microscopic distribution function of the system is, then, the standard generalized canonical distribution[1]

$$W(q,t)=W^{(0)}(q)e^{V(t)\,q/kT}/\langle e^{V(t)\,q/kT}\rangle^{(0)}$$
$$=W^{(0)}(q)e^{\langle q(t)\rangle\,q/kTC}/\langle e^{\langle q(t)\rangle\,q/kTC}\rangle^{(0)}. \quad (36)$$

The equation of motion satisfied by $W(q,t)$ is obtained by differentiating with respect to t.

$$\dot{W}(q,t)=(1/kTC)W(q,t)[\langle\dot{q}(t)\rangle q-\langle\dot{q}(t)\rangle\langle q(t)\rangle]. \quad (37)$$

Multiplying through by q and averaging over $W^{(0)}(q)$, we obtain an expression for $d\langle q(t)\rangle/dt$ which yields the relation

$$\langle q^2(t)\rangle=\langle q(t)\rangle^2+\langle q^2\rangle^{(0)}. \quad (38)$$

Invoking Eq. (38), a similar calculation of $d\langle q^2(t)\rangle/dt$ yields the relation

$$\langle q^3(t)\rangle=\langle q(t)\rangle^3+3\langle q^2\rangle^{(0)}\langle q(t)\rangle. \quad (39)$$

The results expressed by Eqs. (38) and (39) clearly suffice to reduce the Markoffian equation of motion (31) to the desired form, Eq. (6). Rewritten in terms of the variable q rather than Lax's variable α, Eq. (31) becomes

$$\frac{d}{dt}\langle q(t)\rangle=A-\Lambda'\langle q(t)\rangle-B'\langle q^2(t)\rangle-\Gamma'\langle q^3(t)\rangle+\cdots. \quad (40)$$

Substituting Eqs. (38) and (39), we obtain

$$\frac{d}{dt}\langle q(t)\rangle=[A-B'\langle q^2\rangle^{(0)}]-[\Lambda'+3\Gamma'\langle q^2\rangle^{(0)}]\langle q(t)\rangle$$
$$-B'\langle q(t)\rangle^2-\Gamma'\langle q(t)\rangle^3+\cdots, \quad (41)$$

which is of the form of Eq. (6) with $[A-B'\langle q^2\rangle^{(0)}]=0$, $[\Lambda'+3\Gamma'\langle q^2\rangle^{(0)}]=1/R_0C$, $B'=-R_1/R_0^2C^2$, $\Gamma'=-[(R_2/R_0^2C^3)-(R_1^2/R_0^3C^3)]$. This identification of the coefficients is to be contrasted with that which would result from a direct comparison of Eqs. (6) and (40), ignoring the distinction between $\langle q^n(t)\rangle$ and $\langle q(t)\rangle^n$ [see also the discussion following Eq. (32)].

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Internal Field Emission at Narrow *p-n* Junctions in Indium Antimonide

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An experimental study has been made of the field and temperature dependence of internal field emission in narrow *p-n* junctions in indium antimonide. Relatively good agreement, both qualitative and quantitative, is obtained between the experimental results and the usual expression for the barrier transparency. From studies of Esaki characteristics at low temperatures and from the observed temperature dependence of the tunnelling current, it is confirmed that the tunnelling transitions do not involve phonons. Also, it is shown that the temperature dependence of the barrier transparency is determined by that of the energy gap at $k=0$.

INTRODUCTION

THE theoretical expression for the field dependence of the probability of internal field emission (tunnelling) is dominated by the factor[1]

