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Studies of Low-Lying Levels of Even-Even Nuclei with (*d,p*) and (*d,t*) Reactions*

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Low-lying states of even-even nuclei in the vibrational region were studied by exciting them with (*d,p*) and (*d,t*) reactions. The relative cross sections for exciting ground states (*G*) bear little relationship to whether they are allowed or forbidden by the simple Mayer-Jensen configurations, so that configuration mixing is generally large. Some of the details of this mixing are obtained. The allowed portion of these cross sections are generally quite close to the single-particle values. First and second excited states are much more strongly excited than expected theoretically. In Pd^{104}, Pd^{106}, Pt^{194}, and Pt^{196} searches for the triplet in second excited states indicate that its total spacing must be less than 80 kev. New states were found in the triplet region of Cd^{114} and Cd^{112} bringing the total number of known states in these to 5 and 4, respectively; each includes two 0^+ and two 2^+ states. Higher excited states were studied and in almost all cases they occur below the expected position of the third member of the vibrational band; this gives evidence on the size of the energy gap. The location of all 0^+ levels up to 3 Mev is determined for several nuclei. In two cases the 3^+ states required by Davidov-Filipov theory are found to be 0^+, and in other cases they are not found at all. A number of previously unknown levels are catalogued.

THE low-lying states in even-even nuclei have been studied extensively by beta- and gamma-ray spectroscopy[1] and by Coulomb excitation.[2] However, while these techniques have proved very powerful, they also have limitations. The levels available for study by the first method are the relatively few which happen to be decay products of a parent of suitable half-life and whose excitation is allowed by the very stringent selection rules of beta- and gamma-ray transitions. The second method allows the study only of those few levels which can be excited by an *E*2 transition from the ground state. It is therefore interesting to study these levels with other techniques.

In this paper, we report a study of low-lying levels in even-even nuclei using (*d,p*) and (*d,t*) reactions. This method has the advantage of not being restricted by the overpowering selection rules that characterize the above mentioned methods, so that essentially all levels are excited. In addition, the stripping and pickup reactions are well known to proceed by insertion and removal, respectively, of single particles, and the shell-model states of these particles may be determined by angular distribution measurements. Thus, these studies may be expected to locate essentially all low-lying levels in even-even nuclei, and to determine to some extent their shell-model structure.

EXPERIMENTAL

The experimental method is quite standard and has been described previously.[3–5] Magnetically analyzed 14.9-Mev deuterons from the University of Pittsburgh cyclotron impinge on thin foil targets of the elements being studied; the reaction products are passed through a wedge magnet spectrograph and detected by the tracks they produce in a photographic emulsion located at the focus. In measurements on protons, all other particles are removed by absorbers placed over the emulsions; in measurements on tritons, only the energy region where no other particles have sufficient magnetic rigidity is studied. Some examples of the data obtained are shown in Figs. 1–4

The resolution is about 75 kev except in the few cases where it is limited by poor targets. Good targets are not available in the rare-earth and immediately adjacent regions, so that these nuclei were not studied. The measurements were thus limited to the so-called "vibrational region." The targets needed to produce even-even nuclei by (*d,p*) and (*d,t*) reactions are odd isotopes of even-*Z* elements. In general, there is only

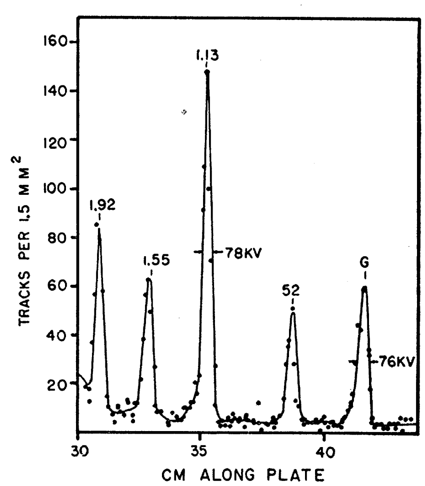

FIG. 1. Energy distribution of protons from $Pd^{105}(d,p)Pd^{106}$ observed at 30°.

* Assisted by the National Science Foundation and the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] Beta- and Gamma-Ray Spectroscopy, edited by K. Siegbahn (North Holland Publishing Company, Amsterdam, 1955).
[2] K. Alder, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Revs. Modern Phys. 28, 432 (1956).
[3] B. L. Cohen, J. B. Mead, R. E. Price, K. S. Quisenberry, and C. Martz, Phys. Rev. 118, 499 (1960).
[4] R. S. Bender, E. M. Reilley, A. J. Allen, J. S. Arthur, and H. S. Hausman, Rev. Sci. Instr. 23, 542 (1952).
[5] B. L. Cohen, Rev. Sci. Instr. 30, 475 (1959).

one or two such isotopes in a given element, and (d,p) and (d,t) reactions have much higher Q values in these isotopes than in the other (even-even) isotopes, so that separated isotope targets were not needed.

The reactions studied are listed in Table I; almost all of them are in the mass region between 90 and 130. The angular distributions from (d,p) and (d,t) reactions in this region have been studied in other experiments.[6] For angular momentum transfers (l_n) less than 3, the peak intensity occurs in the region 20°–30°, and for angles larger than that, the intensity falls off slowly and smoothly, never deviating by more than 20% from

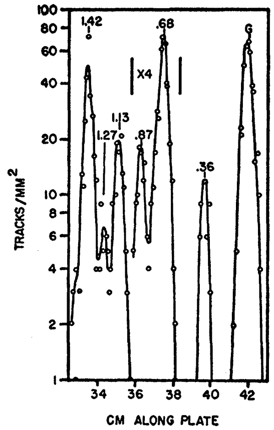

FIG. 3. Energy distribution of tritons from Pd^{105}-$(d,t)Pd^{104}$ observed at 30°. Background in regions where no points are shown has been averaged and is about 0.5 track per mm².

