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X-Ray Yields from μ-Mesonic Atoms*

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The interesting suggestion has been made that the rapid drop in the yield of mesonic K x rays in the light elements may be associated with the capture of μ mesons into the metastable $2s$ state. The mechanisms for making transitions from the $2s$ to the $1s$ state and from various p states into the $2s$ state have been investigated in detail for Li, Be, and B. It is found that the paradoxical reduction of K x rays remains unexplained. (1) Stark mixing of the mesonic $2s$ and $2p$ states by the electric fields of the atomic electrons allows "mixed" Auger-radiative transitions to the $1s$ state to compete favorably with radiationless transitions. These mixed transitions give a high-energy x ray and a relatively negligible (10–50 ev) electronic excitation and so contribute to the observed K_α yield. (2) Even if the above "mixed" transitions are ignored, there is no mechanism which gets a large fraction of μ mesons into the $2s$ state that at the same time does not violently contradict both theoretical estimates and observed K x-ray yields from light π-mesonic atoms.

I. INTRODUCTION

IN condensed matter the long lifetime and weak nuclear interaction of a μ meson presumably ensures its reaching a K orbit before it decays or is captured by a nucleus. But precisely how such mesons are captured into atomic orbits and the details of their subsequent cascade down into the K state has been understood only imperfectly and without certainty. In fact in the case of the lighter elements (Li, Be, B, and C) there is a striking and paradoxical discrepancy between theoretical calculations and various reported data.[1]

These data concern the yields of those x rays which are radiated when a μ meson makes a transition into a K state or into an L state. In addition to such radiation there exists the possibility of Auger transitions in which the energy difference is carried away by a single high-energy electron. For transitions of mesons into the $1s$ state, Auger transitions are not expected to compete appreciably with radiation and a yield of close to 100% should be observed for the K x-ray yield. The measured yields[2] are given in Table I, where only the relative yields have the small quoted error. (Only those x rays are counted which are emitted within about 5×10^{-8} sec of the stopping of the μ meson, but this is very long compared to even the most conservative estimate of the time it could take to reach the $1s$ state.) It is apparent that there is a sharp drop in the K x-ray yield for the very light elements beginning with C. But the argument that radiation should proceed enormously faster than electronic Auger transitions is not a delicate one and is quite convincing. The mesonic orbits in the $n=2$ state have about 1/50 the radius of the surrounding K-state electrons. When a meson makes a $2p\rightarrow 1s$ transition the radiated dipole electric field is, as far as the electrons are concerned, identical to that of a point dipole located at the nucleus. An electronic Auger transition can begin to compete with a radiative transition only when the electron is much closer to the dipole than the wavelength of the emitted radiation, i.e., when $kR_e\ll1$ where $k=2\pi/\lambda$ and R_e is the expected radius of the electronic K state. With a μ meson in the $n=2$ state around a nucleus of charge Z, the K electrons see an effective nuclear charge $Z-1$ and

$$kR_e\approx\frac{3}{8}\left(\frac{\alpha\mu}{m}\right)\frac{Z^2}{Z-1}=0.6\frac{Z^2}{Z-1}. \quad (1)$$

Here μ/m is the ratio of μ meson to electron mass and α is the fine structure constant. Even for Li, $kR\sim3$ and we are enormously far from satisfying the criterion that Auger transitions reduce the K x-ray yield. A more detailed calculation confirms that the theoretical Auger

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[1] A recent and comprehensive review of experimental and theoretical work on mesonic atoms is that of D. West, *Reports on Progress in Physics* (The Physical Society, London, 1958), Vol. 21, p. 271. Further references to work before 1958 are contained here.

[2] M. B. and M. Stearns, Phys. Rev. **105**, 1573 (1957).

transition probabilities are about 300 times too small to explain the K x-ray yields reported in Table I.[2,3] A qualitatively similar difficulty exists for mesonic L x-ray yields where the calculated Auger transition rates are about 35 times too small to offer an explanation of the measured reduction in yields.[2] Such greatly reduced yields have been reported in both π[4,5] and μ atoms. If the Auger rate for transferring energy to the nearby K electrons is too small then transitions involving the more distant electrons in higher orbits or on neighboring atoms should be entirely negligible. Bernstein and Wu[6] have indeed shown that collisional deexcitation of the mesonic atom[7] cannot compete with radiation from the $2p$ state. The large energy released in transitions of a meson into the $1s$ state would certainly seem to rule out any mechanisms, other than radiation or Auger transitions, in which a single electron carries off the energy. The energy is much too high for collective electronic phenomena to be relevant while a single nucleus would have to acquire over 15 Mev/c momentum to absorb that much energy.

Since μ mesons will not be immediately captured from a low s state the absence of μ x-rays would seem to offer less possibility of explanation and we shall concern ourselves mostly with the μ-meson yields. Recently Krall and Gerjuoy[8] have made the very interesting suggestion that, as far as the K yields of μ mesons are concerned, perhaps the μ meson may be trapped in a $2s$ state from which a single quantum radiation is forbidden. This can happen only in the light elements where, because of the effect of vacuum polarization, the $2s$ state is slightly lower than the $2p$. For higher Z the finite nuclear size is more important than the vacuum polarization in splitting the mesonic $2p$ and $2s$ levels and the $2s$ energy is pushed higher than that of the $2p$ level.[9] In Table II various energy splittings are summarized. For Li, Be, and B the $2s_{\frac{1}{2}}$ state is the lowest $n=2$ state. (When the hyperfine splitting is included one of the $2s_{\frac{1}{2}}$ states of B^{11} is raised by 1 volt; the other is lower by 1.5 volts. Both s states are still lower than the lowest p state.) For C the $2p_{\frac{1}{2}}$ state is lowest and a

TABLE I. Yield of K x rays from μ-mesonic atoms.

Element	Li	Be	B	C	N
K_α energy (kev)	19	33	52	75	102
Yield	<0.16	0.33±0.03	0.46±0.04	0.60±0.04	0.82±0.07
Ratio K_α to all K x rays	···	0.78	0.78	0.80	0.77

TABLE II. Energy differences (in ev) between pairs of $n=2$ states in μ-mesonic atoms.[a]

	Li^7	Be^9	B^{11}	C^{12}
$2p_{\frac{1}{2}}-2s_{\frac{1}{2}}$	3	4.6	1.5	−7
$2p_{\frac{3}{2}}-2s_{\frac{1}{2}}$	3.7	7	7.3	+5
$2s_{\frac{1}{2}}$ hyperfine splitting	0.2	0.6	2.5	0

[a] The first two rows include only vacuum polarization and finite nuclear size effects.[9] The maximum p-state hyperfine splittings are less than $\frac{1}{3}$ those of the s state.

μ meson could not be trapped in the $2s$ state for long times[10]; but we shall postpone consideration of these atoms.

In the following sections two conclusions will be drawn about the significance of the metastability of the $2s$ state of μ mesons for the K x-ray yields in Li, Be, and B. First, even if all μ mesons did get into the $2s$ state, a kind of "mixed" radiative-Auger transition takes place with high probability and is experimentally indistinguishable from a simple radiative transition. This *alone* gives more apparent K x-ray yield in, for example, Li than has been observed. Second, the assumption that a majority of μ mesons get into the $2s$ state leads to a number of independent contradictions with calculated rates, and with the reported x-ray yields from π atoms.