$$\exp(-\alpha\epsilon^{\frac{3}{2}}/E), \quad (1)$$

where, for direct transitions,

$$\alpha=\pi(m^*)^{\frac{1}{2}}/2e\hbar,$$

and for indirect transitions,

$$\alpha=4(2m^*)^{\frac{1}{2}}/3e\hbar,$$

ϵ is the energy gap (direct or indirect, as appropriate), E is the electric field, m^* is an effective mass, e is the electron charge, and $\hbar$ is $h/2\pi$, where h is Planck's constant. Chynoweth et al.[2] have recently made an experimental study of tunnelling in narrow silicon and germanium junctions and in particular, they have verified that Eq. (1) satisfactorily describes, both qualitatively and quantitatively, the field dependence of the tunnel current at a given temperature. They also investigated the temperature dependence of the tunnel current. The form of the temperature dependence depends on whether the tunnelling transitions are direct or indirect, the latter requiring the absorption or emission of phonons. In semiconductors where the minimum energy gap lies at $k=0$, tunnelling of carriers between the two bands occurs by direct transitions. In this case the temperature dependence of the tunnel current is determined primarily by that of the energy gap. In those materials where the minimum energy gap does not occur at $k=0$, tunnelling may occur by indirect transi-

[1] F. V. Keldysh, J. Exptl. Theoret. Phys. (U.S.SR.) **33**, 994 (1957), and **34**, 962 (1958) [translations: Soviet Phys. JETP **6**, 763 (1958), and **7**, 665 (1958), respectively].

[2] A. G. Chynoweth, W. L. Feldmann, C. A. Lee, R. A. Logan, G. L. Pearson and P. Aigrain (in press).

tions. The temperature dependence of the tunnelling current is then determined by the combined effects of the temperature variation of the energy gap and the phonon density, as described by Keldysh.[1] Chynoweth et al.[2] showed that the case of direct transitions being dominant held for narrow junctions formed in arsenic-doped germanium while in silicon, tunnelling was by indirect transitions. They also evaluated the quantity, $\alpha\epsilon^{\frac{3}{2}}$, which appears in Eq. (1), finding it to be in relatively close agreement (to within less than a factor of 2) with that expected on the basis of the known effective mass and energy gap.

It was of interest to make a similar investigation of the tunnelling probability for indium antimonide because, for this material, the effective mass and the energy gap are both considerably less than for germanium and silicon. Furthermore, for this material the minimum energy gap occurs very definitely at $k=0$. It is the purpose of this paper to describe a study which has given additional confirmation of Eq. (1) and, as expected, has supported the view that tunnelling in indium antimonide is by direct transitions.

EXPERIMENTS

Indium antimonide p-n junctions were formed by alloying cadmium into n-type crystals with excess donor densities of 1×10^{16} and 3×10^{16} cm^{-3}. The cadmium was derived from small pellets (about 0.003 inch in diameter) of cadmium-indium alloys with various compositions in the range 0.004% to 1.2% of cadmium, using a quick heat cycle in the alloying process. There was no obvious correlation between the rectification characteristics and the alloy compositions. There was thus some uncertainty as to the acceptor concentrations formed in the p-type regrowth layer though they were estimated to be in the range 10^{17} cm^{-3} to 10^{19} cm^{-3}. Contact to the n-type material was made using indium solder while a pressure point contact was used to the alloyed dot.

The rectification characteristic that is shown in Fig. 1 was obtained at 77°K and is typical of the junctions used in these investigations. It shows a normal forward characteristic and the reverse characteristic, though rather "soft," exhibits a considerable amount of current

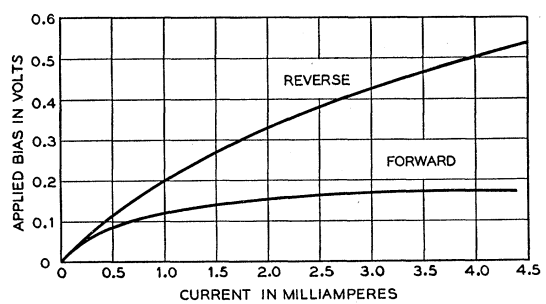

FIG. 1. The voltage-current characteristics of a typical InSb diode at 77°K.

