticated approximation to arrive at the proper form of $\delta I/\delta_e\langle A\rangle|_{A=0}$: We must include the scattering corrections insofar as it is these terms which are primarily responsible for the resonance which occurs in both photomeson production and photodisintegration. The occurrence of the analogous terms in the expansion of the RA is expected to be of much less importance since here it can be shown that they do not retain their resonant behavior. For further details note: A. Klein and B. C. McCormick, Phys. Rev. **104**, 1747 (1950), and H. Miyazawa, Phys. Rev. **104**, 1741 (1956).

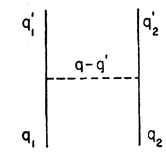

FIG. 1. Feynman diagram for the two nucleon interactions considered.

In Eq. (48) $F_{\mu\nu}$ is the electromagnetic field tensor, $\mu_p'(\mu_N')$ is the proton (neutron) anomalous moment, and in (49), $\mathcal{T}_3$ is the matrix

$$\mathcal{T}_3=\begin{pmatrix}0&i&0\\-i&0&0\\0&0&0\end{pmatrix}, \qquad (50)$$

defined by the equation

$$\boldsymbol{\tau}^{(1)}\mathcal{T}_3\boldsymbol{\phi}=\tfrac{1}{2}[\tau_3^{(1)},\boldsymbol{\tau}\cdot\boldsymbol{\phi}]. \qquad (51)$$

From the approximate forms (48) and (49) we obtain

$$\boldsymbol{\Gamma}_5^{(i)}(p,q)=(f/\mu)(\gamma_5\gamma)^{(i)}q\boldsymbol{\tau}^{(i)}, \qquad (52)$$

$$\Delta(q)=(q^2+\mu^2)^{-1}, \qquad (53)$$

and

$$G_i(q)=(\gamma^{(i)}q+m)^{-1}. \qquad (54)$$

Accordingly

$$\langle q_1q_2|I|q_1'q_2'\rangle=[i/(2\pi)^4](\gamma_5\gamma)^{(2)}(q-q')(\gamma_5\gamma)^{(2)}\times(q-q')\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\Delta(q-q')\delta^4(Q-Q'). \qquad (55)$$

To initiate the iteration procedure we assume first that the interaction is static, i.e.,

$$\begin{aligned}(q_1q_2|I|q_1'q_2'\rangle&=I(q,q';Q)\delta^4(Q-Q')\\&=\mathcal{I}(\mathbf{q},\mathbf{q}';Q)\delta^4(Q-Q'). \qquad (56)\end{aligned}$$

Realizing that[18]

$$\Lambda_+^{(1)}(q_1)G_1(q_1)\gamma_0^{(1)}=-\Lambda_+^{(1)}(q_1)(\tfrac{1}{2}Q_0-E_{q1}-q_0+i\epsilon)^{-1}, \qquad (57)$$

we obtain after integrating over q_0 and q_0' in Eq. (42)

$$\varphi_p^{(-)}(\mathbf{q})^\dagger=\varphi_p^{(0)}(\mathbf{q})^\dagger+\int\bar{\varphi}_p^{(-)}(\mathbf{q}')\mathcal{I}(\mathbf{q}',\mathbf{q};P)\times(-2\pi i)(P_0-E_{q1}-E_{q2})^{-1}d\mathbf{q}'. \qquad (58)$$

wherein we have

$$\mathbf{q}_1=\tfrac{1}{2}\mathbf{Q}+\mathbf{q}=\tfrac{1}{2}\mathbf{P}+\mathbf{q},\quad \mathbf{q}_2=\tfrac{1}{2}\mathbf{Q}-\mathbf{q}=\tfrac{1}{2}\mathbf{P}-\mathbf{q}. \qquad (59)$$

To proceed it is first necessary to note that $\bar{\chi}_p^{(0)}(q)^\dagger$ can be written in the readily verified form

$$\bar{\chi}_p^{(0)}(q)=i(2\pi)^{-\frac{1}{2}}(\mathfrak{F}_1(q)+\mathfrak{F}_2(q))\bar{\varphi}_p^{(0)}(\mathbf{q}), \qquad (60)$$

where

$$\begin{aligned}\mathfrak{F}_1(q)&=(\tfrac{1}{2}P_0-E_{q1}+q_0)^{-1},\\\mathfrak{F}_2(q)&=(\tfrac{1}{2}P_0-E_{\ 2}-q_0)^{-1}.\end{aligned} \qquad (61)$$

[18] In the Feynmann definition we add a small negative imaginary part of the mass.

Inserting (58) and (60) into (42) and again demanding that the interaction be static, we find that

$$\bar{\chi}_p^{(-)}(q)=i(2\pi)^{-\frac{1}{2}}(\mathfrak{F}_1(q)+\mathfrak{F}_2(q))\bar{\varphi}_p^{(-)}(\mathbf{q}). \quad (62)$$

To continue the iteration procedure we insert (62) back into (42), which yields

$$\bar{\chi}_p^{(-)}(q)=\bar{\chi}_p^{(0)}(q)+i\int \bar{\varphi}_p^{(-)}(\mathbf{q}')(\mathfrak{F}_1(q')+\mathfrak{F}_2(q')) \times I(q,q';P)\gamma_0^{(1)}\gamma_0^{(2)}\mathfrak{F}_1(q)\mathfrak{F}_2(q)(2\pi)^{-\frac{1}{2}}d\mathbf{q}'. \quad (63)$$

The right-hand side of Eq. (63) is readily separated into the sum $\bar{\chi}_{0p}^{(-)}(q)+\bar{\chi}_{1p}^{(-)}(q)$. However, before doing so we make one further assumption, namely, that we can neglect the fourth component of the meson nucleon vertex since it is a recoil correction to the spatial part of the vertex operator.[19] Consequently, inserting the resulting interaction, (63) takes the form

$$\bar{\chi}_p^{(-)}(q)=\bar{\chi}_p^{(0)}(q)+(2\pi)^{-\frac{3}{2}}(f^2/\mu^2) \times\int \bar{\varphi}_p^{(-)}(\mathbf{q}')(\mathfrak{F}_1(q')+\mathfrak{F}_2(q')) \times\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}') \times\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}((\mathbf{q}-\mathbf{q}')^2+\mu^2)^{-1} \times\mathfrak{F}_1(q)\mathfrak{F}_2(q)\gamma_0^{(1)}\gamma_0^{(2)}dq'. \quad (64)$$

Integrating the r.h.s. of (64) over q_0' we then obtain

$$\bar{\chi}_p^{(-)}(q)=\bar{\chi}_p^{(0)}(q)+i(2\pi)^{-7/2}(f^2/\mu^2) \times\int \bar{\varphi}_p^{(-)}(\mathbf{q}')\{(\tfrac{1}{2}p_0-E_{\mathbf{q}_1'}-\omega+q_0)^{-1} +(\tfrac{1}{2}p_0-E_{\mathbf{q}_2'}-\omega-q_0)^{-1}\}(2w)^{-1} \times\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}') \times\mathfrak{F}_1(q)\mathfrak{F}_2(q)\gamma_0^{(1)}\gamma_0^{(2)}d\mathbf{q}', \quad (65)$$

where

$$\omega=((\mathbf{q}-\mathbf{q}')^2+\mu^2)^{\frac{1}{2}}. \quad (66)$$

Finally, by some straightforward algebraic manipulation, (65) can be cast into the desired form

$$\bar{\chi}_p^{(-)}(q)=i\bar{\varphi}_p^{(-)}(\mathbf{q})(\mathfrak{F}_1(q)+\mathfrak{F}_2(q))(2\pi)^{-\frac{1}{2}} +i(2\pi)^{-7/2}(f^2/\mu^2)\bar{\varphi}_p^{(-)}(\mathbf{q}') \times\{(P_0-E_{\mathbf{q}_1'}-E_{\mathbf{q}_2}-\omega)^{-1} \times(\tfrac{1}{2}P_0-E_{\mathbf{q}_1'}-\omega+q_0)^{-1}\mathfrak{F}_1(q) +(P_0-E_{\mathbf{q}_2'}-E_{\mathbf{q}_1}-\omega)^{-1} \times(\tfrac{1}{2}P_0-E_{\mathbf{q}_2'}-\omega-q_0)\mathfrak{F}_2(q)\}(2\omega)^{-1} \times\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}') \times\gamma_0^{(1)}\gamma_0^{(2)}d\mathbf{q}'. \quad (67)$$

[19] Essentially this involves ω/m which is a v_N/c correction.

The integral equation which $\bar{\varphi}_p^{(-)}(\mathbf{q})$ satisfies can be easily determined from Eq. (65) and is seen to be

$$\bar{\varphi}_p^{(-)}(\mathbf{q})=\bar{\varphi}_p^{(0)}(\mathbf{q})+\int \bar{\varphi}_p^{(-)}(\mathbf{q}')\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}') \times\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')\gamma_0^{(1)}\gamma_0^{(2)}\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}(2\omega)^{-1} \times\{(P_0-E_{\mathbf{q}_1'}-E_{\mathbf{q}_2}-\omega)^{-1} +(P_0-E_{\mathbf{q}_2'}-E_{\mathbf{q}_1}-\omega)^{-1}\} \times(P_0-E_{\mathbf{q}_1}-E_{\mathbf{q}_2})^{-1}(f^2/\mu^2)(2\pi)^{-3}d\mathbf{q}'. \quad (68)$$

To obtain the analogous results for $\chi^D(q)$ we need only replace P by P' and interchange the order of all operators. To complete the description of the reduction of the RA all that is necessary is to determine the appropriate normalization of the single time amplitude. The need for such a consideration may be understood as follows: It has long been known that the single time amplitude which we have obtained is roughly equivalent to the two nucleon no meson amplitude of a Fock space expansion of the state vector. We must therefore anticipate that the norm associated with this amplitude be less than unity. We determine the appropriate normalization within the framework of our method from a consideration of the matrix element of the nucleon "number" operator[20]

$$\int\bar{\chi}_\alpha(12)\langle 12|\Gamma_0^{(1)}(\xi)G_2^{-1}+\Gamma_0^{(2)}(\xi)G_1^{-1} +\delta I/\delta\langle V_0(\xi)\rangle|_{V=0}|1'2'\rangle\chi_\alpha(1'2')dx_1\cdots dx_2', \quad (69)$$

where $V_\mu(\xi)$ is an external vector field which is coupled to the nucleon "particular current," $:\bar{\psi}(x)\gamma_\mu\psi(x):$.[21] Examining the normalization of a single nucleon amplitude, it can be shown that to a sufficient approximation[22]

$$\Gamma_0=\gamma_0, \quad (70)$$

whereas from a consideration of the linear coupling, $V_\mu(\xi):\bar{\psi}(\xi)\gamma_\mu\psi(\xi):$, we realize that the meson propagator depends on $\langle V_\mu(\xi)\rangle$ in a manner completely analogous to its dependence on $\langle A_\mu(\xi)\rangle$ except that $\mathcal{T}_3$ is replaced by the unit matrix. Consequently, for a single meson exchange, recalling the approximate form, (49), we have

$$\langle q_1q_2|\delta I/\delta\langle V_0(\kappa)\rangle|_{V=0}|q_1'q_2'\rangle =2i(2\pi)^{-4}(f^2/\mu^2)\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'+\tfrac{1}{2}\boldsymbol{\kappa})\boldsymbol{\sigma}^{(2)} \cdot(\mathbf{q}-\mathbf{q}'-\tfrac{1}{2}\boldsymbol{\kappa})(q_0-q_0')\Delta(q-q'+\tfrac{1}{2}\kappa) \times\Delta(q-q'-\tfrac{1}{2}\kappa)\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\delta^{(4)}(Q-Q'-\kappa), \quad (71)$$

