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Gamma Rays from Deuteron Stripping Reactions

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The distorted-wave Born approximation is used to calculate the p-γ angular correlation from several deuteron-stripping (d,p) reactions. One $l=2$ and four $l=1$ captures are considered. Optical potentials with rounded edges are used to distort the wave functions. In some cases the correlation is considerably changed from the pattern predicted by the plane wave Born approximation, and the distortion effects are strongly dependent on the direction of the emitted proton, and on the type of distortion assumed. We include a general discussion of the theory of the $(d,p\gamma)$ correlation.

INTRODUCTION

THE distorted wave Born approximation (DWBA) has been used recently for a numerical evaluation of deuteron stripping cross sections and polarizations,[1,2] showing that a considerable improvement in the fit to experimental data can be obtained compared to the simple plane wave, or Butler, theory.[3] The DWBA theory also modifies the predictions of the simple theory for the angular distribution of any γ rays emitted by the residual nucleus.[4,5] The study of such departures from the simple theory can give valuable information as to the validity of the distorted wave theory. Several of such experiments have been reported.[6]

We present here the results of $(d,p\gamma)$ angular correlations for several particular reactions. For each reaction four different forms of stripping theory were tried: (a) plane wave Born approximation with radial cutoff (Butler theory), (b) Coulomb wave Born approximation with radial cutoff, (c) optical potential wave Born approximation with radial cutoff, (d) optical potential wave Born approximation without radial cutoff. Our calculations were based on stripping amplitudes previously calculated by Tobocman.[2] Preliminary to the presentation of our results we will give a description of the theory of the $(d,p\gamma)$ correlation which represents an elaboration of the approach of Huby, Refai, and Satchler.[5]

GENERAL FEATURES

When plane waves are used to describe the reaction $A(d,p)B$, the probability of a neutron being captured with component m of orbital angular momentum l vanishes unless $m=0$, when m is measured along the direction of the recoil momentum $\mathbf{k}=\mathbf{k}_d-(M_I/M_F)\mathbf{k}_p$ ($\mathbf{k}_d$ and $\mathbf{k}_p$ are the center-of-mass wave vectors for the deuteron and proton, respectively). That is, the vector $\mathbf{l}$ is perpendicular to $\mathbf{k}$, and equally distributed around it. So the residual nucleus is oriented as though it had captured neutrons from a plane wave incident along $\mathbf{k}$.[7] Then (provided they are measured in coincidence with protons along $\mathbf{k}_p$ so that $\mathbf{k}$ is defined) the angular distribution of any ensuing γ rays is the same as for a resonant (n,γ) reaction, with $\mathbf{k}$ an axis of azimuthal as well as back-forward symmetry.

These simple results are modified by any distortion of the proton or deuteron waves. The Butler theory completely neglects refraction and reflection of the waves, while attempting to simulate absorption within the nucleus by ignoring contributions from inside some cutoff radius. Such effects introduce some shadowing which, crudely speaking, favors one "side" of the nucleus as a source of protons emitted in a given direction.[8] This is reflected in the capture of neutrons with nonzero components m of orbital angular momentum $\mathbf{l}$ along the recoil direction $\mathbf{k}$. No longer is $\mathbf{l}$ always perpendicular to $\mathbf{k}$, or equally distributed around it. Rather, it becomes preferentially aligned parallel or

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[1] W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955); H. C. Newns and M. Y. Refai, Proc. Phys. Soc. (London) **A71**, 627 (1958); H. A. Weidenmüller, Z. Physik **150**, 389 (1958); J. L. Richter and E. V. Ivash, Phys. Rev. **111**, 245 (1958).

[2] W. Tobocman, Phys. Rev. **115**, 99 (1959).

[3] S. T. Butler, Phys. Rev. **106**, 272 (1957), and other references contained there.

[4] L. C. Biedenharn, K. Boyer, and R. A. Charpie, Phys. Rev. **88**, 517 (1952); G. R. Satchler and J. A. Spiers, Proc. Phys. Soc. (London) **A65**, 980 (1952); J. Horowitz and A. M. L. Messiah, J. phys. radium **15**, 142 (1954); G. R. Satchler, *Comptes Rendue Congres International de Physique Nucléaire Interactions aux Basses Energies et Structure des Noyaux, Paris, 1958*, edited by P. Guggenberger (Dunod, Paris, 1958).

[5] R. Huby, M. Y. Refai, and G. R. Satchler, Nuclear Phys. **9**, 94 (1958).

[6] K. W. Allen, B. Collinge, B. Hird, B. C. Maglic, and P. R. Orman, Proc. Phys. Soc. (London) **A69**, 705 (1956); S. A. Cox and R. M. Williamson, Phys. Rev. **105**, 1799 (1957); H. A. Hill and J. M. Blair, Phys. Rev. **111**, 1142 (1958); R. Taylor, Phys. Rev. **113**, 1293 (1959); J. P. Martin and K. S. Quisenberry, Bull. Am. Phys. Soc. **4**, 404 (1959); J. A. Kuehner, E. Almqvist, and D. A. Bromley, Bull. Am. Phys. Soc. Ser. II, **5**, 56 (1960). The last two references include specific attempts to test the predictions of the DWBA.

[7] The discussion here is equally applicable to (d,n) reactions, although of course the outgoing neutrons do not experience any Coulomb repulsion.

[8] N. Austern, S. T. Butler, and C. Pearson, Phys. Rev. **112**, 1227 (1958); H. C. Newns, Proc. Phys. Soc. (London) **A66**, 477 (1953).

antiparallel to $\mathbf{k}_d \times \mathbf{k}_p$, and no longer is there a simple axis of alignment for the residual nuclei in general. This polarization of the capture process allows the outgoing protons to be polarized, and also tends to destroy the symmetry of the γ-ray distribution about the recoil direction. In particular, for $l=1$ captures the effect is to rotate the γ-ray symmetry axis away from $\mathbf{k}$, and to introduce some azimuthal anisotropy about this axis, while for captures with higher l there is, in general, no axis of symmetry for the γ rays.

The distorted waves we use here are those given by an optical model, adjusted so as to give a reasonable description of the observed elastic scattering. Effects of spin orbit coupling on the scattering have been neglected; they are expected to be small, but will be investigated in future calculations.

THEORY

The general angular correlation for γ rays emitted in a direction (θ,ϕ) following a (d,p) reaction, without any assumption as to the mechanism of the reaction, takes the form[5]

$$W(\theta\phi)=\sum_{kq} a_{kq} C_{kq}(\theta\phi), \tag{1}$$

where $C_{kq}(\theta\phi)=(4\pi)^{\frac{1}{2}}(2k+1)^{-\frac{1}{2}}Y_k{}^q(\theta\phi)$ is the spherical harmonic normalized so that $C_{00}=1$ and $C_{k0}(\theta\phi)=P_k(\cos\theta)$. The angles θ and φ refer to the center-of-mass system of coordinates. The coefficients a_{kq} depend upon the nuclear spins and γ-ray multipolarity, and of course the direction of emission of the protons. Since W is real they must have the symmetry $a_{kq}=(-)^q a_{k-q}{}^*$, and if the γ transition is between states of definite parity only a_{kq} with even k are nonzero. The γ distribution has symmetry of reflection through the reaction plane containing $\mathbf{k}_d$ and $\mathbf{k}_p$, so if the x and z axes are chosen in the reaction plane we require the a_{kq} to be real to give $W(\theta,\phi)=W(\theta,-\phi)$. Alternatively, if we took the z axis along $\mathbf{k}_d\times\mathbf{k}_p$ we would require q to take only even values so that $W(\theta,\phi)=W(\pi-\theta,\phi)$.