II. "MIXED" TRANSITION RATE FOR $2s \rightarrow 1s$ TRANSITIONS

A μ meson in a $2s$ state around a point nucleus cannot get to the ground state by single photon radiation. Since in the light nuclei ($Z<6$) the $2p$ level is some few electron volts higher in energy than the $2s$ level, transitions to the $2p$ are not possible here. The two-photon transition rate is quite negligible in comparison to other processes which we shall consider. ($R_{2s\rightarrow 1s}{}^{2\gamma} \sim 1.6\times 10^3 Z^6$ sec^{-1}.)

The extra nuclear electrons may well immobilize the mesonic atom in the lattice and probably partially screen the μ orbit from the electric fields of extra atomic electrons and nuclei. The presence of these electrons in the μ mesonic atom suggest two mechanisms for the $2s\rightarrow 1s$ transition. One is the possibility of conventional Auger transitions which are discussed in the next section. The other is a kind of mixed Auger and radiative transition which, in light atoms, gives a large contribution to the $2s\rightarrow 1s$ transition rate. These "mixed" transitions result in a photon which carries away most ($\sim 99\%$) of the energy and a transfer of one unit of angular momentum together with a small amount of energy to the surrounding electrons. For the light elements these compete favorably with the canonical Auger effect only for $2s\rightarrow 1s$ transitions since the latter are greatly reduced in this case.

[3] G. R. Burbidge and A. H. de Borde, Phys. Rev. **89**, 189 (1953); M. Demeur, Nuclear Phys. **1**, 516 (1956).
[4] M. B. Stearns, M. Stearns, and L. Leipuner, Phys. Rev. **108**, 445 (1957).
[5] M. Camac, M. L. Halbert, and J. B. Platt, Phys. Rev. **99**, 905 (1955).
[6] J. Bernstein and Ta-You Wu, Phys. Rev. Letters **2**, 404 (1959).
[7] T. Day and P. Morrison, Phys. Rev. **107**, 912 (1957).
[8] N. Krall and E. Gerjuoy, Phys. Rev. Letters **3**, 142 (1959).
[9] L. Foldy and E. Eriksen, Phys. Rev. **95**, 1048 (1954).
[10] B. L. Ioffe and I. Ya Pomeranchuk. J. Exptl. Theoret. Phys. (U.S.S.R.) **23**, 123 (1952).

To calculate the "mixed" transition rate we consider a μ meson in a $2s$ state around a nucleus of charge Z; surrounding the meson and nucleus are n electrons whose combined wave function is designated $\psi_0(\mathbf{r}_1,\mathbf{r}_2,\cdots,\mathbf{r}_n)$. The interaction between the μ meson and the electrons is included in ψ_0 only in taking the effective nuclear charge seen by the electrons as $Z-1$, instead of Z. The residual perturbation is

$$H_1=e^2\sum_{1=j}^{n}\left(\frac{1}{|\mathbf{r}_j-\mathbf{r}_\mu|}-\frac{1}{|\mathbf{r}_j|}\right). \qquad (2)$$

The perturbation H_1 causes some small amount of mixing among the various states which a μ meson may have when the electrons are neglected (or even when their average charge distribution has been taken into account). The mixing is largest with those states whose energy is not too far from that of the unperturbed state. In particular the $2s$ state of the μ meson will mix appreciably with the $2p$ state which differs from it by an enormously smaller energy than that between the $2s$ and states with different n. Thus Δ_{2s-2p}, the energy difference between $2s$ and $2p$ for the μ atom, is only a few electron volts in the lighter atoms while $\Delta_{2s-3p}\sim 800Z^2$ ev.

The mixing of some $2p$ state with the $2s$ state of the μ (accompanied, of course, by mixing of excited electron states with ψ_0 to conserve angular momentum) modifies the selection rule which forbids a single quantum to be radiated in the transition to the $1s$ state.

The single quantum emission rate is simply the mixed-in fraction of $2p$ state times the normal $2p\rightarrow 1s$ radiation rate for the μ meson. This simple result follows from the excellent accuracy of two approximations. The average energy of excitation of the electron states accompanying the $2p$ μ-meson state is so much smaller (by roughly the electron-muon mass ratio) than the emitted photon energy that we may, with impunity, neglect its effect in slightly reducing the photon energy. Secondly, the mixture of the final $1s$ state with other states, as well as the mixture of the initial $2s$ state with states other than the $2p$ state give a quite negligible (less than 1%) correction to the calculated transition rate. The fraction of $2p$ mixing is given by

$$f=\sum_{m\neq 0}\frac{|\langle\psi_m,2p|H_1|\psi_0,2s\rangle|^2}{(E_m-E_0)^2}, \qquad (3)$$

where ψ_m is the mth excited state of the n electrons and $|\psi_0,2s\rangle$ is the state function for the μ meson in a $2s$ state and the electrons in the state ψ_0. Such a state function is an eigenfunction of the entire Hamiltonian minus H_1 and factors into the product $|\psi_0\rangle|2s\rangle$. Exactly similar considerations hold for $\langle\psi_m,2p|$. In Eq. (3) the $2s$ and $2p$ states are treated as exactly degenerate. For the cases of interest $E_m-E_0\geq 10(Z-1)^2$ ev which is much greater than the $2s-2p$ energy difference. Since $|\mathbf{r}_j|$ for any of the electrons is generally over 100 times larger than $|\mathbf{r}_\mu|$ we may use the expansion

$$\frac{1}{|\mathbf{r}_j-\mathbf{r}_\mu|}=\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{(l-|m|)!}{(l+|m|)!}P_l^m(\cos\theta_j)P_l^m(\cos\theta_\mu)\times e^{im(\varphi_j-\varphi_\mu)}\frac{|\mathbf{r}_\mu|^l}{|\mathbf{r}_j|^{l+1}}. \qquad (4)$$

After the substitution of Eq. (4) into Eq. (2) and Eq. (2) into Eq. (3) the result for f can be simplified to

$$e^4|\langle 2p|z|2s\rangle_\mu|^2\sum_{m\neq 0}\frac{\left|\left\langle\psi_m\left|\sum_{j=1}^{n}\frac{\mathbf{r}_j}{|\mathbf{r}_i|^3}\right|\psi_0\right\rangle\right|^2}{(E_m-E_0)^2}, \qquad (5)$$

where $|\langle 2p|z|2s\rangle_\mu|^2$ is the square of the matrix element of z between the $2s$ state and the $m=0$ $2p$ state for the μ meson and

$$|\langle 2p|z|2s\rangle_\mu|^2=(9/Z^2)(m/\mu)^2a_0^2, \qquad (6)$$

where a_0 is the electronic Bohr radius.

