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Polarization in Pion-Nucleon Scattering and the Second and Third Pion-Nucleon Resonances*

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The polarization of the recoil nucleon in pion-nucleon scattering is studied from the point of view of providing a means of distinguishing among the various angular momentum assignments proposed for the higher pion-nucleon resonances. It is shown that polarization in this reaction is just as useful a guide as polarization in photoproduction. In particular, a measurement of the polarization at and in the neighborhood of 90° in the energy range between the first and third resonances should give a fairly convincing verification of one or the other of the prevailing assignments.

I. INTRODUCTION

A CONSIDERABLE amount of work has been done by now on the angular momentum assignment[1–3] of the second and third resonances which appear to exist in the pion-nucleon interaction, and various schemes have been proposed[4–6] to verify such assignments. In this paper we investigate the polarization of the recoil nucleon in pion-nucleon scattering from the point of view of providing clues to the angular momentum assignment of these higher resonances. It is shown that such polarization experiments can give just as reliable information in this respect as do the similar polarization experiments in pion photoproduction. In fact, perhaps the situation in scattering is even more favorable because there are fewer states (the considerations are not complicated by the photon multipoles) and because the magnitudes of the amplitudes are uniquely related to the phase shifts, while in photoproduction only the phases of the amplitudes are determined by the phase shifts.

Before going into details, however, it might be worth pointing out that none of these schemes, not excluding the present one, provides an unambiguous proof for any assignment. It is always possible to also achieve a given prediction by a sum of small terms from many angular momentum states. The only really unambiguous way of verifying an assignment would be a complete phase-shift analysis with a unique solution for scattering, and a corresponding amplitude analysis for photoproduction. As we go to higher and higher energies, this becomes increasingly difficult, and in fact one begins to question the usefulness of talking about angular momentum states in the first place. The situation is further complicated by inelastic channels, making the phases complex. In the absence of a unique way to verify assignments, therefore, one has to contend with plausibility and consistency arguments such as given in this paper.

II. FORMULAS FOR POLARIZATION

The derivation of the expressions for the polarization is straightforward and has been discussed, for S and P waves, e.g., by Bethe.[7] Using his notation (Bethe's e is usually denoted by P), we have

$$P\equiv e\equiv (I_+-I_-)/(I_++I_-), \tag{1}$$

with

$$I_\pm=|f_\alpha\mp if_\beta|^2=(|f_\alpha|^2+|f_\beta|^2\pm 2\,\mathrm{Im}f_\alpha{}^*f_\beta), \tag{2}$$

where f_α and f_β are the no-spin-flip and spin-flip amplitudes, the latter taken at $\varphi=0$. A generalization of the argument given in this reference gives immediately for $f_\alpha(l)$ and $f_\beta(l)$, the contributions to the two amplitudes in the l angular momentum state,

$$f_\alpha(l)=[(l+1)a_{l+}+la_{l-}]P_l(x), \tag{3}$$

and

$$e^{-i\varphi}f_\beta(l)=\sin\theta[a_{l-}-a_{l+}]P_l'(x). \tag{4}$$

Here a_{l+} and a_{l-} are the amplitudes in the $J=l+\frac{1}{2}$ and $J=l-\frac{1}{2}$ states, respectively; $P_l(x)$ is the lth Legendre polynomial, and $P_l'(x)$ its derivative with respect to x, both being functions of $x=\cos\theta$, where θ is the scattering angle in the c.m. system. Just as in the other schemes trying to distinguish between competing assignments, we will neglect the inelastic channels altogether.

Since $I_++I_-\sim\sigma$, where σ is the unpolarized cross section, we have

$$e\sigma\sim\mathrm{Im}f_\alpha{}^*f_\beta, \tag{5}$$

so that we will study the behavior of $\mathrm{Im}f_\alpha{}^*f_\beta$ for the various assignments for the resonances.