If we now assume the stripping mechanism (in DWBA, but still neglecting the small spin coupling in the distorted waves) and only one value of l takes part, the a_{kq} separate into two factors

$$a_{kq}=g_k d_{kq}, \tag{2}$$

of which g_k contains the dependence on nuclear spins and the *total* angular momentum $j=l\pm\frac{1}{2}$ of the captured neutron.[9] If the target of spin J_A absorbs a neutron to form a state of spin J_B which then decays to a final state of spin J_c,

$$g_k=\left[\sum_{jj'}\theta_{jl}\theta_{j'}\,\eta_k(jj'J_AJ_B)\right] \times\left[\sum_{LL'}C_LC_{L'}F_k(LL'J_cJ_B)\right], \tag{3}$$

which we may write symbolically as $g_k=\eta_kF_k$.[10] As usual, we have $k\leqslant 2l$, $j+j'$, $L+L'$, and $2J_B$. θ_{jl} is the reduced width for capture with $j=l\pm\frac{1}{2}$, and C_L the reduced matrix element for a 2^L-pole γ ray. If we normalize to $\sum_j \theta_{jl}{}^2=\sum_L C_L{}^2=1$, then $g_0=1$. The coefficients η_k [11] and F_k [12] have been tabulated. If one prefers to work with a channel spin representation (vector sum of target spin and captured neutron spin $S=J_A\pm\frac{1}{2}$) we replace $g_k=\eta_kF_k$ by $g_k=Z_kF_k$, where

$$Z_k=\sum_S \theta_{Sl}{}^2(-)^{S-J_B}(2J_B+1)^{-\frac{1}{2}}\bar{Z}(lJ_BlJ_B,Sk), \tag{4}$$

so $Z_0=1$ if $\sum_S \theta_{Sl}{}^2=1$. Tables of $Z=i^k\bar{Z}$ are available.[13] The dynamics of the stripping mechanism, including the effects of distorted waves, are described by the d_{kq}, which are essentially the statistical tensors $\rho_{kq}(ll)$ described in detail in reference 5,

$$d_{kq}=\rho_{kq}(ll)(-)^l\left[(l0,l0|k0)(2l+1)^{\frac{1}{2}}\rho_{00}(ll)\right]^{-1}, \tag{5}$$

so that $d_{00}=1$. (This normalization is only possible with even k, since $(l0,l0|k0)$ vanishes if k is odd.) In the plane wave, Butler, theory the d_{kq} are unity if $q=0$ and zero otherwise, if the z axis is chosen along the recoil $\mathbf{k}$, so that the distribution (1) becomes

$$W_0(\theta)=\sum_k g_kP_k(\cos\theta). \tag{6}$$

In the general case it is instructive to study the correlation function (1) referred to two sets of axes. First, if we choose the x and z axes to be in the reaction plane, so that the d_{kq} are real, the correlation becomes

$$W(\theta\phi)=\sum_k g_k\left\{d_{k0}P_k(\cos\theta) +2\sum_{q>0}(-)^q\left[\frac{(k-q)!}{(k+q)!}\right]^{\frac{1}{2}}P_k{}^q(\theta)d_{kq}\cos(q\phi)\right\}. \tag{7}$$

The existence of a symmetry axis for W would imply that the z-axis could be chosen so that all d_{kq} with odd q (and hence odd powers of $\sin\theta$) vanished. This can always be done when $k=0$ and 2 only, but not otherwise, except in the plane wave limit (6). Alternatively,

[9] Although spin-orbit coupling is assumed to have negligible effect on the distortion of the deuteron and proton waves, it is implied for the captured neutron in so far as the neutron reduced widths depend on j. However, this dependence enters the g_k, and does not affect the dynamical factors d_{kq}.

[10] The discussion of this section is also appropriate to other stripping reactions such as (He^3,d), etc. In particular Eqs. (2), (3), (4), and (5) give the $b-\gamma$ correlation coefficients for any such reaction $A(a,b)B$ in which a single nucleon, initially in an S state within a, is captured with j and l by the target A. The exchange of a cluster, again if initially in an S state within a, such as in (α,d) if viewed as a stripping reaction, will lead to a different coupling factor η_k in (3) if the cluster spin is not $\frac{1}{2}$, although the Z_k of (4) will be unchanged. The $a-\gamma$ correlation from the inverse pickup reactions $B(b,a)A$ is also described by the same equations (with appropriate interchange of suffices A with B and a with b) with the coefficient d_{kq} for pickup being equal to the stripping coefficient $(-)^{k-q}d_{kq}{}^*$ of (5) and reference 5. The only consequence of this is that the phase angles α_{kq} or ϕ_0, discussed below, change sign. Thus the results reported here and in reference 2 give a qualitative idea of distorted wave effects in these reactions also.

[11] G. R. Satchler, Proc. Phys. Soc. (London) **A66**, 1081 (1953).

[12] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953); M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL-5324 (unpublished).

[13] L. C. Biedenharn, Oak Ridge National Laboratory Report ORNL-1501 (unpublished).

TABLE I. Parameters characterizing the cases calculated.[a]

	$B^{10}(d,p)B^{11}$ (i)	$B^{10}(d,p)B^{11}$ (ii)	$Ca^{44}(d,p)$-Ca^{45}	$Pb^{207}(d,p)$-Pb^{208}	$T^{48}(d,p)$-Ti^{49}
E_D(Mev)	8.1		7.01	15.1	2.6
Q(Mev)	9.24		3.30	5.41	4.46
l	1		1	1	1 2
$R(10^{-13}$ cm)	5.41		6.15	8.74	6.18
V_{ID}(Mev)	−60	−50	−50	−50	−44
W_{ID}(Mev)	−17	−14	−14	−15	−13
$a_{ID}(10^{-13}$ cm)	0.70	0.68	0.68	0.70	0.70
$R_{ID}(10^{-13}$ cm)	3.66	3.23	5.30	8.60	5.34
V_{FP}(Mev)	−50	−50	−60	−60	−60
W_{FP}(Mev)	−11	−8	−10	−10	−7
$a_{FP}(10^{-13}$ cm)	0.40	0.40	0.40	0.40	0.45
$R_{FP}(10^{-13}$ cm)	2.9	2.9	4.26	7.12	4.36

[a] See reference 2.

if we choose the z axis along $\mathbf{k}_d\times\mathbf{k}_p$ (the axes chosen in reference 5, and use a tilde to denote quantities referred to these axes, the $\tilde{d}_{kq}$ are now complex, but vanish for odd q. Let us put

$$\tilde{d}_{kq}=|\tilde{d}_{kq}|\exp(-iq\alpha_{kq}),$$

so the correlation (1) becomes

$$W(\tilde{\theta}\tilde{\phi})=\sum_k g_k\bigg\{\tilde{d}_{k0}P_k(\cos\tilde{\theta})+2\sum_{q>0}(-)^q \times\bigg[\frac{(k-q)!}{(k+q)!}\bigg]P_k^q(\tilde{\theta})|\tilde{d}_{kq}|\cos q(\tilde{\phi}-\alpha_{kq})\bigg\}. \quad (8)$$

Now the existence of a symmetry axis requires that we can find an x axis which makes all the phases $(q\alpha_{kq})=0$ or π. Again, generally this is only possible if $k=0$ and 2 only, or in the plane wave limit with x along $\mathbf{k}$.