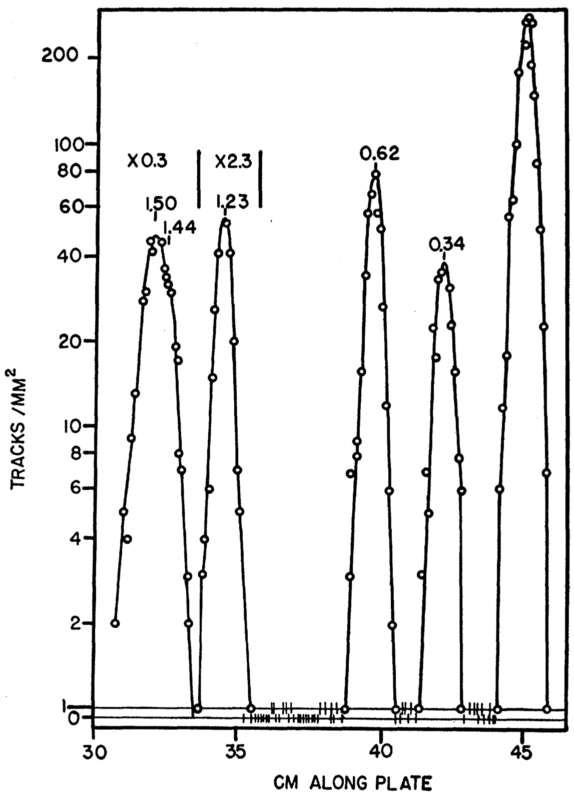

FIG. 2. Energy distribution of tritons from $Pt^{195}(d,t)Pt^{194}$ observed at 60°. Scale is logarithmic except that zero is included. Different types of points for ordinates equal to zero or one are to simplify drawing

a smooth, monotonically decreasing curve. There is very little dependence on Q value (there is little predicted by Butler theory), and even the dependence on l_n is not large, but there is good independent evidence that the stripping mechanism is predominant at all angles.[3] At angles below 20°, differences between different l_n's become apparent, but even in this region, there is little detectable dependence on Q value.

One of the principal purposes of this paper is to compare the strengths of (d,p) and (d,t) reactions leading to various energy levels. In comparing cross sections for different reactions with the same l_n, the simplest procedure is to compare intensities at a single angle in a region where angular distributions are varying slowly and have little dependence on Q value, and where intensities are near their maxima. For this purpose, 30° was chosen. A large fraction of the total cross section is concentrated near this angle, and it has the additional experimental advantage of giving about the optimum energy resolution.[5] In comparing cross sections for different values of l_n, an empirical intensity ratio must be determined (no treatments of the stripping reaction, even in light elements, have been able to do more than this). Experience in other studies has shown that, in the large angle region, the intensity decreases by about a factor of two per unit increase in l_n.

This single angle method differs from the usual treatment in light elements where angular distributions are measured and fitted to Butler theory to determine reduced widths. That procedure is somewhat impractical in heavy elements since the fits to Butler theory are very poor. The closest approach to it would be to fit the Butler theory on a convenient peak in the angular distribution; this was done in a few cases. The principle difference between this procedure and determining intensities at a single angle is that the Butler theory predicts a dependence of intensity on Q value,

TABLE I. Reactions studied.

Target	Final nuclide (d,p)	Final nuclide (d,t)
Zn	Zn^{68}	Zn^{66}
Se	Se^{78}	Se^{76}
Zr	Zr^{92}	Zr^{90}
Mo	Mo^{96}, Mo^{98}	Mo^{94}, Mo^{96}
Pd	Pd^{106}	Pd^{104}
Cd	Cd^{112}, Cd^{114}	Cd^{110}, Cd^{112}
Cd^{111}	Cd^{112}	Cd^{110}
Cd^{113}	Cd^{114}	Cd^{112}
Sn	Sn^{118}, Sn^{120}	Sn^{116}, Sn^{113}
Sn^{117}	Sn^{118}	Sn^{113}
Sn^{119}	Sn^{120}	Sn^{118}
Te	Te^{124}, Te^{126}	Te^{124}
Pt	Pt^{196}	Pt^{194}

[6] B. L. Cohen and R. E. Price (to be published).

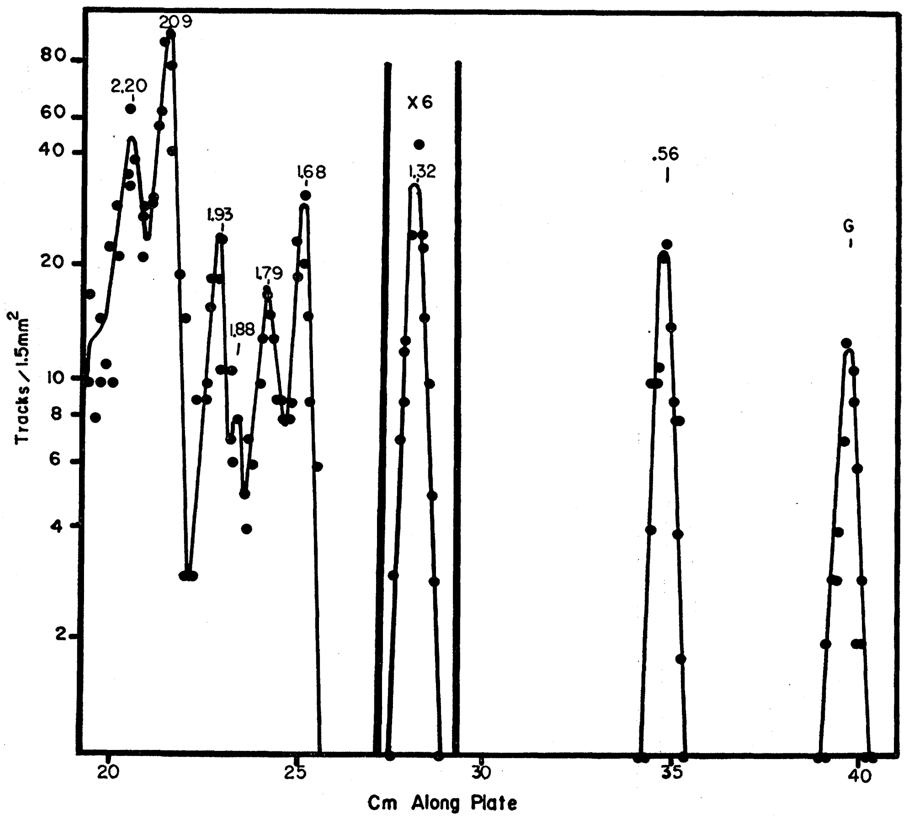

FIG. 4. Energy distribution of protons from Pt^{195}-$(d,p)Pt^{196}$ observed at 60°. Background in regions of 38–39 and 40–41 cm has been averaged and is about 0.4 track/mm².

so that the results of the two methods when comparing levels of different Q values differ by the ratio of the Butler predictions. For a Q-value difference of 3 Mev, which is about the largest encountered in this experiment, this ratio is about 1.4. Where this correction is significant, it will be pointed out in the discussion.