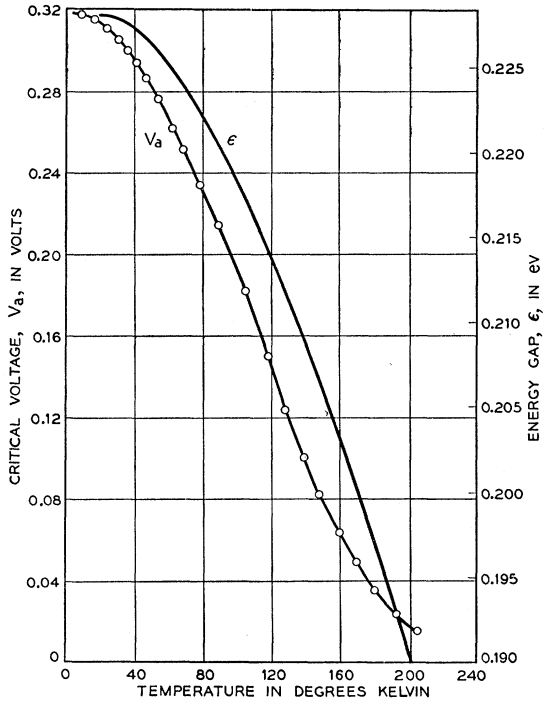

FIG. 2. A plot of the critical voltage, V_a, versus the absolute temperature. The temperature dependence of the energy gap (from the data of Roberts and Quarrington) is shown for comparison.

at applied biases less than 0.3 volt. From analysis of the forward characteristics it was established that the reverse currents were many orders of magnitude greater than the normal junction saturation currents and furthermore, they were relatively insensitive to temperature. The reverse currents cannot be regarded, therefore, as primarily thermal in origin. Nor can they be due to multiplication and avalanche processes for the following reason. Though as is usual in narrow junctions it is difficult to define an actual breakdown voltage, it can safely be said to be of the order of 0.3 volt. Now it is known that the threshold energy for electron-hole pair production by energetic carriers is about 0.5 ev in indium antimonide.[3] In our junctions the built-in voltage lies between 0.2 and 0.3 volt, so that up to about 0.2 to 0.3 volt applied potential there can be no charge multiplication by injected minority carriers and considerably higher potentials would be required before avalanching could become important.

For the purposes of the analysis described later, the reverse characteristics were measured in detail over the current range 1 μa to 10 ma.

Tunnelling was studied as a function of temperature in the same way as has been done previously for silicon and germanium[2]; the temperature dependence of the reverse bias required to maintain a constant current was measured. The actual value chosen for the con-

[3] J. Tauc, J. Phys. Chem. Solids 8, 219 (1959).

stant current was not critical as long as the resulting bias lay in the range where tunnelling was dominant. A typical curve of the critical bias, V_a, versus the temperature, T, is shown in Fig. 2, the data being obtained from the same junction as that used for Fig. 1. For comparison purposes, the temperature variation of the energy gap is also shown[4] and it will be noted that there is considerable similarity between the two curves. Qualitatively, the V_a vs T curve resembles that previously observed for germanium rather than silicon. This is to be expected since the minimum band gap occurs at $k=(000)$, so resulting in direct transitions.

For applied voltages in excess of about 0.5 volt, the shape of the curve differs markedly from that for ϵ, requiring a greater bias at a particular temperature to maintain a constant current. This is to be expected if a significant portion of the current is now due to charge multiplication since this will probably exhibit an opposite temperature dependence to that of the tunnelling process.[5]

Additional confirmation that the tunnelling transitions are direct ones was derived from the Esaki characteristics of sufficiently narrow junctions at 4.2°K. For forward biases ranging between 0 and 0.050 volt (the latter bias being approximately that at which the peak current occurred), there was no evidence of any structure in the current-voltage curve, and in particular, pronounced discontinuities of the sort that have been observed in silicon and germanium were not present.[6,7]

ANALYSIS

(a) Reverse Characteristics

The method of analyzing the reverse characteristics has been given before.[2] To a fair approximation, the measured current, I_m, can be written

$$I_m = A V_a^p E^q P, \tag{2}$$

where A is a constant for a given temperature, V_a is the applied voltage raised to some power p (usually about unity), E is the field in the junction raised to the power q (usually between 1 and 3), and P is the tunnelling probability given by Eq. (1). For small V_a, it is easy to show that

$$d(\ln I_m)/d(\ln V_a) \simeq p + (\alpha\epsilon^{\frac{3}{2}} W_1/4V_i^{\frac{3}{2}})V_a, \tag{3}$$

where W_1 is the width of the space charge region for a total potential drop of 1 volt, and V_i is the built-in voltage. Thus a plot of $d(\ln I_m)/d(\ln V_a)$ against V_a should be a straight line whose slope yields a value for $\alpha\epsilon^{\frac{3}{2}}$ which can be compared with the theoretical value.

