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Parity Nonconserving Internucleon Potentials*

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The general form that a parity nonconserving internucleon potential must take because of invariance requirements is obtained. A detailed calculation is then made of the parity nonconserving potential arising from a self-interacting current description of weak interactions. If the polar vector part of the current ($J_\mu{}^V$) is conserved, then parity nonconservation of the order 1 part in 10^7 ($\mathfrak{F}\approx 10^{-7}$) is to be expected in nuclear processes. Failure to observe such an effect would indicate either that $J_\mu{}^V$ is not conserved or that the self-interacting current description is incorrect.

1. INTRODUCTION

THERE have recently been a number of attempts[1–4] to establish the extent to which parity is conserved in nuclear reactions and nuclear electromagnetic transitions. Such experiments are aimed at setting an upper limit to the magnitude of the parity nonconserving part of the internucleon potential, although there is some ambiguity with experiments involving electromagnetic transitions since any observed parity breakdown could also be attributed to the electromagnetic interaction. However, there has so far been no indication of parity nonconservation and the point remains academic. Nevertheless, the experimental limits on parity nonconservation in nuclear processes continue to decrease and it becomes important to investigate theoretically the extent to which parity nonconservation should be expected in such processes.[5]

The purpose of the present work is to obtain some idea of the form and order of magnitude of the parity nonconserving (PNC) internucleon potential which may be present as a result of interactions between nucleons through the now well established parity nonconserving weak interactions. In Sec. 2 the general form a PNC internucleon potential must take is discussed from the viewpoint of invariance requirements and in Sec. 3 a detailed calculation is made of the PNC internucleon potential resulting from the universal four-fermion theory of weak interactions.

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[1] N. Tanner, Phys. Rev. **107**, 1203 (1957).

[2] R. E. Segel, J. V. Kane, and D. H. Wilkinson, Phil. Mag. **3**, 204 (1958).

[3] D. H. Wilkinson, Phys. Rev. **109**, 1603, 1611, 1615 (1958).

[4] F. Boehm and U. Hauser, Bull. Am. Phys. Soc. Ser. **II**, 4, 460 (1959).

[5] It is usual to indicate the magnitude of the parity nonconservation by a factor $\mathfrak{F}$ [T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956)] which is a measure of the relative strengths of parity nonconserving and parity conserving interactions. Thus for a nuclear state of predominantly one parity, $\mathfrak{F}^2$ represents the fractional weight of admixed states possessing opposity parity.

2. FORM OF THE PNC INTERNUCLEON POTENTIAL FROM INVARIANCE REQUIREMENTS

Eisenbud and Wigner[6] have shown how to construct the most general *parity conserving* internucleon potential from invariance arguments. Their assumptions are that for two nucleons 1 and 2, the potential must be a symmetric function of the operators $\mathbf{r}_{12}$, $\mathbf{p}_{12}$, $\boldsymbol{\sigma}^{(1)}$, $\boldsymbol{\sigma}^{(2)}$, $\boldsymbol{\tau}^{(1)}$, and $\boldsymbol{\tau}^{(2)}$ where $\mathbf{r}_{12}$ is the spatial separation of the particles, $\mathbf{p}_{12}$ their relative momentum, $\boldsymbol{\sigma}^{(1)}$, $\boldsymbol{\sigma}^{(2)}$ their spin operators, and $\boldsymbol{\tau}^{(1)}$, $\boldsymbol{\tau}^{(2)}$ their isotopic spin operators. This function is required to transform as a spatial scalar quantity, to be invariant under time reversal and to commute with the total charge operator $1+\frac{1}{2}(\tau_z{}^{(1)}+\tau_z{}^{(2)})$. Furthermore, since the potential concept only has significance at low nucleon velocities, $\mathbf{p}_{12}$ is considered to appear at most linearly. These requirements then considerably limit the form that the internucleon potential can take.