As a consequence of the experimental method and the rather crude status of Butler theory in heavy elements, the quantitative intensity ratios given in this paper should be considered to be unreliable by at least 25%, and perhaps by as much as a factor of two. On the other hand, the theoretical questions studied are generally of a qualitative rather than a quantitative nature.

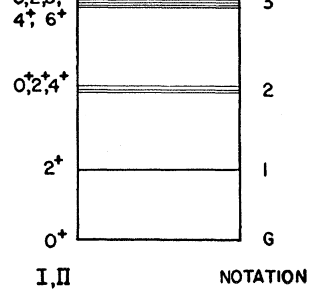

FIG. 5. Typical level scheme for an even-even nucleus in the vibrational region. The notation shown to the right is used in the text referring to these levels or groups of levels.

In some cases, it is of interest to determine l_n by angular distribution measurements. For this purpose, the systematics of angular distributions for various values of l_n and Q was studied using known transitions.[6] The only very striking effect is that for angles smaller than 14°, $l_n=0$ angular distributions rise rapidly with decreasing angle, whereas $l_n=2$, 4, and 5 angular distributions fall rapidly. Thus, by comparing intensities at 8° and 14°, an $l_n=0$ transition can be very easily detected. This procedure was checked for many cases, and always proved adequate.

CORRECTIONS

Before applying intensity ratios to questions of nuclear structure, two corrections must be applied to remove factors which arise from the stripping theory. Firstly, intensities from (d,p) reactions must be multiplied by the ratio R,

$$R=(2J_i+1)/(2J_f+1), \qquad (1)$$

where J_i and J_f are the spins of the initial and final nuclei, to correct for the inverse of this factor appearing in the expression for the stripping cross section.[7] Secondly, French[8] has shown that, to correct for the fractional parentage coefficients, the reduced widths for

[7] A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953); J. B. French and B. J. Raz, Phys. Rev. **104**, 1411 (1956).
[8] J. B. French (to be published).

TABLE II. σ_{oe}/σ_{eo} in (d,p) reactions.

Odd to even		Even to odd		σ_{oe}/σ_{eo}		
Target	M-J transition	Target	M-J transition	Measured	Corrected	Class[a]
Zn^{67}	$(f_{5/2})^5 \rightarrow (f_{5/2})^6$	Zn^{64}	$(f_{5/2})^2 \rightarrow (f_{5/2})^3$	~0.06	~0.25	
Zr^{91}	$d_{5/2} \rightarrow (d_{5/2})^2$	Zr^{90}	$0 \rightarrow d_{5/2}$	0.05	0.9	
Mo^{95}	$(d_{5/2})^3 \rightarrow (d_{5/2})^4$	Mo^{98}	$0 \rightarrow s_{1/2}$	0.022	0.07	A/A
Mo^{97}	$(d_{5/2})^5 \rightarrow (d_{5/2})^6$	Mo^{98}	$0 \rightarrow s_{1/2}$	0.075	0.23	
Pd^{105}	$(d_{5/2})^5 \rightarrow (d_{5/2})^6$	Pd^{106}	$0 \rightarrow s_{1/2}$	0.10	0.20	
Se^{77}	$(g_{9/2})^2 p_{1/2} \rightarrow (g_{9/2})^4$	Se^{80}	$0 \rightarrow p_{1/2}$	0.10	0.24	
Cd^{111}	$(g_{7/2})^6 s_{1/2} \rightarrow (g_{7/2})^8$	Cd^{114}	$0 \rightarrow s_{1/2}$	0.10	0.20	
Cd^{113}	$s_{1/2} \rightarrow (h_{11/2})^2$	Cd^{114}	$0 \rightarrow s_{1/2}$	0.10	0.20	
Sn^{117}	$(h_{11/2})^2 s_{1/2} \rightarrow (h_{11/2})^4$	Sn^{118}	$0 \rightarrow s_{1/2}$	0.27	0.54	
Sn^{119}	$(h_{11/2})^4 s_{1/2} \rightarrow (h_{11/2})^6$	Sn^{118}	$0 \rightarrow s_{1/2}$	0.27	0.54	F/A
Te^{123}	$(h_{11/2})^6 s_{1/2} \rightarrow (h_{11/2})^8$	Te^{130}	$0 \rightarrow s_{1/2}$	0.66	1.3	
Te^{125}	$(h_{11/2})^8 s_{1/2} \rightarrow (h_{11/2})^{10}$	Te^{130}	$0 \rightarrow s_{1/2}$	0.30	0.60	
Pt^{195}	$(i_{13/2})^6 p_{1/2} \rightarrow (i_{13/2})^8$	Pt^{194}	$0 \rightarrow p_{1/2}$	0.20	0.40	

[a] A = allowed; F = forbidden.

(d,p) and (d,t) reactions connecting members of a $j-j$ coupling subshell of angular momentum j should be proportional to S where

$$S(n \rightleftharpoons n-1) = n \quad \text{for } n \text{ even}$$
$$= 1 - [(n-1)/(2j+1)] \quad \text{for } n \text{ odd.} \tag{2}$$

The inverse of this factor is therefore applied as a correction to intensity ratios.

RESULTS AND DISCUSSIONS

A typical level scheme for an even-even nucleus in the vibrational region is shown in Fig. 5. In the discussion, the various levels will be referred to as *G*, *1*, *2*, and *3* as labelled in that figure.

A. Ground States (*G*)

The simplest to measure, as well as one of the most interesting quantities to determine about (d,p) and (d,t) reactions leading to the ground states of even-even nuclei is the ratio of this cross section, σ_{oe}, to the cross section for the same reaction going from the ground state of one of the even-even isotopes of that element to a known level in the resulting odd nucleus; we call the latter cross section σ_{eo}. One reason for interest in this ratio is that, in accordance with Mayer-Jensen (M-J) shell-model configurations,[9] some of these transitions should be forbidden and others allowed. The results for the ratio σ_{oe}/σ_{eo} are shown for (d,p) reactions in Table II, and for (d,t) reactions in Table III.