[20] We use a method due to A. Klein and C. Zemach, reference 5.
[21] The double dot surrounding the operator indicates the "normal" order.
[22] See A. Klein and C. Zemach, reference 5.

the variational derivative of the vertex[23] giving a small additional relativistic correction which we shall ignore (and wherein we have again limited ourselves to the spatial part of the vertex operator). Incorporating the results of (69)–(71) we find that for the deuteron state

$$2(2\pi)^{-3}\text{ volume}$$
$$=(N_1+N_2)(2\pi)^{-3}\text{ volume}+2(2\pi)^{-4}(2\pi)^{-3}\text{ volume}\int\chi^D(q)\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\times\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')(q_0-q_0')\times[\Delta(q-q')]^2\chi^D(q')dqdq', \quad (72)$$

where

$$N_1=-i\int\chi^D(q)G_1^{-1}(q_1)\gamma_0^{(2)}\chi^D(q)dq. \quad (73)$$

Owing to the fact that the integrand on the r.h.s. of (72) is odd in the variable (q_0-q_0') the corresponding integral vanishes, and we find that

$$N_1+N_2=2. \quad (74)$$

In the present approximation in which

$$\chi=\chi_0+\chi_1 \quad (75)$$

as given by (67) converted to the deuteron state, we are led to the result

$$1=\int\varphi^D(\mathbf{q})^\dagger\varphi^D(\mathbf{q})d\mathbf{q}+(2\pi)^{-3}(f^2/\mu^2)\times\int(2\omega_{(\mathbf{q}-\mathbf{q}')})^{-1}\varphi^D(\mathbf{q}')^\dagger\{(P_0'-E_{\mathbf{q}1}-E_{\mathbf{q}2'}-\omega_{(\mathbf{q}-\mathbf{q}')})^{-1}+(P_0-E_{\mathbf{q}2}-E_{\mathbf{q}1'}-\omega_{(\mathbf{q}-\mathbf{q}')})^{-1}\}\times\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')\times\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\varphi(\mathbf{q}')d\mathbf{q}d\mathbf{q}'. \quad (76)$$

Finally taking the adiabtic limit, i.e., setting

$$P_0'-E\mathbf{q}-E\mathbf{q}'-\omega=-\omega, \quad (77)$$

which means essentially that we neglect the relative kinetic energies $\mathbf{q}^2/m$, $(\mathbf{q}')^2/m$ compared with μ, the meson mass, we arrive at the expression

$$1=\int\varphi^D(\mathbf{q})^\dagger\varphi^D(\mathbf{q})d\mathbf{q}+(2\pi)^{-3}(f^2/\mu^2)\times\int\varphi^D(\mathbf{q})^\dagger(\omega_{(\mathbf{q}-\mathbf{q}')})^{-2}\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')\times\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}'. \quad (78)$$

[23] This involves the matrix element of γ_5 and contributes a relativistic correction to the surviving meson term.

If we take the adiabatic limit of the integral equation which $\varphi^D(\mathbf{q})$ satisfies, as determined from (68) and use the result in the second term on the r.h.s. of Eq. (78), we obtain a crude estimate for the latter,

$$1\cong\int\varphi^D(\mathbf{q})^\dagger\varphi^D(\mathbf{q})d'\mathbf{q}+(\bar{\omega})^{-1}\int\varphi^D(\mathbf{q})(P_0'-E_{\mathbf{q}1}-E_{\mathbf{q}2})\varphi^D(\mathbf{q})d\mathbf{q}, \quad (79)$$

where $\bar{\omega}$ is an average value of $\omega_{(\mathbf{q}-\mathbf{q}')}$. Consequently we see that indeed the normalization correction $\langle\mathbf{q}^2/m\bar{\omega}\rangle$ is of the order of μ/m as the relative kinetic energy, $\mathbf{q}^2/m$, is about 20 Mev and $\bar{\omega}$ is approximately $\mu=140$ Mev. It should be remarked that had we retained the temporal part of the vertex operator it would have contributed a normalization correction of order $(\mu/m)^2$. A detailed calculation of the second term on the r.h.s. of (79), shows it to be even smaller than the order of magnitude value we have attained.[24] We shall not take this correction into account in our numerical results, since it involves effects of an order of magnitude that will be neglected throughout our work.

Performing the calculation for the continuum state we achieve the result analogous to Eq. (79),

$$(2\pi)^{-3}\text{ volume}=\int\varphi_p^{(-)}(\mathbf{q})^\dagger\varphi_p^{(-)}(\mathbf{q})d\mathbf{q}+(\bar{\omega})^{-1}\times\int\varphi_p^{(-)}(\mathbf{q})^\dagger(P_0-E_{\mathbf{q}1}-E_{\mathbf{q}2})\varphi_p^{(-)}(\mathbf{q})d\mathbf{q}, \quad (80)$$

where, in this case the adiabatic limit states that the difference in the kinetic energies of the free and interacting states, $(\mathbf{p}^2/m-\mathbf{q}^2/m)$ is negligible compared to the meson mass. Examining (80) we see that the l.h.s. and the first term on the r.h.s. go as the volume of the system whereas the second term on the r.h.s. is independent of the volume and consequently negligible. This is to be expected since the continuum state is non-normalizable. However, the second term on the r.h.s. arises from the one meson bound state which exists only when the two nucleons are no greater than a pion compton wavelength apart and consequently should be finite and independent of the size of the system.

IV. WORKING FORM OF THE MATRIX ELEMENT; PROOF OF STEGERT'S THEOREM

We are now in a position to exhibit the explicit form of the S matrix. Previously, we have shown that

$$S_{\alpha\beta}=S^{(1)}+S^{(2)}, \quad (32)$$

[24] J. Bernstein and A. Klein, Phys. Rev. **99**, 966 (1955).

with

$$S^{(1)}=(e^2/4\pi k_0)^{\frac{1}{2}}\bar{\chi}_p^{(-)}(x)e^{-iPR}$$
$$\times\langle x_1x_2|G^{(2)-1}\Gamma_\mu^{(1)}(\xi)+G^{(1)-1}\Gamma_\mu^{(2)}(\xi)|x_1'x_2'\rangle$$
$$\times\chi^D(x')e^{iP'R'}\epsilon_\mu e^{ik\xi}(2\pi)^{-4}dx_1\cdots dx_2'd\xi, \quad (33)$$

and

$$S^{(2)}=(e^2/4\pi k_0)^{\frac{1}{2}}\int\bar{\chi}_p^{(-)}(x)e^{-iPR}$$
$$\times\langle x_1x_2|\delta I/\delta_e\langle A_\mu(\xi)\rangle|_{A=0}|x_1'x_2'\rangle\chi^D(x')$$
$$\times e^{iP'R'}\epsilon_\mu e^{ik\xi}(2\pi)^{-4}dx_1\cdots dx_2'd\xi. \quad (34)$$

In re-exhibiting Eqs. (33)–(34) we have changed to the total and relative coordinate dependence of the RA as defined by Eqs. (37) and (38). Also we have assumed the usual normalization for the photon wave function.

$$A_{k(\mu)}(\xi)=(2\pi)^{-\frac{3}{2}}(2k_0)^{-\frac{1}{2}}\epsilon_\mu e^{ik\xi}. \quad (81)$$

To perform the evaluation of (32) we make a transformation from the photon coordinate, to $\xi-R$, the coordinate relative to the center of momentum. The value of this transformation is obvious since it is only meaningful to make a multipole expansion in the relative photon coordinate.

To rewrite (32) we proceed as follows: taking the fourier components of the matrix elements in (33) and (34) we obtain

$$S^{(1)}=(e^2/4\pi k_0)^{\frac{1}{2}}\int e^{-i(P-Q)R}\bar{\chi}_P^{(-)}(q)$$
$$\times\langle q_1q_2|\Gamma_\mu^{(1)}(\kappa)G_2^{-1}+\Gamma_\mu^{(2)}(\kappa)G_1^{-1}|q_1'q_2'\rangle$$
$$\times x^D(q')e^{i(P'-Q')R'}e^{-i\kappa\xi}e^{ik\xi}\epsilon_\mu dq_1\cdots dq_2'$$
$$\times dRdR'd\kappa d\xi(2\pi)^{-12}, \quad (82)$$

and

$$S^{(2)}=(e^2/4\pi k_0)^{\frac{1}{2}}\int e^{-i(P-Q)R}\chi_p^{(-)}(q)$$
$$\times\langle q_1q_2|\delta I/\delta_e\langle A_\mu(\kappa)\rangle|_{A=0}|q_1'q_2'\rangle\chi^D(q')$$
$$\times e^{i(P'-Q')R'}e^{-i(\kappa-k)\xi}\epsilon_\mu(2\pi)^{-12}dq_1\cdots$$
$$\times dq_2'dRdR'd\kappa d\xi, \quad (83)$$

where

$$\chi(q)=(2\pi)^{-2}\int\chi(x)e^{iqx}dx, \quad (84)$$

and (for other than the wave-functions)

$$f(k)=\int f(\xi)e^{ik\xi}d\xi, \quad (85)$$

define the fourier transforms. Noting that

$$\langle q_2|G_2^{-1}|q_2'\rangle=G_2^{-1}(q_2)\delta^4(q_2-q_2'), \quad (86)$$

and

$$\langle q_1|\Gamma_\mu^{(1)}(\kappa)|q_1'\rangle=\langle q_1|-\delta G^{-1}/\delta_e\langle A_\mu(\kappa)\rangle|_{A=0}|q_1'\rangle$$
$$=\Gamma_\mu(q_1,\kappa)\delta^4(q_1-q_1'-\kappa), \quad (87)$$

and defining

$$\langle q_1q_2|\delta I/\delta_e\langle A_\mu(\kappa)\rangle|_{A=0}|q_1'q_2'\rangle$$
$$=\mathbf{I}_\mu(q,q';\kappa)\delta^4(Q-Q'-\kappa), \quad (88)$$

(82) and (83) can be written as

$$S^{(1)}=\alpha k_0^{-\frac{1}{2}}\int\bar{\chi}_p^{(-)}(q)[G_2^{-1}(q_2)\Gamma_\mu^{(1)}(q_1,\kappa)\chi^D(q-\tfrac{1}{2}\kappa)$$
$$+G_1^{(1)-1}(q_1)\Gamma_\mu^{(2)}(q_2,\kappa)\chi^D(q+\tfrac{1}{2}\kappa)]e^{i(k-\kappa)\eta}$$
$$\times\epsilon_\mu dqd\kappa d\eta(2\pi)^{-4}\delta^4(P-P'-k), \quad (89)$$

$$S^{(2)}=\alpha k_0^{-\frac{1}{2}}\int\bar{\chi}_p^{(-)}(q)\mathbf{I}_\mu(q,q';\kappa)\chi^D(q')e^{-i(\kappa-k)\eta}\epsilon_\mu$$
$$\times dqdq'd\kappa d\eta(2\pi)^{-4}\delta^4(P-P'-k), \quad (90)$$

where,

$$\eta=\xi-R, \quad \alpha=(e^2/4\pi)^{\frac{1}{2}}. \quad (91)$$

Recalling the definition of the S matrix as given by (30) we now have

$$S_{\alpha\beta}=-\int j_\mu(\eta)A_{k(\mu)}(\xi)d\eta, \quad (92)$$

with

$$j_\mu(\eta)=e(2\pi)^{-4}\int d\kappa\int\bar{\chi}_p^{(-)}(q)[\{G_2^{-1}(q_2)\Gamma_\mu^{(1)}(q_1,\kappa)$$
$$\times\chi^D(q+\tfrac{1}{2}\kappa)+G_1^{-1}(q_1)\Gamma_\mu^{(2)}(q_2,\kappa)\chi^D(q-\tfrac{1}{2}\kappa)\}dq$$
$$+\mathbf{I}_\mu(q,q';\kappa)\chi^D(q')dqdq']e^{-i\kappa\eta}. \quad (93)$$