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We will consider $\pi^- - p$ elastic scattering, where both isotopic spin states are present. The amplitude for this process in fact consists of two-thirds $T=\frac{1}{2}$ contribution and one-third $T=\frac{3}{2}$ contribution. Thus, e.g., denoting by $a_{31}(P)$ the amplitude in the P states with $T=\frac{3}{2}$ and $J=\frac{1}{2}$, we have the contributions of the S, P, D, and F states to f_α and f_β for the process $\pi^- + p \rightarrow \pi^- + p$ as follows:

$$\begin{aligned} f_\alpha = &\tfrac{2}{3}a_{11}(S)+\tfrac{1}{3}a_{31}(S) \\ &+\{2[\tfrac{2}{3}a_{13}(P)+\tfrac{1}{3}a_{33}(P)]+[\tfrac{2}{3}a_{11}(P)+\tfrac{1}{3}a_{31}(P)]\}x \\ &+\{3[\tfrac{2}{3}a_{15}(D)+\tfrac{1}{3}a_{35}(D)]+2[\tfrac{2}{3}a_{13}(D)+\tfrac{1}{3}a_{33}(D)]\} \\ &\times(\tfrac{3}{2}x^2-\tfrac{1}{2})+\{4[\tfrac{2}{3}a_{17}(F)+\tfrac{1}{3}a_{37}(F)] \\ &\qquad +3[\tfrac{2}{3}a_{15}(F)+\tfrac{1}{3}a_{35}(F)]\}(\tfrac{5}{2}x^3-\tfrac{3}{2}x), \end{aligned} \tag{6}$$

and

$$\begin{aligned} f_\beta e^{-i\varphi}(\sin\theta)^{-1} &\\ =\{&[\tfrac{2}{3}a_{11}(P)+\tfrac{1}{3}a_{31}(P)]-[\tfrac{2}{3}a_{13}(P)+\tfrac{1}{3}a_{33}(P)]\} \\ +\{&[\tfrac{2}{3}a_{13}(D)+\tfrac{1}{3}a_{33}(D)]-[\tfrac{2}{3}a_{15}(D)+\tfrac{1}{3}a_{35}(D)]\}3x \\ +\{&[\tfrac{2}{3}a_{15}(F)+\tfrac{1}{3}a_{35}(F)]-[\tfrac{2}{3}a_{17}(F)+\tfrac{1}{3}a_{37}(F)]\} \\ &\qquad\times[(15/2)x^2-\tfrac{3}{2}]. \end{aligned} \tag{7}$$

Here the a's are related to the phase shifts by

$$a_{ij}(l)=e^{i\delta_{ij}(l)}\sin\delta_{ij}(l). \tag{8}$$

We will be interested in those two terms in the resulting expression for $\sin\theta^{-1}\,\mathrm{Im}(f_\alpha^* f_\beta)$ that are proportional to 1 and x, respectively. These are given by

$$\begin{aligned} A\equiv \mathrm{Im}\{&[\tfrac{2}{3}a_{11}(S)+\tfrac{1}{3}a_{31}(S)-a_{15}(D)-\tfrac{1}{2}a_{35}(D)-\tfrac{2}{3}a_{13}(D) \\ &-\tfrac{1}{3}a_{33}(D)]^*[\tfrac{2}{3}a_{11}(P)+\tfrac{1}{3}a_{31}(P)-\tfrac{2}{3}a_{13}(P)-\tfrac{1}{3}a_{33}(P) \\ &-a_{15}(F)-\tfrac{1}{2}a_{35}(F)+a_{17}(F)+\tfrac{1}{2}a_{37}(F)]\} \end{aligned} \tag{9}$$

for the absolute term, and

$$\begin{aligned} B\equiv \mathrm{Im}\{&[\tfrac{4}{3}a_{13}(P)+\tfrac{2}{3}a_{33}(P)+\tfrac{2}{3}a_{11}(P)+\tfrac{1}{3}a_{31}(P) \\ &-4a_{17}(F)-2a_{37}(F)-3a_{15}(F)-\tfrac{3}{2}a_{35}(F)]^* \\ &\times[\tfrac{2}{3}a_{11}(P)+\tfrac{1}{3}a_{31}(P)-\tfrac{2}{3}a_{13}(P)-\tfrac{1}{3}a_{33}(P) \\ &-a_{15}(F)-\tfrac{1}{2}a_{35}(F)+a_{17}(F)+\tfrac{1}{2}a_{37}(F)] \\ &+[\tfrac{2}{3}a_{11}(S)+\tfrac{1}{3}a_{31}(S)-a_{15}(D)-\tfrac{1}{2}a_{35}(D)-\tfrac{2}{3}a_{13}(D) \\ &-\tfrac{1}{3}a_{33}(D)]^*[2a_{13}(D)+a_{33}(D)-2a_{15}(D)-a_{35}(D)]\} \end{aligned} \tag{10}$$

for the coefficient of the x term. Finally, let us remark that

$$\mathrm{Im}a_u^* a_w=\sin\delta_u\,\sin\delta_w\,\sin(\delta_w-\delta_u), \tag{11}$$

whose geometrical meaning is illustrated in Fig. 1. The absolute value of the expression in Eq. (11) gives twice the area of the triangle formed by the two amplitudes, and its sign depends on the sign of $\delta_w-\delta_u$.