In Table II, the first group are cases in which both transitions are allowed according to the M-J configurations. For Zr, which is at a closed shell, the theoretical prediction for the corrected ratio $\sigma_{oe}/\sigma_{eo}=1.0$ is well approximated; further evidence for the validity of the simple M-J configuration in Zr will be noted later. For Mo^{95}, Mo^{97}, and Pd^{105}, the transitions being compared are $l_n=2$ and $l_n=0$. The corrected ratios are in rough agreement with the aforementioned factor of two decrease in intensity per unit change in l_n. The result for the ratio of Zn^{67} to Zn^{64} seems very anomalous, especially since the l_n values are the same in the two transitions and the M-J configurations would seem to

TABLE III. σ_{oe}/σ_{eo} in (d,t) reactions.

Odd to even		Even to odd		σ_{oe}/σ_{eo}		
Target	M-J transition	Target	M-J transition	Measured	Corrected	Class
Zn^{37}	$(f_{5/2})^5 \rightarrow (f_{5/2})^4$	Zn^{68}	$(f_{5/2})^6 \rightarrow (f_{5/2})^5$	0.48	8.6	
Zr^{91}	$d_{5/2} \rightarrow 0$	Zr^{96}	$(d_{5/2})^6 \rightarrow (d_{5/2})^5$	0.22	1.36	
Zr^{91}	$d_{5/2} \rightarrow 0$	Zr^{94}	$(d_{5/2})^4 \rightarrow (d_{5/2})^3$	0.37	1.50	
Zr^{91}	$d_{5/2} \rightarrow 0$	Zr^{92}	$(d_{5/2})^2 \rightarrow d_{5/2}$	0.89	1.78	A/A
Mo^{95}	$(d_{5/2})^3 \rightarrow (d_{5/2})^2$	Mo^{98}	$(d_{5/2})^6 \rightarrow (d_{5/2})^5$	0.39	3.5	
Mo^{97}	$(d_{5/2})^5 \rightarrow (d_{5/2})^4$	Mo^{98}	$(d_{5/2})^6 \rightarrow (d_{5/2})^5$	0.30	5.4	
Pd^{105}	$(d_{5/2})^5 \rightarrow (d_{5/2})^4$	Pd^{106}	$(d_{5/2})^6 \rightarrow (d_{5/2})^5$	0.18	3.2	
Se^{77}	$p_{1/2} \rightarrow 0$	Se^{80}	$(g_{9/2})^8 \rightarrow (g_{9/2})^6 p_{1/2}$	0.27	0.54	
Cd^{111}	$s_{1/2} \rightarrow 0$	Cd^{112}	$(g_{7/2})^8 \rightarrow (g_{7/2})^6 s_{1/2}$	0.90	1.8	
Cd^{113}	$s_{1/2} \rightarrow 0$	Cd^{112}	$(g_{7/2})^8 \rightarrow (g_{7/2})^6 s_{1/2}$	0.74	1.5	
Sn^{117}	$s_{1/2} \rightarrow 0$	Sn^{118}	$(h_{11/2})^4 \rightarrow (h_{11/2})^2 s_{1/2}$	0.76	1.5	A/F
Sn^{119}	$s_{1/2} \rightarrow 0$	Sn^{118}	$(h_{11/2})^4 \rightarrow (h_{11/2})^2 s_{1/2}$	0.67	1.3	
Te^{125}	$s_{1/2} \rightarrow 0$	Te^{128}	$(h_{11/2})^{12} \rightarrow (h_{11/2})^{10} s_{1/2}$	0.7	1.4	
Pt^{195}	$p_{1/2} \rightarrow 0$	Pt^{194}	$(i_{13/2})^6 \rightarrow (i_{13/2})^4 p_{1/2}$	0.16	0.32	

[9] M. Mayer and J. H. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley and Sons, New York, 1955).

be realistic. If the result is taken at face value, it could only be explained by assuming that Zn^{68}, with 38 neutrons, has very little $(f_{5/2})^6$ or $(f_{5/2})^4$ in its configuration. Since Zn^{67}, Zn^{65}, and probably even Zn^{63} have spin 5/2, this seems very unlikely. A possible explanation is that the correction (2) is too large—it is a factor of nine in this case. There is further evidence for this from Table III as will be discussed below.

The second group of Table II should be considered together with the second group of Table III, as these are essentially inverse reactions of each other. These are cases where, according to the M-J scheme of high orbital angular momentum shells filling in pairs,[9] σ_{oe} should be forbidden in (d,p) and allowed in (d,t) reactions, and the opposite should be true for σ_{eo}. Thus σ_{oe}/σ_{eo} should be very large in (d,t) reactions and very small in (d,p). The results do show a slight trend in this direction, but not nearly as strong as expected.[10] We may thus conclude that the M-J configurations are very far from pure. If the alternative method of determining reduced widths by dividing by the Butler predictions had been used, σ_{oe}/σ_{eo} would be increased by a factor of 1.4 in (d,p) reactions and decreased by the same factor in (d,p) reactions, so that the above effect would be accentuated. The Sn and Te isotopes would then have $(s_{1/2})^2$ in about half of their configurations. This is in agreement with recent calculations of the effects of pairing forces.[11] In Cd, the $s_{1/2}$ shell seems to be mostly filled.

In the first group of Table III, both competing reactions are allowed, and the l_n values are the same. The corrected results for Zr are somewhat larger than the expected value, unity; the situation would be only slightly improved if the Butler theory corrections for dependence on Q value were inserted. This would seem to indicate that the ground states of Zr^{92}, Zr^{94}, and Zr^{96} are not pure $(d_{5/2})^{2n}$, but contain admixtures of $(g_{7/2})^2$, $(s_{1/2})^2$, etc. While such admixtures are expected from pairing theory, the large deviation of Zr^{92} from $(d_{5/2})^2$ is somewhat surprising.

TABLE IV. $d\sigma/d\Omega$ at 30° for excitation of ground states of even-even nuclei.

(d,p) reactions			(d,t) reactions		
	$d\sigma/d\Omega$(mb/sr)			$d\sigma/d\Omega$(mb/sr)	
Target	Observed	Corrected	Target	Observed	Corrected
Zr^{91}	0.14	0.42	Zr^{91}	1.33	1.33
Mo^{95}	0.18	0.27	Mo^{95}	0.40	0.60
Mo^{97}	0.55	0.55	Mo^{97}	0.30	0.90
Pd^{105}	0.18	0.18	Pd^{105}	0.39	1.17
Cd^{111}	0.40	0.40	Cd^{111}	1.00	1.00
Cd^{113}	0.40	0.40	Cd^{113}	0.80	0.80
Sn^{117}	0.82	0.82	Sn^{117}	1.30	1.30
Sn^{119}	0.86	0.86	Sn^{119}	1.15	1.15

[10] In applying correction (2) it was assumed that the final nucleus has a configuration that makes the transition allowed; otherwise the corrected ratios would be zero or infinity.

