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Pion-Pion Scattering in the φ^4 Theory*

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Pion-pion scattering has been calculated using the determinantal method, assuming that a relativistic $(\lambda/4)(\varphi_i\varphi_i)^2$ coupling is responsible for the interaction. The scattering amplitude for the individual partial waves is expressed as a ratio of two power series and terms through λ^3 have been kept in each series. Numerical results for the S and P waves have been obtained. λ is adjusted by attempting to fit the electromagnetic structure of nucleons. The best value of λ obtained by this fit is unfortunately so large that the validity of the determinantal approximation is doubtful.

I. INTRODUCTION

Of the various elementary particle interactions, the π-π interaction is one of the most basic theoretically, and at the same time one of the most nebulous experimentally. It is evident that this interaction affects, more or less directly, all processes involving pions and nucleons, or, in fact, pions and anything else. Nevertheless, it is very difficult to isolate these effects in any clean-cut way.

It has been mentioned many times that scattering of the two outgoing pions in the processes $\pi+N\rightarrow\pi+\pi+N$ and $\gamma+N\rightarrow\pi+\pi+N$ may produce important effects in the cross sections for these processes, but a reliable theoretical calculation of these effects is difficult to make. It seems that a strong π-π interaction is not inconsistent with present experimental and theoretical uncertainties.[1] The fact that the various models of pion-nucleon scattering agree quite well with experiment without including a π-π coupling has been suggested as evidence that the π-π coupling is weak, but no one knows what effects, if any, a strong π-π interaction would produce in the $\pi+N\rightarrow\pi+N$ reaction.

The first suggestion which may allow one to pick out the π-π scattering process explicitly from a possible experiment has come from the observation[2] that there should be a pole in the nonphysical region for the reaction $\pi+N\rightarrow\pi+\pi+N$ which has a residue related to the π-π cross section. If the pion production could be measured sufficiently well to permit an extrapolation to the pole position, one might hope to see π-π scattering.

Another, considerably less precise, way in which a handle might be obtained experimentally on π-π scattering is through the nucleon-electromagnetic form factors. Theoretical analyses of this problem[3] show that in the so-called two-pion approximation the isotopic vector form factors should depend on the pion electromagnetic form factor, which can in turn be expressed as an integral over the $J=1$, $T=1$ phase shift for π-π scattering. The accuracy of the two-pion approximation is, of course, an unknown quantity, but the nucleon form factors are fairly well known experimentally, so that at present this might be the place to try to pin down the π-π interaction. It seems that, if the two-meson approximation is accepted, the requirement that the P-wave π-π scattering go through a resonance at a one-pion center-of-mass energy of about two-pion rest masses produces agreement with the measured nucleon structure.

In view of the above discussion, it would appear that the most sensible approach to take at present toward the π-π problem is a purely theoretical one; that is, to guess what the coupling producing the π-π interaction is, and to try to make a reasonable estimate of the consequences of this coupling. Once such a purely theoretical guess about π-π scattering is made, then one could try to check its validity by comparison with some of the experimental handles which have just been mentioned.

The first question to which an answer must be given is what kind of a coupling is responsible for the π-π interaction. If one believes in the standard γ_5 meson theory, there are contributions to π-π scattering from Feynman graphs of the type shown in Fig. 1(a), for example. However, in order to avoid infinite results in computing such diagrams, it is necessary to introduce a point π-π interaction of the form $\frac{1}{4}\lambda_0(\varphi_i\varphi_i)^2$ where φ_i are the three Hermitian components of the pion field. In lowest order this interaction produces a π-π scattering as shown in Fig. 1(b). The coupling constant λ_0 is adjusted to remove the infinite contribution of the graph of Fig. 1(a). For example, in lowest order, if the graph of Fig. 1(a) is called $I(E,\theta)$, then the amplitude represented by Figs. 1(a) and 1(b) is $I(E,\theta)+\lambda_0$.

This is rewritten as

$$I(E,\theta)-I(E_0\theta_0)+I(E_0\theta_0)+\lambda_0=I_c(E,\theta)+\lambda,$$

which defines the renormalized coupling constant λ and the finite remainder $I_c(E,\theta)$.