When the value of k is restricted to 0 and 2, the correlation can be rewritten in a simple way.[5] From (1) or (8) we can write quite generally

$$W(\tilde{\theta}\tilde{\phi})=1+A_2^0P_2(\cos\tilde{\theta})+A_2^2P_2^2(\tilde{\theta})\cos 2(\tilde{\phi}-\phi_0). \quad (9)$$

If we call $A_2^2/A_2^0=-\frac{1}{2}\lambda$, then in the reaction plane, $\tilde{\theta}=\pi/2$, the correlation becomes

$$W=1+\alpha\cos^2(\tilde{\phi}-\phi_0),\quad \alpha=\frac{-6A_2^0\lambda}{2-A_2^0(1-3\lambda)}, \quad (10)$$

while in the plane perpendicular to the symmetry axis $\tilde{\phi}=\phi_0$, $\tilde{\theta}=\pi/2$,[14]

$$W=1+\beta\cos^2\tilde{\theta},\quad \beta=\frac{3A_2^0(1-\lambda)}{2-A_2^0(1-3\lambda)}, \quad (11)$$

so that $\beta/\alpha=(\lambda-1)/2\lambda$. The equations (9), (10), and (11) are independent of the reaction mechanism. The stripping assumption identifies $\phi_0=\alpha_{22}$ and makes definite predictions for the coefficients; in the plane wave theory it gives $A_2^0=-\frac{1}{2}g_2$ and $\lambda=1$. For $l=1$ captures in particular,[15] stripping imposes the restriction $0\leqslant\lambda\leqslant 1$ and further predicts that $A_2^0=-\frac{1}{2}g_2$ independent of proton angle (so that A_2^0 is unaffected by the distortions for $l=1$). The restriction on λ for $l=1$ implies β/α is always negative or zero; the anisotropy in azimuth about the symmetry axis $\tilde{\phi}=\phi_0$ is of opposite sign to that in the reaction plane. The λ plays the role of an attenuation coefficient when $l=1$, since if we define the anisotropy $\epsilon=[W(0)-W(\pi/2)]/[W(0)+W(\pi/2)]$, then in the reaction plane $\epsilon/\epsilon_0=\lambda$; the anisotropy is reduced to λ times its value ϵ_0 in the plane wave, Butler, theory. (This simple result does not hold for other l values because then $\tilde{d}_{20}$ and thus A_2^0 depend on the distortions; in general $\epsilon/\epsilon_0=2\lambda\tilde{d}_{20}(1+\frac{1}{2}g_2)/(g_2\tilde{d}_{20}-1)$. Also, λ may be greater than one so that β/α need not be negative; see Fig. 11.)

The proton polarization is proportional to $(1-\lambda^2)^{\frac{1}{2}}$ for $l=1$ captures,[5] but there is no such simple relation for other l values. However, when the proton polarization reaches its maximum allowed in this theory ($\frac{1}{3}$ if $j=l-\frac{1}{2}$, $\frac{1}{3}l/(l+1)$ if $j=l+\frac{1}{2}$) then $\lambda=0$ for all l and the γ rays are isotropic in the reaction plane.[5]

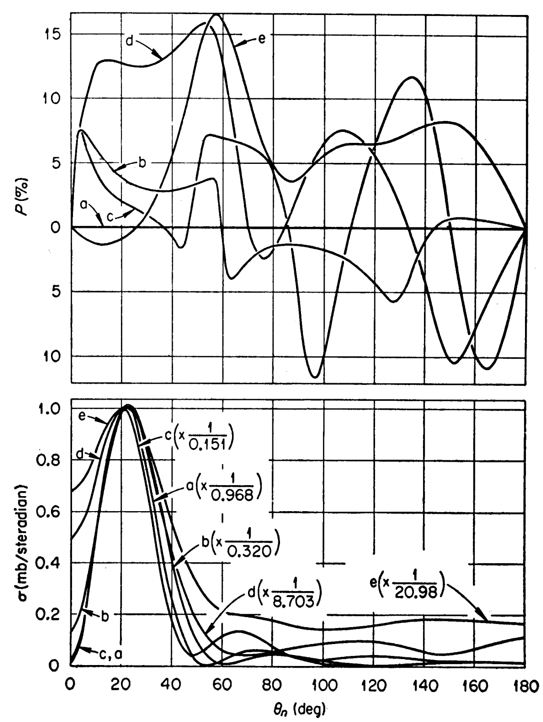

FIG. 1. The cross section σ and proton polarization P (in the c.m. system) for $B^{10}(d,p)B^{11}$. The curves were calculated for $l=1$ capture and (a) Butler theory, (b) Butler theory modified to include Coulomb effects, (c) the cutoff DWBA using optical potentials (i) of Table I, (d) DWBA using potentials (i), (e) DWBA using potentials (ii).

[14] The words "perpendicular to the symmetry axis" were omitted from reference 5, following Eq. (6.7).

[15] If the restriction $k\leq 2$ is not already imposed by the values of J_B or L, but is found experimentally, then there is already some indication of the selection rule $k\leq 2l$ (or $k\leq 2j$) and thus some evidence for the stripping mechanism. A reaction proceeding through a compound nucleus would not impose this restriction in general.

