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*Note added in proof.*—While this paper was in press

an article by Kaneko on the uniform field polarizability of rare gas atoms has appeared.<sup>48</sup> Kaneko has carried out a numerical calculation on helium using both the exact first order perturbed Hartree-Fock equation and Sternheimer's equation. He finds that Sternheimer's approximation gives a value 12.5% too large.

<sup>48</sup> S. Kaneko, *J. Phys. Soc. Japan* **14**, 1600 (1959).

## Elastic Scattering of Alpha Particles by $O^{16}\dagger$

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Absolute cross sections for the elastic scattering of alpha particles by  $O^{16}$  have been measured in a differentially pumped gas scattering chamber. The measurements were made for laboratory energies from 3.7 to 6.5 Mev, corresponding to 7.7- to 9.9-Mev excitation in  $Ne^{20}$ , at center-of-mass angles of  $168.9^\circ$ ,  $149.4^\circ$ ,  $140.8^\circ$ ,  $125.3^\circ$ , and  $90.0^\circ$ .

Narrow resonances were observed at bombarding energies of 5.002, 5.11, 5.190, 5.432, 5.532, and 6.030 Mev. The data were analyzed in terms of Wigner-Eisenbud dispersion theory to find the spins, parities, resonant energies, widths, reduced widths, and characteristic energies of the levels. The resonances observed correspond to states in  $Ne^{20}$  with the following excitation energies, spins, and parities: 8.755( $1^-$ ), 8.84( $5^-$ ), 8.905( $1^-$ ), 9.099( $4^+$ ), 9.179( $3^-$ ), and 9.577 Mev ( $2^+$ ). In order to obtain a good fit to the data, it was necessary to assume the existence of two broad overlapping resonances, one at  $\sim 8.7$ -Mev excitation ( $0^+$ ) and the other at  $\sim 8.8$  Mev ( $2^+$ ). There is also some evidence for the presence of a broad  $4^+$  level at an energy higher than 9.9-Mev excitation in  $Ne^{20}$ .

### I. INTRODUCTION

**T**HEORETICAL developments in the last few years have aroused interest in the nuclei with atomic number just above that of the doubly closed shell nucleus  $O^{16}$ . Shell model calculations for nuclei of masses 17, 18, and 19<sup>1</sup> and collective model calculations for nuclei of masses 19<sup>2,3</sup> and 25<sup>4</sup> have been performed. Both types of calculations have been very successful in predicting the spins and parities of the lower excited states.

An experimental determination of the spins and parities of the states of  $Ne^{20}$  would be of the greatest importance in further testing the theoretical predictions of the proposed models. The elastic scattering of alpha particles by  $O^{16}$  provides an effective method for the investigation of the  $T=0$  levels of  $Ne^{20}$  above 5-Mev excitation. Since the spins of both particles are zero, the only possible combinations of total angular momentum

and parity for the levels which can be seen are even  $J$ -even parity and odd  $J$ -odd parity. Elastic scattering and capture are the only energetically possible processes in the energy region covered. The  $(\alpha, \gamma)$  process is negligible compared with the probability of particle emission. For the case of spin zero on spin zero scattering with no reactions present, the partial wave analysis of excitation curves taken at several angles and the subsequent interpretation according to dispersion theory are, in principle, relatively simple.

The elastic scattering of alpha particles by  $O^{16}$  from 0.94- to 4.0-Mev bombarding energy has previously been investigated by Cameron.<sup>5</sup> Five levels in  $Ne^{20}$  were found in the region of excitation from 5.5 to 7.9 Mev. The present experiment extends the energy range studied from 4.0- to 6.5-Mev bombarding energy. This same energy range had been studied much earlier by Ferguson and Walker,<sup>6</sup> who used  $RaC'$  as an alpha-particle source. They found two resonances, one at 5.5-Mev and the other at 6.5-Mev bombarding energy. A probable assignment of  $1^-$  was made for both of them.

### II. EXPERIMENTAL APPARATUS

The scattering chamber and associated equipment used for the measurement of the absolute cross sections

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<sup>1</sup> J. P. Elliott and B. H. Flowers, *Proc. Roy. Soc. (London)* **A229**, 536 (1955).

<sup>2</sup> E. B. Paul, *Phil. Mag.* **2**, 311 (1957).

<sup>3</sup> G. Rakavy, *Nuclear Phys.* **4**, 375 (1957).

<sup>4</sup> A. E. Litherland, E. B. Paul, G. A. Bartholomew, and H. E. Gove, *Phys. Rev.* **102**, 208 (1956).

<sup>5</sup> J. R. Cameron, *Phys. Rev.* **90**, 839 (1953).

<sup>6</sup> A. J. Ferguson and L. R. Walker, *Phys. Rev.* **58**, 666 (1940).