[11] R. Sorenson and L. Kisslinger (private communication).

In the other cases listed in the first group of Table III, σ_{oe}/σ_{eo} is many times larger than unity. These cases cannot be explained by impure configurations as such impurities would affect both cross sections. Furthermore, the results are quite insensitive to reasonable changes in the assumed configurations; for example, if the ground states in Mo and Pd were $(d_{5/2})^4$ instead of $(d_{5/2})^6$, the result would change by only a factor of 1.5. If corrections for the Butler theory dependence on Q value were applied, the discrepancies would be reduced, but only by about 40%.

One possible explanation is that the correction (2) for odd n may be too small. The assumptions used in deriving (2) are only that the states involved are of lowest seniority and are unique, both of which seem reasonable in considering ground states. Another possibility is that the states involved contain small admixtures of unexpected configurations which become important because of the correction factor. For example, if an odd $\rightarrow$ even transition usually considered to be $[(d_{5/2})^5]_{5/2} \rightarrow [(d_{5/2})^4]_0$ consisted partly of

$$[(d_{5/2})^4 d_{3/2}]_{5/2} \rightarrow [(d_{5/2})^3 d_{3/2}]_0,$$

the correction (2) would accentuate the latter relative to the former by a factor of 12. It should be pointed out, however, that a configuration $[(d_{5/2})^3 d_{3/2}]_0$ is most unexpected in the ground state of an even-even nucleus.

The absolute differential cross sections at 30° for exciting ground states of even-even nuclei (i.e., σ_{oe}) are listed in Table IV. The uncertainties in the absolute values are about 50%, and even the relative uncertainties among the group are about 30% as the method of taking data (e.g., slit arrangements) was changed several times during the experiment without making more than rough calculated estimates of the effects. The grouping of nuclei in the table and the shell model configurations assumed in the corrections are taken from Tables II and III.

In the first group of (d,p) reactions, the corrected cross section for Zr may be assumed to be the single-particle value. On this basis, it would seem that the $(d_{5/2})^5$ and $(d_{5/2})^6$ configurations in Mo^{97} and Mo^{98}, respectively, are quite pure, whereas the configurations assumed in the $Mo^{95}(d,p)Mo^{96}$ and $Pd^{105}(d,p)Pd^{106}$ reactions are not.

In the first group of (d,t) reactions, the $Zr^{91}(d,t)$ reaction may be considered to have the single-particle cross section. The $Pd(d,t)$ reaction has essentially the same corrected cross section, and in the Mo isotopes, it is not very much smaller. This indicates that in all these cases, there is strong overlap between the ground state of the target nucleus, and the ground state of the residual nucleus plus a $d_{5/2}$ neutron. The large value in Pd is somewhat surprising as one might expect a large part of the Pd^{104} configuration to be $(d_{5/2})^6$ for which the reaction is forbidden. The result indicates that Pd^{104} has largely $(d_{5/2})^4$; if the same is true of Pd^{106},

TABLE V. Excitation of vibrational levels. K is the ratio of intensities for reactions with $l_n=l+2$ and $l_n=l$. It is roughly 1/4. $(1-c)$ is the coefficient of terms with $(p_{3/2})^4$ in the configuration; c is roughly 0.4 in Se and Pt. q is the coefficient of terms with $(s_{1/2})^2$ in the original nucleus; q is about 0.2 in the Pd and Mo isotopes.

Target nuclide	$I-\pi$	(d,p) reactions $1/G$ Meas.	Cor.	$2/G$ Meas.	Cor.	(d,t) reactions $1/G$ Meas.	Cor.	$2/G$ Meas.	Cor.
Zn^{67}	5/2−	0.7	0.7K	1.0	1.0K	1.1	0.1K	1.0	0.09K
Se^{77}	1/2−	0.7	0.6/c	5.7	5/c	0.9	0.25	1.1	0.30
Zr^{91}	5/2+	5.6	1.1						
Mo^{97}	5/2+					1.2	0.3K/q		
Pd^{105}	5/2+	0.73	0.87K	1.8	2.2K	1.3	0.3K/q	0.4	0.1K/q
Cd^{111}	1/2+	0.21	0.17/K	0.26	0.26	0.3	0.08/K	0.3	0.3
Cd^{113}	1/2+	0.35	0.28/K	0.25	0.25	0.35	0.09/K	0.14	0.14
Sn^{117}	1/2+	0.13	0.11/K			0.41	0.10/K		
Sn^{119}	1/2+	0.13	0.11/K			0.26	0.07/K		
Te^{125}	1/2+	0.09	0.08/K	0.06	0.06	0.16	0.04/K	0.06	0.06
Pt^{195}	1/2−	0.16	0.13/c	0.16	0.13/c	0.11	0.03	0.23	0.06

the relatively small cross section for $Pd^{105}(d,p)$ is explained.

A comparison may be made between the Sn and Zr cross sections if the former are multiplied by about two to correct for the fact that they are half forbidden (see above) and divided by a factor of about four to correct for the difference between $l_n=0$ and $l_n=2$; the net correction is thus to divide the Sn cross sections by about two. The result is that the (d,p) reactions in the Sn isotopes have about the single-particle value, while the (d,t) cross sections are about half of the single-particle expectation.

This calculation is, of course, very crude, and no account was taken of the possible change of single-particle cross sections with mass number, but the general conclusion is that the Sn cross sections are not far below the single-particle value. Independent evidence has recently been obtained which indicates that they are very close to it.[12] The Cd cross sections for both (d,p) and (d,t) are smaller than the corresponding Sn cross sections, but not by a large factor.

The general conclusion from Table IV is, therefore, that all of the (d,p) and (d,t) cross sections listed there are within a factor of two of the single-particle values, and are in most cases considerably closer. This indicates a relatively high degree of overlap between the wave functions of the target nuclei and the wave functions of the final nuclei plus or minus a neutron.