The effective contribution of Fig. 1(a) is then its value minus its value at some fixed energy—thus if the energy variation of Fig. 1(a) were slow, the diagram would in general have a rather small effect. One might

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[1] L. Rodberg, Phys. Rev. Letters **3**, 58 (1959).
[2] G. Chew and F. Low, Phys. Rev. **113**, 1640 (1958).
[3] P. Federbush, M. L. Goldberger, and S. B. Treiman, Phys. Rev. **112**, 642 (1958). W. Fraser and J. Fulco, Phys. Rev. **117**, 1609 (1960).

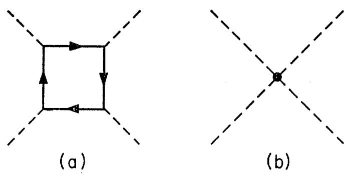

FIG. 1. Lowest order Feynman graphs for π-π scattering (a) due to pion-nucleon coupling and (b) due to direct π-π coupling.

expect this to be the case in regions where the total energy is much less than the nucleon mass. In such a case, one would be left only with the (renormalized) point coupling of Fig. 1(b).

The above discussion is not intended to be a real justification of the use of only a $(\lambda/4)(\varphi_i\varphi_i)^2$ coupling in calculating π-π scattering; nevertheless, it does suggest that the predictions of a pure $(\lambda/4)(\varphi_i\varphi_i)^2$ theory will be interesting, and could even more or less represent physical reality. Calculations of π-π scattering based on the Mandelstam representation[4] also effectively omit the contributions of diagrams like Fig. 1(a) except for the renormalization effects, although such an approach does not need to specifically discuss what kind of interaction Hamiltonians are to be used, and is not conveniently expressed in terms of Feynman graphs.

The next point which must be taken up is how to calculate with a $(\lambda/4)(\varphi_i\varphi_i)^2$ coupling. Perturbation theory is attractively simple, but likely to be wildly inaccurate if λ is of any appreciable size. An approximation scheme which is essentially no more complicated than perturbation theory is the determinantal method.[5] This method is based on the observation that the accuracy of a power series expansion depends on the properties of the quantity to be expanded; thus a power series expansion of the scattering amplitude itself (perturbation theory) may be poor, but it may yet be possible to expand successfully some other quantity from which the scattering amplitude can be calculated. Thus one would like to express the scattering amplitude in terms of a quantity which is an entire function of the coupling constant. Such a quantity can be found in potential theory. It is possible to write the scattering amplitude, for a particular partial wave, as

$$T(E)=(1/\pi)\sin\delta(E)e^{i\delta(E)}=r(E)/D(E), \tag{1}$$

where

$$D(E)=1-\int_0^\infty \frac{r(E')dE'}{E'-E-i\epsilon}. \tag{2}$$

$r(E)$ is given by a power series expansion in the potential strength λ which converges in the entire λ plane for a wide class of potentials. This solution of the potential scattering problem is equivalent to the Fredholm solution of the scattering integral equation. The positions E_B of the bound states can also be determined from the equation $D(E_B)=0$. In field theory it is possible to define a quantity analogous to $r(E)$, but nothing can be proved regarding the validity of its power series expansion. Nevertheless the use of this approximation scheme in the static meson theory produces good results. It may be hoped, therefore, that using the determinantal method to compute the consequences of the $(\lambda/4)(\varphi_i\varphi_i)^2$ theory may be reasonably reliable.

II. CALCULATION

For the relativistic field theory, Eqs. (1) and (2) are conveniently replaced by

$$T(\omega)=(1/\pi)\sin\delta(\omega)e^{i\delta(\omega)}=r(\omega)/D(\omega), \tag{1'}$$

$$D(\omega)=1-(\omega^2-\omega_0{}^2)\int_{\mu^2}^\infty \frac{r(\omega')d\omega'^2}{(\omega'^2-\omega_0{}^2)(\omega'^2-\omega^2-i\epsilon)}, \tag{2'}$$

where ω is the center-of-mass energy of a single meson in the π-π scattering process, and $\delta(\omega)$ is the phase shift in a given angular momentum and isotopic spin state. ω_0 is the energy at which the renormalization discussed earlier is to be carried out. The specific choice of ω_0 is a matter of convenience; it has been taken to be $\omega_0{}^2=\mu^2/3$ for reasons to be indicated later. Finally, the relativistic invariant variable ω^2 replaces the energy E of the nonrelativistic problem.