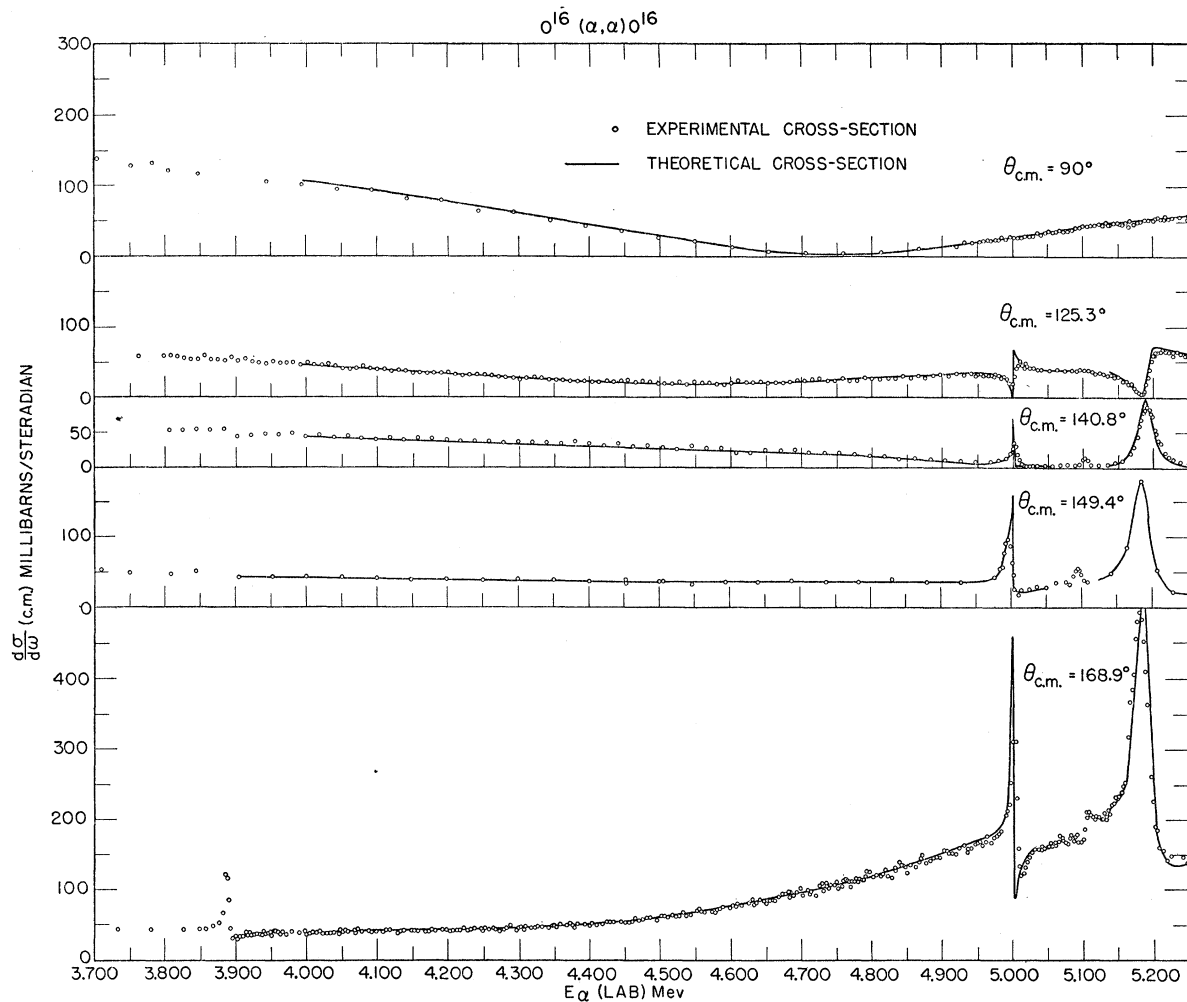


FIG. 1. Experimental and theoretical differential cross sections from 3.70 Mev to 5.25 Mev.

have been described in detail by Smotrich et al.<sup>7</sup> In the present experiment the target gas was not recirculated. Otherwise, the equipment was identical.

For the purpose of this experiment the amount of impurities and other oxygen isotopes present is considered negligible. The isotopic abundance of  $O^{16}$  is 99.76%. The target gas was supplied from a tank of Airco commercial grade oxygen which has a purity of 99.5%. Condensable impurities were removed with a liquid nitrogen cooled trap so that very small amounts of nitrogen and argon were the major contaminants.

### III. DATA

The experimental absolute differential cross sections, together with the theoretical fits, are shown in Figs. 1 and 2. The angles chosen were the maximum back angle possible and the four angles in the center-of-mass system at which the Legendre polynomials of orders 1,

2, 3, and 4 vanish. The size of the energy steps was chosen after the general shape of the excitation curve was first determined. The target thickness was varied between 2 and 5 kilovolts by adjusting the gas pressure in the scattering chamber. Absolute differential cross sections and bombarding energies were calculated with the aid of an IBM 650 digital computer.