B. First Excited States (*1*)

The ratio $1/G$; of the strengths with which the first excited states (*1*) and the ground states (G) are excited are listed in Table V. Applying correction (2) to the excitation of *1* is not completely straightforward, but estimates can be made by assuming reasonable transitions as will now be described:

(d,p)−Zn. Elwyn and Shull[13] measured the angular distribution and found $l_n=1$, so that the transition is probably $(f_{5/2})^5 \rightarrow (f_{5/2})^6 p_{1/2}$ for which $S=1$. There is probably a smaller contribution from $(p_{3/2})^2 \rightarrow (p_{3/2})^3$ for which $S=1/2$; a two to one weighted average was taken.

(d,p)−Se, Pt. They are probably $(p_{3/2})^{2n} p_{1/2} \rightarrow (p_{3/2})^{2n+1} p_{1/2}$ where $n=0$ or 2, in which cases $S=1$ or 1/2, respectively. However, the most probable configuration for the original nuclei is $(p_{3/2})^4 p_{1/2}$ for which the transition is forbidden. If the amount of this configuration is $(1-C)$; a correction of $1/C$ should then be applied; as a very rough estimate, C might be about 0.4. The results of Table V indicate that C may be considerably higher for Se.

(d,p)−Zr. An angular distribution measurement indicated $l_n=2$ so that the transition is $(d_{5/2}) \rightarrow [(d_{5/2})^2]_2$. This is quite interesting as $l_n=0$ is allowed from selection rules.

(d,p)−Pd. The angular distributions were measured for Pd, and it was found that $l_n=0$ ($l_n=2$ for G). The transition must therefore be $(d_{5/2})^5 \rightarrow (d_{5/2})^5 s_{1/2}$ for which $S=1$.

(d,p)−Cd, Sn, Te. They may be $s_{1/2}(d_{3/2})^{2n} \rightarrow s_{1/2}(d_{3/2})^{2n+1}$ where for $n=0$, $S=1$ and for $n=1$, $S=1/2$; or they may be $s_{1/2}(d_{5/2})^4 \rightarrow s_{1/2}(d_{5/2})^5$ for which $S=1/3$. A straight average of the three cases was used.

(d,t)−Zn. They are probably $(p_{3/2})^4 p_{1/2} \rightarrow (p_{3/2})^3 p_{1/2}$ for which $S=4$.

(d,t)−Se, Pt. They are probably $(p_{3/2})^{2n} p_{1/2} \rightarrow (p_{3/2})^{2n-1} p_{1/2}$ where $n=1$ or 2 for which $S=2$ or 4, respectively; a weighted average with $(n=2)/(n=1)=2$ was taken.

(d,t)−Pd, Mo. They are probably $(d_{5/2})^5 (s_{1/2})^2 \rightarrow (d_{5/2})^5 s_{1/2}$, for which $S=2$. However, one would expect very little $(s_{1/2})^2$ in the original nuclei; taking its amount as q, a correction of $1/q$ should be applied. The results indicate that q must be at least about 0.2 which is perhaps surprisingly large.

(d,t)−Cd, Sn, Te. Likely transitions are $(d_{5/2})^{2n} s_{1/2} \rightarrow (d_{5/2})^{2n-1} s_{1/2}$ where $n=3$ or 2 for which $S=6$ or 4, respectively; or $(d_{3/2})^2 s_{1/2} d_{3/2} s_{1/2}$ for which $S=2$. A straight average was taken.

In several cases, l_n is different for G and *1*, so that a correction for this must be applied; this correction is

[12] B. L. Cohen and R. E. Price (to be published).
[13] F. B. Shull and A. J. Elwyn, Phys. Rev. **112**, 1667 (1958).

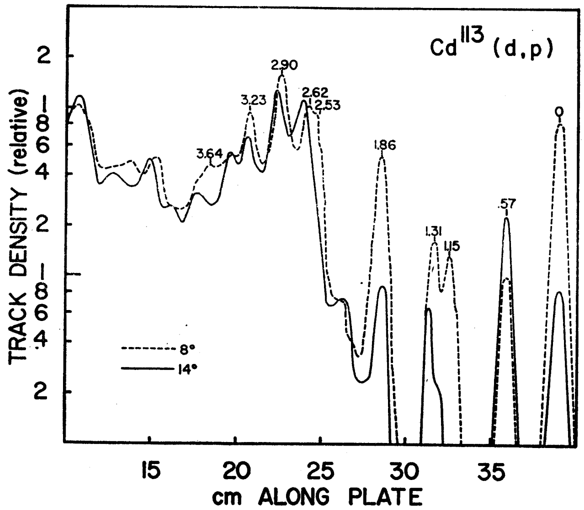

FIG. 6. Energy distributions of protons emitted from $Cd^{113}(d,p)Cd^{114}$ at 8° and 14°. Energies shown are excitation energies of levels in Cd^{114}.

indicated by K in Table V. As a rough approximation, K may be taken as 1/4.

For $Zr^{91}(d,p)$, the ratio close to unity as well as the above-mentioned angular distributions are as predicted by the simple shell-model configurations $(d_{5/2})_0{}^2$ and $(d_{5/2})_2{}^2$ for G and 1, respectively. For all other cases shown, the situation is quite different; while the uncertainties and fluctuations are large, the ratio $1/G$ averages out to roughly 1/3.

In the usual quadrupole vibration theories,[14] *1* and *G* are identical particle-wise, differing only in that the nucleus is vibrating in *1* but not in *G*. The very fact that there are frequently different l_n values for *1* and *G* is contradictory with the theory in the strictest sense. However, in a broader sense, this would be possible if the excitation of *1* in $Cd(d,p)$, for example, proceeded as follows: (a) the neutron enters with $l_n=2$, (b) it transfers two units of angular momentum to a vibrational motion leaving it with $l_n=0$, and (c) it fits into the nucleus as an $s_{1/2}$ particle. However, the excitation of *G* can proceed by steps (a) and (c) alone, omitting (b). Thus the excitation of *1* is a higher order process than the excitation of *G* and should be very much less probable. Satchler[15] has treated this problem in detail, and estimates that the ratio should be about 1/10; evidence presented below would reduce his estimate by about a factor of two. This is considerably lower than the experimental value of 1/3. The discrepancies with Satchler's calculation are even more striking in the case of second excited states, as will be discussed in the next section.

From the shell-model point of view, *1* is seen to be a very complicated mixture of configurations rather than just a pair recoupled to $J=2$. For example, terms such as $(sd)_2$ and $(pf)_2$ are important.