The approximation now requires obtaining the power series expansion in λ of $r(\omega)$ to some given order. The mechanism for obtaining this expansion is as follows. Expand $T(\omega)$ to the required order—that is, carry out ordinary perturbation theory to the order desired. Thus

$$T(\omega)=T^{(1)}(\omega)+T^{(2)}(\omega)+T^{(3)}(\omega)+\cdots.$$

Then one clearly gets

0th order:

$$r^{(0)}(\omega)=0 \quad D^{(0)}(\omega)=1,$$

1st order:

$$r^{(1)}(\omega)=T^{(1)}(\omega),$$

$$D^{(1)}(\omega)=-(\omega^2-\omega_0{}^2)\int_{\mu^2}^\infty \frac{r^{(1)}(\omega'^2)d\omega'^2}{(\omega'^2-\omega_0{}^2)(\omega'^2-\omega^2-i\epsilon)},$$

2nd order:

$$r^{(2)}(\omega)=T^{(2)}(\omega)+T^{(1)}(\omega)D^{(1)}(\omega),$$

$$D^{(2)}(\omega)=-(\omega^2-\omega_0{}^2)\int_{\mu^2}^\infty \frac{r^{(2)}(\omega')d\omega'^2}{(\omega'^2-\omega_0{}^2)(\omega'^2-\omega^2-i\epsilon)},$$

3rd order:

$$r^{(3)}(\omega)=T^{(3)}(\omega)+T^{(2)}(\omega)D^{(1)}(\omega)+T^{(1)}(\omega)D^{(2)}(\omega),$$

$$D^{(3)}(\omega)=-(\omega^2-\omega_0{}^2)\int_{\mu^2}^\infty \frac{r^{(3)}(\omega')d\omega'^2}{(\omega'^2-\omega_0{}^2)(\omega'^2-\omega^2-i\epsilon)}.$$

If third order is sufficient, one stops here and computes the scattering from

$$\frac{1}{\pi}\sin\delta(\omega)e^{i\delta(\omega)}=T(\omega)=\frac{r^{(1)}(\omega)+r^{(2)}(\omega)+r^{(3)}(\omega)}{1+D^{(1)}(\omega)+D^{(2)}(\omega)+D^{(3)}(\omega)}.$$

[4] G. F. Chew and S. Mandelstam, Phys. Rev. (to be published).
[5] M. Baker, Ann. Phys. 4, 271 (1958).

The calculation of T to third order requires the evaluation of the Feynman graphs shown in Fig. 2. The resulting amplitudes must then be broken up into given angular momentum and isotopic spin states. We will confine our attention to the two S-states $T=0$ and $T=2$ and the P-state $T=1$. As mentioned earlier, the P-wave $T=1$ phase shift is the one of primary interest for the nucleon electromagnetic structure.

The lowest order graph, Fig. 2(a), produces only S-wave scattering, with the $T=0$ and the $T=2$ amplitudes in the ratio 5 to 2. In second order there is one graph, 2(b)I, which again gives only S-wave scattering, and two graphs, 2(b)II and 2(b)III, which give rise to scattering in all angular momentum states. If the meson four momenta are labelled q_1 to q_4, corresponding to the mesons 1 to 4, then diagram 2(b)I is the same function of $(q_1+q_2)^2$ that 2(b)II is of $(q_1-q_3)^2$ that 2(b)III is of $(q_1-q_4)^2$. Now when the renormalization is performed, the value of each graph at a specific point is to be subtracted and those subtracted pieces are to be used to renormalize the coupling constant of the lowest order graph. It can be seen by looking at the isotopic spin dependence that if the subtracted parts of the three second order graphs are all equal, the 5 to 2 ratio of the $T=0$ to $T=2$ parts of the lowest order graph will be preserved, that is, the renormalization of the $T=0$ and $T=2$ coupling constants is identical. This requires, then, that the renormalization point be chosen so that

$$(q_1+q_2)^2=(q_1-q_3)^2=(q_1-q_4)^2.$$

In the c.m. system, this means the choice $\omega_0^2=\mu^2/3$. One may easily convince oneself that with this renormalization point the 5 to 2 ratio at ω_0 is maintained in all orders.