The sum in Eq. (5) can be performed exactly. We note first that n electrons interacting among themselves and with a fixed point charge $Z-1$ obey the equation

$$\sum_{j=1}^{n}m\frac{d^2\mathbf{r}_j}{dt^2}=(Z-1)e^2\sum_{j=1}^{n}\frac{\mathbf{r}_j}{|\mathbf{r}_j|^3}. \qquad (7)$$

In such a sum the Coulomb interaction among the electrons cancels exactly. From Eq. (7) we have[11]

$$\left\langle\psi_{m'}\left|\sum_{j=1}^{n}\frac{\mathbf{r}_j}{|\mathbf{r}_j|^3}\right|\psi_0\right\rangle=\frac{m}{(Z-1)e^2}\left\langle\psi_{m'}\left|\sum_{j=1}^{n}\frac{d^2\mathbf{r}_j}{dt^2}\right|\psi_0\right\rangle$$
$$=\frac{(E_{m'}-E_0)^2}{(Z-1)e^2}m\langle\psi_{m'}|\sum_j\mathbf{r}_j|\psi_0\rangle. \qquad (8)$$

When Eq. (8) is combined with Eq. (5) the energy denominators cancel and the sum over m is quite trivial. Equations (5), (6), and (8) then give

$$f=\frac{9}{Z^2}\left(\frac{m}{\mu}\right)^2a_0^2\frac{e^2m}{(Z-1)}\left\langle\psi_0\left|\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{\mathbf{r}_i\cdot\mathbf{r}_j}{|\mathbf{r}_i|^3}\right|\psi_0\right\rangle. \qquad (9)$$

In a Hartree approximation for the ground-state wave function in Eq. (9) the expectation value of $\mathbf{r}_i\cdot\mathbf{r}_j/|\mathbf{r}_i|^3$ vanishes unless $i=j$. With this simplification Eq. (9) becomes

$$f=\frac{9}{Z^2}\left(\frac{m}{\mu}\right)^2\frac{1}{(Z-1)}\sum_{j=1}^{n}\left\langle\psi_0\left|\frac{a_0}{|\mathbf{r}_j|}\right|\psi_0\right\rangle. \qquad (10)$$

[11] The transformation of the matrix element of a point dipole field when taken between Coulomb wave functions has been pointed out to me by Dr. Gian Carlo Wick to whom it was communicated by Dr. E. Guth. I am happy to thank both of them.

The $2p \rightarrow 1s$ radiation rate,

$$R_{2p\rightarrow 1s}{}^{r} = 1.4\times 10^{10} Z^4 \text{ sec}^{-1}. \qquad (11)$$

The rate for the mixed $2s \rightarrow 1s$ transition of a μ meson $R_{2s\rightarrow 1s}{}^{m}$, is the product of the fraction (9) or (10) and the rate (11).

III. AUGER TRANSITION RATE FOR $2s \rightarrow 1s$ TRANSITIONS

Auger transitions in μ-mesonic atoms follow the same selection rules as those for electric dipole radiation but in a qualitative rather than absolute way. Thus while a μ meson in a $2s$ state is rigorously forbidden to accomplish the jump to the $1s$ state solely by radiation of a single quantum, the Auger transition probability, while two orders of magnitude smaller than that for a $2p \rightarrow 1s$ Auger transition, is not negligible. Whenever one of the atomic electrons appears *inside* of the relatively small μ orbit the transfer of energy from the μ meson to the electron becomes possible, but since the radius of the $2s$ μ orbit is about 1/50 that of a K electron the probability for finding the electron in such a position is only of order 10^{-4}. This probability is quite unaffected by the particular environment in which the entire mesonic atom happens to be.[12]

For a μ meson in a $2s$ orbit around a nucleus of atomic number Z the rate for an Auger transition to the ground state is given by

$$R_{2s\rightarrow 1s}{}^{a} = \frac{\sqrt{3}\,2^{21}}{3^{12}Z^3}\left|\frac{\psi_{Z-1}(0)}{\psi_1(0)}\right|^2 \frac{me^4}{\hbar^3}\left(\frac{m}{\mu}\right)^{\frac{7}{2}} C^2. \qquad (12)$$

This rate includes a factor of two for the Auger effect of two K electrons. Here m and μ are again the electron and μ-meson masses, respectively, and

$$\left|\frac{\psi_{Z-1}(0)}{\psi_1(0)}\right|^2$$

is the probability for finding a given one of the K electrons at the Z nucleus relative to this same probability in atomic hydrogen. If the partial screening of one K electron by the other were to be neglected this ratio would be $(Z-1)^3$ because the effective charge seen by the electron is the nuclear charge minus the μ-meson charge. Actually it is slightly less. The coefficient C^2 is unity in that approximation which assumes that the ejected Auger electron's wave function is a plane wave. For the particular Auger transition considered here the ejected K electron is given more than 150 times its binding energy and this is expected to be a rather accurate assumption. Burbidge and De Borde[3] have calculated this same transition rate without such an approximation, assuming only that the ejected electron sees only a nuclear charge Z'. A comparison of their result with Eq. (12) shows that

$$C^2 = \frac{\pi y \exp(\pi y)}{\sinh \pi y} C_3{}^2, \qquad (13)$$

where $y = (4m/3\mu)^{\frac{1}{2}} Z'/Z$ and $C_3{}^2$ is a "correction factor of order one," but which is in general less than one. Since for the purposes of our argument only an upper limit for the transition rate is needed we shall take $C^2 \cong \exp(\pi y)$, $Z' = Z-1$, and $\pi y = 0.25(Z-1/Z)$. Then the Auger rate, assuming two K electrons able to participate in the transition, satisfies

$$R_{2s\rightarrow 1s}{}^{a} \leqslant 2.0\times 10^9 \frac{\exp(\pi y)}{Z^3}\left|\frac{\psi_{Z-1}(0)}{\psi_1(0)}\right|^2 \text{ sec}^{-1}. \qquad (14)$$

with the correct value only slightly less than the limit.

For two K electrons which see a nucleus plus μ meson of net charge $Z-1$, the approximate effect of the screening of the electrons by each other has been estimated by the Ritz variation method with a simple product of exponentials as a trial function. The resulting wave function[13] is that which would obtain if each K electron sees an effective charge of $Z-1-5/16$. Then

$$\frac{1}{Z^3}\left|\frac{\psi_{Z-1}(0)}{\psi_1(0)}\right|^2 = \left(1-\frac{21}{16Z}\right)^3. \qquad (15)$$

[The case of $Z-1=2$ has been calculated more accurately in the Hartree approximation by Wilson and Linsen.[14] Then find $|\psi_{Z-1}(0)/\psi_1(0)|^2 = 4.6$ while Eq. (15) gives 4.8.]

With this same approximation

$$\left\langle \psi_0 \left| \sum_{i,j=1}^{2} \frac{a_0 \mathbf{r}_i\cdot\mathbf{r}_j}{|\mathbf{r}_j|^3} \right| \psi_0 \right\rangle = 2Z\left(1-\frac{21}{16Z}\right). \qquad (16)$$

IV. COMPARISON OF "MIXED" AND AUGER $2s \rightarrow 1s$ TRANSITION RATES

From Eqs. (9), (11), and (14) the ratio of mixed to Auger transitions is slightly greater than

$$r = \frac{0.012Z^2 \exp(-\pi y) a_0}{(1-21/16Z)^3(Z-1)} \left\langle \psi_0 \left| \sum_{i,j=1}^{n} \frac{\mathbf{r}_i\cdot\mathbf{r}_j}{|\mathbf{r}_j|^3} \right| \psi_0 \right\rangle. \qquad (17)$$

If, because of previous Auger transitions, both K-electron states were not filled, then Eq. (17) would give a lower limit for r. The mixed transition depends essentially on the expectation value of $1/r$ for the electron cloud around the nucleus and so gives a considerable contribution even if no K electrons are present. The

[12] The K electrons alone give almost the entire probability. But even for the electrons in higher orbits which are much more affected by the atomic environment there is abundant evidence from Knight shift measurements that the probability for finding an electron at the nucleus is about the same in the free atom as in a solid.