The errors in the measurement of the absolute cross sections and bombarding energies were similar to those discussed by Smotrich et al.<sup>7</sup> with the exception of the error due to gas impurities. In the present experiment it is estimated that impurities caused no greater error than  $\pm 0.5\%$ . The over-all uncertainty in the cross section is about  $\pm 4.0\%$ , while the maximum experimental error in the energy scale is a little over  $0.1\%$ .

### IV. ANALYSIS OF THE DATA

#### A. General Considerations

The differential scattering cross section for elastic scattering of spin-zero target particles by spin-zero

<sup>7</sup> H. Smotrich, K. W. Jones, L. C. McDermott, and R. E. Benenson (to be published).

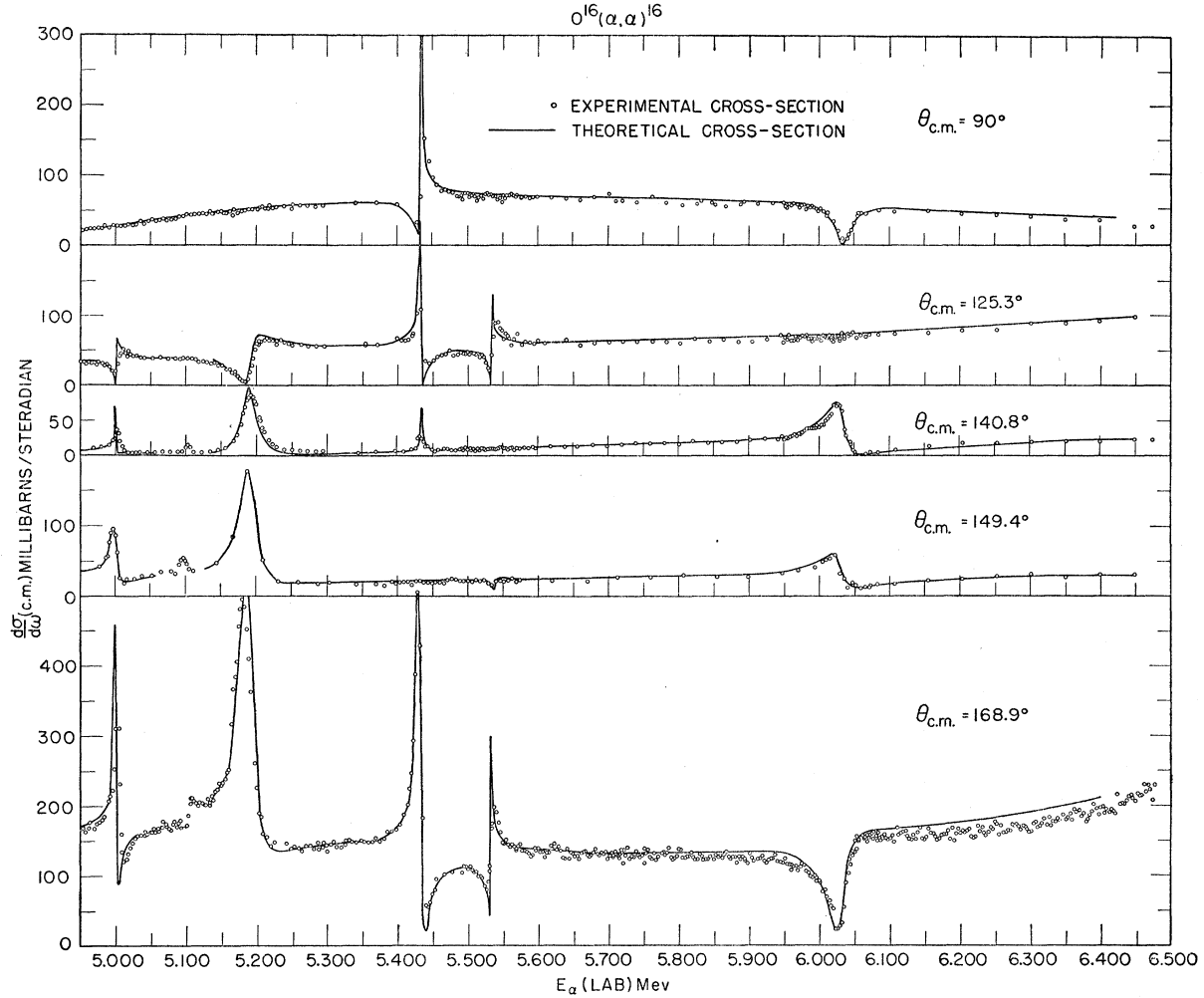


FIG. 2. Experimental and theoretical differential cross sections from 4.95 Mev to 6.50 Mev.