In the present case, essentially the same requirements obtain except that now the PNC potential must transform as a *pseudoscalar* rather than a *scalar*. The same types of argument as presented by Wigner and Eisenbud can then be used and it is found that the most general (subject to the prescribed restrictions) PNC potential must have the following form:

Static Terms (i.e., independent of p_{12})

$$\begin{aligned}\mathcal{V}_{\text{static}}=\mathbf{r}_{12}\cdot(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})[&V_{\text{I}}(r_{12})+\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}V_{\text{II}}(r_{12})\\&+\tau_z{}^{(1)}\tau_z{}^{(2)}V_{\text{III}}(r_{12})+(\tau_z{}^{(1)}+\tau_z{}^{(2)})V_{\text{IV}}(r_{12})]\\&+\mathbf{r}_{12}\cdot(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})(\boldsymbol{\tau}^{(1)}\times\boldsymbol{\tau}^{(2)})_zV_{\text{V}}(r_{12}),\end{aligned}\quad(1)$$

where V_{I}, V_{II}, V_{III}, V_{IV}, and V_{V} are unspecified functions of r_{12}.

Velocity Dependent Terms

Here the situation is more complicated and there is much more arbitrariness. Thus, the following symmetric spatial terms

$$(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot\mathbf{p}_{12};$$
$$(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot\mathbf{p}_{12}-\tfrac{1}{3}(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\cdot\mathbf{r}_{12}\mathbf{r}_{12}\cdot\mathbf{p}_{12}/r_{12}{}^2$$

[6] L. Eisenbud and E. Wigner, Proc. Nat. Acad. Wash. **27**, 281 (1941).

can be combined with any of the following isotopic spin terms

$$1,\quad \boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)},\quad \tau_z^{(1)}\tau_z^{(2)},\quad (\tau_z^{(1)}+\tau_z^{(2)}),$$

each combination being associated with some unspecified function of r_{12}. Similarly, the following antisymmetric spatial terms

$$(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot\mathbf{p}_{12};$$
$$(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot\mathbf{p}_{12}-\tfrac{1}{3}(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot\mathbf{r}_{12}\mathbf{r}_{12}\cdot\mathbf{p}_{12}/r_{12}^2$$

can be combined with $(\tau_z^{(1)}-\tau_z^{(2)})$ and some unspecified function of r_{12}.

It is to be noticed that no terms corresponding to a spin-orbit coupling appear since such terms can only appear in the form of a scalar quantity.

One final point has to be discussed, namely, whether a one particle PNC potential can be constructed. This has some interest in that it might be required, for example, to add a parity nonconserving part to the nuclear shell-model potential. It turns out that a static one-particle potential cannot be constructed. Thus, although $\boldsymbol{\sigma}\cdot\mathbf{r}V(r)$ is a pseudoscalar, it is not invariant under time reversal and can only be made invariant by combining it with either τ_+ or τ_-; the resulting term, however, then no longer commutes with the nucleon isotopic spin. On the other hand, $\boldsymbol{\sigma}\cdot\mathbf{p}V(r)$ is a perfectly well-behaved PNC one-particle potential and is the only possibility.

We now consider in detail the form of the PNC potential resulting from the four-fermion description of weak interactions.

3. PNC INTERNUCLEON POTENTIAL FROM WEAK INTERACTIONS

One of the most profitable ways of regarding the weak interactions is to interpret them as resulting from a self-interacting current[7] J_μ. Thus the interaction Hamiltonian can be written

$$H_{\mathrm{int}}=-\tfrac{1}{2}[J_\mu\tilde{J}_\mu+\mathrm{H.c.}]$$

with

$$\tilde{J}_\mu=[J_1^\dagger, J_2^\dagger, J_3^\dagger, -J_4^\dagger],\tag{2}$$

where the current operator is the sum of a polar vector part $J_\mu{}^V$ and an axial vector part $J_\mu{}^A$ and may include lepton, nucleon, pion, and strange particle terms. Thus

$$J_\mu=G^{\frac{1}{2}}\{i\bar{N}\tau_+\gamma_\mu(1+\gamma_5)N-i\sqrt{2}(\pi_+\partial_\mu\pi_0-\pi_0\partial_\mu\pi_+)+i[\bar{\nu}\gamma_\mu(1+\gamma_5)e]+\cdots\},\tag{3}$$

with $G\approx1\times10^{-49}$ erg cm^3, where G is determined from β- and μ-decay data. In (3), N, π, ν, $e\cdots$ are field functions for the nucleon, pion, neutrino, electron.