III. THE HIGHER RESONANCES

In arguing about the assignments for the higher resonances we will encounter three kinds of products of the a's. We might have the situation where two a's belonging to small phase shifts are multiplied. Their contribution will be very small, because both the two sides of the triangle *and* the angle between them will be small. The second situation arises when a resonant-state amplitude is multiplied by a small amplitude. This arrangement will have a small contribution, because one side of the amplitude triangle is small, but it will nevertheless be much larger than the first type of term, because the other side of the triangle *and* the angle will be large. Finally, the third kind of expression will consist of the product of two medium-sized amplitudes, which, if the two phases are in different quadrants, gives the largest contribution. We shall consider only the largest of such contributions and neglect any other types, although, just as in photoproduction, such "smaller" terms might alter the quantitative picture considerably.

The arguments distinguishing between the various assignments are qualitative and rely on the sign and magnitude of e. For instance, Peierls[2] has argued that in the energy region between the first and second resonances the two important phases are $\delta_{33}(P)$ and $\delta_{13}(D)$, and they differ by an angle of the order of 90°. This resulted in large polarization in the case of photoproduction.[4] In our case, assuming the Peierls assignment, the dominant term of A in this region is

$$\mathrm{Im}\{[-\tfrac{2}{3}a_{13}^*(D)][-\tfrac{1}{3}a_{33}(P)]\} = (2/9)\,\mathrm{Im}a_{13}^*(D)a_{33}(P), \tag{12}$$

which tends to be large in terms of the above classification. If, however, the second resonance is in a P state, as Wilson[1] claimed, then there would be no such "large" term in A, since there are no terms consisting of the products of two P amplitudes. As far as sign goes, for the Peierls assignment, $\mathrm{Im}a_{13}^*(D)a_{33}(P)$ would be positive, and hence, by Eq. (5), e would be positive.

Similar results for B, as well as for the third resonance, are tabulated in Tables I and II. These tables assume that the lower resonance state has a phase

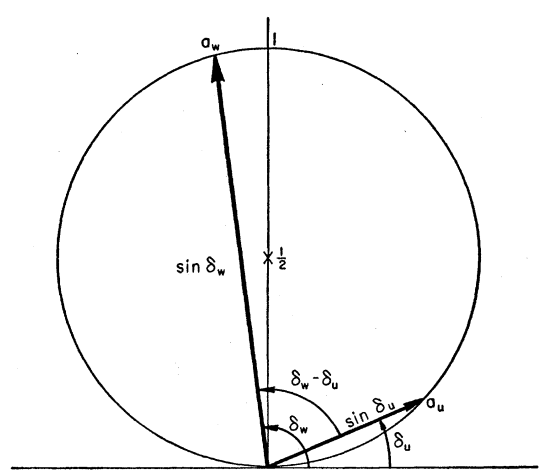

FIG. 1. Illustration of $\mathrm{Im}a_u^* a_w$, whose absolute value is twice the area of the triangle defined by the two amplitudes.

TABLE I. Sign and magnitude of A and B [defined by Eqs. (9) and (10) in the region between the first and second resonances]. S=small, L=large.

Assignment of second resonance	$P(\frac{1}{2},\frac{3}{2})$	$D(\frac{1}{2},\frac{3}{2})$
A	S	$+L$
B	S	S

TABLE II. Sign and magnitude of A and B [defined by Eqs. (9) and (10) in the region between the second and third resonances]. S=small, L=large.