bombarding particles may be expressed as<sup>8-10</sup>

$$\left(\frac{d\sigma}{d\omega}\right)_{\text{e.m.}} = \frac{1}{k^2} \left| - (1/2)\eta \csc^2(\theta/2) \exp[i\eta \ln \csc^2(\theta/2)] + \sum_l (2l+1) P_l(\cos\theta) e^{i\alpha_l} e^{i\delta_l} \sin\delta_l \right|^2,$$

in which  $k = \mu v / \hbar$ ,  $\eta = ZZ'e^2 / \hbar v$ , and  $\theta$  is the center-of-mass scattering angle. The reduced mass of the system is given by  $\mu$  and the relative velocity of incident and target particles by  $v$ .  $\delta_l$  is the phase shift of the partial wave with orbital angular momentum  $l\hbar$ . The Coulomb phase shift  $\alpha_l$  is given by

$$e^{i\alpha_l} = \frac{(l+i\eta) \cdots (1+i\eta)}{(l-i\eta) \cdots (1-i\eta)}, \quad l > 0;$$

and  $e^{i\alpha_0} = 1$ .

<sup>8</sup> E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).

<sup>9</sup> R. G. Sachs, *Nuclear Theory* (Addison-Wesley Publishing Company, Inc., Reading, 1953).

<sup>10</sup> A. M. Lane and R. G. Thomas, Revs. Modern Phys. **30**, 257 (1958).

The phase shift  $\delta_l$  consists of two parts, a resonant term  $\beta_l$ , and a potential term  $\phi_l$ , such that

$$\begin{aligned} \delta_l &= \beta_l - \phi_l, \\ \phi_l &= \tan^{-1}(F_l/G_l)_{r=a}, \end{aligned}$$

in which  $F_l$  and  $G_l$  are the regular and irregular Coulomb wave functions. If no two levels of the same spin and parity lie close together, the single-level approximation for  $\beta_l$  may be used:

$$\beta_l = \tan^{-1} \left( \frac{\gamma_{\lambda l}^2 k / A_l^2}{E_{\lambda} + \Delta_{\lambda l} - E} \right),$$

in which  $A_l^2 = F_l^2 + G_l^2$ .

$E_{\lambda}$  is the characteristic energy corresponding to the solution of the internal eigenvalue problem for the compound nucleus, and  $\gamma_{\lambda l}^2$  is the reduced width.  $\Delta_{\lambda l}$  is the level shift and depends on the choice of boundary conditions. The expression for  $\Delta_{\lambda l}$  used in this analysis

is

$$\Delta_{\lambda l} = -\frac{\gamma_{\lambda l}^2}{a} \left( \frac{d \ln A_l}{d \ln \rho} + l \right)_{r=a},$$

in which  $\rho = kr$ . The resonant energy is defined as the energy at which  $E_{\lambda} + \Delta_{\lambda l} - E = 0$ . For a narrow single resonance  $k$ ,  $A_l^2$ , and  $\Delta_{\lambda l}$  may be taken as constants over the resonance. In this case the resonant phase shift may be expressed as

$$\beta_l = \tan^{-1}[(1/2)\Gamma_{\lambda l}/(E_R - E)],$$

in which  $\Gamma_{\lambda l} = 2k\gamma_{\lambda l}^2/A_l^2$ .

If two levels of the same spin and parity must be taken into consideration, it can be shown that

$$\beta_l = \tan^{-1} \left[ \frac{(k/A_l^2)_a}{\left( \frac{\Delta_{\lambda l}}{\gamma_{\lambda l}^2} \right) + \left[ \frac{\gamma_{\lambda l1}^2}{E_{\lambda l1} - E} + \frac{\gamma_{\lambda l2}^2}{E_{\lambda l2} - E} \right]^{-1}} \right],$$

in which the subscripts "1" and "2" refer to the two levels.

The interaction radius "a" is largely arbitrary and is chosen to give the best fit to the data. Its choice determines the values of  $\beta_l$  and  $\phi_l$  but should not affect  $\delta_l$ . It is customary in this type of analysis to use a value of the radius roughly equal to the sum of the radii of the two particles involved in the collision. The value chosen for this experiment is

$$r = 1.30(16^{\frac{1}{2}} + 4^{\frac{1}{2}}) \times 10^{-13} \text{ cm} = 5.34 \times 10^{-13} \text{ cm}.$$

An equally good fit was obtained over a large part of the energy region with the use of a factor of 1.40 instead of 1.30.

The order of magnitude of the reduced width of a particular level gives some indication of the character of its wave function. Teichmann and Wigner<sup>11</sup> have derived a sum rule which requires that  $\gamma_{\lambda l}^2 \leq 3\hbar^2/2\mu a$ . If the reduced width of a level is close to this limit, the level is expected to be characterized by single-particle wave functions.

As can be seen from the formula for the differential scattering cross section effects due to the  $l$ th partial wave are not present at the center-of-mass angle at which the  $l$ th Legendre polynomial vanishes. The absence of a narrow resonance in the cross-section data at a particular angle of observation immediately makes possible a tentative assignment of the spin and parity of that resonance. The angles of observation in the center-of-mass system which were chosen for the present experiment were 168.9°, the maximum angle allowed by the scattering chamber, 149.4°, 140.8°, 125.3°, and 90.0°. The maximum back angle permitted all Legendre polynomials of interest to approach their maximum value sufficiently closely so that the corresponding resonances were all displayed prominently. The re-

maining angles represented the positions at which, respectively, the Legendre polynomials of orders  $l=4$ ,  $l=3$ ,  $l=2$ , and all odd  $l$  values vanish.