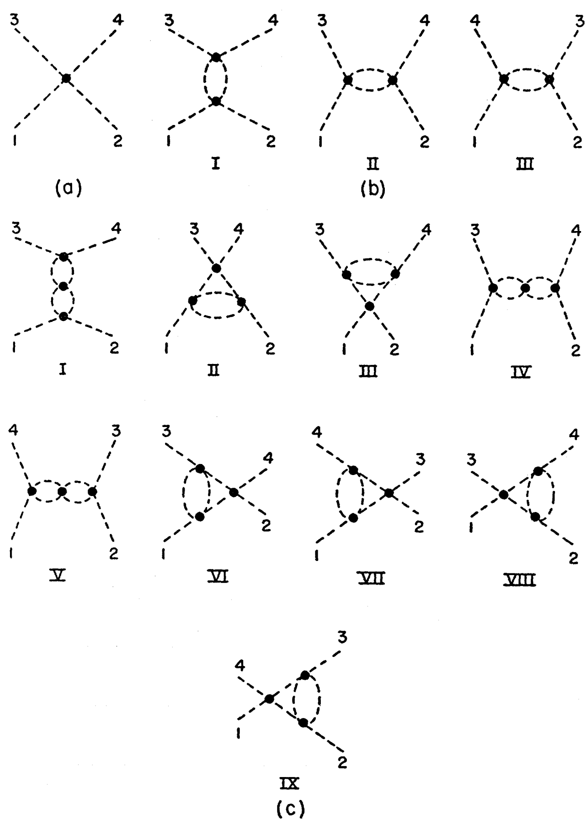

FIG. 2. Feynman graphs for π-π scattering in $\lambda\varphi^4$ theory. (a) order λ, (b) order λ^2, (c) order λ^3.

The calculation of the first and second order Feynman graphs and the necessary S- and P-wave projection of the graphs of Figs. 2(b)II and 2(b)III are straightforward, and the answers may be obtained analytically. The third order graphs like Fig. 2(c)VI are somewhat more complicated. One way to compute them is to note that they satisfy a dispersion relation. The absorptive part of the graph is just given by an angular integral over the appropriate first and second order matrix elements, which can be evaluated analytically. The final result can therefore be expressed as a single dispersion integral. The S- and P-wave parts can be picked out analytically, leaving one integral to be done numerically.

Altogether, then, the calculation of T to third order is relatively straightforward; $r^{(3)}$ is then obtained directly from T and the second order D. The third order is then evaluated by integrating $r^{(3)}$. This integration must be carried out numerically. The most complicated thing required, then, is the evaluation of some double numerical integrals.

III. RESULTS

The results depend on one parameter, the renormalized coupling constant λ. To determine λ it is necessary to attempt to use the π-π results obtained above to fit some experiment. As explained in the introduction, however, it is very difficult to find an experimental situation in which the π-π scattering is clearly discernible.

At present, perhaps the most reasonable way to evaluate λ is to attempt to use the π-π interaction to achieve a fit to the electromagnetic form factors of nucleons. This determination of λ, of course, rests on a number of somewhat dubious theoretical assumptions, particularly the assumption that it is valid to neglect all but the two-pion intermediate state in evaluating the form factors.

The determination has been made by setting the imaginary part of the isotopic vector moment form factor equal to its Born approximation value times the absolute square of the pion form factor, an approximation which is believed to be reasonably good.[3] To the extent of the validity of the two-pion approximation, the pion form factor is precisely $eD_{11}(0)/D_{11}(\omega)$ where 11 means $J=1$ and $T=1$. Figure 3 shows the resulting nucleon isotopic vector moment form factor, obtained from its dispersion relation, for $\lambda/4\pi=2$ and $\lambda/4\pi=2.5$; the corresponding pion form factors are shown in Fig. 4. The curves in Fig. 3 are all normalized to unity at $q^2=0$, so