The presence of the pion term is by no means certain. Further, as will be seen later, the most important contribution to the PNC internucleon potential resulting from this self-interacting current does not appear if this term is absent. It was originally introduced by Feynman and Gell-Mann[7] in order that $J_\mu{}^V$ should be conserved so that the apparent equality of the polar vector coupling in β decay and μ decay could easily be accounted for.

With J_μ given by (3), H_{int} then consists of a sum of 4-fermion, 2-fermion—2-boson, and 4-boson terms. Of course a PNC internucleon potential will arise from the ordinary β-decay interaction,

$$G[\bar{N}\gamma_\mu(1+\gamma_5)\tau_+N][\bar{e}\gamma_\mu(1+\gamma_5)\nu],$$

through the interchange of an electron-neutrino pair. However, this potential would be of order G^2 and therefore very small. On the other hand, with the self-interacting current description there are terms of the form

$$H_1=G[\bar{N}\gamma_\mu(1+\gamma_5)\tau_+N][\bar{N}\gamma_\mu(1+\gamma_5)\tau_-N],\tag{4}$$

and

$$H_2=G\sqrt{2}\{[\bar{N}\gamma_\mu(1+\gamma_5)\tau_+N]\times[\pi_-\partial_\mu\pi_0-\pi_0\partial_\mu\pi_-]+\mathrm{H.c.}\},\tag{5}$$

both of which lead to PNC internucleon interactions of order G. The former corresponds to a contact interaction while the latter has to be considered in conjunction with the usual strong nucleon-pion interaction and leads to a potential of finite range.

Taking the four fermion interaction H_1, evaluating for plane wave nucleons and retaining only terms of first order in the nucleon momentum, the following expression is obtained for the internucleon potential

$$\begin{aligned}\mathcal{V}=(G/4)(1-\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)})(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}-\tau_z^{(1)}\tau_z^{(2)})\delta(\mathbf{r}_{12})\\-(G/4Mc)(\boldsymbol{\sigma}^{(1)}-\boldsymbol{\sigma}^{(2)})\\\cdot\mathbf{p}_{12}(\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}-\tau_z^{(1)}\tau_z^{(2)})\delta(\mathbf{r}_{12}).\end{aligned}\tag{6}$$

It is to be noticed that the static terms are parity conserving. The parity nonconserving term is velocity dependent and is a combination of two of the possible terms listed in Sec. 2. Because of its velocity dependence, however, it is not expected to be very important and its importance is likely to be even further diminished because of the inhibiting effect of the repulsive core nature of the strong internucleon potential on a contact interaction.

The PNC potential arising from H_2 will result, in lowest order, from diagrams of the type shown in Fig. 1. In the static (nucleon) limit

$$H_2=\sqrt{2}Gi[\tau_+\boldsymbol{\sigma}\cdot(\pi_-\nabla\pi_0-\pi_0\nabla\pi_-)+\tau_-\boldsymbol{\sigma}\cdot(\pi_0\nabla\pi_+-\pi_+\nabla\pi_0)],\tag{7}$$

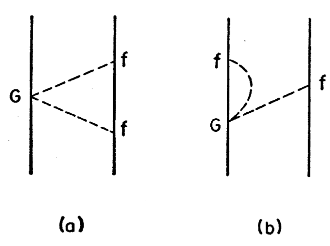

FIG. 1. Diagrams contributing to a PNC internucleon potential.