At a particular energy there is a practical limit to the order of the polynomial which must be included in the sum appearing in the cross-section formula. Both the potential and resonant phase-shift contributions to  $\delta_l$  depend on the magnitude of the Coulomb wave functions,  $F_l$  and  $G_l$ . At the energies involved,  $\phi_l$  is negligible for  $l \geq 5$ . For  $l \geq 7$ , the maximum value of the reduced width (taken to be of the order of the Wigner limit) yields an experimental width  $\Gamma_{\lambda}$  which is too small to be resolved. Values of  $\phi_l$ ,  $A_l^2$  and  $\Delta_{\lambda l}/\gamma_{\lambda l}^2$  were calculated as functions of energy for  $l=0$  through  $l=4$  with the use of Breit's tables.<sup>12</sup> Values of  $A_l^2$  for  $l=5$ , 6, and 7 were also calculated for use in substantiating the arguments for the assignment of the probable 5-resonance which was not resolved.

The analysis of the data divides naturally into two parts. The first deals with the analysis of regions not including the sharp resonances. The second is concerned with the analysis of the narrow levels.

## B. Broad Resonances

An examination of the excitation curves in the energy region from 4.0 to 5.0 Mev reveals an energy dependence in the cross section which is not associated with clearly defined sharp resonances. There is a large dip at 90° at about 4.7 Mev, where the cross section drops to about 5 millibarns/steradian. This value is much smaller than that given by Rutherford and potential scattering. The narrow resonances are too far away to have much effect on the cross section at this energy. There is at the same time a steady rise at 168.9°, although the Rutherford contribution is decreasing with energy as  $1/E^2$ . From these preliminary observations, the existence of at least one, and possibly more, broad resonances in this energy region may be postulated.

The analysis was begun with an attempt to fit the data between 4.0 and 5.0 Mev. The simplest assumption which could be made, the existence of a single broad resonance, did not prove adequate for a detailed fit. A fit to the data at 90° requires the presence of an even parity resonance since odd parity levels have no effect at this angle. Unsuccessful attempts were made to extract an  $s$ - or  $d$ -wave phase shift at several energies that would fit the data at all the angles. At 5.0 Mev the laboratory width of an  $l=4$  resonance with a reduced width equal to the Wigner single-particle limit is about 150 kev. The reduced width of an  $l=4$  resonance sufficiently broad to fit the data would be several times as large as the Wigner limit. The possibility of an  $l=4$  resonance was therefore eliminated. Nevertheless, the extraction of a  $g$ -wave phase shift was also tried without success.

<sup>11</sup> T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).

<sup>12</sup> I. Bloch, M. H. Hull, A. A. Broyles, W. G. Bouricius, B. E. Freeman, and G. Breit, Revs. Modern Phys. **23**, 147 (1951).

The second most simple assumption which can be made is the existence of two broad overlapping resonances in this region. All possible combinations of  $l=0, 1, 2,$  and  $3$  which involve at least one even parity resonance were tried. An  $l=0$  resonance combined with an odd parity resonance could not account for the low  $90^\circ$  cross section at  $4.7$  Mev. An  $l=2$  resonance together with an  $l=1$  resonance fit the data at  $4.0$  Mev but not at  $4.7$  Mev. It was not possible to obtain a fit to the data at any energy with a combination of  $l=2$  and  $l=3$ . Since sufficiently broad resonances with  $l \geq 4$  were excluded, the only remaining possibility was the combination of  $l=0$  and  $l=2$ .  $s$ - and  $d$ -wave shifts were extracted from the data at 100-kilovolt intervals from  $4.0$  to  $5.0$  Mev. The phase shifts obtained in this manner exhibited the energy dependence predicted by theory for both  $l$  values. The choice of  $l$  values of the two broad levels necessary for a good fit was insensitive to small changes in the interaction radius. Values of  $\delta_0$  and  $\delta_2$  in the range from  $4.0$  to  $5.0$  Mev were obtained which were capable of fitting the data to within  $8\%$  at all angles at each energy.

It is possible that a combination of three broad resonances, not including both  $l=0$  and  $l=2$ , could be found which would fit the data. Throughout the analysis, however, the simplest assumptions leading to a fit of the data were made until contradictions with these assumptions necessitated modifications.

A fit to within  $5\%$  of the data between  $4.0$  and  $5.0$  Mev became possible only when the effect of the  $10$  kev wide  $3^-$  level at  $3.045$  Mev found by Cameron<sup>5</sup> was explicitly included. At the energies involved in this experiment,  $A_3^2$  is a rapidly decreasing function of energy. The result of this energy dependence is a contribution by this resonance to the  $l=3$  phase shift of several degrees. Instead of gradually approaching  $180^\circ$ ,  $\beta_3$  decreases with energy in the energy range considered.