With $l=2$ captures there will in general be $k=4$ terms also. It turns out for $l=2$ that the phase angle $2\alpha_{42}=2\alpha_{22}+\pi$ (though this will not be true for other l values), but we still have the other angle α_{44}. Then in the reaction plane (8) takes the form (with $\phi_0=\alpha_{22}$, $\phi_1=\alpha_{44}$)

$$W=b_0+b_2\cos2(\tilde\phi-\phi_0)+b_4\cos4(\tilde\phi-\phi_1), \qquad (12)$$

where $\phi_0\neq\phi_1$ except in the plane wave limit when both become the angle of the recoil $\mathbf{k}$. With $l=2$ we also

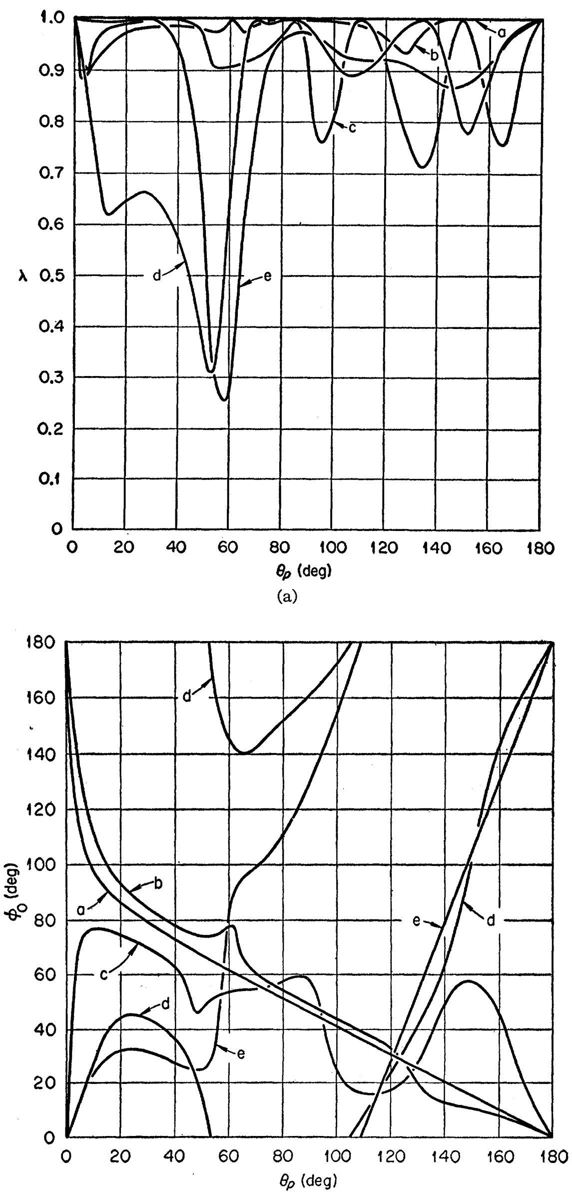

FIG. 2. (a) The p-γ correlation attenuation factor λ and (b) the symmetry axis angle ϕ_0 (measured from the deuteron beam) (in the c.m. system) for $B^{10}(d,p)B^{11}$. The curves are labeled as in Fig. 1.

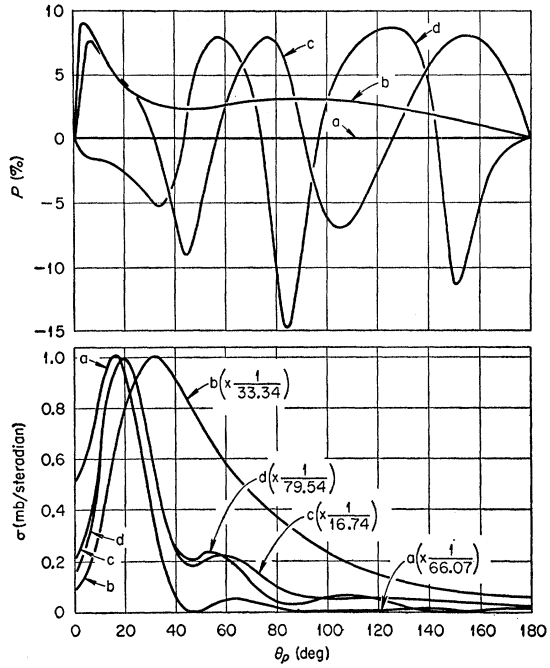

FIG. 3. Cross section σ and proton polarization P (in the c.m. system) for $l=1$ capture in $Ca^{44}(d,p)Ca^{45}$. (a) Butler, (b) Butler with Coulomb effects, (c) cutoff DWBA, (d) DWBA.

have the relationship with the proton polarization P,

$$(2j+1)P=\pm(5)^{-\frac{1}{2}}[(5/3)(1-\tilde{d}_{20})^2-(96/7)|\tilde{d}_{44}|^2].$$

These various relations and restrictions form tests of the DWBA theory. They are essentially unaltered by inclusion of the D state in the deuteron internal wave function; there is no interference between it and the dominant S state, so the D state can contribute at most a few percent to the γ correlation. Spin-orbit coupling in the distortions would produce some effect by making the d_{kq} depend on j as well as l, but this is thought to be small.

It has been suggested[16] that measurement of the angular distribution of a particular group of protons (or neutrons in a (d,n) reaction) would sometimes be facilitated if they were detected in coincidence with de-excitation γ rays emitted perpendicular to the reaction plane. In the plane wave theory the results of such a measurement, as a function of the angle between the protons and incident deuterons, would be the same as the differential cross section for protons not in coincidence provided only one l value contributes. In the distorted wave theory it turns out that this is still true for captures with $l=0$ (because the γ's are isotropic) and with $l=1$. For other l values the proton distribution is changed when measured in coincidence. It is given most simply by (8) with $\tilde\theta=0$,

$$W(\tilde\theta=0)=\sum_k g_k\tilde{d}_{k0}, \qquad (12)$$

[16] A. E. Litherland and A. J. Ferguson, Bull. Am. Phys. Soc., Ser. II, 5, 45 (1960).

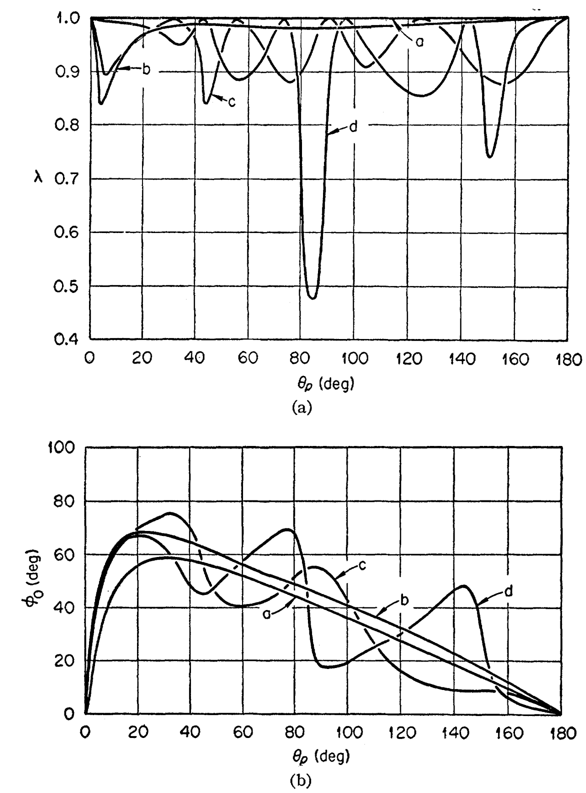

FIG. 4. (a) Attenuation λ and (b) angle ϕ_0 (in the c.m. system)for $Ca^{44}(d,p)Ca^{45}$. Curves labeled as in Fig. 3.

and only for $l=0$, $l=1$ (when $\tilde{d}_{20}=-\frac{1}{2}$), is this independent of the proton angle. Otherwise the coincidence proton angular distribution is given by the noncoincidence one modulated by the angle dependent function (12), which thus provides a convenient way of measuring the $\tilde{d}_{k0}$ as a function of the proton angle.