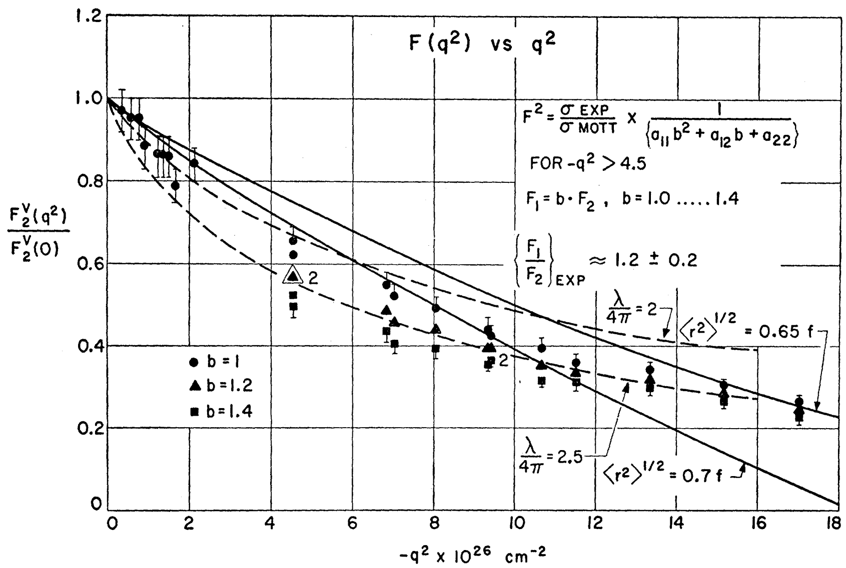

FIG. 3. The nucleon isovector moment form factor. The dashed lines are the theoretical results on the basis of the two-pion approximation using the π-π scattering phase shift obtained here for $\lambda/4\pi=2$ and $\lambda/4\pi=2.5$. The theoretical curve is normalized to unity at $q^2=0$. The two solid curves are attempted fits with zero π-π scattering with one extra subtraction made and the subtraction constant $\langle r^2\rangle^{\frac{1}{2}}$ used as a parameter. The low $q^2(-q^2<2)$ experimental points are measurements of F_1, not F_2. The remaining points are experimental values of F_2.

that we have only attempted to fit the shape of the experimental curve. Because of the experimental uncertainties and the theoretical approximations involved in this comparison, only a rough determination of λ can be made. Values of $\lambda/4\pi$ between 2 and 2.5 produce good agreement with the experimental shape. Without inclusion of the pion structure, i.e., if $\lambda=0$, the theoretical curves lie considerably above the experimental data. Even when an extra subtraction is made and the subtraction constant is treated as a parameter, the shape of the $\lambda=0$ curve cannot be fitted to the experimental points (see solid curves of Fig. 3). The predicted value of the isotopic vector anomalous magnetic moment $F_2^v(q^2)|_{q^2=0}$ is rather insensitive to λ for $\lambda/4\pi$ between 2 and 2.5. For λ in this range the theoretical result for $F_2^v(0)\approx 0.8e/2M$, compared to the experimental value of $1.85e/2M$.

Altogether then, we can say that our P-wave π-π phase shift improves the general situation in the electromagnetic structure problem and the remaining discrepancies might be due to the inadequacy of the 2-meson approximation or of our P-wave phase shift, or both. There is, of course, also the possibility that the limited agreement we have obtained is fortuitous.

Figure 5 shows the S-wave $T=0$ and $T=2$ phase shifts, and Fig. 6 shows the P-wave $T=1$ phase shift for $\lambda/4\pi=1$, 2 and 2.5. We have not investigated what implications our S-wave π-π phase shifts might have on other experiments.

Now we come to the question of the accuracy of our calculation of these phase shifts. At $\lambda/4\pi=2$, the third order contribution to the P-wave r is about three times as big as the second order contribution. The low-energy S waves are not so bad, but the third order pieces are still comparable to the second order for $\lambda/4\pi=2$. There is, therefore, no reason to believe that the results would not be changed considerably by going to higher order. For the S states an additional difficulty occurs. Since

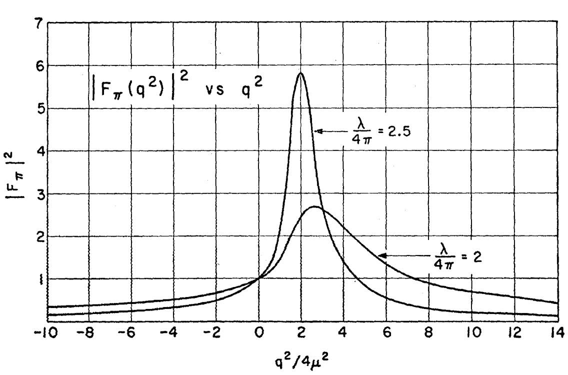

FIG. 4. The pion form factor predicted here with $\lambda/4\pi=2$ and $\lambda/4\pi=2.5$.