Following a method due to Foldy,[25] we can separate the S-matrix element into its contributions from electric and magnetic multipole radiation,

$$S_{\alpha\beta}=[S_{\alpha\beta}(E)+S_{\alpha\beta}(M)]\delta^4(P-P'-k), \quad (94)$$

where

$$S_{\alpha\beta}(E)=-i(k_0/4\pi)^{\frac{1}{2}}\int_0^1 ds\ j_0(\eta)e^{ik\eta s}\eta\cdot\epsilon d\eta, \quad (95)$$

and

$$S_{\alpha\beta}(M)=i(4\pi k_0)^{-\frac{1}{2}}\textstyle\sum_i\int_0^1 ds\int j_i(\eta)$$
$$\times(\boldsymbol{\eta}\times(\mathbf{k}s\times\boldsymbol{\epsilon}))_i e^{ik\eta s}d\eta, \quad (96)$$

with

$$k\eta s=\mathbf{k}\cdot\boldsymbol{\eta}s-k_0\eta_0. \quad (97)$$

We now need only determine the current density, defined by the equation

$$j_\mu(\kappa)=\int\bar{\chi}_p^{(-)}(q)J_\mu(q,q',\kappa)\chi^D(q')dqdq', \quad (98)$$

[25] L. J. Foldy, Phys. Rev. **92**, 178 (1953).

since we have already determined an approximate representation of the RA. For the electromagnetic vertex operator, first note that to our approximation

$$\Gamma_\mu^{(1)}(q_1,\kappa) = \gamma_\mu^{(1)}\tfrac{1}{2}(1+\tau_3^{(1)})+i(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\kappa})/2e[\tfrac{1}{2}\mu_p'(1+\tau_3^{(1)}) +\tfrac{1}{2}\mu_n'(1-\tau_3^{(1)})], \quad (99)$$

in consequence of Eqs. (87) and (48). We also require the form of $\mathbf{I}_\mu(q,q';\kappa)$ defined by Eq. (88). According to (46), we can write

$$\langle q_1q_2|\delta I/\delta_e\langle A_\mu(\kappa)\rangle|_{A=0}|q_1'q_2'\rangle = -i(2\pi)^{-4}\langle q_1q_2|(\delta/\delta_e\langle A_\mu(\kappa)\rangle) \times\int\cdot\boldsymbol{\Gamma}_5^{(1)}(\lambda)\Delta(\lambda,\lambda')\cdot\boldsymbol{\Gamma}_5^{(2)}(-\lambda') \times d\lambda d\lambda'|_{A=0}|q_1'q_2'\rangle. \quad (100)$$

Carrying through the functional differentiation, we obtain, as described in more detail below, the form

$$\begin{aligned}\mathbf{I}_\mu(q,q';\kappa) &= -i(2\pi)^{-4}(f^2/\mu^2)\{[(\gamma_5\gamma_\mu)^{(1)}(\gamma_5\boldsymbol{\gamma})^{(2)} \\ &\times(q-q'-\tfrac{1}{2}\kappa)\Delta(q-q'-\tfrac{1}{2}\kappa)+(\gamma_5\gamma_\mu)^{(2)}(\gamma_5\boldsymbol{\gamma})^{(1)} \\ &\times(q-q'+\tfrac{1}{2}\kappa)\Delta(q-q'+\tfrac{1}{2}\kappa)](\boldsymbol{\tau}^{(1)}\mathcal{T}_3\boldsymbol{\tau}^{(2)}) \\ &-[2(q-q')_\mu(\gamma_5\boldsymbol{\gamma})^{(1)}(q-q'+\tfrac{1}{2}\kappa)(\gamma_5\boldsymbol{\gamma})^{(2)} \\ &\times(q-q'-\tfrac{1}{2}\kappa)\Delta(q-q'+\tfrac{1}{2}\kappa)\Delta(q-q'-\tfrac{1}{2}\kappa)] \\ &\times(\boldsymbol{\tau}^{(1)},\mathcal{T}_3\boldsymbol{\tau}^{(2)})+(\mu/f)\textstyle\sum_j[(\gamma_5\boldsymbol{\gamma})^{(1)} \\ &\times(q-q'+\tfrac{1}{2}\kappa)\tau_j^{(1)}\Delta(q-q'+\tfrac{1}{2}\kappa) \\ &\times\mathbf{V}_\mu^{(2)}(q-q'+\tfrac{1}{2}\kappa,\ j;\ \kappa)+(1\leftrightarrow 2;\ q\leftrightarrow q')]\};\end{aligned} \quad (101)$$

where $\mathbf{V}_\mu^{(i)}(p,j;\kappa)$, the p-wave meson part of the "photon-nucleon-meson" vertex operator, is defined by the relation

$$\left\langle q\left|\frac{\delta\Gamma_5^{(1)}(p)_j}{\delta_e\langle A_\mu(\kappa)\rangle}\right|_{A=0}\left|q'\right\rangle\right. = -\left\langle q\left|\frac{\delta^2G^{-1}}{\delta_e\langle A_\mu(\kappa)\rangle\delta(\phi_j(p))}\right|_{A=\phi=0}\left|q'\right\rangle\right. = [-(f/\mu)(\gamma_5\gamma_\mu)^{(1)}\tfrac{1}{2}[\tau_3^{(1)},\tau_j^{(1)}] +\mathbf{V}_\mu^{(1)}(p,j;\kappa)]\delta^4(q+p-q'-\kappa). \quad (102)$$

The first term here follows from the approximate form of G^{-1} given by Eq. (48) and is recognized to be the Kroll-Ruderman, s-wave contribution,[26] whereas the second term, which is responsible for the resonance in photomeson production is represented by the Feynman diagrams in Fig. 2. Thus the matrix element (102) describes photomeson production (off the energy shell) exclusive of two contributions, namely that of the

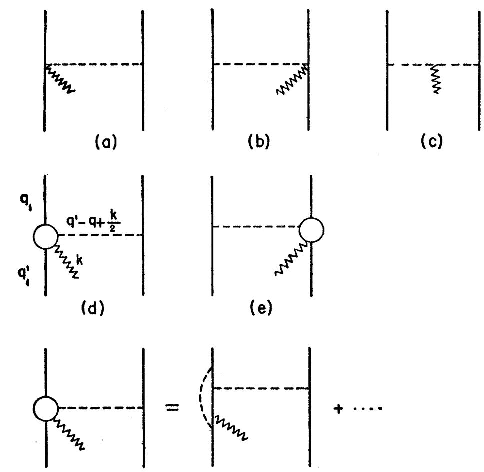

FIG. 2. Feynman diagrams obtained from $\delta I/\delta_e\langle A_\mu\rangle|_{A=0}$. Dashed lines represent mesons and wavy lines represent photons.

nucleon current transitions proceeding via one nucleon intermediate states and that of the meson current. The analog of the latter accounts for the second term of (101). It is characteristic that the dominant effects of the former (the "Born Approximation") do not occur explicitly in the current density, since they are in fact absorbed into the definition of the nuclear wave function. The terms depending on the operator $V_\mu^{(i)}(q,j;\kappa)$ will be seen to be decisive for understanding the high-energy photodisintegration.

In determining the current operator, $j_\mu(\eta)$, as defined by (93) we consider the RA, in the approximate form

$$\chi=\chi_0+\chi_1, \quad (103)$$

defined by Eq. (67). Utilizing Eq. (103) we realize that

$$j_\mu(\kappa)=j_\mu^{(0)}(\kappa)+j_\mu^{BO}(\kappa)+j_\mu^{(1)}(\kappa), \quad (104)$$

where the quantities in (104) are defined by the following equations:

$$\begin{aligned}j_\mu^{(0)}(\kappa) = -e\int\bar{\chi}_{0p}^{(-)}(q)[G_2^{-1}(\tfrac{1}{2}p-q) &\times\Gamma_\mu^{(1)}(\tfrac{1}{2}p+q,\ \kappa)\chi_0^D(q-\tfrac{1}{2}\kappa)+G_2^{-1}(\tfrac{1}{2}p+q) \\ &\times\Gamma_\mu^{(2)}(\tfrac{1}{2}p-q,\ \kappa)\chi_0^D(q+\tfrac{1}{2}\kappa)dq,\end{aligned} \quad (105)$$

$$\begin{aligned}j_\mu^{BO}(\kappa) = -e\int\{\bar{\chi}_{0p}^{(-)}(q)[G_1^{-1}(\tfrac{1}{2}p+q) &\times\Gamma_\mu^{(2)}(\tfrac{1}{2}p-q,\ \kappa)\chi_1^D(q+\tfrac{1}{2}\kappa) \\ &+G_2^{-1}(\tfrac{1}{2}p-q)\Gamma_\mu^{(1)}(\tfrac{1}{2}p+q,\ \kappa)\chi_1^D(q-\tfrac{1}{2}\kappa)] \\ &+\bar{\chi}_{1p}^{(-)}(q)[G_1^{-1}(\tfrac{1}{2}p+q)\Gamma_\mu^{(2)}(\tfrac{1}{2}p-q,\ \kappa) \\ &\times\chi_0^D(q+\tfrac{1}{2}\kappa)+G_2^{-1}(\tfrac{1}{2}p-q) \\ &\times\Gamma_\mu^{(1)}(\tfrac{1}{2}p+q,\ \kappa)\chi_1^D(q-\tfrac{1}{2}\kappa)]\}dq,\end{aligned} \quad (106)$$

$$j_\mu^{(1)}(\kappa) = -e\int\bar{\chi}_{0p}(q)\mathbf{i}_\mu(q,q';\kappa)\chi_0^D(q')dqdq'. \quad (107)$$

[26] N. M. Kroll and M. A. Ruderman, Phys. Rev. 93, 233 (1954).

It will be seen presently that $j_\mu^{(0)}(\boldsymbol{\kappa})$ is the standard contribution to the current, whereas $j_\mu^{BO}(\boldsymbol{\kappa})$ is that part of the current containing the residual aspects of the Born approximation to photomeson production.