[13] H. A. Bethe and E. E. Salpeter, *Handbuch der Physik* (Springer-Verlag, Berlin, 1957), pp. 232–4.

[14] W. Wilson and R. Linsen, Phys. Rev. **47**, 681 (1935).

Table III. Radiative rates and upper limits for Auger rates of μ mesons in various states of light atoms.

Element / Transition	Li	Be	B	Li	Be	B
	Radiative rate (sec^{-1})			Maximum Auger rate (sec^{-1})		
$3p \to 2s$	4×10^{11}	1×10^{12}	3×10^{12}	7×10^{11}	1.5×10^{12}	2×10^{12}
$3p \to 1s$	3×10^{12}	9×10^{12}	2×10^{13}	1×10^{10}	2×10^{10}	3×10^{10}
$4p \to 3s$	5×10^{10}	2×10^{11}	4×10^{11}	4×10^{12}	6×10^{12}	8×10^{12}
$4p \to 2s$	2×10^{11}	5×10^{10}	1×10^{12}	1×10^{11}	2×10^{11}	2×10^{11}
$4p \to 1s$	1×10^{12}	4×10^{12}	9×10^{12}	3×10^{9}	4×10^{9}	5×10^{9}

pure Auger rate, on the other hand, is proportional to the square of the electron wave function *at the nucleus* which is almost completely that of the K electrons. However, in Li, Be, and B the μ meson remains in the $2s$ state of the light atoms for longer than 10^{-10} sec. In a solid an ejected electron should be replaced and the K shell filled (by electronic Auger transitions) in much less time that this.[15]

We shall evaluate the sum in Eq. (17) only for the two K electrons. Then if the Hartree approximation is even qualitatively reliable for the determination of ψ_0, it follows from Eq. (10) that the omission of other electrons under estimates the ratio (17).

From Eqs. (17), (15), and (16) we obtain

$$r=\frac{0.024Z^3\exp(-\pi y)}{(Z-1)(1-21/16Z)^2}. \qquad (18)$$

For Li, Be, and B, Eq. (18) gives minimum K_α yields of 42%, 48%, and 52%, respectively. In the case of Li this is over twice the observed yield. But there are sufficiently delicate points in the calculation that it may be felt to be insufficiently convincing. Therefore we turn now to an entirely different argument to show that the metastability of the $2s$ level cannot possibly suppress the K_α yield sufficiently to agree with the reported experimental data.

V. TRANSITIONS OF μ-MESONIC ATOMS INTO THE 2s STATE

The lower bounds for the K_α x-ray yields of Li, Be, and B of Sec. IV are based upon the assumption that 100% of the μ mesons pass through the $2s$ state. However, strong arguments can be offered which suggest that only a small fraction of mesons would be expected to do so. Even if all the $2s$ mesons made Auger transitions to the $1s$ states, (disregarding the result of Sec. IV), the remaining mesons would give a K x-ray yield far in excess of those which have been reported.

This conclusion depends upon two considerations: the specific mechanisms which exist for a meson to get into the $2s$ state from higher states and the $2p \to 2s$ transition rate. We begin our argument with the first of these subjects.

For those mesonic states whose radii are much smaller than that of a K electron, Auger transitions and electromagnetic transitions rather accurately obey the same dipole selection rule, $\Delta l=\pm1$. Thus transitions into the $2s$ state occur only from an np state, $n\geqq2$. Table III gives various μ-meson transition probability estimates from the $3p$ and $4p$ levels.

It is clear that Auger transitions with $\Delta n\geqq2$ are negligible. The $2s$ can be appreciably populated only by Auger transitions from the $3p$ or $2p$ levels or by radiative transitions. But from any p state the radiative transition probability is always much larger for a transition directly to the $1s$ state than for a transition to the $2s$ state. Moreover the radiative $3p \to 1s$ transition is much faster than the $3p \to 2s$ Auger transition by about a factor of 4, 6, and 10 in Li, Be, and B. Therefore we reject the supposition that mesons enter the $2s$ state by transitions from $3p$, $4p$ or higher p states. Not only would too few mesons make transitions into the $2s$ state even if they all started from the $3p$ state, but we would also obtain a large yield of K_β x rays from the $3p$ state in contradiction to the observation that the yield of K_β x rays is at most only about 20% of the K_α yield. We are thus led to the assumption that low-energy $2p \to 2s$ Auger transitions are what populate the $2s$ state.

It is then necessary that the $2p \to 2s$ transition rate be much faster than the $2p \to 1s$ radiation rate since the latter transitions will always contribute to the K x-ray yield. In Li, Be, and B the radiation rates are 1.1×10^{13} sec^{-1}, 3.4×10^{13} sec^{-1}, and 8.2×10^{13} sec^{-1}, respectively. Even if the argument of Sec. IV were to be ignored the $2p \to 2s$ transition rates must be greater than those of Table IV in order to keep to the $2p \to 1s$ x-ray yield from exceeding the observed values.

For the energies of Table II the $2p \to 2s$ Auger transitions would not be possible in the free μ-mesonic atom. Rather it is the perturbation of the $n=2$ electron states into a band that permits small energies to be transferred to these electrons. (The transfer of five or 10 electron volts to a neighboring nucleus without electronic excitation is much less probable because of the enormous momentum transfer that would accompany it.)

The minimum transition rates of Table IV are enormously larger that simple theoretical estimates. The case of μ-mesonic Li is particularly striking. When a μ meson gets inside the K shell of Li the effective nuclear charge seen by the electrons is reduced to two.

TABLE IV. Minimum $2p \to 2s$ transition rates to suppress $2p \to 1s$ x rays in μ-mesonic atoms.

Element	Li	Be	B
Minimum rate (sec^{-1})	6×10^{13}	8×10^{13}	1.0×10^{14}

[15] E. H. S. Burhop, *The Auger Effect* (Cambridge University Press, New York, 1952).