So far we have discussed γ rays detected in coincidence with the protons. We may ask how far is the angular correlation "smeared out" when the protons are not observed. If θ is the angle between the γ ray and a z axis chosen parallel to the incident deuterons, the noncoincident γ-angular distribution is given by

$$W(\theta)=\sum_k g_k\langle d_{k0}\rangle P_k(\cos\theta), \tag{13}$$

where $\langle d_{k0}\rangle$ is d_{k0} averaged over the proton angular distribution. (The d_{kq} with $q\neq 0$ give zero when averaged in azimuth around the incident beam.) This cannot be evaluated in any simple way even in the plane wave limit, but in this limit $\langle d_{k0}\rangle=\langle P_k(\cos\theta_R)\rangle$, where θ_R is the angle the recoil $\mathbf{k}$ makes with the deuteron beam. So if the proton distribution is strongly peaked, we may obtain a rough idea of $\langle d_{k0}\rangle$ by evaluating $P_k(\cos\theta_R)$ at the θ_R corresponding to the proton peak. But if, as usually happens, the proton distribution is still quite strong at angles well away from the main peak, the average $\langle d_{k0}\rangle$ would have to be evaluated numerically from the DWBA values.

RESULTS AND DISCUSSION

The parameters characterizing the transitions considered are summarized in Table I. Although most of these figures correspond to ground-state transitions in the nuclei named, so involving no γ rays, this is irrelevant for our purpose. Other reactions leading to excited states, but with similar values of the parameters, would show similar behavior. We merely wish to give some preliminary idea of the effects of distorted waves on the p-γ correlation.

Four cases are considered of $l=1$ captures, and the calculated cross sections, proton polarizations, attenuation coefficients λ, and symmetry axis angles ϕ_0 are plotted as a function of the proton detector angle θ_p in Figs. 1–8. The λ are related to the proton polarizations P of reference 2 by

$$(2j+1)P=\pm\tfrac{2}{3}(1-\lambda^2)^{\frac{1}{2}}, \tag{14}$$

while the symmetry axis angle ϕ_0 measured from $\mathbf{k}_d$ is given in terms of the matrix elements B^m defined in reference 2,

$$\tan 2\phi_0=\sqrt{2}\,\mathrm{Re}(B^0B^{1*})(|B^1|^2-\tfrac{1}{2}|B^0|^2). \tag{15}$$

This determines ϕ_0 to within $\pm\pi/2$; the correct ϕ_0 is given by knowing $\cos 2\phi_0$ has the same sign as $\frac{1}{2}|B^0|^2-|B^1|^2$. In the plane wave limit ϕ_0 is the angle θ_R between $\mathbf{k}_d$ and the recoil $\mathbf{k}$.

In addition one $l=2$ capture was considered, (a) in the plane wave limit and (b) with optical model distorted waves with no radial cutoff. The differential

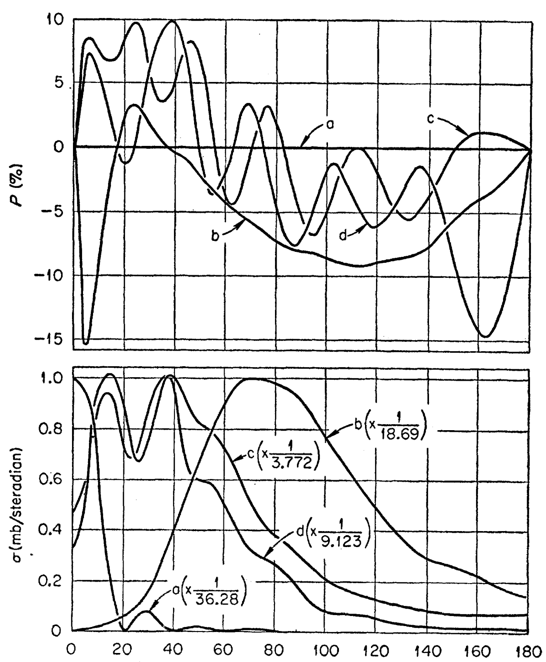

FIG. 5. Cross section σ and proton polarization P (in the c.m. system) for $l=1$ capture in $Pb^{207}(d,p)Pb^{208}$. (a) Butler, (b) Butler with Coulomb effects, (c) cutoff DWBA, (d) DWBA.

cross section and proton polarization are repeated in Fig. 9, and the p-γ correlation coefficients d_{kq} of Eq. (7) (referred to z axis along $\mathbf{k}_d$, y axis along $\mathbf{k}_d \times \mathbf{k}_p$) are given in Fig. 10. Further, as discussed above, when only values $k=0$ and 2 are allowed, the correlation is conveniently given in terms of $\tilde{d}_{20}$, λ and ϕ_0, and these are plotted in Fig. 11. The expressions for λ, ϕ_0 and the d_{kq} in terms of the B^m matrix elements are given in the Appendix.

Perhaps the most striking feature of the results is the same as that found for the differential cross sections and polarizations,[2] namely the importance of the contributions from the interior of the nucleus even when optical model distorted waves are used. Inclusion of the Coulomb effects, and even of the optically distorted waves when the interior contribution is neglected, does not make the results deviate very far from the plane wave theory predictions. It had been anticipated that the deuterons, at least, would be sufficiently strongly absorbed within the nucleus to make the interior contributions negligible. The optical model fits to the deuteron elastic scattering[2,17] require imaginary parts corresponding to a mean free path in

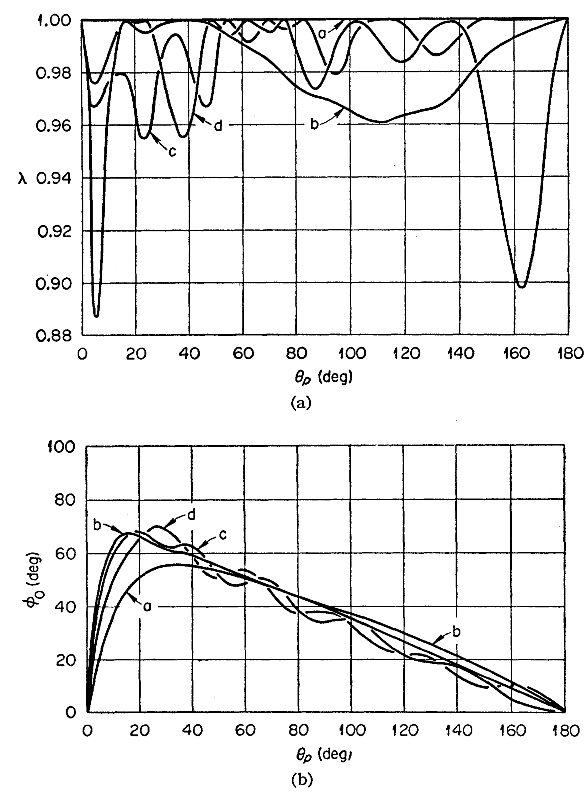

FIG. 6. (a) Attenuation λ and (b) angle ϕ_0 (in the c.m. system) for $Pb^{207}(d,p)Pb^{208}$. Curves labeled as in Fig. 5.