We now turn to the evaluation of Eqs. (105)–(107). Inserting the pertinent definitions, namely Eqs. (67), (88), and (99), we obtain for (105) and (106).

$$j_\mu^{(0)}(\kappa)=ie\int \varphi_p^{(-)}(\mathbf{q})^\dagger[R_\mu^{(1)}(q,\kappa)\varphi^D(\mathbf{q}-\tfrac{1}{2}\boldsymbol{\kappa}) +R_\mu^{(2)}(-q,\kappa)\varphi^D(\mathbf{q}+\tfrac{1}{2}\boldsymbol{\kappa})]dq, \quad (108)$$

and

$$j_\mu^{BO}(\kappa)=\frac{ie}{(2\pi)^3}\frac{f^2}{\mu^2}\int \varphi_p^{(-)}(\mathbf{q})^\dagger\Big[\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\boldsymbol{\kappa})}{P_0-E_\mathbf{q}-E_{\mathbf{q}'+\frac{1}{2}\kappa}-\omega_-}\times\frac{\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}}{2\omega_-}\times\frac{R_\mu^{(1)}(q',\kappa)}{P_0'-E_\mathbf{q}-E_{\mathbf{q}'-\frac{1}{2}\kappa}-\omega_-}$$
$$+\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\mathbf{k})\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\boldsymbol{\kappa})}{P_0-E_\mathbf{q}-E_{\mathbf{q}'-\frac{1}{2}\kappa}-\omega_+}\times\frac{\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}}{2\omega_+}\times\frac{R_\mu^{(2)}(-\mathbf{q}',\kappa)}{P_0'-E_\mathbf{q}-E_{\mathbf{q}'+\frac{1}{2}\kappa}-\omega_+}+\frac{R_\mu^{(1)}(q,\kappa)}{P_0-E_\mathbf{q}-E_{\mathbf{q}'+\frac{1}{2}\kappa}-\omega_-}$$
$$\times\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})}{P_0'-E_\mathbf{q}-\kappa-E_{q'}+\frac{1}{2}\kappa-\omega_-}\times\frac{\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}}{2\omega}+\frac{R_\mu^{(2)}(-q,\kappa)}{P_0-E_\mathbf{q}-E_{\mathbf{q}'-\frac{1}{2}\kappa}-\omega_+}\times\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\boldsymbol{\kappa})\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\boldsymbol{\kappa})}{P_0'-E_{\mathbf{q}+\kappa}-E_{\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa}}-\omega_+}$$
$$\times\frac{\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}}{2\omega_+}\times\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}', \quad (109)$$

wherein

$$R_\mu^{(i)}(q,\kappa)=\{(\mathbf{q}/m)\tfrac{1}{2}(1+\tau_3^{(i)})+[i(\boldsymbol{\sigma}^{(i)}\times\boldsymbol{\kappa})/2em](\gamma_p\tfrac{1}{2}(1+\tau_3^{(i)})+\gamma_n\tfrac{1}{2}(1-\tau_3^{(i)});\gamma_0^{(i)}\tfrac{1}{2}(1+\tau_3^{(i)}))\}, \quad (110)$$

$$\omega_\pm=[(\mathbf{q}-\mathbf{q}'\pm\tfrac{1}{2}\kappa)^2+\mu^2]^{\frac{1}{2}}, \quad (111)$$

with $\gamma_p(\gamma_n)$ the gyromagnetic ratio of the proton (neutron) and

$$\mathbf{P}=0,\quad P_0=E_{\mathbf{p}1}+E_{\mathbf{p}2}=2E_\mathbf{p}, \quad (112)$$

the center-of-momentum coordinates for the continuum state. Equation (108) is now recognized to be the standard current operator. Consequently, in the adiabatic limit, namely, for

$$P_0-E_\mathbf{q}-E_{\mathbf{q}'}-\omega=\tfrac{1}{2}k_0-\omega,\quad P_0'-E_\mathbf{q}-E_{\mathbf{q}'}-\omega=-\tfrac{1}{2}k_0-\omega, \quad (113)$$

we obtain for the complete one meson contribution to the current, the addition to the standard result (108),

$$j_\mu^{BO}(\kappa)=\frac{ie}{(2\pi)^3}\frac{f^2}{\mu^2}\int\varphi_p^{(-)}(\mathbf{q})\Bigg\{\Bigg[\Bigg[\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})}{2\omega_-(\omega_-^2-\frac{1}{4}k_0^2)}\times\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}R_\mu^{(1)}(q',\kappa)+\left(1\leftrightarrow2\quad\begin{matrix}\mathbf{q}\leftrightarrow-\mathbf{q}\\ \mathbf{q}'\leftrightarrow-\mathbf{q}'\end{matrix}\right)\Bigg]$$
$$+\Bigg[R_\mu^{(1)}(q,\kappa)\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\frac{\sigma^{(1)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})\sigma^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})}{2\omega_-(\omega_+^2-\frac{1}{4}k_0^2)}+\left(1\leftrightarrow2\quad\begin{matrix}\mathbf{q}\leftrightarrow-\mathbf{q}\\ \mathbf{q}'\leftrightarrow-\mathbf{q}'\end{matrix}\right)\Bigg]\Bigg]\Bigg\}\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}', \quad (114)$$

and

$$j_\mu^{(1)}(\kappa)=\frac{ie}{(2\pi)^3}\frac{f^2}{\mu^2}\int\varphi_p^{(-)}(\mathbf{q})^\dagger\Bigg[-\Bigg(\frac{i(\gamma_5\gamma_\mu)^{(1)}\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})}{\omega_-^2-\frac{1}{4}k_0^2}+\frac{i(\gamma_5\gamma_\mu)^{(2)}\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\boldsymbol{\kappa})}{\omega_+^2-\frac{1}{4}k_0^2}$$
$$-\frac{2(\mathbf{q}-\mathbf{q}';0)\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\boldsymbol{\kappa})\sigma^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})}{(\omega_-^2-\frac{1}{4}k_0^2)(\omega_+^2-\frac{1}{4}k_0^2)}\Bigg)\times(\boldsymbol{\tau}^{(1)},T_3\boldsymbol{\tau}^{(2)})+\frac{i\mu}{f}\sum_j\Bigg(V_\mu^{(1)}(\mathbf{q}-\mathbf{q}'-\tfrac{1}{2}\boldsymbol{\kappa},\tfrac{1}{2}\kappa_0,j;\kappa)$$
$$\times\frac{\tau_j^{(2)}\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\boldsymbol{\kappa})}{\omega_-^2-\frac{1}{4}k_0^2}+(1\leftrightarrow2,\ \mathbf{q}\leftrightarrow\mathbf{q}')\Bigg)\Bigg]\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}', \quad (115)$$

wherein once again we have dropped the temporal part of the meson-nucleon vertex function. Also we note that the fourth component of the meson current, the second term of Eq. (115) is identically zero in the adiabatic limit.

The currents (108), (114), and (115) are now to be inserted into the appropriate electric and magnetic parts of the S matrix given by Eqs. (95) and (96). The standard contributions require no additional discussion at present and will be found recorded below in Eqs. (128) and (129). We then turn to a consideration of the additional electric effects to show that they conform with Siegert's theorem.[2] From (114) and (115) as inserted in (95) we find that

$$S_1^{BO}(E)=-\frac{i\alpha k_0^{\frac{1}{2}}}{(2\pi)^3}\frac{f^2}{\mu^2}\int_0^1 ds\,\boldsymbol{\varepsilon}\cdot\nabla_{\mathbf{k}s}\int\varphi_p^{(-)}(\mathbf{q})^\dagger\bigg[\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\mathbf{k}s)\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\mathbf{k}s)}{2\omega_-(\omega_-^2-\frac{1}{4}k_0^2)}\{\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)},\tfrac{1}{2}(1+\tau_3^{(1)})\}_+ +\begin{pmatrix}1\leftrightarrow 2\\ \mathbf{k}\leftrightarrow-\mathbf{k}\end{pmatrix}\bigg]\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}', \quad (116)$$

$$S_1^{(1)}(E)=\frac{-i\alpha k_0^{\frac{1}{2}}}{(2\pi)^3}\frac{f^2}{\mu^2}\int_0^1 ds\,\boldsymbol{\varepsilon}\cdot\nabla_{\mathbf{k}s}\int\varphi_p^{(-)}(\mathbf{q})^+\bigg\{\bigg[\frac{i\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\mathbf{k}s)}{\omega_+^2-\frac{1}{4}k_0^2}(\gamma_5\gamma_0)^{(2)}-\frac{i\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\mathbf{k}s)}{\omega_-^2-\frac{1}{4}k_0^2}(\gamma_5\gamma_0)^{(2)}\bigg]$$
$$\times(\boldsymbol{\tau}^{(1)},\mathcal{T}_3\boldsymbol{\tau}^{(2)})+i\frac{\mu}{f}\bigg[V_0^{(1)}(\mathbf{q}-\mathbf{q}'-\tfrac{1}{2}\mathbf{k}s,\tfrac{1}{2}k_0,j;\mathbf{k}s,k_0)\frac{\tau_j^{(2)}\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\mathbf{k}s)}{\omega_-^2-\frac{1}{4}k_0^2}+\frac{\tau_j^{(1)}\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'+\frac{1}{2}\mathbf{k}s)}{\omega_+^2-\frac{1}{4}k_0^2}$$
$$\times V_0^{(2)}(\mathbf{q}-\mathbf{q}'+\tfrac{1}{2}\mathbf{k}s,\tfrac{1}{2}k_0,j;\mathbf{k}s,k_0)\bigg]\bigg\}\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}' \quad (117)$$

wherein $\boldsymbol{\kappa}$ has been replaced by $\mathbf{k}s$ and $\{\ ,\ \}$ indicates that the anticommutator be taken.

Examining the first term on the r.h.s. of the latter equation, we note that it is proportional to the matrix element of γ_5 which in one case is equal to $\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}'-\frac{1}{2}\mathbf{k}s)/m$ and consequently, since it is a correction of order ω/m compared to the analogous term arising from (116), we can neglect it. A similar argument allows us to drop the contribution from $V_0^{(i)}$ since it too can be shown to be at best a correction of order ω/m compared with the terms of Eq. (116).[27] We now examine the $E1$ contribution from (116) given by

$$S_1^{BO}(E1)=\frac{-i\alpha(k_0)^{\frac{1}{2}}}{(2\pi)^3}\frac{f^2}{\mu^2}\int\varphi_p^{(-)}(\mathbf{q})^\dagger\bigg\{-\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')(\mathbf{q}-\mathbf{q}')\cdot\boldsymbol{\varepsilon}}{\omega(\omega^2-\frac{1}{4}k_0^2)}$$
$$\times\left(\frac{2}{\omega^2-\frac{1}{4}k_0^2}+\frac{1}{\omega^2}\right)+\frac{\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\varepsilon}\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')+\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\varepsilon}\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')}{2\omega(\omega^2-\frac{1}{4}k_0^2)}\bigg\}\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}', \quad (118)$$

wherein we have removed the isotopic spin dependence. Had we evaluated the electric effect from the original form of the S matrix, Eq. (92), it would have been evident that the mesonic correction given by (118) is a recoil effect of order v/c compared to the former: thus we have a proof of the Siegert theorem.