As far as its electron cloud is concerned μ-mesonic Li acts like a He atom inserted into metallic Li. If both K electrons are present there is no large piling up of band electron wave function around the neutral core (Li nucleus, μ^- and $2K$ electrons) as there is near all of the Li^+ cores which form the periodic lattice. Therefore in the neighborhood of μ-mesonic Li we may expect the band electrons to behave much like a degenerate gas of free electrons with Fermi energy of about 4 volts characteristic of metallic Li. For this model the $2p \to 2s$ transition rate for Li is

$$R_{2p\to 2s}=2\pi\int_l^{k_f}\frac{d\mathbf{k}\,k'md\Omega'}{(2\pi)^6}\left|\int d\tau\frac{z}{r^3}\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}]\right|^2 \times e^4|\langle 2p|z|2s\rangle_\mu|^2, \quad (19)$$

with the notation of Eq. (5). The upper limit of the integral is the Fermi momentum k_f. The lower limit l depends upon the energy of the transition, Δ:3, or 3.7 ev; $l=(k_f^2-2M\Delta)^{\frac{1}{2}}$ if $\Delta<k_f^2/2M$ and $l=0$ if $\Delta\geqq k_f^2/2M$. The momentum k' is $(k^2+2M\Delta)^{\frac{1}{2}}$. In the case of Li $l\sim 0$ and the transition rate is

$$R_{2p\to 2s}=\frac{16m\Delta}{3\pi}\int_0^{k_f/(2m\Delta)^{\frac{1}{2}}}xdx\ln\left[\frac{(x^2+1)^{\frac{1}{2}}-x}{(x^2+1)^{\frac{1}{2}}+x}\right] \times me^4|\langle 2p|z|2s\rangle_\mu|^2\sim 2\times 10^{11}\text{ sec}^{-1}. \quad (20)$$

This result is more than two orders of magnitude less than that of Table IV; it is expected to be approximately valid if both K electrons are present during the mesonic $2p-2s$ transition. Whether this is a reliable assumption depends upon the detailed mechanisms for filling holes in the electron K shell.

In the lighter elements when a μ-mesonic Auger transition ejects a K electron the hole is filled chiefly by electronic Auger transitions between two L shell (band) electrons. There exist experimental values for this rate in carbon and heavier elements which agree reasonably well with various theoretical calculations[15,16]; the observed and the calculated rates are found to be about 5×10^{14} sec^{-1} with no Z dependence for all elements with $Z\geqq 6$. If this were valid also for Li then presumably the K shell would be full when the meson makes a transition from the $2p$ state. As far as the band electrons are concerned μ-mesonic Li with one K electron missing has the same charge as all the other "normal" Li atoms in the lattice. It takes around $2m\hbar/k_f^2$ for a band electron or hole to move from one atom to the next and so this is about the time it takes for the band to adjust to the altered charge when a K electron is ejected. In Li this is 2×10^{-16} sec, much shorter than all other relevant transition times. This means that, on the average, there is *one* L electron close to a μ Li when a single K electron is missing. In this case (but probably not for Be or B) the 5×10^{14} sec^{-1} rate may greatly over estimate the rate at which K shell holes are filled. And so we must examine the $2p\to 2s$ Auger transition rate for μ mesons in Li when one of the Li K electrons is missing. (The lifetime for a $3d$ μ meson in Li is more than 10^{-12} sec which should allow time for both K-electron states to be occupied before the transition to the $2p$ state; then only a single K electron is missing immediately after an Auger transition from the $3d$ state. Moreover even if we suppose that two K electrons were missing, μ-mesonic Li would average two L electrons in its neighborhood so that the fast 5×10^{14} sec^{-1} transition rate for filling one of the K-shell holes would probably obtain qualitatively even for Li.)

For this case of mesonic Li with a missing K electron we represent the band electrons by the Bloch function

$$\psi_{\mathbf{k}}(\mathbf{r})=\exp(i\mathbf{k}\cdot\mathbf{r})u_{\mathbf{k}}(\mathbf{r})\Omega^{\frac{1}{2}}. \quad (21)$$

The function $u_{\mathbf{k}}(\mathbf{r})$ is periodic in the lattice and for small $\mathbf{k}$ significantly large only close to one of the Li or μ-Li atoms. The normalization is such that $\int_\Omega d\mathbf{r}|u_{\mathbf{k}}(\mathbf{r})|^2=1$ where Ω, the region of integration, is the atomic volume surrounding a given Li atom. The $2p\to 2s$ transition rate is given by Eq. (19) with $\exp[i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}]$ replaced by $\psi_{\mathbf{k}}(\mathbf{r})\psi_{\mathbf{k}'}^*(\mathbf{r})$.

To take into account, and even exaggerate, the increased amplitude for finding a conduction electron near a mesonic lithium ion, we shall first describe the conduction electrons by a superposition of $2s$ and $2p$ atomic wave functions:

$$\psi_{\mathbf{k}}=\sum_{\mathbf{a}}[a(k)\varphi_{2s}(\mathbf{r}-\mathbf{a})+b(k)\varphi_{2p}(\mathbf{r}-\mathbf{a})]\Omega^{\frac{1}{2}}, \quad (22)$$

with $a^2+b^2=1$. φ_{2s}, φ_{2p} are the wave functions of valence electron in its ground ($2s$) and excited ($2p$) state in the normal Li atom. The particular $2p$ state is that which is a pure $m=0$ state with respect to the $\mathbf{k}$ direction. The summation is over all positions of Li nuclei.

For low $\mathbf{k}$ and rather large atomic separations both of which obtain for the conduction band in Li metal Eq. (22) is expected to be qualitatively reasonable near the ions and the mesonic $2p\to 2s$ transition rate depends upon the matrix element of z/r^3.

The contribution to the $2p\to 2s$ transition rate from terms in Eq. (22) with $\mathbf{a}\neq 0$ is much smaller than the rate of Eq. (20) and will be neglected.

Therefore

$$\left|\left\langle\psi_{\mathbf{k}'}\left|\frac{z}{r^3}\right|\psi_{\mathbf{k}}\right\rangle\right| \cong\left|\left\langle\varphi_{2s}(r)\left|\frac{z}{r^3}\right|\varphi_{2p}(r)\right\rangle[a(k)b^*(k')+a^*(k')b(k)]\frac{k_z}{k}\right|\Omega \leqslant\left|\left\langle\varphi_{2s}(r)\left|\frac{z}{r^3}\right|\varphi_{2p}(r)\right\rangle\frac{k_z}{k}\right|\Omega. \quad (23)$$

[16] A summary of data from numerous experiments is given by C. D. Broyles, D. A. Thomas, and S. K. Haynes, Phys. Rev. 89, 715 (1953).

The argument which gave Eq. (8) can also be applied here to give

$$|\langle\varphi_{2s}|z/r^3|\varphi_{2p}\rangle|=[\delta^2 m/e^2(Z-1)]|\langle\varphi_{2s}|z|\varphi_{2p}\rangle|, \quad (24)$$

where δ is the $2p-2s$ energy difference in the atom $Li^{+++}+\mu^-+$a single K electron+an $n=2$ electron. We shall overestimate this splitting by using the $2p-2s$ energy difference of the normal atom: $\delta=1.6$ ev. The matrix element is conveniently evaluated by using for φ_{2s} and φ_{2p} the Slater wave functions[17] for Li, $(b^5/3\pi)^{\frac{1}{2}}r \times\exp(-br)$ and $(c^5/\pi)^{\frac{1}{2}}r\exp(-cr)\cos\theta$, respectively, where $b=c=0.65$ and lengths are measured in units of a_0. The bracket in Eq. (24) is then $2.2a_0$ and for Li

$$|\langle\psi_{k'}|z/r^3|\psi_k\rangle|\leq(3\times10^{-3}/a_0^2)\Omega. \quad (25)$$

A direct estimate of the L.H.S. of Eq. (24) with these same Slater wave functions gives

$$|\langle\psi_{k'}|z/r^3|\psi_k\rangle|\approx(8\times10^{-2}/a_0^2)\Omega. \quad (26)$$

This latter bound is larger than that of the inequality (25) because the matrix element of z/r^3 between degenerate electron wave functions in an atom should vanish according to Eq. (8) while this fails to be true for the approximate Slater wave functions. The matrix element of z/r^3 is expected to be much smaller than casual estimates because of this particular property of the electron wave functions. Nevertheless, even with the value of Eq. (26) the mesonic Auger transition rate $2p\rightarrow2s$ is tremendously less than that of Table IV. For Li Eqs. (26) and (19) give about 10^{11} sec^{-1} while Eqs. (25) and (19) give about 10^9 sec^{-1}. Both rates are smaller than the minimum rate of Table IV by more than two orders of magnitude.