C. Second Excited States (*2*)

The strength with which *2* are excited is also listed in Table V. Since this state is believed to be a 0^+, 2^+, 4^+ triplet, the corrections are almost impossible to apply accurately. However, in all cases except those where the target nucleus is $1/2^+$, one would expect the 2^+ state to be the most strongly excited as its l_n value is minimum, and the $(2J_f+1)$ factor gives it strong preference to the 0^+ state, so that the corrections are the same as for $1/G$. In cases where the target nucleus is $s_{1/2}$, the l_n value favors the 0^+ state; if that state is assumed to be excited, there is no obvious correction to the ratio $2/G$, so that none is applied. It is clear from Table IV that the ratios $2/G$ are about the same as $1/G$, so that the statements regarding the disagreement with Satchler's prediction also apply here. The discrepancy in this case, is much larger since Satchler predicts $2/G$ to be $(1/G)^2$ or $\approx 1/100$. It may be noted that this discrepancy could be explained if there were other levels nearby with which the states of the triplet could mix; however, as will be shown in Sec. D below, this is usually not the case.

An interesting problem regarding *2* is the question of whether it really is a triplet; experimental evidence on this is sadly lacking except in the single case of Cd^{114}. Since selection rules are not very strong in (d,p) and (d,t) reactions, it was hoped that the three members of the triplet might be seen. Indeed, experience has shown that practically all known levels are found in these experiments.

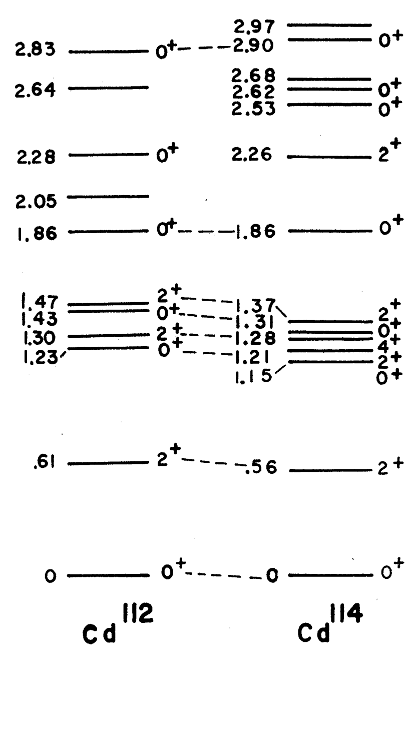

FIG. 7. Level schemes for Cd^{112} and Cd^{114}. Energies are in Mev. Levels found to be 0^+ in these experiments are so labelled, and levels not so labelled are not 0^+. Dashed lines are inserted to point out similarities in level schemes of the two isotopes.

[14] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953); G. Scharff-Goldhaber and J. Weneser, Phys. Rev. **98**, 212 (1955).
[15] G. R. Satchler, Ann. Phys. **3**, 275 (1958).

A rather extensive effort was expended in studying this problem, but with the exception of the cadmium isotopes to be discussed below, no new states were found in this region. Some of the best cases where negative evidence was obtained are shown in Figs. 1–4. For these cases—Pd^{104}, Pd^{106}, Pt^{194}, and Pt^{196}—another state would have been found if it were separated from the one observed by as much as 40 kev assuming it is excited with about equal strength. If it were excited 10 times less strongly, it could be as far away as 80 kev, and if it were excited 50 times less strongly, it would not be seen at all. While intensity ratios of various peaks have been found to vary by factors as large as 50, these large variations are always easily explained, and indeed expected, theoretically. There is no theoretical reason for large intensity ratios among the members of the triplet beyond those due to different l_n's, which can only explain differences of about a factor of four in these cases. It thus seems most unlikely that the triplets for these nuclei are spread by as much as 80 kev.

The situation in Cd^{114}, as studied by $Cd^{113}(d,p)$, is shown in Fig. 6. Energy spectra were measured for $\theta=8°$ and $\theta=14°$; in both cases, there are two peaks in the region of *2*, but they are displaced relative to each other. The peaks at 8° are due to $l_n=0$, and occur at 1.15 Mev and 1.31 Mev. The main peak at 14° is due to $l_n=2$, and occurs at 1.38 Mev. Assuming that Cd^{113} has an $s_{1/2}$ configuration, which seems very likely,[12] the $l_n=0$ levels are almost certainly 0^+. The level scheme for Cd^{114} is shown in Fig. 7; it includes results from Sec. E, and levels known from other sources.

The situation in Cd^{112} as studied by $Cd^{111}(d,p)$ is very similar to that in Cd^{114}. Again there are two 0^+ and two 2^+ levels in the region of *2*. The level scheme for Cd^{112} is also shown in Fig. 7. There seems to be a close correspondence between Cd^{112} and Cd^{114} levels in several cases, as shown by the dashed lines in the figure.

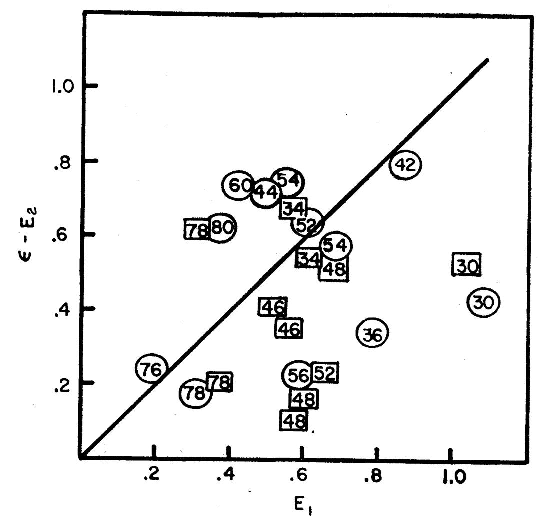

FIG. 8. Energy difference between *2* and next level above *2* vs energy of *1*. Numbers are atomic number of nucleus. Squares are data from this work, and circles are known levels from other sources. The 45° line represents the expected position of *3*.