It was found that relatively good straight lines were obtained for most junctions as long as V_a did not exceed about 0.2 volt. At higher applied voltages deviations occurred which were most likely the result of the approximations used in deriving Eq. (3). Figure 3 gives a typical example of a $d(\ln I_m)/d(\ln V_a)$ vs V_a plot.

To deduce the values of $\alpha\epsilon^{\frac{3}{2}}$ it was necessary to estimate both V_i and W_1. The values of W_1 were extrapolated from experimental data obtained from capacitance studies of p-n junctions in indium antimonide by Lee.[8] As noted, the experimental capacitance measurements in indium antimonide appear to be systematically greater than those predicted by conventional diode theory by a factor of about three. This discrepancy is not yet satisfactorily explained. More serious uncertainties lie in the estimates of V_i because of the lack of any obvious correlation between the rectifier characteristics and the percentage of cadmium in the cadmium-indium alloys used for forming the p-type regrowth layer. Thus, appreciable uncertainty exists concerning the position of the Fermi level on the p-side. Estimates of the energy difference between the Fermi level and the top of the valence band range from less than a tenth of an electron volt for the lightest cadmium dopings to a few tenths of an electron volt for the heaviest dopings.

For the junction used for Fig. 3 the cadmium doping was estimated to be not more than 10^{17} cm^{-3} so that the error in V_i was relatively small. From the known n-type doping concentration and with an energy gap of about 0.22 ev at 77°K, V_i was estimated to be 0.25 ± 0.01 volt. The width constant was estimated to be 8×10^{-6} cm. From the slope of the straight portion of the curve of Fig. 3 it was found that the value of $\alpha\epsilon^{\frac{3}{2}}$ was 1×10^{6} v cm^{-1}, a value that was probably good to within a factor of two. Other junctions gave values

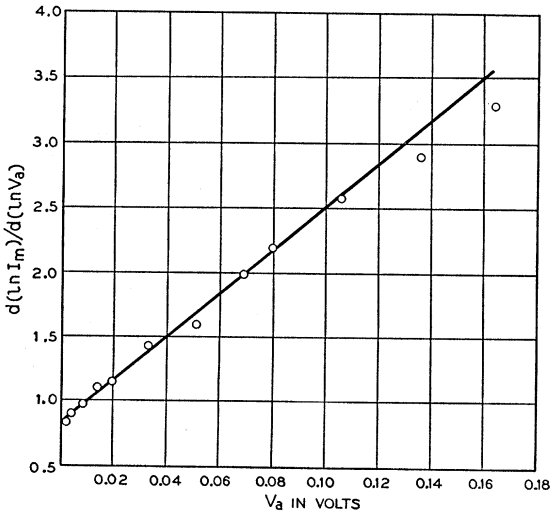

FIG. 3. A plot of $d(\ln I)/d(\ln V_a)$ for a typical InSb diode at 77°K.

[4] V. Roberts and J. E. Quarrington, J. Electronics **1**, 152 (1955).
[5] A. G. Chynoweth and K. G. McKay, Phys. Rev. **106**, 418 (1957).
[6] N. Holonyak et al., Phys. Rev. Letters **3**, 167 (1959).
[7] L. Esaki (private communication).
[8] C. A. Lee and G. Kaminsky, Electrochemical Society Semiconductor Symposium, Columbus, Ohio, October 21, 1959 (unpublished).

for $\alpha\epsilon^{\frac{3}{2}}$ ranging between 1×10^5 and 1×10^6 v cm^{-1} though the scatter could very likely be accounted for by the uncertainties in V_i. However, the junction with the lightest cadmium doping (Fig. 3), yielding the value of 1×10^6 v cm^{-1} was regarded as the most reliable one.