We now turn to a consideration of the magnetic contributions; however, instead of working with Eq. (96) we find it more convenient to extract the magnetic terms directly from Eq. (92). Since we will be concerned only with an evaluation of the $M1$ effect, which is the predominant one-meson phenomenon, we now limit ourselves to that contribution. With this in mind, inserting Eqs. (114) and (115) into Eq. (92) and retaining only the prescribed terms we arrive at

$$S_1(M1)=S_1^{BO}(M1)+S_1^{(1)}(M1), \quad (119)$$

where

$$S_1^{BO}(M1)=\frac{\alpha}{(2\pi)^3}(k_0)^{-\frac{1}{2}}\frac{f^2}{\mu^2}\int\varphi_p^{(-)}(\mathbf{q})^\dagger\bigg[\bigg\{\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')}{2\omega(\omega^2-\frac{1}{4}k_0^2)},\frac{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})}{2m}$$
$$\times\left(\gamma_p\frac{1+\tau_3^{(1)}}{2}+\gamma_n\frac{1-\tau_3^{(1)}}{2}\right)\bigg]_+ +(1\leftrightarrow 2)\bigg\}\varphi^D(q')d\mathbf{q}d\mathbf{q}', \quad (120)$$

and

$$S_1^{(1)}(M1)=i\frac{\alpha}{(2\pi)^3}(k_0)^{-\frac{1}{2}}\frac{f^2}{\mu^2}\int\varphi_p^{(-)}(\mathbf{q})^\dagger\bigg[\bigg\{\frac{(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})}{2(\omega^2-\frac{1}{4}k_0^2)}+\frac{[(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\times(\mathbf{q}-\mathbf{q}')]\cdot[(\mathbf{q}-\mathbf{q}')\times(\mathbf{k}\times\boldsymbol{\varepsilon})]}{(\omega^2-\frac{1}{4}k_0^2)^2}\bigg\}$$
$$\times(\boldsymbol{\tau}^{(1)},\mathcal{T}_3\boldsymbol{\tau}^{(2)})-\sum_j\frac{i\mu}{f}\frac{1}{\omega^2-\frac{1}{4}k_0^2}\{\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\tau_j^{(1)}\mathbf{V}_j^{(2)}(\mathbf{q}-\mathbf{q}',\tfrac{1}{2}k_0;\mathbf{k}\times\boldsymbol{\varepsilon},k_0)$$
$$+\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')\tau_j^{(2)}\mathbf{V}_j^{(1)}(\mathbf{q}-\mathbf{q}',\tfrac{1}{2}k_0;\mathbf{k}\times\boldsymbol{\varepsilon},k_0)\}\bigg]\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}', \quad (121)$$

[27] A consideration of the leading contributions to this term—see A. Klein, reference 18—shows it to be of order ω/m.

with

$$\omega=((\mathbf{q}-\mathbf{q}')^2+\mu^2)^{\frac{1}{2}}. \tag{122}$$

In Eq. (121) $\mathbf{V}_j^{(i)}(\mathbf{q}-\mathbf{q}', k_0; \mathbf{k}\times\boldsymbol{\varepsilon}, k_0)$ is the amplitude, excluding the Born approximation, for the photoproduction of a meson by a $M1$ photon from the ith nucleon and is defined by the relation

$$\boldsymbol{\varepsilon}\cdot V^{(i)}(\mathbf{q}-\mathbf{q}'-\tfrac{1}{2}\mathbf{k}, \tfrac{1}{2}k_0, j; \mathbf{k}\times\boldsymbol{\varepsilon}, k_0)|_{M1}=\mathbf{V}_j^{(i)}(\mathbf{q}-\mathbf{q}', \tfrac{1}{2}k_0; \mathbf{k}\times\boldsymbol{\varepsilon}, k_0), \tag{123}$$

To determine the form of this function it is convenient to examine a representative diagram, viz., Fig. 2(d). We note first that in the adiabatic limit the kinetic energy of nucleon "1" in the final state is about $\frac{1}{2}k_0$. If the use of a fixed source model to describe virtual photomeson production is to have any validity, we must certainly take some account of this final nucleon energy. To first approximation, this can be done by defining an effective meson energy which is the sum of the actual energy of the meson ($\frac{1}{2}k_0$ in the adiabatic limit) and the kinetic energy of the associated nucleon (also $\frac{1}{2}k_0$).[28,4] All that remains then is to relate the above operator, which is off the energy shell, to its well-known form on the energy shell. Within the framework of the fixed source theory this is precisely the problem that has been solved in the study of photo-meson production by Chew and Low.[4] They have shown that the isotopic vector part of the p-wave amplitude for this process in the $M1$ limit, $T_{\gamma m}{}^V$ is related to the amplitude for p-wave meson-nucleon scattering, T_{mn},[29] by the equation

$$\langle \mathbf{q}j|T_{\gamma m}{}^V(k_0)|\mathbf{k}\times\boldsymbol{\varepsilon}3\rangle=\frac{e\mu}{f}\frac{\gamma_p-\gamma_n}{4m}\langle \mathbf{q}j|T_{mn}(k_0)|k\times\boldsymbol{\varepsilon}3\rangle \tag{124}$$

Owing to the simplicity of the fixed source model the above equation holds also for virtual photomeson production.[29] Hence, remembering that the operator eV_j of Eq. (123) is equal to the full amplitude $T_{\gamma m}{}^V$, given by (124), minus the Born approximation, we arrive at the relation

$$\begin{aligned}V_j^{(1)}(\mathbf{q}-\mathbf{q}', \tfrac{1}{2}k_0; \mathbf{k}\times\boldsymbol{\varepsilon}, k_0)=&[\mu(\gamma_p-\gamma_n)/4mf][\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}\times\boldsymbol{\varepsilon}\,\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\tau}_3{}^{(1)}\tau_j{}^{(1)}A(k_0)\\&+(\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}\times\boldsymbol{\varepsilon}\,\tau_2{}^{(1)}\tau_j{}^{(1)}+\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}\times\boldsymbol{\varepsilon}/\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\tau_j{}^{(1)}\tau_3{}^{(1)}\\&\times B(k_0)+\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}\times\boldsymbol{\varepsilon}\,\tau_j{}^{(1)}\tau_3{}^{(1)}C(k_0),\end{aligned} \tag{125}$$

where

$$\begin{aligned}A(k_0)&=\frac{3}{8\pi}\int_\mu^\infty\frac{d\omega_t}{t}\frac{\sigma_{33}(t)}{\omega_t-k_0}+\frac{1}{24\pi}\int_\mu^\infty\frac{d\omega_t}{t}\frac{\sigma_{33}(t)}{\omega_t+k_0},\\B(k_0)&=\frac{1}{8\pi}\int_\mu^\infty\frac{d\omega_t}{t}\sigma_{33}(t)\left(\frac{1}{\omega_t-k_0}+\frac{1}{\omega_t+k_0}\right),\\C(k_0)&=\frac{1}{24\pi}\int_\mu^\infty\frac{d\omega_t}{t}\frac{\sigma_{33}(t)}{\omega_t-k_0}+\frac{3}{8\pi}\int_\mu^\infty\frac{d\omega_t}{t}\frac{\sigma_{33}(t)}{\omega_t+k_0}\end{aligned} \tag{126}$$

with $\sigma_{2J,2T}$ the meson-nucleon cross-section in the state of total angular momentum J and isotopic spin T. In writing down Eq. (126) we have made the approximation

$$\sigma_{31}=\sigma_{13}=\sigma_{11}=0. \tag{127}$$

Inserting (125) and (126) into Eq. (121) we obtain, after removing the isotopic spin dependence,

$$\begin{aligned}S_1{}^{(1)}(M1)=&\frac{\alpha}{(2\pi)^3}(k_0)^{-\frac{1}{2}}\int\varphi_p{}^{(-)}(\mathbf{q})^\dagger\Bigg[-i\left(\frac{(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})}{\omega^2-\frac{1}{4}k_0{}^2}+\frac{2[(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\times(\mathbf{q}-\mathbf{q}')]\cdot[(\mathbf{q}-\mathbf{q}')\times(\mathbf{k}\times\boldsymbol{\varepsilon})]}{(\omega^2-\frac{1}{4}k_0{}^2)^2}\right)\frac{f^2}{\mu^2}\\&-\frac{\gamma_p-\gamma_n}{12m\pi}\int_\mu^\infty\frac{d\omega_t}{t}\times\frac{\sigma_{33}(t)}{\omega_t-k_0}(3\boldsymbol{\sigma}^{(1)}\cdot\mathbf{k}\times\boldsymbol{\varepsilon}\,\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\times\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')(1\leftrightarrow 2))\frac{1}{\omega^2-\frac{1}{4}k_0{}^2}\Bigg]\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}'.\end{aligned} \tag{128}$$

The expression given by Eq. (128), as it now stands, contains an integral over the physical meson-nucleon cross section; to evaluate such a contribution we resort to a form of the meson-nucleon scattering amplitude which

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[28] The success of the low-energy description of photomeson production and meson-nucleon scattering using this theory yields some hope; that such a treatment is valid.

[29] Owing to the separability of the momentum and energy de- pendence of the various amplitudes such an identity can be made —see Chew and Low, reference 4.

is equivalent to that in Eq. (126), namely,

$$
\begin{aligned}
4\pi e^{i\delta_{33}}\sin\delta_{33}&=\frac{4}{3k_0}\frac{f^2}{\mu^2}+\frac{1}{2\pi}\int_\mu^\infty\frac{d\omega_t}{t}\frac{\sigma_{33}(t)}{\omega_t-k_0}+\frac{1}{18\pi}\int_\mu^\infty\frac{d\omega_t}{t}\frac{\sigma_{33}(t)}{\omega_t+k_0},\\
0&=-\frac{2}{3k_0}\times\frac{f^2}{\mu^2}+\frac{2}{9\pi}\int_\mu^\infty\frac{d\omega_t}{t}\frac{\sigma_{33}(t)}{\omega_t+k_0},
\end{aligned}
\tag{129}
$$

wherein we have applied the approximation expressed by (127). In Eq. (129), δ_{33} is the phase for the $\frac{3}{2}\,\frac{3}{2}$ state. Inserting the above result into Eq. (128), we obtain

$$
\begin{aligned}
S_1^{(1)}(M1)=\frac{\alpha}{(2\pi)^3}(k_0)^{-\frac{1}{2}}\int\varphi_p^{(-)}(\mathbf{q})^\dagger\Bigg[&-i\frac{f^2}{\mu^2}\left(\frac{(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})}{\omega^2-\frac{1}{4}k_0^2}+\frac{2[(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\times(\mathbf{q}-\mathbf{q}')]\cdot[(\mathbf{q}-\mathbf{q}')\times(\mathbf{k}\times\boldsymbol{\varepsilon})]}{(\omega^2-\frac{1}{4}k_0^2)^2}\right)\\
&-\frac{\gamma_p-\gamma_n}{12m}\left(\frac{8\pi\sin\delta_{33}e^{i\delta_{33}}}{(k_0^2-\mu^2)^{\frac{3}{2}}}-\frac{3}{k_0}\frac{f^2}{\mu^2}\right)\frac{1}{\omega^2-\frac{1}{4}k_0^2}(3\boldsymbol{\sigma}^{(1)}(\mathbf{k}\times\boldsymbol{\varepsilon})\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')-(1\leftrightarrow 2))\Bigg]\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}'.
\end{aligned}
\tag{130}
$$

Removing the isotopic spin dependence, Eq. (120) can be rewritten as

$$
\begin{aligned}
S_1^{BO}(M1)=-\frac{\alpha}{(2\pi)^3}(k_0)^{-\frac{1}{2}}\frac{f^2}{\mu^2}\Bigg[&\frac{\gamma_p-\gamma_n}{4m}\int\varphi_p^{(-)}(\mathbf{q})_s^\dagger\frac{1}{2\omega}\frac{1}{\omega^2-\frac{1}{4}k_0^2}(3\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')\\
&-\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')-(1\leftrightarrow 2))+\frac{\gamma_p+\gamma_n}{4m}\int\varphi_p^{(-)}(\mathbf{q})_t^\dagger\frac{1}{2\omega}\frac{3}{\omega^2-\frac{1}{4}k_0^2}\\
&\times(\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')+\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\\
&\times\sigma^{(2)}\cdot(\mathbf{q}-\mathbf{q}')+(1\leftrightarrow 2))\Bigg]\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}',
\end{aligned}
\tag{131}
$$

wherein the subscripts t and s on $\varphi_p^{(-)}(\mathbf{q})^\dagger$ refer to the triplet and singlet spin states, respectively.