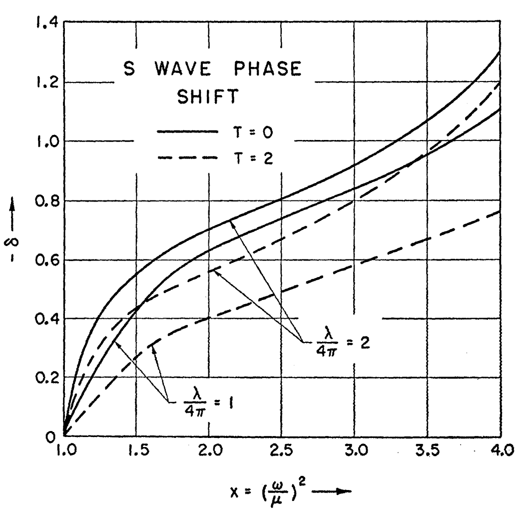

FIG. 5. The S-wave $T=0$ and $T=2$ phase shifts.

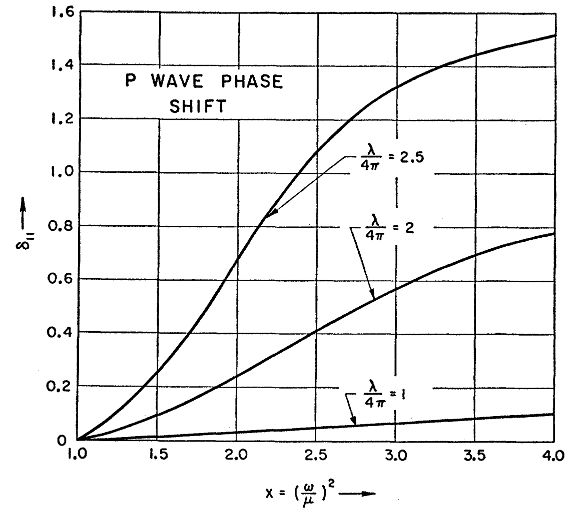

FIG. 6. The P-wave $T=1$ phase shift.

the equation $D(\omega_B^2)=0$ determines the bound-state energies, any consistent approximation to D should not have zeros for negative values of ω^2. For attractive $\lambda/4\pi=-2$ the S-wave $T=0$ D goes through zero at $\omega^2\approx 0.75\mu^2$, predicting an honest bound state. However, for $\lambda/4\pi=+2$ the S-wave D's for both $T=0$ and $T=2$ vanish below $\omega^2=0$, which suggests a further inadequacy of the approximation. For $\lambda/4\pi=1$, the S-wave results converge quite well.

To summarize then, the $(\lambda/4)(\varphi_i\varphi_i)^2$ theory coupled with the determinantal approximation predicts a π-π P phase shift which can be used to improve the agreement of the nucleon electromagnetic structure with experiment. Unfortunately, for values of λ which are needed for this improvement, the determinantal approximation does not look very satisfactory, particularly for $\omega^2\geqq 4\mu^2$. Thus from a theoretical point of view we cannot say to what extent our predictions represent the content of the $(\lambda/4)(\varphi_i\varphi_i)^2$ theory. We must then seek further experimental consequences of our phase shifts, such as their influence on $\pi-N$ phenomena, in order to test their validity.

ACKNOWLEDGMENTS

We should like to thank M. Goldberger for calling our attention to this problem, and S. Mandelstam, W. Frazer, and S. Drell for some interesting conversations.

APPENDIX

Just for the sake of general interest we exhibit here the results of a perturbation calculation of T to third order in λ. The functions r and D for the various angular momentum and isotopic spin states are easily obtained from this, as indicated earlier.

S-wave $T=0$.

$$T_{00}=-\frac{1}{16\pi^2}\left(\frac{q}{\omega}\right)\left[5\lambda+5\left(\frac{\lambda}{4\pi}\right)^2(5I_2+3A_2{}^0)+\frac{5}{4\pi}\left(\frac{\lambda}{4\pi}\right)^3(25I_2{}^2+11A_3{}^0-30I_3-22B_3{}^0)\right],$$

where $q^2=(\omega^2-\mu^2)^{\frac{1}{2}}$, and

$$I_2(\omega)=\frac{q}{\omega}\left[2\cosh^{-1}\left(\frac{\omega}{\mu}\right)-i\pi\right]-c_0-2,$$

$$A_2{}^0(\omega)=2\left\{3+c_0-2\frac{\omega}{\mu}\left(\frac{\sinh^{-1}(q/\mu)}{q/\mu}\right)-\left(\frac{\sinh^{-1}(q/\mu)}{q/\mu}\right)^2\right\},$$