Assignment of the second and third resonances	$P(\frac{1}{2},\frac{3}{2})$ $D(\frac{3}{2},\frac{5}{2})$	$P(\frac{1}{2},\frac{3}{2})$ $F(\frac{3}{2},\frac{5}{2})$	$D(\frac{1}{2},\frac{3}{2})$ $D(\frac{3}{2},\frac{5}{2})$	$D(\frac{1}{2},\frac{3}{2})$ $F(\frac{3}{2},\frac{5}{2})$
A	$+L$	S	S	$-L$
B	S	$+L$	$-L$	S

shift in the second quadrant while the higher one in the first quadrant. The experiment utilizing these tables would consist of measuring e at $\theta=90°$ (which gives A) and also in the neighborhood of this angle (to establish the sign and magnitude of B).

The precise definition of "large" and "small" as used in this paper of course depends on the contributions from the nondominant terms, as well as on the detailed relationship between the phases. As an example, however, if one assumes that only the $a_{33}(P)$ and $a_{13}(D)$ amplitudes contribute at an energy where $\delta_{33}(P)=135°$ and $\delta_{13}(D)=45°$, then one has

$$f_\alpha(90°)=-\tfrac{2}{3}e^{i\pi/4}(\sqrt{2}/2),\qquad f_\beta(90°)=-\tfrac{1}{3}e^{3\pi i/4}(\sqrt{2}/2), \tag{13}$$

so that

$$|f_\alpha|^2=2/9,\quad |f_\beta|^2=1/18,\quad \mathrm{Im}f_\alpha{}^*f_\beta=1/9, \tag{14}$$

which then gives, by Eqs. (1) and (2),

$$e=\frac{\mathrm{Im}f_\alpha{}^*f_\beta}{\frac{1}{2}(|f_\alpha|^2+|f_\beta|^2)}=0.80. \tag{15}$$

IV. ACKNOWLEDGMENTS

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APPENDIX

It might also be useful to summarize briefly the information that can be obtained from a simple differential cross-section measurement, since this is usually a part of a polarization experiment anyway. It will be evident from such a summary that differential cross sections are quite unsuitable for the purpose of distinguishing among the assignments and that polarization experiments are in fact necessary.

If over-all factors are neglected, the differential cross section is given by

$$d\sigma/d\Omega\sim|f_\alpha|^2+|f_\beta|^2, \tag{A.1}$$

with f_α and f_β given by Eqs. (6) and (7).

One conceivable scheme for distinguishing among assignments would be to measure the shape of the angular distribution *at* the resonant energies. If we assume that only the resonant state contributes, then the shapes are given by 1 for the $S(\frac{1}{2})$ and $P(\frac{1}{2})$ states, by $3x^2+1$ for the $P(\frac{3}{2})$ and $D(\frac{3}{2})$ states, by $5x^4-2x^2+1$ for the $D(\frac{5}{2})$ and $F(\frac{5}{2})$ states, by $175x^6-165x^4+45x^2+9$ for the $F(\frac{7}{2})$ and $G(\frac{7}{2})$ states, etc. This scheme has the following handicaps:

(a) As a special case of the Minami[8] ambiguity, the two states with the same J have the same angular distribution and total cross section.

(b) Even the different angular distributions look rather similar (see Fig. 2), so that fairly accurate measurements of the relative differential cross section in the whole angular range would be needed to distinguish between them. The measurement of absolute cross sections, however, can give some information in this case about the resonant state.

(c) The small phase shifts are likely to contribute more here than in the case of polarization, because it is the cosine of the phase-shift differences that enters instead of the sine.

Another conceivable scheme would be to consider

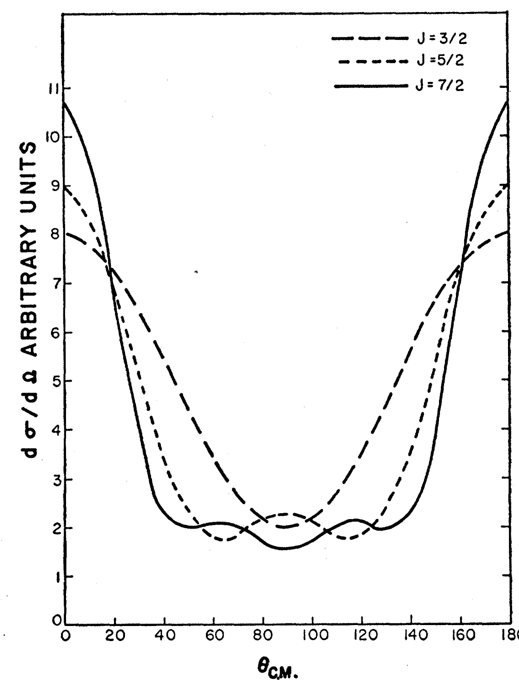

FIG. 2. Shapes of the angular distributions of pion-nucleon scattering in total angular momentum states $J=\frac{3}{2}$, $\frac{5}{2}$, and $\frac{7}{2}$.