We finally note that the matrix element on the r.h.s. of Eq. (131) is inversely proportional to 2ω. Had we used an expression of the symbolic form $\bar{\chi}_0^{(-)}IG^{(1)}\Gamma_\mu^{(1)}\chi_0^D$ which is proportional to the covariant Born approximation, we would have found that such a term is inversely proportional to k_0. Consequently the ratio of that part of the Born approximation as given by (131) to the total effect is of order $k_0/2\omega$.

In summarizing the results of this section we have

$$
S_{\alpha\beta}=S_0(E)+S_0(M)+S_1^{BO}(M1)+S_1^{BO}(E1)+S_1^{(1)}(M1),
\tag{132}
$$

where

$$
S_0(E)=-i\alpha(k_0)^{\frac{1}{2}}\int_0^1 ds\,\boldsymbol{\varepsilon}\cdot\boldsymbol{\nabla}_{ks}\int\varphi_p^{(-)}(\mathbf{q})^\dagger[\tfrac{1}{2}(1+\tau_3^{(1)})\varphi^D(\mathbf{q}-\tfrac{1}{2}\mathbf{k}s)+\tfrac{1}{2}(1+\tau_3^{(2)})\varphi^D(\mathbf{q}+\tfrac{1}{2}\mathbf{k}s)]d\mathbf{q},
\tag{133}
$$

and

$$
\begin{aligned}
S_0(M)=-i\alpha(k_0)^{-\frac{1}{2}}\int_0^1 ds[(\mathbf{k}s\times\boldsymbol{\varepsilon})\times\boldsymbol{\nabla}_{ks}]\cdot\int\varphi_p^{(-)}(\mathbf{q})^\dagger[&(\boldsymbol{\sigma}^{(1)}\times\mathbf{k}s)\{\tfrac{1}{2}(1+\tau_3^{(1)}\gamma_p+\tfrac{1}{2}(1-\tau_3^{(1)})\gamma_N\}\varphi^D(\mathbf{q}-\tfrac{1}{2}\mathbf{k}s)\\
&+(\boldsymbol{\sigma}^{(2)}\times\mathbf{k}s)\{\tfrac{1}{2}(1+\tau_3^{(2)})\gamma_p+\tfrac{1}{2}(1-\sigma_3^{(2)})\gamma_N\}\varphi^D(\mathbf{q}+\tfrac{1}{2}\mathbf{k}s)](2m)^{-1}d\mathbf{q},
\end{aligned}
\tag{134}
$$

are the standard contributions as obtained from a consideration of Eqs. (95), (96), and (108). The remaining terms are defined by Eqs. (118), (130), and (131). In the next and final section we shall compute the cross sections from these expressions and compare the results with experiment.

V. NUMERICAL ANALYSIS AND CONCLUSIONS

In this section we describe the results of the numerical analysis of the various constituents of the single meson terms; we compute the cross sections and discuss their meaning. Finally, we consider in detail the various approximations made and describe their range of applicability. These approximations can be subdivided into two categories, one referring to the assumptions made in arriving at the final form of the matrix elements and the other pertaining to the explicit choice of n–p wave functions.

It is important at this point, however, at least to enumerate some of the simplifying assumptions made with respect to choice of wave functions:

(1) We consider no wave higher than d wave.

(2) In all transitions in which the matrix element is small we shall neglect the tensor interaction in both initial and final states.

To proceed, we consider in detail the contributions from the $E1$ and $M1$ transitions. The former terms are of interest only for the express purpose of exhibiting Siegert's theorem in practice, whereas, as has been previously mentioned, it is the latter terms which are responsible for the resonance. In calculating these latter terms we shall exclude the Born approximation contribution given by Eq. (131) as (see the discussion following Eq. (102)) it contributes a recoil correction compared to the main contribution given by $S_1^{(1)}(M1)$. Since, as has been shown, the considered $E1(M1)$ terms lead to triplet (singlet) final spin states, there will be no interference, and consequently we may sum the individual cross sections.

We now consider the $E1$ effect which is given by Eq. (118), and when written in coordinate space has the form

$$S_1^{BO} = -\alpha(k_0)^{\frac{1}{2}}(f^2/\mu^2)[\mu/(2\pi)^2]\int \varphi_p^{(-)}(\mathbf{x})^\dagger \times\{\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}\hat{x}\cdot\boldsymbol{\varepsilon}K_1(\mu x) - \boldsymbol{\sigma}^{(1)}\cdot\hat{x}\boldsymbol{\sigma}^{(2)}\cdot\hat{x}\,\hat{x}\cdot\boldsymbol{\varepsilon} \times(2K_1(\mu x)+\mu x K_0(\mu x))\}\,\varphi^D(\mathbf{x})d\mathbf{x}, \quad (135)$$

wherein $K_i(\mu x)$ is a modified Bessel function of the second kind. In deriving Eq. (135) we have neglected the kinetic energy of each of the nucleons in the final state ($\frac{1}{2}k_0$). This is certainly a justified approximation at low energies, i.e., $k_0 \ll 2\mu$, and fortunately at the higher energies the over-all electric contribution will prove to be negligible. In addition it is necessary to remark that a singularity arises at threshold for pion production ($\frac{1}{2}k_0=\omega_q$) which is defined in the usual manner (we add a small negative imaginary part to the mass). The resulting imaginary contribution turns out to be insignificant compared to the real part for all matrix elements. Consequently, if we invoke all the prescribed simplifications we arrive at the expression

$$S_1^{BO}(E1) = i\alpha\mu(2\pi)^{-2}(f^2/\mu^2)(k_0/2)^{\frac{1}{2}} \times\int \mathcal{V}_1^3(px)^* e^{-i\delta_1}\{\hat{p}\cdot\boldsymbol{\varepsilon}(3K_1(\mu x) - \mu x K_0(\mu x)) - (\boldsymbol{\sigma}^{(1)}\cdot\hat{p}\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{\varepsilon} + \boldsymbol{\sigma}^{(1)}\boldsymbol{\varepsilon}\boldsymbol{\sigma}^{(2)}\cdot\hat{p}) \times(2K_1(\mu x)+\mu x K_0(\mu x))\}U^D(x)x dx, \quad (136)$$

for the final form of (135). In Eq. (136) $\mathcal{V}_1^3(px)[\delta_1]$ is the wavefunction [phase shift] for the ^3P final state defined by the expansion

$$\varphi_p^{(-)}(\mathbf{x}) = (2\pi)^{-\frac{3}{2}}\chi_1^m \sum_l \mathcal{V}_l^3(px)e^{i\delta_l}P_l(\hat{p}\cdot\hat{x})i^l(2l+1), \quad (137)$$

with χ_1^m the appropriate triplet spin state. In Eq. (136) $U^D(x)$ is the radial part of the S-state deuteron wave function.

For the nuclear wave functions we choose hard core functions for both initial and final states with a core radius of 0.5×10^{-13} cm. The explicit forms of the wave functions are given by

$$U^D(x) = 0 \qquad x < x_c$$
$$= N(e^{-\alpha(x-x_c)} - e^{-\beta(x-x_c)}), \qquad x > x_c \quad (138)$$

and

$$\mathcal{V}_l^3(px) = 0 \qquad x < x_c$$
$$= (j_{-l-1}(px_c)j_l(px) + j_l(px_c)j_{-l-1}(px)), \quad x > x_c \quad (139)$$

wherein $j_l(px)$ is a spherical Bessel function. In Eq. (138) α is the deuteron binding energy and β is fitted to the effective range. The reasons for these particular choices will be clarified when we consider the magnetic contributions. The resulting cross section for the pure mesonic contribution is plotted in Fig. 3, and is evidently insignificant compared to the standard result in its entire range. We have not considered the cross term since we do not have the detailed form of the

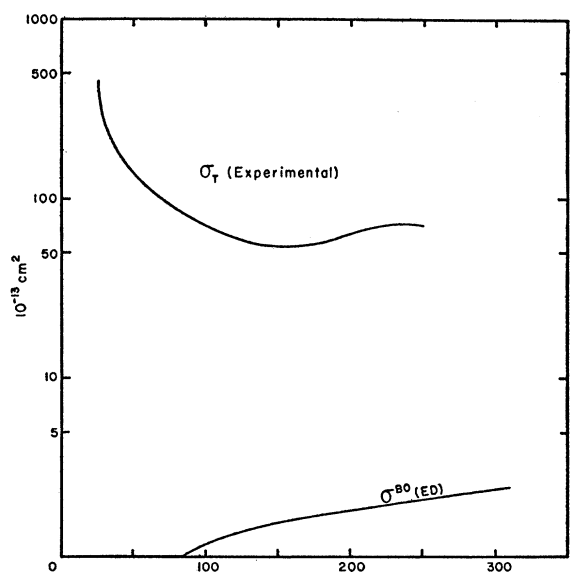

FIG. 3. Total cross section vs photon energy in the center-of-mass system compared with the pure lowest order mesonic correction to the electric dipole cross section.

standard amplitude; however, this cross term, although more significant than the effect considered, is also inconsequential compared to the standard result.

We next turn to a consideration of the magnetic terms. We must evaluate the expression [see Eq. (130)]

$$S_1^{(1)}(M1)=\frac{\alpha}{(2\pi)^3}\int \varphi_p^{(-)}(\mathbf{q})\left\{-i\frac{f^2}{\mu^2}\left[\frac{(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(k\times\boldsymbol{\varepsilon})}{\omega^2-\frac{1}{4}k_0^2}+\frac{2[(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\times(\mathbf{q}-\mathbf{q}')]\cdot[(\mathbf{q}-\mathbf{q}')\times(\mathbf{k}\times\boldsymbol{\varepsilon})]}{(\omega^2-\frac{1}{4}k_0^2)^2}\right]\right.$$
$$-\frac{\Upsilon_p-\Upsilon_N}{12m}F(k_0)[3\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')+\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{q}-\mathbf{q}')\boldsymbol{\sigma}^{(1)}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\boldsymbol{\sigma}^{(2)}\cdot(\mathbf{q}-\mathbf{q}')$$
$$\left.-(1\leftrightarrow 2)]\frac{1}{\omega^2-\frac{1}{4}k_0^2}\right\}\varphi^D(\mathbf{q}')d\mathbf{q}d\mathbf{q}', \quad (140)$$

where $F(k_0)$, which contains the meson-nucleon resonant effect, is given by the formula

$$F(k_0)=8\pi\sin\delta_{33}e^{i\delta_{33}}(k_0^2-\mu^2)^{-\frac{3}{2}}-3k_0f^2/\mu^2. \quad (141)$$

When rewritten in coordinate space, Eq. (140) has the form

$$S_1^{(1)}(M1)=\frac{\alpha}{(2\pi)^{\frac{3}{2}}}\left(\frac{\pi}{2k_0}\right)^{\frac{1}{2}}\int\varphi_p^{(-)}(\mathbf{x})^\dagger\left\{i\frac{f^2}{\mu^2}[-(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})\kappa x+(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot\hat{x}\hat{x}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})(1+\kappa x)]\right.$$
$$\left.+\left[\frac{\Upsilon_p-\Upsilon_n}{12m}F(k_0)(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})(3\boldsymbol{\sigma}^{(1)}\cdot\hat{x}\boldsymbol{\sigma}^{(2)}\cdot\hat{x}-1)\left(\frac{3}{x^2}+\frac{3\kappa}{x}+\kappa^2\right)\right]\right\}(e^{-\kappa x}/x)\varphi^D(\mathbf{x})d\mathbf{x}, \quad (142)$$

wherein $\kappa=(\mu^2-\frac{1}{4}k_0^2)^{\frac{1}{2}}$ and where $\kappa=-i|\kappa|$ for $\frac{1}{2}k_0>\mu$. In arriving at Eq. (142) we have made use of the spin properties of the initial and final state.