The  $s$ - and  $d$ -wave phase shifts extracted from the data were plotted vs energy. Values of  $E_\lambda$  and  $\gamma_\lambda^2$  were chosen that would best reproduce the resonant phase-shift curve deduced from the analysis for each  $l$  value. These values of  $E_\lambda$  and  $\gamma_\lambda^2$  were then used to give rough

estimates for the resonant energy and width of the level involved.

An  $E_\lambda$  of  $3.4$  Mev and a  $\gamma_\lambda^2$  of  $2.9$  Mev-cm were chosen for the  $l=0$  resonance. The phase shifts calculated from the single-level formula are shown in Fig. 3 together with the phase shifts extracted in the analysis. The use of the above values of  $E_\lambda$  and  $\gamma_\lambda^2$  yields a resonant energy of approximately  $4.9$  Mev, a value consistent with that obtained by noting the energy at which  $\beta_0$  crosses  $90^\circ$ . Since there is considerable latitude in the choice of  $E_\lambda$  and  $\gamma_\lambda^2$ ,  $E_R$  is known only to within a few hundred kilovolts. A rough estimate of the order of magnitude of the laboratory width indicates that it is greater than  $1$  Mev.

An  $E_\lambda$  of  $4.2$  Mev and a  $\gamma_\lambda^2$  of  $4.4$  Mev-cm were selected for the  $l=2$  resonance. Because of the existence of a narrow  $2^+$  level at about  $6.03$  Mev, the two-level formula was used to calculate the phase-shift curve shown in Fig. 4 together with the phase shifts extracted in the analysis. The values of  $E_\lambda$  and  $\gamma_\lambda^2$  for the narrow level were deduced by means of the single-level formula with a constant amount of  $\beta_2$  contributed by the broad resonance. The portion of the curve dominated by the narrow  $2^+$  level is not shown. The values of  $E_\lambda$  and  $\gamma_\lambda^2$  for the broad  $l=2$  level give a resonant energy consistent with that obtained by noting the energy at which  $\beta_2$  crosses  $90^\circ$ . The laboratory width for this level is also greater than  $1$  Mev.

The region between  $5.0$  and  $5.6$  Mev is dominated by narrow resonances. The treatment of these is discussed in the next section. Superposition of the narrow resonances on the broad  $0^+$  and  $2^+$  resonances yields a very good fit to the experimental data between  $5.0$  and  $5.6$  Mev.

Between  $5.6$  and  $6.5$  Mev, the theoretical fit to the data based on the previous assumptions deteriorates. The theoretically predicted behavior of the resonant parts of the  $s$ - and  $d$ -wave phase shifts was calculated at 100-kilovolt intervals from the values of  $E_\lambda$  and  $\gamma_\lambda^2$  deduced in fitting the lower energy data. Small changes in these parameters were not sufficient to fit the data above  $5.6$  Mev. In order to maintain the fit to within  $5\%$  of the experimental data between  $5.6$  and  $5.9$  Mev

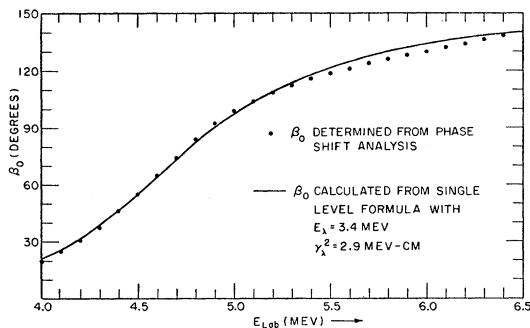


FIG. 3. Values of  $\beta_0$  calculated from  $E_\lambda$  and  $\gamma_\lambda^2$  together with extracted values.

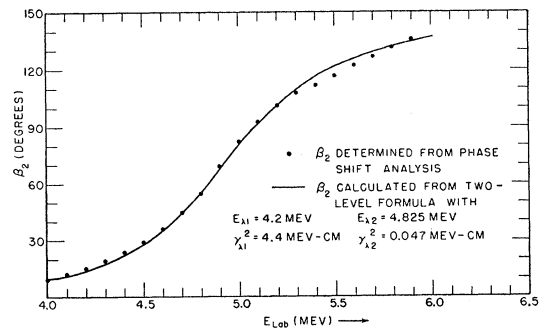


FIG. 4. Values of  $\beta_2$  calculated from  $E_\lambda$  and  $\gamma_\lambda^2$  together with extracted values.

TABLE I. Parameters of excited states in Ne<sup>20</sup> determined from the data of the present experiment.