Another approximation to the $2p\rightarrow2s$ Auger rate follows from the use of the Bloch functions

$$\psi_{\mathbf{k}}(\mathbf{r})=\Omega^{\frac{1}{2}}\exp(i\mathbf{k}\cdot\mathbf{r})\sum_{\mathbf{a}}\varphi_{2s}(\mathbf{r}+\mathbf{a}). \quad (22')$$

Again almost the entire contribution to the matrix element of z/r^3 comes from the term with $\mathbf{a}=0$. With these band wave functions the $2p\rightarrow2s$ rate in Li is (approximately)

$$R_{2p\rightarrow2s}\approx\frac{me^4\Omega^2}{(3\pi)^3}(k_f^2+2m\Delta)^{\frac{1}{2}}\left(\frac{2k_f^5}{5}+\frac{2m\Delta k_f^3}{3}\right)\times|\langle\varphi_{2s}|1/r|\varphi_{2s}\rangle\langle2s|z|2p\rangle_\mu|^2. \quad (27)$$

For the expectation value of $1/r$ we use the already introduced Slater wave function for the $2s$ electron in Li. The rate $R_{2p\rightarrow2s}$ is then approximately 2×10^{11} sec^{-1}.

Thus a variety of different approximation schemes confirm that the expected $2p\rightarrow2s$ transition rate in μ-mesonic Li is not nearly fast enough to compete favorably with radiative transitions to the $1s$ state. We then have no mechanism to get μ-mesons into the $2s$ state which at the same time does not violently contradict either an observed datum or a theoretical estimate.

VI. HIGHER TRANSITIONS AND x-RAY YIELDS OF π-MESONIC ATOMS

The conclusion that there exists no mechanism to appreciably populate the $2s$ state can be drawn from independent arguments based upon the observed yields of K x-rays in π atoms.

A π meson in an ns state of Li will undergo nuclear capture at a rate $R_{\pi ns}{}^{\text{cap}}\approx3\times10^{17}/n^3$ sec^{-1}. For higher Z this capture rate increases roughly like Z^4. This is many orders of magnitude faster than transition rates out of the s state for $n<6$ in the lighter atoms. Thus in Li, Be, or B a π meson which enters into one of the lower s states is lost as far as any possibility of its producing a K x ray is concerned. The experimental situation is as follows: the yield of K x rays for these three π-mesonic atoms is between 15 and 20%; most (about 75%) of the K x rays are K_α.[18,19] Thus we conclude that roughly 15% of the pions make the $2p\rightarrow1s$ transition in these elements.

But a pion can be captured by a nucleus from the $2p$ state. Since this capture rate is not negligible next to the $2p\rightarrow1s$ radiation rate *more* than 15% of pions reach the $2p$ state. If it can be established that most pions do get into the $2p$ state, then this same result is expected for μ mesons.

The radiation rate for a $2p$ π meson is approximately

$$R_{\pi2p}{}^r=1.7\times10^{11}Z^4\text{ sec}^{-1}. \quad (28)$$

The nuclear capture rate from the $2p$ state is proportional to the fraction of the time that a $2p$ pion can be found within the nucleus which is $(ZR/a)^5/120$ where R is the nuclear radius and a is the π-mesonic Bohr radius.[1] For a $1s$ state this fraction is approximately $\frac{4}{3}(ZR/a)^3$. Therefore if we assume that capture is equally probable everywhere within the nuclear volume the ratio of $2p$ to $1s$ direct capture rates is the ratio of these fractions. If capture were preferentially at the edge then this ratio would be 5/3 larger. Therefore a lower limit for the ratio is

$$\frac{R_{\pi2p}{}^{\text{cap}}}{R_{\pi1s}{}^{\text{cap}}}\approx\frac{1}{160}\left(\frac{ZR}{a}\right)^2. \quad (29)$$

The width of the K_α π-mesonic x-ray in Be corresponds to a $1s$ capture rate of $(1.8\pm0.3)\times10^{18}$ sec^{-1}.[1,20] Since the $1s$ rate is proportional to Z^4 we have for the capture rate for π mesons from the $2p$ state

$$R_{\pi2p}{}^{\text{cap}}\approx4.4\times10^{13}Z^6R^2/a^2\text{ sec}^{-1}. \quad (30)$$

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[17] J. Slater, Phys. Rev. **36**, 57 (1930).
[18] M. Stearns and M. B. Stearns, Phys. Rev. **107**, 1709 (1957).
[19] M. Camac, A. D. McGuire, J. B. Platt, and H. J. Schulte, Phys. Rev. **99**, 897 (1955).
[20] D. West and E. Bradley, Phil. Mag. **1**, 972 (1957) and **2**, 957 (1957).

For a nuclear radius $R=(\hbar/m_\pi c)A^{\frac{1}{3}}$ the $2p$ absorption rate satisfies

$$R_{\pi 2p}{}^{\text{cap}} \gtrsim 2.4Z^6A^{\frac{2}{3}}10^9 \text{ sec}^{-1}. \quad (31)$$

In addition there is the mixing of a small amount of $2s$ state with the $2p$ from the Stark type perturbations of the K electrons. This gives rise to a capture rate of approximately $3Z^2\times10^{10}$ sec^{-1} which is much smaller than the radiation rate. Lastly there is the possibility of $2p\rightarrow 2s$ transitions despite the fact that even in Li the nuclear $2s$ interaction pushes that π level hundreds of volts above the $2p$. The width of the level is almost as large as the shift so that the transition is not entirely forbidden; however, its rate is expected to be much less than that of the energetically favorable μ transition and it will be neglected. From Eqs. (28) and (31) the yield of K_α x rays from π atoms equals

$$Y_{K_\alpha(\pi)}\approx f_{2p}(1+1.4A^{\frac{2}{3}}Z^2\times10^{-2})^{-1}, \quad (32)$$

where f_{2p} is the fraction of pions which enter the $2p$ state. For Be, $Z=4$, $A^{\frac{2}{3}}\sim5$ and $Y_{K_\alpha(\pi)}\sim f_{2p}/2$. A comparison with the reported yield gives $f_{2p}\sim\frac{1}{3}$ for Be, $f_{2p}\sim\frac{1}{4}$ for Li and $f_{2p}\sim\frac{1}{2}$ for B. (This is in part additional confirmation of the conclusion of the previous section that for μ mesons which can also populate the $2p$ level by $3s\rightarrow 2p$ transitions, a major part reach the $2p$ level; it is independent of any assumption about mechanisms for going from $n=3$ to $n=2$ levels.)