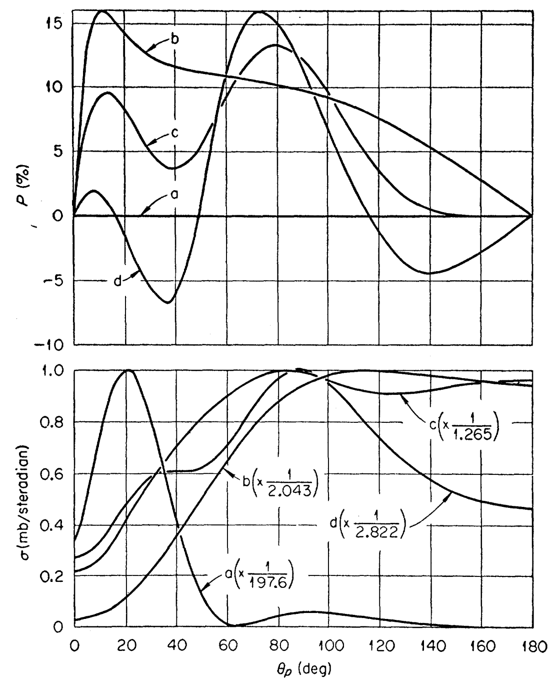

FIG. 7. Cross section σ and proton polarization P (in the c.m. system) for $l=1$ capture in $Ti^{48}(d,p)Ti^{49}$. (a) Butler, (b) Butler with Coulomb effects, (c) cutoff DWBA, (d) DWBA.

nuclear matter of between 1 and 3 fermis. While the latter figure is comparable to the radii of light nuclei, the wave function within a finite nucleus is further attenuated by reflection at the surface. However, one compensating factor is that the distance over which the *amplitude* of the wave is attenuated by $1/e$ is *twice* the mean free path, and it is the amplitude which determines how much the contributions from different regions of the nucleus may interfere. In this way, the differential cross-section, etc., may be much more sensitive to interior contributions than the total cross section in which these interferences average to zero. The effect of the interior is particularly marked in the case of B^{10}, where the results bear little relation to the plane wave theory. This is probably due in part to the smallness of the nucleus, but especially to the large Q value of 9.24 Mev. The latter implies a strongly bound neutron and hence a greatly reduced probability of finding the neutron "outside" the nucleus; this increases the *relative* importance of the interior. Experiments have been carried out on $Be^9(d,p\gamma)Be^{10}$ with deuterons of 4 and 8 Mev, and the results are close to the plane wave theory at the one proton angle used.[6] However, the Q value of this reaction is only 1.22 Mev. It would be of interest to carry out experiments on a light nucleus with a high Q value to see if the p-γ correlation shows the sensitivity our B^{10} calculations imply.

The importance of the interior contributions in the Ti^{48} reaction is also surprising, because at first sight the deuteron energy of 2.6 Mev is well below the

[17] I. Slaus and W. P. Alford, Phys. Rev. **114**, 1054 (1959); M. A. Melkanoff, *Proceedings of the International Conference on the Nuclear Optical Model, Florida State University Studies, No. 32,* edited by A. E. S. Green, C. E. Porter, and D. S. Saxon (Florida State University, Tallahassee, 1959).

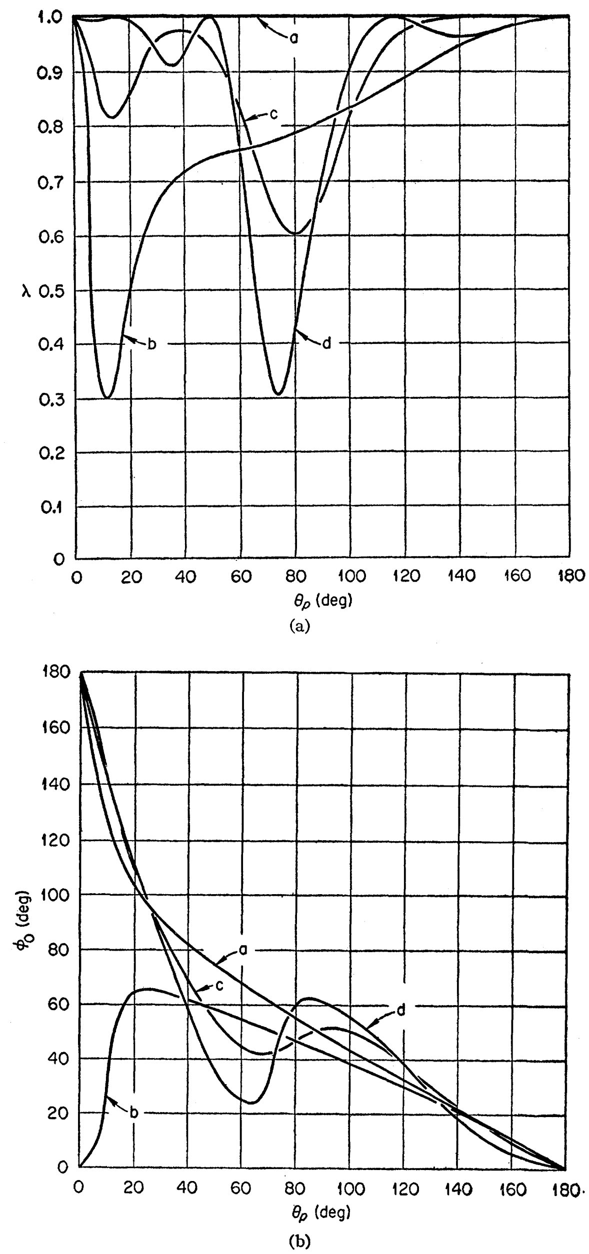

FIG. 8. Attenuation λ and (b) angle ϕ_0 (in the c.m. system) for $Ti^{48}(d,p)Ti^{49}$. Curves labeled as in Fig. 7.

Coulomb barrier (which would be 6 Mev for $R=5.34$). However, the tail of the attractive optical potential considerably rounds off the top of the Coulomb barrier and increases its effective radius; the effect in this case is to give the barrier a peak height of 3.3 Mev at about 9 fermis.

The calculations at present only display the difference between the contributions outside and inside the cutoff radius R. Since the R used are somewhat larger than the nuclear mass and potential radii, it will be of interest to see in future calculations whether the interior contribution is concentrated in the region just inside R, or whether it is distributed throughout the interior. In addition we do not yet know how unique are the choices of optical potential which describe the elastic scattering; for example concentrating the deuteron absorption at the nuclear surface might reduce the sensitivity of the stripping reaction to the interior contribution while still reproducing the elastic scattering. Finally it is not certain that the distorted waves used in the theory have to reproduce the elastic scattering exactly,[18] or even that merely distorting the center-of-mass motion of the deuterons is an adequate representation of their wave function close to the nucleus. However, the distorted wave effects discussed here are undoubtedly present even if there are additional dissociation effects.