It is found that the transition matrix, $S_1^{(1)}(M1)$, is completely dominated by the resonant contribution (the term in the second square bracket on the r.h.s. of Eq. (142); consequently we shall apply our list of approximations to the term in the first square bracket. We thereby arrive at the form

$$S_1^{(1)}(M1)=M(NR)+M(R), \quad (143)$$

where

$$M(NR)=i\alpha N(8\pi^3k_0)^{-\frac{1}{2}}(f^2/\mu^2)\left\{\int\mathcal{V}_0(px)e^{-i\delta_0}\tfrac{1}{3}(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})(1-2\kappa x)e^{-\kappa x}U^D(x)dx\right.$$
$$\left.-\int\mathcal{V}_2(px)e^{-i\delta_2}[(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot\hat{p}\hat{p}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})-\tfrac{1}{3}(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})](1+\kappa x)e^{-\kappa x}U^D(x)dx\right\}$$
$$=i\alpha N(8\pi^3k_0)^{-\frac{1}{2}}(f^2/\mu^2)\{\tfrac{1}{3}(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})J_0(k_0)e^{-i\delta_0}-[(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)}\cdot\hat{p}\hat{p}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})$$
$$-\tfrac{1}{3}(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})]J_2(k_0)e^{-i\delta_2}\}, \quad (144)$$

and

$$M(R)=-\alpha N(8\pi^3k_0)^{-\frac{1}{2}}\left\{\int\mathcal{V}_2(px)e^{-i\delta_2}(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})(3\boldsymbol{\sigma}^{(1)}\cdot\hat{p}\boldsymbol{\sigma}^{(2)}\cdot\hat{p}-1)x^{-2}(3+2\kappa x-\kappa^2x^2)\right.$$
$$\left.\times(U^D(x)-2^{-\frac{1}{2}}W^D(x))+\int\mathcal{V}_0^x(px)e^{-i\delta_0}(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})x^{-2}(3+2\kappa x+\kappa^2x^2)\sqrt{8}W^D(x)\right\}e^{-\kappa x}dxF(k_0)$$
$$\times(\Upsilon_p-\Upsilon_n)/12m$$
$$=-\alpha N\mu(8\pi^3k_0)^{-\frac{1}{2}}F(k_0)(\Upsilon_p-\Upsilon_n)(12m)^{-1}\{(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot(\mathbf{k}\times\boldsymbol{\varepsilon})(3\boldsymbol{\sigma}^{(1)}\cdot\hat{p}\boldsymbol{\sigma}^{(2)}\cdot\hat{p}-1)$$
$$\times[I_{20}(k_0)e^{-i\delta_2}-I_{22}(k_0)e^{-i\delta_2}]+(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot\mathbf{k}\times\boldsymbol{\varepsilon}I_{02}(k_0)e^{-i\delta_0}\}. \quad (145)$$

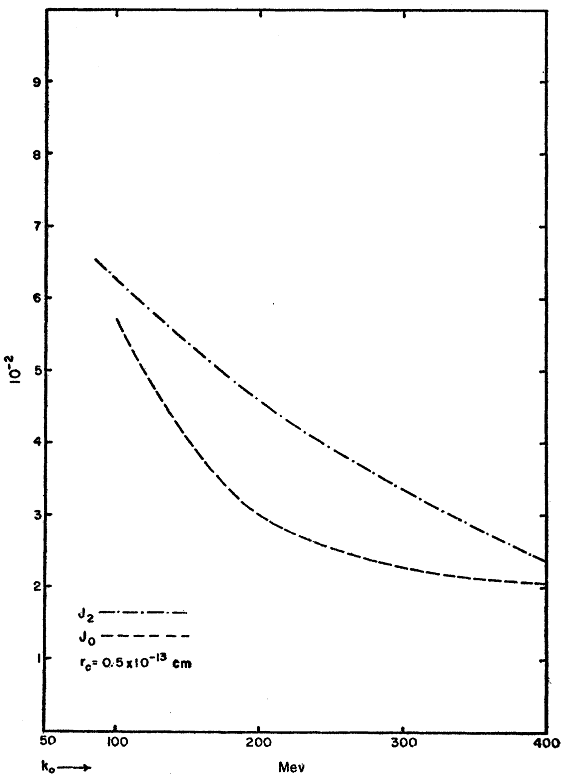

FIG. 4. J_2 and J_0 vs the photon energy in center-of-mass system.

In the above equations, (144) and (145), we have made use of the definition

$$\varphi^D(x)=\frac{N}{(4\pi)^{\frac{1}{2}}}\left\{\frac{U^D(x)}{x}+\frac{S_{12}}{\sqrt{8}}\frac{W^D(x)}{x}\right\}, \qquad (146)$$

and

$$\varphi_p{}^{(-)}(x)=\chi_0{}^0\sum_l(2l+1)i^l v_l(px)e^{i\delta_l}P_l(\hat{p}\cdot\hat{x}). \qquad (147)$$

Once again we use hard core functions with a core radius of 0.5×10^{-13} cm to evaluate (144) with the wavefunctions defined by Eqs. (138) and (139) except that the final state is now in a singlet spin state. In Fig. 4 we have plotted our results for $J_0(k_0)$ and $J_2(k_0)$ in suitable units. To determine its contribution to the cross section we must include as well the interference with the resonant term. The resulting part of the total cross section at the resonance constitutes about a five percent effect.

We shall now discuss in detail the essential properties of the resonant term, Eq. (145). First we note that in the absence of the deuteron D state there is only an s-wave final state, which gives rise to a $2+3\sin^2\theta$ angular distribution. However, the inclusion of the D state will introduce s-wave—d-wave interference with an angular distribution of $2-3\sin^2\theta$. We note that for the greater range of interest (see Fig. 5) the folded angular distribution is approximately $2+\sin^2\theta$;[30] hence we can easily see the need for the interference term. Our fit to this angular distribution, shown in the figure is discussed below.

Next we note that the integrand is proportional to inverse powers of the two-nucleon separation, thereby emphasizing the interior part of the wavefunctions. Thus it is to be expected that our calculation containing the s-wave states should be rather sensitive to the existence and size of a hard core, and this has therefore been essentially taken as one of the parameters of the theory.

There remains now the question of what to do concerning the effect of the attractive part of the nuclear potential on the wavefunctions. For the final scattering state we included its effect only in the phase shifts occurring in the expansion, Eq. (147). We are limited to this approximation simply because at the photon energies of interest, which correspond to n–p laboratory energies of greater than 350 Mev, no adequate quantitative description of the scattering states exist. However, we do expect the behavior at small distances to be most strongly determined by the core. Thus we note that in the angular distribution there is an unknown parameter, namely the relative phase between s- and d-wave states. A phase shift analysis at the appropriate n–p energies should afford a check on our choice.

As for the deuteron wavefunction we examine the consequences of using a Hulthén, a Gartenhaus, and a Hulthén with hard core for the S state; for the D state we assume the Gartenhaus form. We have, of course, chosen the correct normalization for the total deuteron wavefunction.

As can be seen from the plot of $I_{20}(k_0)$ in Fig. 6, the final result does depend most strongly on the radius of

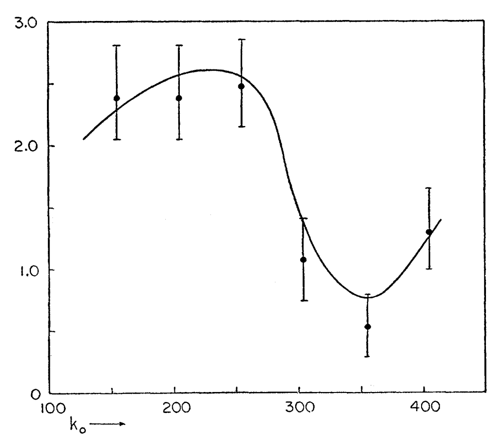

FIG. 5. Folded angular distribution vs photon energy in the center-of-mass system. The solid line represents our fit.

[30] The experimental points were obtained from: E. A. Whalin, B. D. Schriever, and A. D. Hanson, Phys. Rev. **101**, 377 (1956); J. C. Keck and A. J. Tollestrup, Phys. Rev. **101**, 360 (1956); J. C. Keck et al., Phys. Rev. **93**, 827 (1954).

the hard core. In Fig. 7 we plot the value of $I_{22}(k_0)$ and $I_{02}(k_0)$.

We find that agreement between theory and experiment is reasonably good. To produce this agreement in both the total and differential cross sections it is important to include the deuteron D state, and furthermore it was necessary to fix upon a core radius of 0.5×10^{-13} cm, a value which is indicated by the Gammel-Thaler analysis.[31] Had we not included a hard core, our cross sections would be an order of magnitude too large. The values of the relative s-wave—d-wave phase shifts needed to fit the folded angular distribution are plotted in Fig. 8; to arrive at these results we have included the nonresonant effect as well as the extrapolated results of the de Swart-Marshak[32] calculation at the lower energies. We note that this choice is consistent with the Gammel-Thaler prediction.[31] We have already alluded to the resultant fit shown in Fig. 5. Finally we have plotted the total cross section as compared with experiment in Fig. 9; here too we have included the complete matrix element. To obtain a true comparison we have subtracted the results of the de Swart-Marshak calculation.[32]

To sum up, our calculation shows that in order to achieve agreement with experiment we must include

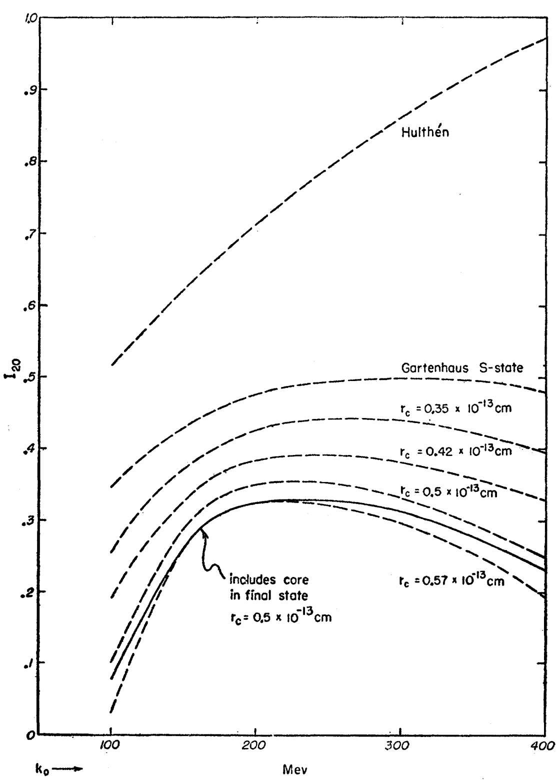

FIG. 6. I_{20} vs the photon energy in the center-of-mass sysstem for the various choices of initial and final state wave function.