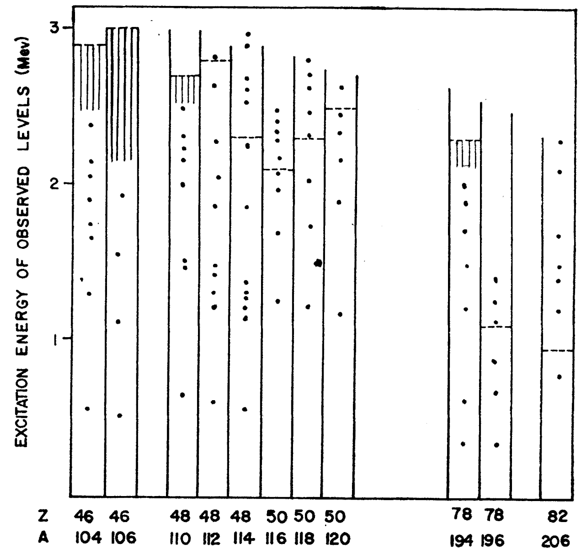

FIG. 9. Positions of observed levels in various nuclei. Dashed lines show value of Δ defined in text. Shading indicates unexplored regions.

In the Sn isotopes, the region of *2* is not defined, as there are several levels below twice the energy of *1*. In Te^{126} and Te^{124}, only single levels were found in the region of *2*, but the resolution was poor and intensities were low, so that no conclusive results were obtained. In Mo, the region of *2* was obscured by levels from other isotopes. In Se, intensity difficulties were severe as the isotopic abundance of the odd isotope is low, and beam currents were restricted by the low melting point of the target.

D. Higher Excited States and the Size of the Energy Gap

Any vibrational theory predicts that a 0^+, 2^+, 3^+, 4^+, 6^+ quintet should occur at roughly three times the energy of *1*. In Figs. 1 to 4 it is seen that there is usually a considerable spacing between *2* and the next level; this suggests that these next levels may be members of this quintet. The magnitudes of these spacings, $(\epsilon-E_2)$, for all cases studied are shown plotted vs E_1, the energy of *1*, in Fig. 8 (squares). Data from other sources is also shown in Fig. 8 (circles), although it should be noted that these are essentially upper limits on $(\epsilon-E_2)$, as there may be lower undiscovered levels.

It is apparent from Fig. 8 that in almost all cases where fairly complete data is available, the energy of the next level, ϵ, is less than (E_1+E_2). This indicates that these levels are probably not members of the three phonon oscillation band, and their occurrence signals the top of the famous "energy gap" in even-even nuclei. A better perspective on this gap may be obtained from Fig. 9 which gives the level schemes found in this work

TABLE VI. The energy gap in even-even nuclei.

A	Z	Element	ϵ(Mev)	Δ(Mev)
66	30	Zn	2.40	3.5
104	46	Pd	1.66	2.9
106	46	Pd	1.54	3.0
110	48	Cd	2.00	2.7
112	48	Cd	1.43	2.8
114	48	Cd	1.31	2.3
116	50	Sn	1.70	2.1
118	50	Sn	1.75	2.3
120	50	Sn	1.89	2.5
126	52	Te	1.65	2.4
196	78	Pt	0.87	1.1

for the cases where the most complete data were obtained.

From Figs. 8 and 9, the energy ϵ, of the lowest excited state which is apparently not a member of the vibrational band structure may be determined; the values are listed in Table VI. In accordance with recent developments in pairing force theory, the lowest lying levels of this type should be grouped about an energy Δ, which may be defined as the difference between the binding energy of the last neutron in that nucleus and the average of the binding energies of the last neutrons in the nuclei with one more and one less neutron.[11] Values of Δ are listed in Table VI, and shown by the dashed lines in Fig. 9. In all cases, ϵ is less than Δ, and in many cases it is much less.

It should perhaps be pointed out that if the multiplet of levels composing the three phonon oscillation were very widely spaced, this would generally explain the rather large number of levels found near and below its expected location. This explanation would be difficult in Cd^{112} and Cd^{114}, however, as there are too many 0^+ levels. It would also be somewhat surprising in view of the evidence from Sec. C that the spacing of the triplet second excited state is quite small.

E. Location of 0^+ Levels

From angular distribution studies of (d,p) reactions, levels for which $l_n=0$ are easily located. In cases where the target nucleus is $1/2^+$, this means that the levels observed have positive parity and spin 0 or 1. However, it seems very likely that the ground-state configurations of $1/2^+$ nuclei are almost purely $s_{1/2}$. Since $(s_{1/2})^2$ cannot couple to one, it therefore seems reasonable to believe that the $l_n=0$ transitions lead to 0^+ states of the even-even nucleus. Under this assumption, the 0^+ states in Sn^{118}, Sn^{120}, Cd^{112}, and Cd^{114} are listed in Table VII, and the appropriate levels in the cadmium isotopes are labelled in Fig. 7. Similar attempts with (d,t) reactions were unsuccessful due to background difficulties at small angles.

In the reaction $Pd^{105}(d,p)Pd^{106}$, the angular distributions of the 1.56- and 1.94-Mev levels were strongly indicative of $l_n=2$. Since the ground state of Pd^{105} is probably $(d_{5/2})^5$, these states are probably $(d_{5/2})^6$ which must couple to zero.

It is interesting to note that the 1.86-Mev levels in Cd^{112} and Cd^{114} are 0^+ rather than 3^+ as required by the Davidov-Filipov theory.[16] The previous evidence for this

TABLE VII. Location of 0^+ levels in even-even nuclei. Energies are in Mev.

Sn^{118}—0, 1.75, 2.03, 2.48
Sn^{120}—0, 1.89, 2.16, 2.62
Cd^{112}—0, 1.23, 1.43, 1.86, 2.28, 2.83
Cd^{114}—0, 1.15, 1.31, 1.86, 2.53, 2.62, 2.90, 3.23, 3.64
Pd^{106}—0, 1.13, 1.56(?), 1.94(?)

level being 3^+ was the occurrence of an 0.57-Mev gamma ray in $Cd^{113}(n,\gamma)$ which was assigned by Motz[17] as a transition from the 1.86- to the 1.28-Mev level. However, with the discovery here of the 1.15-Mev 0^+ state, the 0.57-Mev gamma ray is expected from the transition of this to the 0.57 Mev first excited state. The fairly definite absence of a state at exactly the energy E_1+E_2 in the Pd and Pt isotopes might also be cited as evidence against the Davidov-Filipov theory.

The very large gap between the second and third excited states induced by the reaction $Pt^{195}(d,t)Pt^{194}$ (see Fig. 2) is very difficult to understand on the basis of any of the usual theories.

ACKNOWLEDGMENTS

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[16] A. S. Davidov and G. P. Filipov, Nuclear Phys. **8**, 237 (1958).
[17] H. T. Motz, Phys. Rev. **104**, 1353 (1956).