The dependence of λ, the attenuation factor for $l=1$ captures, on the proton angle θ_p emphasizes the importance of carrying out p-γ correlation experiments with the proton counter set at a number of angles. Experimental angular resolution will tend to wash out the effects of sharp spikes such as seen in the λ curve for Ca^{44}, Fig. 4, but it is of value to search for the variations implied, for example, by Figs. 2 and 8 [in particular, for $l=1$ in conjunction with proton polarization measurements to test the relation (14)]. These curves also remind one that the experimental observation of a near-isotropic γ correlation with protons away from the main peak of the differential cross section need not imply these protons did not originate in a stripping reaction. The fact that in a given case the proton angular distribution is only slightly changed by distortion effects seems to be no

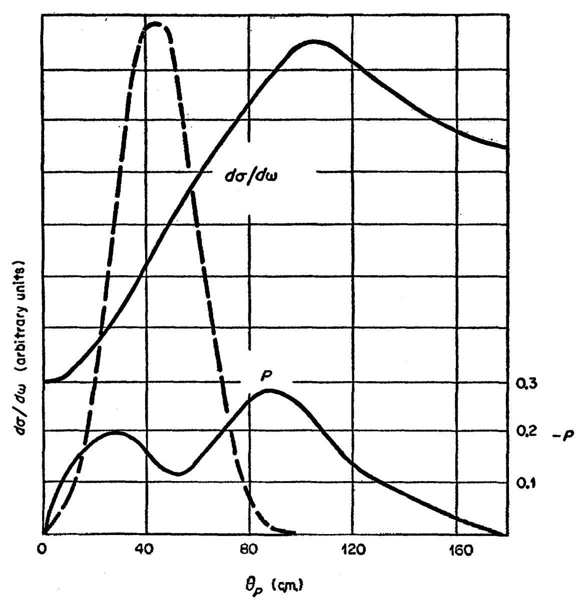

FIG. 9. Cross section $d\sigma/d\omega$ and proton polarization P (in the c.m. system) for $l=2$ capture in $Ti^{48}(d,p)Ti^{49}$. Broken line—Butler theory, full line—DWBA.

[18] T. K. Fowler, Phys. Rev. Letters **1**, 371 (1958).

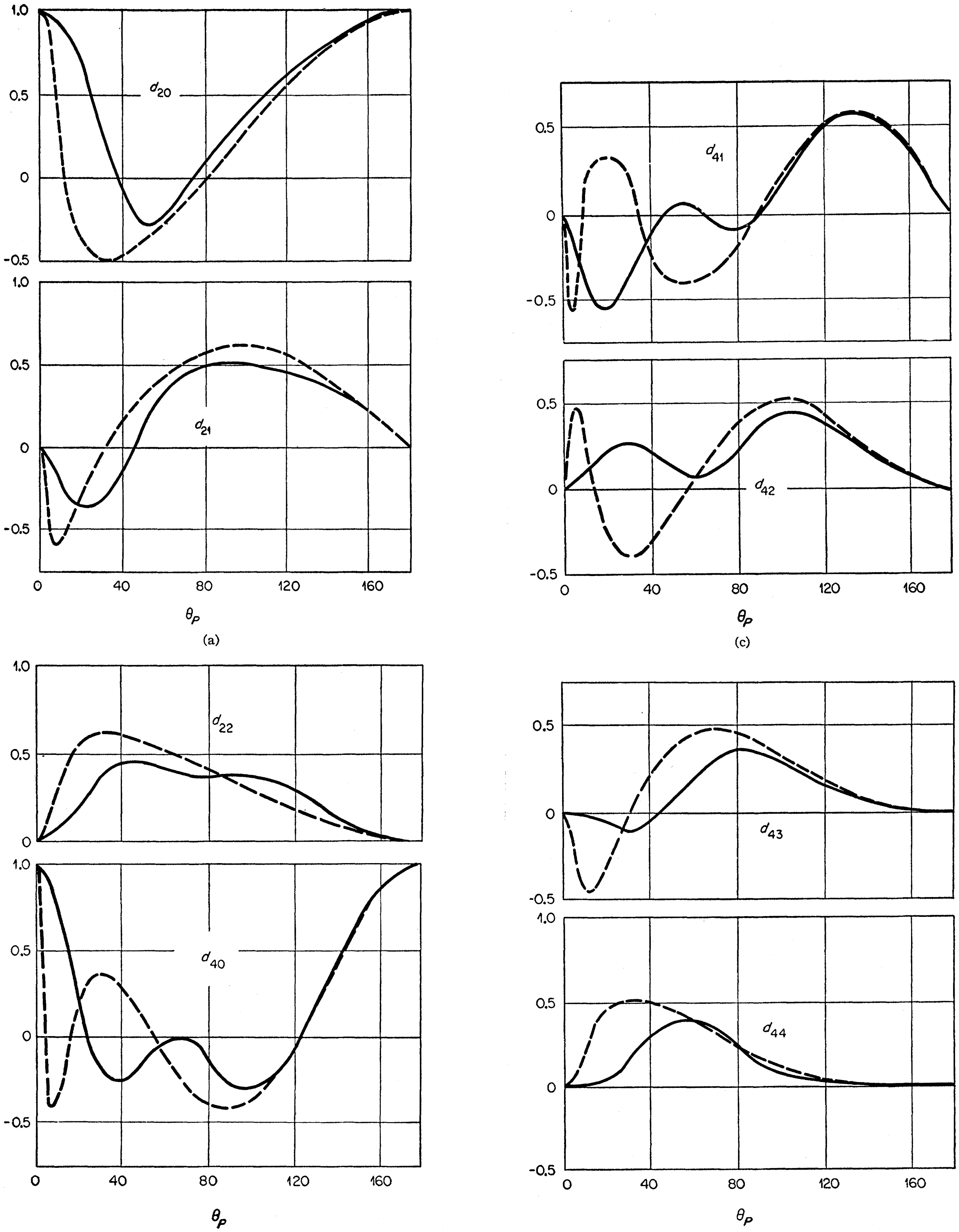

FIG. 10. The p-γ correlation coefficients d_{kq} (in the c.m. system) for $l=2$ capture in $Ti^{48}(d,p)Ti^{49}$, referred to $\mathbf{k}_d$ as z axis, $\mathbf{k}_d \times \mathbf{k}_p$ as y axis. Broken line—Butler theory; full line—DWBA.