[8] S. Minami, Progr. Theoret. Phys. (Kyoto) **11**, 213 (1954).

the interference terms between neighboring resonances at an energy somewhere *between* the two resonant energies. These interference terms would produce odd powers of x in the angular distribution if the two resonances have different parities, but only even powers if the parities are the same. Thus a measurement of the differential cross section at two angles symmetric about 90° could give some information. This scheme, however, is not very promising either, since if the two resonant-state amplitudes differ in phase by an amount of the order of 90°, as it is conjectured, then these interference terms are likely to be very small and could be easily masked by contributions of the small phases.

It appears, therefore, that polarization measurements, although more difficult to carry out, provide much better information about the assignment of the resonances than do the simple differential cross-section experiments.

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Some Cross Sections for the Production of Radio-Nuclides in the Bombardment of C, N, O, and Fe by Medium Energy Protons

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A number of nuclide formation cross sections have been measured, using the Berkeley 184-in. cyclotron, to assist in the interpretation of the data on cosmic-ray-produced nuclides in the atmosphere and in iron meteorites.

Cross sections of H^3 and Be^7 have been measured in bombardments of organic targets containing nitrogen and oxygen by protons of energy 225–730 Mev. Semicarbazide (CH_5N_3O) targets were used to obtain cross sections in air nuclei. The targets were prepared by mixing with a few percent of aluminum dust to permit reliable monitoring of the beam. Polyethylene, aluminum lactate, and Plexiglas targets provided elementary cross sections in carbon and oxygen.

The cross sections for the production of the long lived isotopes Cl^{36} (3×10^5 yr) and Al^{26} (8×10^5 yr) at 730 Mev, and of a number of short lived radionuclides at 500 and 730 Mev, in iron bombardment by protons have been measured. These data and those of earlier workers suggest some modifications in empirical relations used for predicting spallation cross sections in the case of nuclides close to stability.

I. INTRODUCTION

THIS paper describes the results of a selective study of cross sections for the production of some isotopes in the bombardment of carbon, nitrogen, oxygen, and iron by protons of energy between 225 and 730 Mev. The targets and the isotopes have been chosen with the aim of evaluating the data on cosmic-ray-produced isotopes in the atmosphere and in iron meteorites.

The natural rate of production of tritium in the atmosphere by cosmic rays have been calculated by Currie et al.[1] and Fireman and Rowland[2] using the formation cross sections of tritium in nitrogen and oxygen by protons of energy, $E \geqslant 450$ Mev. However, substantial isotope production in the atmosphere occurs due to nucleons of lower energy (Lal et al.[3]) and it becomes essential to know the cross sections at lower energies. Similarly, in the case of the isotope Be^7, the only measurements in nitrogen and oxygen are those by Benioff[4] at a proton energy of 5.7 Bev. We have studied the cross sections for the formation of H^3 and Be^7 in bombardments of organic targets containing nitrogen and oxygen by protons of energy between 225 and 730 Mev. The targets were chosen to permit measurements of the average H^3 and Be^7 cross sections for a mixture of nitrogen and oxygen corresponding to that of the atmosphere. Elementary cross sections in carbon, and in some cases in nitrogen and oxygen, have been given.

In iron bombardments we have studied the cross sections for the production of long-lived isotopes, Al^{26} and Cl^{36}, and short-lived isotopes, Be^7, Na^{22}, $Cl^{34m,38,39}$, $K^{42,43}$, and $Mn^{52,54}$ at proton energies 500 and 730 Mev. These measurements were made in order to interpret the results of Shedlovsky et al.[5] and of Honda[6] in this laboratory, who have studied cosmic-ray-produced radio-nuclides Be^{10}, Al^{26}, Cl^{36}, K^{40}, and Mn^{53} in some iron meteorites.

The energy range studied was limited by practical considerations, but it provides for the present a rea-

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