[7] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

and for the purposes of the calculation we take for the strong pion-nucleon interaction

$$H_{N\pi}=(4\pi)^{\frac{1}{2}}(f/\mu)\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\boldsymbol{\tau}\cdot\boldsymbol{\pi}, \quad (8)$$

where μ is the pion Compton wavelength and f the renormalized pseudovector coupling constant ($f^2\approx 0.08$). It is then a straightforward matter to calculate the internucleon potential resulting from diagrams of the type 1(a) and 1(b). It turns out that there is no contribution from diagrams 1(b) and that diagrams 1(a) lead to

$$\mathcal{V}=\frac{-Gf^2}{2\pi\hbar c}\left[\frac{1}{r^4}+\frac{2}{\mu r^5}+\frac{1}{\mu^2 r^6}\right]e^{-2\mu r}\mathbf{r} \cdot(\boldsymbol{\sigma}^{(1)}\times\boldsymbol{\sigma}^{(2)})[\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}-\tau_z^{(1)}\tau_z^{(2)}], \quad (9)$$

where $\mathbf{r}=\mathbf{r}_{12}$. Thus in Eq. (1)

$$V_{\mathrm{I}}=V_{\mathrm{IV}}=V_{\mathrm{V}}=0;$$

$$V_{\mathrm{II}}=-V_{\mathrm{III}}=\frac{-Gf^2}{2\pi\hbar c}\left[\frac{1}{r^4}+\frac{2}{\mu r^5}+\frac{1}{\mu^2 r^6}\right]e^{-2\mu r}.$$

This potential has been deduced with no cutoff in momentum space. Properly we should have used $k_{\max}\approx M$. However, the error introduced by taking $k_{\max}=\infty$ only appreciably affects the properties of the potential for values of $r<\hbar/Mc$ and it is just in this region that the strong internucleon potential has a large repulsive core. Thus, no appreciable error is introduced provided that $\mathcal{V}_{12}$ is used in conjunction with a nuclear wave function having a correlation function $p(\mathbf{r})$ which reflects the effects of the repulsive core and vanishes for $r\lesssim\hbar/Mc$.

4. DISCUSSION

The first point to be noted about the PNC internucleon potential (9) and also (6) is that its isotopic spin dependence, which can be written $\frac{1}{2}(\tau_-^{(1)}\tau_+^{(2)}+\tau_+^{(1)}\tau_-^{(2)})$, is such that the potential only has nonvanishing matrix elements when a neutron and proton are interacting with one another; there is no PNC interaction between like particles. This follows at once from the nature of the primary process H_2 responsible for the interaction.

Secondly, although the PNC potential was calculated using the four fermion contact theory of weak interactions, it is clear that essentially the same result would be obtained if the four fermion type interactions stemmed from an intermediate charged boson theory. The only effect would be a slight modification in the radial dependence of the potential.

Finally, we have to estimate the order of magnitude of this potential compared to the usual strong internucleon potential. Perhaps the most significant comparison to make is with the lowest order (f^2) contribution V_{12} to the central part of the strong potential. The comparison is then between contributions calculated to the same order in f and made with the same approximations (e.g., no recoil). We therefore consider the orders of magnitude of V_{12} and $\mathcal{V}_{12}$ at $r=\bar{r}=1.8\times 10^{-13}$ cm, the average distance between nucleons in nuclear matter.

$$\langle\mathcal{V}_{12}\rangle\approx Gf^2/\hbar c\bar{r}^3; \quad \langle V_{12}\rangle\approx f^2/\bar{r},$$

and the ratio

$$\mathfrak{F}=\langle\mathcal{V}_{12}\rangle/\langle V_{12}\rangle\approx G/\hbar c\bar{r}^2\approx 10^{-7}.$$

This estimate is, of course, very crude since spin and isotopic factors, averaging over wave functions, etc., could introduce factors of the order 10 or more. However, this figure is sufficiently close to the most recent limits[4] ($\mathfrak{F}\leqslant 8\times 10^{-6}$) on $\mathfrak{F}$ to warrant more detailed investigation of the way such a potential will manifest itself in nuclear processes; such an investigation will be the subject of a subsequent paper. Suffice it to say here that if experiments should show that $\mathfrak{F}\lesssim 10^{-7}$, then this will be a strong indication either that the vector current $J_\mu{}^V$ is not conserved or that the concept of a self-interacting current or intermediate boson theory of weak interactions is incorrect.

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