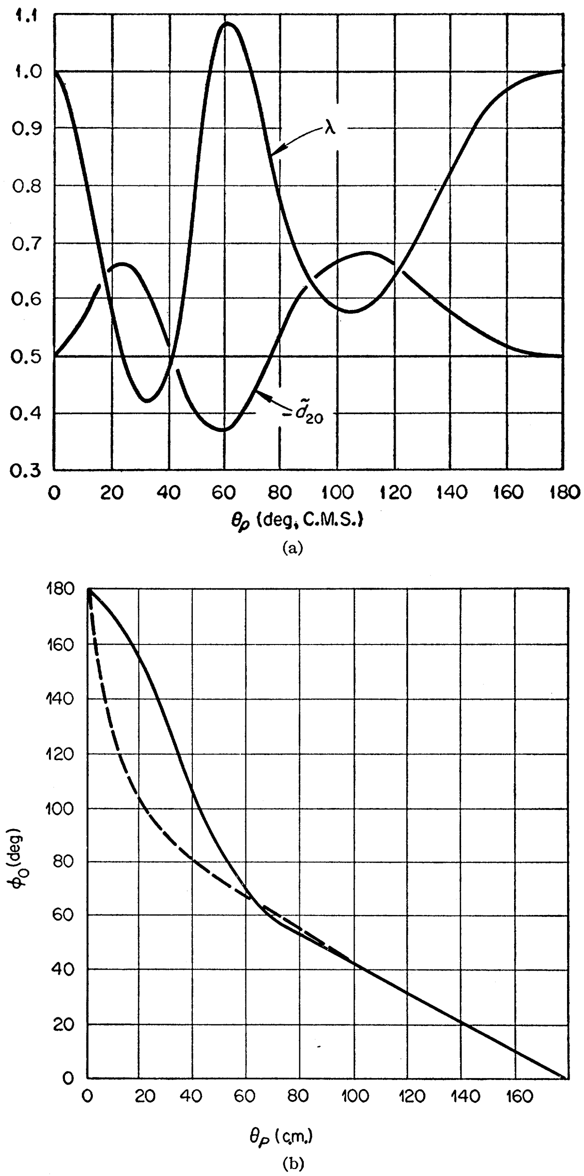

FIG. 11. (a) The parameters λ and $\tilde{d}_{20}$ and (b) the angle ϕ_0 (in the c.m. system) for $l=2$ capture in $Ti^{48}(d,p)Ti^{49}$. Broken line—Butler theory; full line—DWBA. In Butler theory, $\lambda=1$ and $\tilde{d}_{20}=-\frac{1}{2}$.

guarantee that the change in the p-γ correlation will be small. On the other hand, we have one case (Fig. 6) where distortion has an important effect on the angular distribution but very little effect on the angular correlation.

The oscillation of ϕ_0 about the recoil angle may probably be understood in terms of the opposing effects of the deuteron and proton distortions. It was argued on semiclassical grounds that the two distortions tended to produce proton polarizations of opposite sign, and this may be demonstrated analytically when $\mathbf{k}_d$ and $\mathbf{k}_p$ have similar magnitudes (which is the usual experimental situation).[19] Under the same circumstances one may also show that the phase angles α_{kq} in (8), and in particular $\phi_0=\alpha_{22}$, are of opposite sign according to whether we distort the deuteron or proton waves, while the magnitudes of the d_{kq}, and hence of λ, are the same. Hence in favorable cases the shift of the γ-symmetry axis away from the recoil direction may be small even though the attenuation of the anisotropy is considerable.

APPENDIX

The matrix elements B^m described by Tobocman[2] differ only in normalization from the B_{lm} defined by Huby, Refai, and Satchler,[5]

$$B^m=i^l[(2l+1)2M_{IN}R]^{\frac{1}{2}}B_{lm}/\hbar\zeta_l(R).$$

We give the expressions for the coefficients d_{kq} for $l=2$ in terms of the B^m, referred to a z axis along $\mathbf{k}_d$, y axis along $\mathbf{k}_d\times\mathbf{k}_p$. (The corresponding expressions in reference 5 were referred to a z axis along $\mathbf{k}_d\times\mathbf{k}_p$.) We have $B^m=(-)^m B^{-m}$ in these axes. We let $d_{kq}=D_{kq}/D_{00}$, where

$$D_{00}=2|B^2|^2+2|B^1|^2+|B^0|^2,$$
$$D_{20}=|B^0|^2+|B^1|^2-2|B^2|^2,$$
$$D_{21}=\mathrm{Re}\,[(6)^{\frac{1}{2}}B^1B^{2*}+B^0B^{1*}],$$
$$D_{22}=(\tfrac{3}{2})^{\frac{1}{2}}|B^1|^2-2\,\mathrm{Re}\,(B^0B^{2*}),$$
$$D_{40}=\tfrac{1}{3}|B^2|^2-\tfrac{4}{3}|B^1|^2+|B^0|^2,$$
$$D_{41}=\mathrm{Re}\,[(10/3)^{\frac{1}{2}}(B^0B^{1*})-(5/9)^{\frac{1}{2}}(B^1B^{2*})],$$
$$D_{42}=(5/3)^{\frac{1}{2}}\,\mathrm{Re}\,(B^0B^{2*})+(10/9)^{\frac{1}{2}}|B^1|^2,$$
$$D_{43}=(35/9)^{\frac{1}{2}}\,\mathrm{Re}\,(B^1B^{2*}),$$
$$D_{44}=(35/18)^{\frac{1}{2}}|B^2|^2.$$

They obey the relations

$$d_{41}+(7)^{\frac{1}{2}}d_{43}-(10/3)^{\frac{1}{2}}d_{21}=0,$$
$$d_{20}-d_{40}-(\tfrac{2}{3})^{\frac{1}{2}}d_{22}-(8/5)^{\frac{1}{2}}d_{42}+(14/5)^{\frac{1}{2}}d_{44}=0,$$
$$(10/3)d_{20}-d_{40}+(70)^{\frac{1}{2}}d_{44}=7/3.$$

Referred to $\mathbf{k}_d\times\mathbf{k}_p$ as z axis, $\mathbf{k}_d$ as x axis, the $\tilde{d}_{2q}$ are

$$\tilde{d}_{20}=-(\tfrac{3}{2})^{\frac{1}{2}}d_{22}-\tfrac{1}{2}d_{20},$$
$$\tilde{d}_{22}=-\tfrac{1}{2}d_{22}+(\tfrac{3}{8})^{\frac{1}{2}}d_{20}+id_{21},$$

so that the parameter λ and symmetry axis angle ϕ_0 are given by

$$\lambda=|(\tfrac{3}{2})^{\frac{1}{2}}D_{20}-D_{22}+2iD_{21}|/(3D_{22}+(\tfrac{3}{2})^{\frac{1}{2}}D_{20}),$$
$$\tan 2\phi_0=2D_{21}/[D_{22}-(\tfrac{3}{2})^{\frac{1}{2}}D_{20}].$$

We also have in these axes,

$$12\tilde{d}_{40}=7+5\tilde{d}_{20},\quad (12)^{\frac{1}{2}}\tilde{d}_{42}=-5^{\frac{1}{2}}\tilde{d}_{22}.$$

[19] M. Y. Refai (private communication from H. C. Newns).