We can now show that, in Be for example, the assumption that the $2p\rightarrow 2s$ transition rate for μ mesons is greater than that of Table IV is incompatible with a majority of pions reaching the $2p$ state. The assumption of rapid $2p\rightarrow 2s$ transitions with splittings energetically comparable to those of Table II predicts still other pion transitions in which the angular momentum l but not the principle quantum number n decreases. These will lead to a π meson ultimately getting into one of the higher s states from which it is captured or into the $3p$ state from which it radiates a K_β x ray or goes to the $2s$ state and is annihilated; in neither case is there any possibility of a contribution to the K_α yield.

As long as $n^2\ll280$ the meson orbit radius is much smaller than the normal Bohr radius for a K electron. When a meson in such an orbit makes a very low-energy exothermal dipole transition $n, l\rightarrow n, l-1$, the energy difference is transferred to radiation, band electrons, other atoms, etc. Because of the very small meson orbit radius compared to the inverse of the momentum transfer, the distance to the nearest neighboring atom, the wavelength of the band electrons, etc., it is a good approximation to keep only terms linear in $\mathbf{r}_\mu$ or $\mathbf{r}_\pi$ when calculating such transition rates. In other words, the transition electric field associated with a low-energy $n, l\rightarrow n, l-1$ transition is well approximated by assuming a dipole at the nucleus whose strength is $\langle n,l|\mathbf{r}_{\pi,\mu}|n, l-1\rangle$ and whose frequency is the energy difference $E_{n,l}-E_{n,l-1}$. Therefore whenever an $n, l\rightarrow n, l-1$ transition has about the same energy as a $2p\rightarrow 2s$ transition then the only difference in the transition rates arises from the different dipole transition matrix element of the meson, no matter what the detailed mechanism of the transition is. There are higher n for which $E_{n,l}-E_{n,l-1}$ for a π meson is closely comparable to $E_{2,1}-E_{2,0}$ for a μ meson. In these cases we can predict the minimum rate for $n, l\rightarrow n, l-1$ transitions in terms of the rates of Table IV and the ratio of dipole matrix elements. A comparison of the rate for these "sliding" transitions $n, l\rightarrow n, l-1$ with those of the "normal" transitions $n, l\rightarrow n-1, l-1$ will give a picture of the cascade of π mesons from higher to lower n-values which is incompatible with the large fraction which reaches the $2p$ level.

There are two not dissimilar effects which play the major roles in giving an $n, l-1$ state a lower energy than an n, l state ($l\neq1$). One is vacuum polarization whose effect decreases with increasing n. The other is the partial screening of the nuclear charge by those electrons whose wave function is not entirely negligible within the meson orbits: K electrons and to a much less extent $2s$ electrons. This latter effect grows rapidly with increasing n.

Vacuum polarization gives the main contribution to the $3d-3p$ splitting for π mesons. Despite the higher n this splitting need not be smaller than that between the $2p$ and $2s$ states for μ mesons. Level shifts for vacuum polarization increase sharply when the μ mass is replaced by a π mass. But more significantly for $r_\mu\ll\hbar/mc$ the splitting is very small relative to the shift of all the states in an n-multiplet, but for $r_\pi\sim\hbar/mc$ the two are comparable. Therefore although the average level shifts certainly decrease rapidly with increasing n, a particular splitting may not. Finally, almost the entire $3d-3p$ shift comes from vacuum polarization while much of the large $2p-2s$ shift is cancelled by the opposite splitting caused by the finite nuclear size.

The $3d$ level shift for π-mesons in Be is negligible but the $3p$ level is considerably lowered because of the appreciable probability (about 15%) for finding a $3p$ in the region within the first node of this wave function. Almost the entire contribution to the $3p$ level shift comes from the expectation value of the vacuum polarization potential in this region. For Li, Be, and B the $3d-3p$ level splittings from vacuum polarization are very approximately 1.5 volts, 5 volts, and 10 volts, respectively. A comparison with Table II shows that $3d-3p$ π-mesonic splittings do not differ qualitatively from the μ-mesonic $2p-2s$ splittings. It is higher in B, approximately equal in Be, and smaller in Li. Thus for Be in particular we can estimate the π-mesonic $3d\rightarrow 3p$ rate from the assumed $2p\rightarrow 2s$ rates of Table IV and the known square of the dipole matrix element. For a transition from an n, l to an $n, l-1$ state we have[13] (in units of the meson Bohr radius)

$$\sum_{m'}|r_{n,l,m}{}^{n,l-1,m'}|^2=\frac{9ln^2(n^2-l^2)}{4(2l+1)}. \quad (33)$$

The $3d \rightarrow 3p$ π-matrix element is also slightly reduced relative to a μ-matrix element by the inverse mass ratio. From Eq. (33), the minimum rates of Table IV, and the $\pi-\mu$ mas ratio we then have the lower limit for a π-meson making a $3d \rightarrow 3p$ transition in Be,

$$R_{\pi 3d\rightarrow 3p}{}^{\text{Be}} \geq 2\times 10^{14}\ \text{sec}^{-1}. \tag{34}$$

This must be compared to the radiation rate and the Auger rate for $3d \rightarrow 2p$ transitions. The former is 4×10^{12} sec^{-1}; the Auger rate for π mesons, assuming both K electrons present and overestimating it slightly by neglecting the partial screening of one electron by the other, is 2×10^{12} sec^{-1}. Thus a π meson in the $3d$ state of Be would be expected to make a transition to the $3p$ state more than 33 times more often than to the $2p$ state. A $3p$ π meson can radiate a K_β x ray or be absorbed after a transition to the $2s$ state but it cannot ever contribute a K_α x ray. Therefore we are at least an order of magnitude short of explaining how $\frac{1}{3}$ of the π mesons could ever get into the $2p$ state. Moreover since a $3p$ π meson in Be is expected to radiate to the $1s$ state at five times the rate of radiative and Auger transitions to the $2s$ state, the observed paucity of K_β x rays is an even stronger datum that indicates that $3d \rightarrow 2p$ is faster than $3d \rightarrow 3p$ (by a factor of four). Again we are led to the conclusion of Sec. V that the $2p \rightarrow 2s$ μ meson rates of Table IV, are too large by around two orders of magnitude at least.