The value of $\alpha\epsilon^{\frac{3}{2}}$ was estimated from Eq. (2), putting $\epsilon=0.22$ ev and using a reduced effective mass, m_R, where

$$m_R=m_-m_+/(m_-+m_+)\simeq m_-,$$

where m_-, m_+ are the density of states masses for the electrons and holes, respectively.[9] In this way, $\alpha\epsilon^{\frac{3}{2}}$ was predicted to be 9×10^5 cm^{-1}, a value which is in excellent agreement with that found experimentally. The values of $\alpha\epsilon^{\frac{3}{2}}$ in indium antimonide are between 1 and 2 orders of magnitude lower than those for silicon and germanium so that the good agreement again obtained between experiment and theory is an indication of the validity of the method of analysis and of Eq. (1).

(b) Temperature Characteristics

As the temperature is changed, variations in the band gap and the effective mass cause P to change. The manner of taking the temperature data, holding I_m constant and determining V_a as a function of T, therefore requires that, essentially, P remains constant in Eq. (2) unless V_a becomes very small. In the region where V_a is sufficiently large, we thus have the condition,[2]

$$\alpha\epsilon^{\frac{3}{2}}(V_a+V_i)^{-\frac{1}{2}}=\text{constant}.$$

For the more lightly doped junctions it is sufficient to put $V_i\simeq\epsilon$. Further, the effective mass contained in α can be put roughly proportional to ϵ. Thus, a plot of ϵ^2 against $(V_a+V_i)^{\frac{1}{2}}$ should be a straight line. Such a plot is shown in Fig. 4, where the temperature variation of the direct energy gap was taken from the curves given by Roberts and Quarrington.[4] It was found that very good straight lines, such as the one in Fig. 4, were given by all the junctions studied. (The deviation from linearity occurs in the data at high temperatures where V_a is small and comparable to kT.) The temperature data, therefore, are further evidence in favor of the expression for the tunnelling probability and of direct tunnelling transitions.

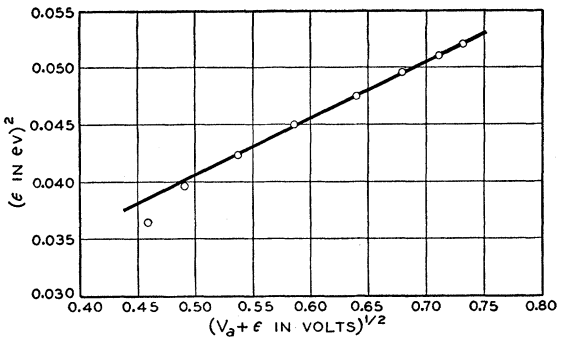

FIG. 4. The square of the energy gap plotted against the square root of the sum of the energy gap and the critical voltage.

CONCLUSIONS

Internal field emission has been studied in narrow indium antimonide p-n junctions as a function of field and temperature by methods similar to those previously used for silicon and germanium junctions. The junctions were sufficiently narrow for the total potential drop across them to be less than that required for any charge multiplication or avalanching to occur. As with silicon and germanium, good agreement has again been attained, both qualitative and quantitative, between the experimental results and the expression for the barrier transparency. This continued agreement is particularly significant because of the very different values for the effective mass and the energy gap in indium antimonide compared with those in silicon and germanium. The temperature data show that the tunnelling in indium antimonide is by direct transitions and that the temperature variation of the tunnelling probability is determined almost completely by that of the energy gap.

ACKNOWLEDGMENTS

The authors are indebted to R. L. Batdorf and C. A. Lee for much valuable information concerning indium antimonide junctions, and to A. J. Peters for fabricating many of the junctions used in this work.

[9] H. J. Hrostowski, F. J. Morin, T. H. Geballe, and G. H. Wheatley, Phys. Rev. **100**, 1672 (1955).