$E_R(\text{Mev})_{\text{lab}}$	$\sim 4.9$	5.002	$\sim 5.1$	5.11	5.190	5.432	5.532	6.030
$E_{\text{exc}}(\text{Mev})_{\text{c.m.}}$ in Ne <sup>20</sup> (a)	$\sim 8.7$	8.755	$\sim 8.8$	8.84	8.905	9.099	9.179	9.577
$J^\pi$	0 <sup>+</sup>	1 <sup>-</sup>	2 <sup>+</sup>	(5 <sup>-</sup> )	1 <sup>-</sup>	4 <sup>+</sup>	3 <sup>-</sup>	2 <sup>+</sup>
$\Gamma_\lambda(\text{Mev})_{\text{lab}}$	$> 1.0$	0.003	$> 1.0$	$< 0.001$	0.023	0.004	0.002	0.030
$E_\lambda(\text{Mev})_{\text{c.m.}}$ in Ne <sup>20</sup> (a)	$\sim 8.1$	8.755	$\sim 9.0$	...	8.905	9.104	9.180	9.578
$\gamma_{\lambda\text{c.m.}}^2 \times 10^{18}(\text{Mev-cm})$	$\sim 2.9$	0.005	$\sim 4.4$	...	0.010	0.053	0.007	0.047
$\gamma_\lambda^2/(3\hbar^2/2\mu a)$ (%)	$\sim 80$	0.15	$\sim 120$	...	1.0	1.4	0.19	1.3

<sup>a</sup> The values of  $E_{\text{exc}}$  and  $E_\lambda$  are referred to the ground state of Ne<sup>20</sup>. The mass values used are those given by F. Ajzenberg and T. Lauritsen [Nuclear Phys. 11, 1 (1959)].

at 90°, it was necessary to introduce a small amount of  $g$ -wave phase shift. The amount of  $g$ -wave phase shift needed increases slowly with energy as would be expected if there were a broad  $l=4$  resonance above the limit in energy of the experimental data. Without this  $g$ -wave contribution, a fit to the data above 6.0 Mev requires a  $d$ -wave phase shift which differs markedly from its theoretically predicted value. Use of the predicted  $d$ -wave phase shift, on the other hand, leads at 90° to a theoretical cross-section curve that deviates from the experimental one towards the end of the range studied by more than 100%. Some  $g$ -wave contribution also helps the fit to the rising cross section at 168.9° and 125.3°. Because of the weighting factor  $(2l+1)$  in the cross-section formula, a small amount of  $g$ -wave phase shift has a large effect at angles where  $P_4$  is not too small. At 6.4 Mev approximately 12° of resonant  $g$ -wave phase shift has been included in obtaining the fit to the data shown in Fig. 2. The existence of another broad level above the range of the experimental data is not implausible when one examines the rapid fall of the 90° data, the steady rise of the 125.3° data, and the more rapid rise of the 168.9° data. The 140.8° and 149.4° data show little change in this range. These observations are all consistent with the possible existence of a broad  $l=4$  resonance above the range of the experimental data.

Between 6.0 and 6.5 Mev, there is an increasing divergence between the experimental and theoretical cross sections. This discrepancy could easily arise from the existence of narrow levels just above the upper end of the data. The existence of such unknown levels would not only affect the fit to the experimental cross section, but would also lead to errors in the choice of the broad level phase shifts. These errors would in turn affect the shapes of the phase-shift curves when plotted against energy. Because of inadequate knowledge of the region above the limit of the data, no additional assumptions have been introduced in the analysis. At the highest energy analyzed, the fit is within 15% at all angles.

### C. Narrow Resonances

The analysis of well-separated narrow levels is a much simpler and far less ambiguous process than that of overlapping broad levels. The following tentative assignments for the narrow resonances observed may

be made from an inspection of the data. The resonances which appear at about 5.00 Mev and 5.19 Mev appear at all angles except 90°. The disappearance at 90° immediately indicates an odd-parity resonance. Because this angle is the only one at which these resonances vanish, the assignment of 1<sup>-</sup> is suggested. The resonance at 5.43 Mev can be seen at 90° and therefore must have even parity. Since it vanishes at 149.4° an assignment of 4<sup>+</sup> may be made. The resonance at 5.53 Mev is not present at 90° or 140.8°. The disappearance at both 90° and 140.8° labels it as a 3<sup>-</sup> level. The resonance at 6.03 Mev vanishes only at 125.3° and thus an assignment of 2<sup>+</sup> is the only suitable one. The small resonance at 5.11 Mev appears very weakly at 168.9°, 149.4°, and 140.8°. It is too large to be due to the presence of isotopes other than O<sup>16</sup> or to impurities. The disappearance at 90° indicates an odd parity resonance. The Legendre polynomial for  $l=5$  vanishes at approximately 123°, which is very near to 125.3°. The fact that the resonance does not appear at either 90° or 125.3° but appears at the other angles suggests 5<sup>-</sup> as the lowest possible assignment. It is clear that this resonance was not resolved by the apparatus since the observed maximum cross sections at the different angles are much smaller than the calculated values for a 5<sup>-</sup> level.