Finally, the rates of Table IV can again be shown to be a great overestimate by considering the possible history of a π meson in a higher orbit ($n\sim 10$) of Be or B. Only Auger transitions which transfer energy to L electrons can occur because K electrons are too tightly bound; the meson radiation rate is quite negligible here. De Borde[21] has calculated such Auger rates assuming a complete shell of $2s$ and $2p$ electrons which overestimates the rate for π-mesonic Be. He finds a rate always less than 10^{15} sec^{-1} for $9\leq n\leq 12$ so that a reasonable upper limit for the Auger transition $n=12$, $l=11 \rightarrow n=11$, $l=10$ is 10^{15} sec^{-1}. Each such transition from a high n state will introduce a hole in the conduction band near the atom. Such a hole will move on to another atom in a time of order $\hbar/E_f$ where E_f is the bandwidth; this is of order 5×10^{-17} sec for Be and so the band electron equilibrium is very rapidly restored near the π atom well before the next transition. We may also have "sliding" transitions $n, l \rightarrow n, l-1$ since $\Delta_{n,l} \equiv E_{n,l}-E_{n,l-1} \geqq 0$. An approximation to this splitting follows from the expectation value with respect to the Coulombic meson wave functions of the average potential from two K electrons which see a nuclear charge $Z-1$. As long as $n^2 \ll m_\pi/m=280$, it is legitimate to approximate the K-electron density within the pion orbit by $|\psi_{Z-1}(0)|^2$. Then

$$\Delta_{n,l-n,l-1}{}^{l=n-1} = 1.4\times 10^{-3}(Z-1)^3 n^2(n-1)/Z^2\ \text{ev}. \tag{35}$$

For B, $\Delta_{n,n}\sim 1.5$ ev for $n=8$ and $\Delta_{n,n}\sim 7.3$ ev for $n=13$. Although these results are not quantitatively accurate especially for higher n we may expect that, for $n\sim 8$ to 15, $\Delta_{n,n}$ becomes comparable to the splittings of Table II for B. For large n and $l=n-1$ the transition rate for "sliding" transitions in B follows from Table IV and Eq. (33):

$$R_{n,l\rightarrow n,l-1}{}^{\text{B},l=n-1} \geq 1.4\times 10^{13}(n-1)n^2\ \text{sec}^{-1}. \tag{36}$$

For $n\sim 10$ this rate is over an order of magnitude greater than the expected $n, l \rightarrow n-1, l-1$ transition rate. For π-meson states with n values in this region, this argument based upon the rates of Table IV predicts that "sliding" $n \rightarrow n$ transitions will dominate over conventional $n \rightarrow n-1$ transition. Even if only a smaller fraction of mesons were to "slide" in each n-level the effect continues over many n-levels. Thus the high n circular orbits will be depopulated in favor of more and more elliptic ones with ever smaller angular momentum. But only those mesons in circular orbits have a good chance of reaching the $2p$ state. A change of n via an Auger transition obeys the selection rule $\Delta n=-1$ in addition to $\Delta l=\pm 1$. Moreover the matrix elements for $\Delta l=-1$ are generally much larger than those for $\Delta l=+1$. Thus the mesons in elliptic orbits will ultimately land in an ns state with $n>1$ and be absorbed; they will have only a very small probability of getting to the $2p$ state in contradiction to the experimental observation and interpretation of the K_α yield from π-mesonic atoms.[22] Again we conclude that the $2p \rightarrow 2s$ transition rates of Table IV must be a great overestimate. This last argument unlike the first two does not involve any discussion of the mechanism for either $2p \rightarrow 2s$ transitions or for $n=3 \rightarrow n=2$ transitions.

VII. CONCLUSION

It would appear from the foregoing that we are still without any theoretical explanation of the drop in K x-ray yield from the lighter μ-mesonic atoms. Moreover evidence of a paradox is apparent even without any detailed calculation. Even if an explanation of the K x-ray yield were possible in terms of the metastable $2s$ state such an explanation could have no relevance to the observation that the L x-ray yield has almost disappeared when Z is less than 8, in strong contradiction

[21] A. H. de Borde, Proc. Phys. Soc. (London) A67, 55 (1954).

[22] For $Z>6$ the $3p$ level is lower than the $3d$ because of vacuum polarization and also lower than the $3s$ because of the effect of finite nuclear size on the latter. Therefore a μ meson which gets into the $3p$ state cannot get out except by an Auger transition to the $2s$ state or radiation to $1s$ state. For oxygen the latter is about ten times more probable and so we would expect the observed reduction of the L x-ray yield but also that most of the K yield would be K_β. For $Z<6$ the $3s$ state is lower than the $3p$ state and the L_α yield might suddenly jump. The difficulty with such an explanation of the L-yield drop is the identification of the large (75%) K_α yield in reference 2. The measured energies of K_α μ-mesonic transitions for 0 and F have also be compared to L_α transitions in heavier π atoms where the transition energies are predicted to be almost identical: M. Stearns, M. B. Stearns, S. DeBenedetti, and L. Leipuner, Phys. Rev. **97**, 240 (1955). The agreement is excellent.

to calculated Auger rates. There is no metastable state except for $2s$; the possibility of direct $n=3 \rightarrow n=1$ radiative transitions ($3p \rightarrow 1s$) is ruled out by the observation that more than $\frac{2}{3}$ of the K x rays are K_α[22]; it is impossible to suppose that the μ meson doesn't reach the low n levels because the yield of K x rays is close to 100% between $Z=14$ and $Z=8$ where the L x-ray yield is rapidly falling.[2] Finally there exist direct measurements of the Auger yield in C when negative μ mesons are stopped in photographic emulsions. In Ag or Br the meson is almost always captured by a nucleus before it decays and the high-energy electron which is the signature of a μ decay is not seen. Whenever it is observed it is almost always from a μ meson which is captured into an orbit of C, N, or O. Capture of a meson by H is rare. Roughly $\frac{1}{3}$ of those μ^- mesons which give the characteristic high-energy electron are expected to be captured in C. But from Table I we see that in C more than $\frac{1}{4}$ of the μ^- mesons do not give the characteristic K x rays. If we assume that there exists some mechanism to transfer this 75 kev to an electron then this electron should be visible in a photographic emulsion in at least $\frac{1}{12}$ of those cases where a μ^- gives off its characteristic high-energy decay electron. In an analysis of 800 such μ^- events Pevsner and Madansky[23] see one certain 75-kev electron and perhaps at most two more when over 60 would be expected from the K x-ray yield for μ-mesonic C. Thus we are again faced with the paradoxical situation that the μ meson somehow gets rid of at least 75 kev without either radiating it or transferring it to a single electron.

In view of the difficulty in understanding this result it is perhaps significant that all of the unexplained reduced yields of K x rays in μ-mesonic Li, Be, B, and C, of the K x ray in π-mesonic Li where $2p$ direct capture is not important, of L x rays from μ atoms with $8 \leq Z \leq 14$, and of L x rays from π atoms with $5 \leq Z \leq 12$ can all be described by the *same* yield curve when these yields are plotted not as a function of Z but as a function of the x-ray energy. This is done in Fig. 1. For reasons that are not yet understood, all of these data can then be summarized in a single remark: when the energy of x rays from mesonic atoms gets below 75 kev the expectation for detecting them begins to drop rapidly.

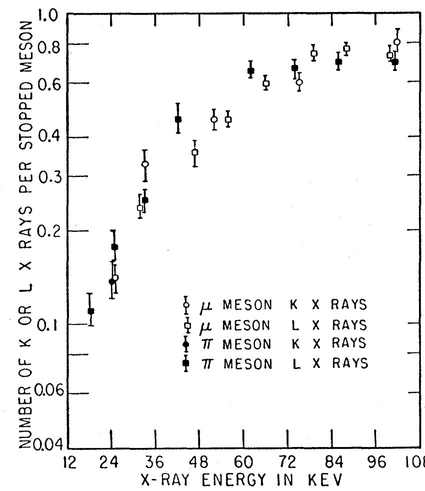

FIG. 1. K and L x-ray yields from μ mesonic and π-mesonic atoms as a function of x-ray energy.[2,4,5]

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[23] Private communication.