$$A_3{}^0(\omega)=\int_{-1}^{1}d\alpha\left\{2\left[\left(1+\frac{2\mu^2}{q^2(1-\alpha)}\right)^{\frac{1}{2}}\right]\times\sinh^{-1}\left[\left(\frac{q^2(1-\alpha)}{2\mu^2}\right)^{\frac{1}{2}}\right]-2-c_0\right\}^2,$$

$$I_3(\omega)=\left(\omega^2-\frac{\mu^2}{3}\right)\times\int_{\mu^2}^{\infty}\frac{d\omega'^2}{(\omega'^2-\mu^2/3)(\omega'^2-\omega^2-i\epsilon)}\frac{q'}{\omega'}A_2{}^0(\omega'),$$

$$B_3{}^0(\omega)=2\int_{\mu^2}^{\infty}d\omega'^2\frac{q'}{\omega'}A_2{}^0(\omega')\times\left\{\frac{1}{\omega'^2-\mu^2/3}-\frac{1}{q^2}\ln\left(1+\frac{q^2}{\omega'^2}\right)\right\}$$

$$c_0=2[\sqrt{2}\sin^{-1}(1/\sqrt{3})-1].$$

S-wave $T=2$.

$$T_{02}=-(1/16\pi^2)(q/\omega)[2\lambda+(\lambda/4\pi)^2(4I_2+9A_2{}^0)+(1/4\pi)(\lambda/4\pi)^3(8I_2{}^2+43A_3{}^0-36I_3-56B_3{}^0)].$$

For completeness we also list below r for the S-wave states, in third order.

$$r_{00}=-(1/16\pi^2)(q/\omega)[5\lambda+15(\lambda/4\pi)^2A_2{}^0+(5/4\pi)(\lambda/4\pi)^3[11A_3{}^0-22B_3{}^0-15(I_3+I_2A_2{}^0)]],$$

$$r_{02}=-(1/16\pi^2)(q/\omega)\{2\lambda+9(\lambda/4\pi)^2A_2{}^0+(1/4\pi)(\lambda/4\pi)^3[43A_3{}^0-56B_3{}^0-18(I_3+I_2A_2{}^0)]\}.$$

Note that the combination $I_3+I_2A_2{}^0$ has a particularly simple form—namely

$$I_3+I_2A_2{}^0=\left(\omega^2-\frac{\mu^2}{3}\right)\int_{\mu^2}^{\infty}\frac{d\omega'^2}{(\omega'^2-\mu^2/3)(\omega'^2-\omega^2)}\frac{q'}{\omega'}\times[A_2{}^0(\omega')-A_2{}^0(\omega)].$$

To a given order r is considerably simpler than T. There are no singular integrals appearing in the expression for r; the singularities in $T^{(3)}$ being canceled by $D^{(2)}T^{(1)}+T^{(2)}D^{(1)}$ etc.

P-wave $T=1$:

$$T_{11}=-(1/16\pi^2)(q/\omega)[5(\lambda/4\pi)^2A_2{}^1+(5/4\pi)(\lambda/4\pi)^3(7A_3{}^1-4B_3{}^1)],$$

where

$$A_2{}^1(\omega)=-\left(\frac{\mu}{q}\right)^2\left[\left(\frac{q}{\mu}\right)^2-1+2\frac{\omega}{\mu}\left(\frac{\sinh^{-1}(q/\mu)}{q/\mu}\right)-\left[1+2\left(\frac{q}{\mu}\right)^2\right]\left(\frac{\sinh^{-1}(q/\mu)}{q/\mu}\right)^2\right],$$

$$A_3{}^1(\omega)=\int_{-1}^{1}\alpha d\alpha\left\{2\left[\left(1+\frac{2\mu^2}{q^2(1-\alpha)}\right)^{\frac{1}{2}}\right]\times\sinh^{-1}\left[\left(\frac{q^2(1-\alpha)}{2\mu^2}\right)^{\frac{1}{2}}\right]-2-c_0\right\}^2,$$

$$B_3{}^1(\omega)=-\frac{2}{q^2}\int_{\mu^2}^{\infty}d\omega'^2\frac{q'}{\omega'}A_2{}^0(\omega')\times\left\{\left(1+\frac{2\omega'^2}{q^2}\right)\ln\left(1+\frac{q^2}{\omega'^2}\right)-2\right\}.$$

For the P-wave, to order λ^3, $r=T$ since $T=0$ in first order in λ.