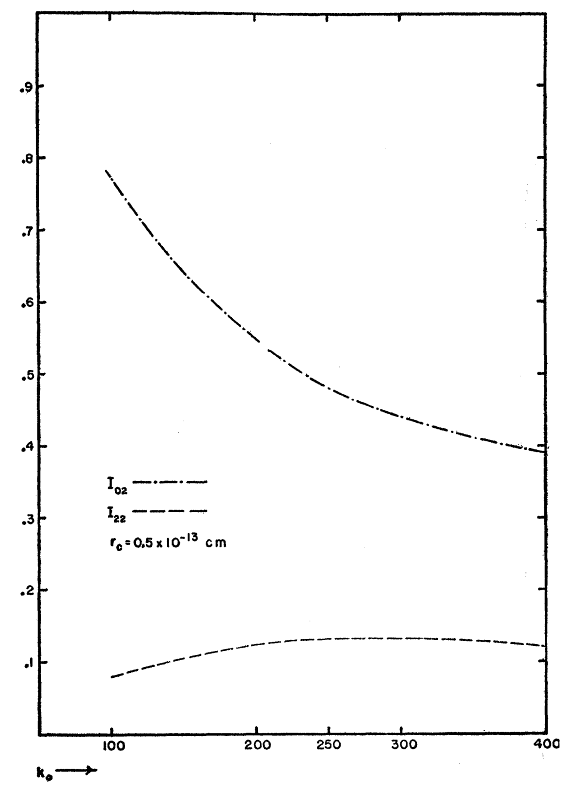

FIG. 7. I_{02} and I_{22} vs the photon energy in the center-of-mass system.

the following effects: (1) a hard core ($r_c=0.5\times10^{-13}$ cm), (2) an attractive nuclear potential, (3) a tensor interaction in the initial state (deuteron).

It is also of interest to note that for the resonant term our results have the form of Wilson's,[33] namely it is essentially a product of two probabilities, one the probability of photoproduction into a $(\frac{3}{2},\frac{3}{2})$ state and the other, the probability of absorption by the remaining nucleon. However, in our case the probability for absorption is dependent on the nucleon relative momentum. Furthermore, photodisintegration via s-wave mesons does not have this simple form. Thus the similarity is actually quite superficial.

It is of more interest to compare our work with Zachariasen's calculation. Although the underlying formalisms were quite different, the two calculations differ, in a practical sense, as follows:

(1) the treatment of the nuclear wave functions,
(2) the technique of introducing the resonant meson-nucleon interaction,
(3) the treatment of higher order exchanges, i.e., Zachariasen includes a piece of the two meson exchange.

As for (1) it seems somewhat surprising that Zachariasen achieved such relatively good results without including any details of the nuclear potential

[31] J. Gammel and R. Thaler, Phys. Rev. **107**, 291 (1957).
[32] See J. J. de Swart and R. E. Marshak, reference 1.
[33] R. R. Wilson, Phys. Rev. **104**, 218 (1956).

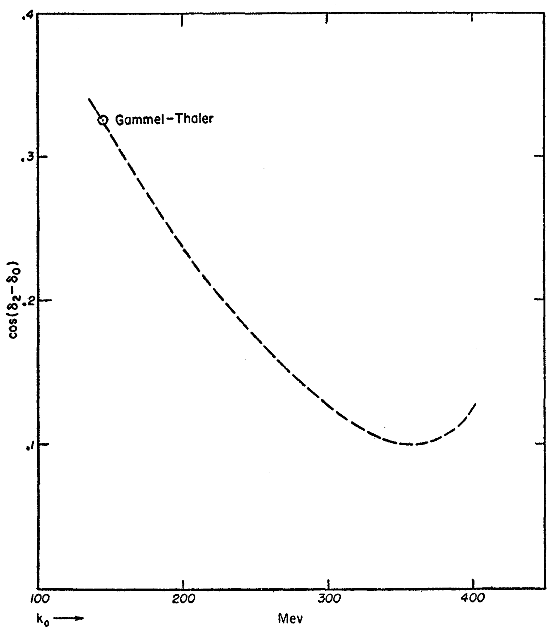

FIG. 8. $\cos(\delta_2-\delta_0)$ vs the photon energy in the center-of-mass system.

in the wave functions. Especially so since our results depend so sensitively on such effects.

Concerning (2), the method of going off the energy shell, both methods assume that the major effect is just the $J=\frac{3}{2}$, $T=\frac{3}{2}$ transition. However, the two calculations differ in the technique of handling this term. Basically, once we have made the fixed source approximation we are led unequivocably to an expression which is dictated by exact pion-nucleon scattering in the prescribed state. Zachariasen, on the other hand, makes a one-meson approximation to the Tamm-Dancoff method to arrive at the resonant meson-nucleon interaction which he handles in an approximate manner. This major difference has the effect of introducing a much larger resonant effect, which in our calculation results in the great sensitivity to the qualitative form of the nuclear wavefunctions.

We shall now discuss the sources and relative importances of the errors made in the explicit evaluation of the formal solution of the S matrix. These can be subdivided into the following classes: (1) The low-energy approximations on the vertex operators; (2) The neglect of recoil; (3) The method of going off the energy shell; and (4) The exclusion of higher order meson exchanges.

Concerning (1), we mean that in making the low-energy approximations we have neglected both finite size effects as well as all consequences arising from the fact that these operators are off the energy shell. It is unfortunate that within the framework of the present theory of strong interactions we have little detailed idea as to the actual form of these neglected effects. However, it is strongly believed that such effects are decreased by a factor of at least v_N/c, the nucleon recoil, relative to the terms retained.

By "recoil effects" we mean those terms of relative order v_N/c which can be determined explicitly. Since their neglect involves an error of the same order of magnitude as that made in the inevitable low-energy approximations, we have not tried to evaluate these terms. We note that this nucleon recoil effect arises in three possible ways; two of these, which we now consider, namely, the temporal part of the vertex functions and the nonadiabatic wavefunction corrections are true corrections of order v_N/c. Since, however, there is one dominant term (the resonant contribution) recoil can only be of qualitative importance in this term. Moreover, in this term recoil enters as a factor in the energy denominator which lends justification to our procedure of going off the energy shell. Thus we have accounted for recoil semiadequately in a way dictated by the fixed source model.

As for the error incurred in going off the energy shell, we can make no rigorous comments since to do so would entail a detailed knowledge of its form off the energy shell, an unknown factor at present. Hence, our only justification is the success or failure of such a procedure.

Finally, as regards the higher order exchanges, we can offer no justification for its neglect. An examination of the two-meson exchange terms should indicate the seriousness of such a deletion.

A consequence of our calculation is the overwhelming importance of the hard core, i.e., its deletion would result in a cross section which is a factor of ten too

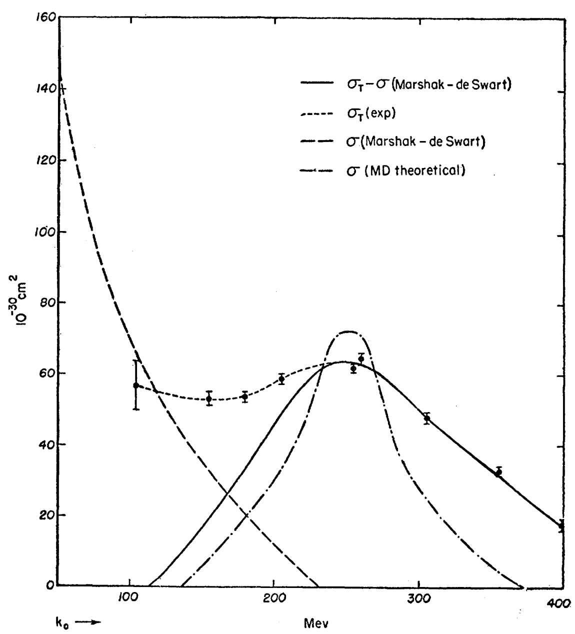

FIG. 9. Total cross section vs the photon energy in the center-of-mass system.

large. Hence if we assume that the higher order meson exchanges are unimportant we can view our theory both as an independent conformation of the hard core as well as an indication that our method of going off the energy shell is qualitatively correct. Furthermore, our work shows the necessity of using qualitatively accurate wave functions. Thus we believe that the major unknown effect is the higher order exchanges, recoil being completely overshadowed by the hard core. An extension into two meson exchange effects should determine whether the above conclusions are meaningful.

Finally we remark that at the lower energies our work indicates that the entire effect is essentially a classical one; there are no mesonic contributions of importance. Thus we may conclude that the various classical calculations[1] account for the low-energy phenomenon in a manner which is consistent with first principles.

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Spin States Associated with Neutron Resonances in In^{115}

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By polarizing both the neutron beam and the nuclear sample, the spin states of the first three slow neutron resonances in the target nucleus In^{115} have been measured. These were obtained by observing the direction of change in the transmitted intensity upon reversing the polarization of the neutrons with respect to the target nuclei. The spin states associated with the resonances at 1.46 ev, 3.86 ev, and 9.10 ev were found to be $J=5$, 4, and 5, respectively. These spin assignments are consistent with measurements of other parameters of these resonances.

INTRODUCTION

THERE has been much recent interest in the spin states of levels in compound nuclei formed by interaction with slow neutrons. In particular, the information desired is the relationship of the spin state to the other resonance parameters such as the radiation width, neutron scattering width, level spacing, capture gamma-ray spectrum, and the distribution of the two possible spin states among the resonances in an isotope. We are considering only cases where the neutron orbital angular momentum is zero. Sailor[1] has made a survey of the measured spin states and concludes that the often made assumption that the two possible spins are equally distributed among the levels may not be valid, since there are many more levels with measured spin states $J=I+\frac{1}{2}$ than with $J=I-\frac{1}{2}$, where I is the spin of the target nucleus ground state.

Generally speaking, the spin states of these compound nucleus levels have eluded measurement because the methods commonly used to obtain the resonance parameters are rather insensitive to the spin state. In principle, the combination of good measurements of total and scattering or capture cross sections would yield all the resonance parameters, including J. Several laboratories [2–5] are presently using this method to obtain J values, although partial cross-section measurements to the necessary degree of accuracy are difficult to make and results obtained in different laboratories have not always been in agreement. In some cases, spin assignments could be made on the basis of total or scattering cross-section measurements alone[6–8] (see also cases cited in reference 1). In the kilovolt region, Hibdon[9] has reported J values for several resonances in aluminum from analysis of total cross-section curves. Other investigators[10,11] have measured spin states by looking for the presence of a ground-state transition. This technique is applicable only when $I=\frac{1}{2}$.

The technique adopted in this investigation is to polarize both the neutron beam and the target. The spin state of the compound nucleus can then be obtained directly by observing the direction of change in the transmitted intensity upon reversing the relative orientation of the neutrons and the nuclei. The transmission of a polarized neutron beam through a polarized

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