A geometric interpretation of the cross-section formula described by Laubenstein and Laubenstein<sup>13</sup> provides a very effective method of analysis for the relatively sharp resonance levels. This procedure was applied in detail at all angles to every sufficiently resolved narrow resonance included in the data. The assignments based on visual examination of the data were verified by this method for five of the narrow resonances. The parameters deduced from this analysis are included in Table I. Since this procedure cannot be applied to the unresolved resonance at 5.11 Mev, the assignment 5<sup>-</sup> has not been confirmed analytically. An assignment of 7<sup>-</sup> or higher odd parity is ruled out because the low value of the penetrability at this energy yields a value of the reduced width several times the Wigner single-particle limit. The width in the laboratory system of the level is estimated to be less than a kilovolt.

<sup>13</sup> R. A. Laubenstein and M. J. W. Laubenstein, Phys. Rev. 84, 18 (1951).

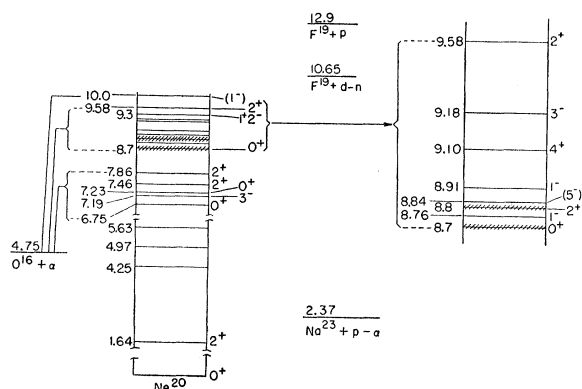


FIG. 5. Energy level diagram of  $Ne^{20}$ . The levels found in the present experiment are shown in an expanded scale on the right.

### V. RESULTS AND CONCLUSIONS

As a result of this experiment, eight previously unobserved states in  $Ne^{20}$  in the region of excitation between 7.9 and 9.9 Mev have been found and identified. Six of the resonances are relatively sharp, while the two others overlap and are very broad. Interpretation of a phase-shift analysis of the data in the context of Wigner-Eisenbud dispersion theory has resulted in the assignment of the spins, parities, resonant energies, and widths of the levels. The characteristic energies and alpha-particle reduced widths have also been deduced and the latter have been compared

to the Wigner single-particle limit of 3.66 Mev-cm. In addition to the broad  $0^+$  and  $2^+$  levels, an additional broad resonance in the region above the energy range of the data seems likely. The results of the analysis are shown in Table I and also in Fig. 5, which is an energy level diagram of  $Ne^{20}$  up to 10-Mev excitation. The five levels between 6.75 and 7.86 Mev were investigated by Cameron.<sup>5</sup> The positions of the four lowest excited states have been obtained from the work of Buechner and Sperduto.<sup>14</sup> The 9.2-Mev level reported by Ferguson and Walker<sup>6</sup> as  $1^-$  does not appear.

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<sup>14</sup> W. W. Buechner and A. Sperduto, Phys. Rev. **106**, 1008 (1957).

## Isomerism of Silver-108

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A long-lived isomer of  $Ag^{108}$  has been detected in old  $Ag^{110m}$  samples. The isomer decays with a half-life  $\geq 5$  years. Gamma- and beta-ray spectrometer data show that 90% of the disintegrations proceed by electron capture followed by a cascade of three gamma rays of 616-, 722-, and 434-kev energy, while 10% go by isomeric transition to  $Ag^{108}$ . New values are given for the branching ratios of 2.4-minute  $Ag^{108}$ .

### I. INTRODUCTION

BASED on the data available for the even mass number isotopes of rhodium, silver, and indium, systematics indicate an isomer should be expected for  $Ag^{108}$  with a half-life comparable to the 270-day  $Ag^{110m}$  and with the isomeric level at  $\sim 100$  kev above the ground state. This isomer has been observed recently in a sample of  $Ag^{110m}$  obtained in 1951 from the Isotope Division of Oak Ridge, the  $Ag^{110m}$  activity having decayed by now to  $\sim 0.05\%$  of its initial value. A difference in neutron energy dependence for production of these two long-lived silver isomers could account for the discrepancies reported in the half-life of  $Ag^{110m}$ .

### II. EXPERIMENTAL

The silver activities were studied with a one hundred-channel pulse-height analyzer and associated equipment described elsewhere.<sup>1</sup> The gamma-ray detector was a 3-in.  $\times$  3-in. NaI(Tl) crystal whose calculated photopeak efficiencies were checked by  $4\pi$  beta counting.

The irradiations to obtain the 2.4-minute  $Ag^{108}$  were performed using the pneumatic tube facilities of the Ford Nuclear Reactor at the University of Michigan.<sup>1</sup> An irradiation time of two minutes at a thermal neutron flux of  $\sim 1.4 \times 10^{12}$  n  $cm^{-2}$   $sec^{-1}$  was used and the shorter-

<sup>1</sup> W. W. Meinke, Nucleonics **17**, No. 9, 86 (1959).