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Foldy Transformation in the Pion-Hyperon System*

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A unitary transformation, which plays the same role as the Foldy transformation in the pion-nucleon system, is constructed for the case where the pion interacts with both Σ and Λ hyperons through γ_5 couplings. The transformation function and the transformed Hamiltonian are very similar to those of the Foldy transformation, in spite of the complexity of our system in isotopic spin space. The application to practical problems is not considered in this paper.

We consider a system where the pion interacts with both Σ and Λ hyperons through charge independent γ_5 couplings. The interaction Hamiltonian is

$$H_I=H_{\Sigma\Sigma\pi}+H_{\Sigma\Lambda\pi}, \quad (1)$$

with

$$H_{\Sigma\Sigma\pi}=if_\Sigma\int d\mathbf{x}\ \boldsymbol{\psi}_\Sigma{}^*\rho_2\times\boldsymbol{\psi}_\Sigma\cdot\boldsymbol{\varphi},$$

$$H_{\Sigma\Lambda\pi}=f_\Lambda\int d\mathbf{x}\ (\psi_\Lambda{}^*\rho_2\boldsymbol{\psi}_\Sigma\cdot\boldsymbol{\varphi}+\boldsymbol{\psi}_\Sigma{}^*\rho_2\psi_\Lambda\cdot\boldsymbol{\varphi}).$$

Here the bold faced letters represent vectors in isotopic spin space.

We wish to find a unitary transformation,

$$H'=e^{iS}(H_{\text{free}}+H_I)e^{-iS}, \quad (2)$$

which eliminates the ρ_2 components completely from the sum $H_I+H_{\text{h.m.}}$, where H_I represents the interaction and $H_{\text{h.m.}}$ the hyperon-mass terms in the free Hamiltonian H_{free}. According to Foldy[1] such a transformation corresponds to a certain rotation around the ρ_1 axis in ρ space. Then, we assume[2]

$$\begin{aligned}S&=S_1+S_2,\\ S_1&=-i\int d\mathbf{x}\ \boldsymbol{\psi}_\Sigma{}^*\rho_1\times\boldsymbol{\psi}_\Sigma\cdot\boldsymbol{\varphi}[\chi(\phi)/\phi],\\ S_2&=\int d\mathbf{x}\ \boldsymbol{\psi}_\Sigma{}^*\cdot\boldsymbol{\varphi}\rho_1\psi_\Lambda[\omega(\phi)/\phi]+\text{H.c.},\end{aligned} \quad (3)$$

with $\phi=(\boldsymbol{\varphi}\cdot\boldsymbol{\varphi})^{\frac{1}{2}}$. Here χ and ω are odd functions of ϕ only and correspond to the angle of rotation in ρ space. Using formulas (A-6) in the Appendix, we see that

$$\begin{aligned}e^{iS}[H_{\text{h.m.}}+H_I]e^{-iS}&=-i\int d\mathbf{x}\ \boldsymbol{\psi}_\Sigma{}^*\rho_2\times\boldsymbol{\psi}_\Sigma(m_\Sigma\sin2\chi-f_\Sigma\phi\cos2\chi)\\&\quad+\left\{\int d\mathbf{x}\ \boldsymbol{\psi}_\Sigma{}^*\cdot\boldsymbol{\varphi}\rho_2\phi^{-1}\times[\tfrac{1}{2}(m_\Sigma+m_\Lambda)\sin2\omega-f_\Lambda\phi\cos2\omega]\psi_\Lambda+\text{H.c.}\right\}\\&\quad+\text{terms proportional to }\rho_3.\end{aligned}$$

* Supported by a grant from the National Science Foundation.
† On leave of absence from Hokkaido University, Sapporo, Japan.
[1] L. L. Foldy, Phys. Rev. 84, 168 (1951). J. M. Berger, L. L. Foldy, and R. K. Osborn, Phys. Rev. 87, 1061 (1952).

[2] In (3) we may add one more independent term;

$$S_3=\int(\boldsymbol{\psi}_\Sigma{}^*\rho_1\boldsymbol{\varphi})(\boldsymbol{\varphi}\boldsymbol{\psi}_\Sigma)\chi'd\mathbf{x},$$

where χ' is an even function of ϕ. Since this term gives no change in the final results, we omit it for the simplicity.