Pion Production in Muon-Nucleon Collisions

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A relation between the differential cross sections for certain angles and energies of inelastic electron scattering and inelastic muon scattering is given. The asymmetry of the pion production by longitudinal polarized muons has been calculated using Fubini-Nambu-Wataghin matrix elements.

1. INTRODUCTION

THE work of Panofsky and Allton[1] has shown that the production of pions in electron-nucleon collisions offers a new experimental tool for exploring the nucleon structure. Electroproduction of pions can be regarded as a modification of photoproduction, where the external photon is replaced by the intermediate photon emitted by the electron. Because the intermediate photon is not restricted to the mass shell and can have longitudinal components, electroproduction gives new information about the photoproduction matrix elements. A similar process is expected if the electron is replaced by a muon, which, to our present knowledge, also interacts only electromagnetically with the nucleon and pion (weak interactions will be neglected).

Due to the different rest masses of the electron and the muon, the kinematics are different for electroproduction and for pion production in muon-nucleon collisions. The deviation of the intermediate photon momentum $k_\mu = s_\mu - s_\mu'$ (s_μ and s_μ' stand for the initial and final four-momenta of the muon) from the mass shell is invariantly given by the square of the "imaginary mass" λ of the photon:

$$\lambda^2 = \mathbf{k}^2 - k_0^2 = 2s_0(s_0-k_0) - 2\mu^2 - 2\cos\theta[(s_0^2-\mu^2)((s_0-k_0)^2-\mu^2)]^{\frac{1}{2}}, \quad (1)$$

where μ is the muon mass and θ the scattering angle of the muon. For forward scattering ($\theta = 0°$) and given initial energy, the largest value for λ, λ_0 is obtained when the muon gives all its kinetic energy to the photon, that means if $s_0 - k_0 = \mu$. From Eq. (1) we have:

$$\lambda_0^2 = 2k_0\mu.$$

Whereas the corresponding value for electroproduction, $\lambda_0^2 = 2k_0 m$ (where m is the electron mass) reaches $\lambda_0 = 100$ Mev only for $k_0 \approx 10$ Gev, with muons in forward scattering one can obtain $\lambda_0 = 200$ Mev with $k_0 \approx 190$ Mev [see curves (a) and (b) of Fig. 1]. But the fact that λ_0 increases only proportional $k_0^{\frac{1}{2}}$ and that λ falls rapidly if not all of the kinetic energy of the muon is transferred to the photon makes the use of the forward direction inconvenient for higher values of λ. For large angles λ is almost equal for inelastic electron and inelastic muon scattering if the total incident energy and the energy loss are the same, as shown in Fig. 1.

2. INELASTIC SCATTERING FOR FIXED λ^2 AND k_0

If one supposes that the muon and the electron differ only in their rest mass, the question arises whether there is a relation between the cross sections for electroproduction and pion production by muons.[2] The matrix elements for both processes involve the photoproduction matrix elements off the mass shell which have to be calculated from meson theory and of course are only approximately known. (The best known approach, using dispersion relations, is due to Fubini, Nambu, and Wataghin.[3]) Therefore one is interested in a relation between the two processes which does not require knowledge of the photoproduction matrix elements.

If we restrict ourselves to the lowest order of electromagnetic interaction, but treat the photoproduction

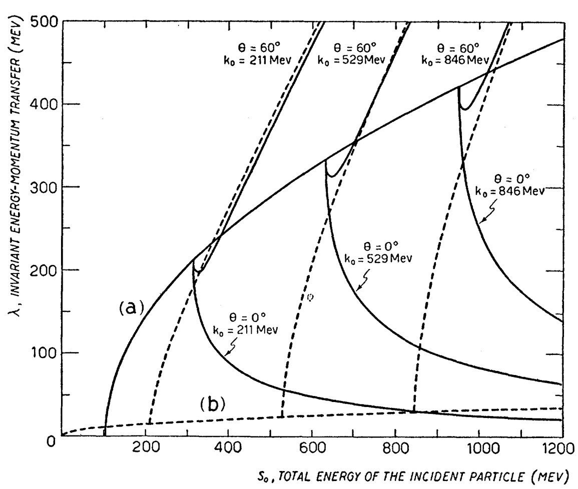

FIG. 1. Energy-momentum transfer in the inelastic scattering of electrons (dashed lines) and muons (solid lines) as functions of the total incident energy of the electron and muon, respectively. Curves (a) and (b): Maximal energy-momentum transfer λ_0 in forward scattering. Other lines: Energy-momentum transfer for fixed energy loss k_0 and fixed scattering angle θ.

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[1] W. K. H. Panofsky and E. A. Allton, Phys. Rev. **110**, 1155 (1958).

[2] The author is indebted very much to Professor E. Amaldi for drawing his attention to this question.

[3] S. Fubini, Y. Nambu, and V. Wataghin, Phys. Rev. **111**, 329 (1958).

vertex j_μ generally, the differential cross section for the production of a pion of momentum q_μ' with the nucleon of initial momentum r_μ, recoiling with a momentum r_μ' and final muon momentum s_μ', is given by (M is the nucleon mass):

$$d\sigma=\frac{e^2}{(2\pi)^5}\frac{1}{4}\sum\frac{\mu^2M}{s_0s_0'r_0'2q_0'}\frac{s_0}{s}\frac{|\langle q'r'|j_\mu|r\rangle\bar{u}(\mathbf{s})\gamma_\mu u(\mathbf{s})|^2}{\lambda^4}$$
$$\times\delta(k_\mu+r_\mu-r_\mu'-q_\mu')d^3s'd^3r'd^3q'. \quad (2)$$

For electroproduction we have the same expression, only with the muon four-momenta s_μ and s_μ' replaced by electron four-momenta p_μ, p_μ', and m instead of μ. The unknown matrix elements $\langle q'r'|j_\mu|r\rangle$ depend on the lepton variables only through $k_\mu=s_\mu-s_\mu'$ or $k_\mu=p_\mu-p_\mu'$, respectively. Therefore, if we choose p_μ and p_μ' such that $p_\mu-p_\mu'=s_\mu-s_\mu'$ we can use the same $\langle q'r'|j_\mu|r\rangle$ in both cross sections. Now we restrict ourselves to the important case that only the final muon and electron are observed. Then it is sufficient to have equal $\mathbf{k}^2$ and k_0 and from (1) we get the condition (φ is the deflection angle of the electron in electroproduction):

$$s_0(s_0-k_0)-\mu^2-\cos\theta[(s_0^2-\mu^2)((s_0-k_0)^2-\mu^2)]^{\frac{1}{2}}$$
$$=p_0(p_0-k_0)-m^2-\cos\varphi$$
$$\times[(p_0^2-m^2)((p_0-k_0)^2-m^2)]^{\frac{1}{2}}, \quad (3)$$

which for a given k_0 relates the initial energies to the deflection angles.

Following Dalitz and Yennie[4] we use gauge invariance and sum over the muon spins to replace in (2)

$$\mu^2\sum_{\text{spins}}|\langle q'r'|j_\mu|r\rangle\bar{u}(\mathbf{s}')\gamma_\mu u(\mathbf{s})|^2$$

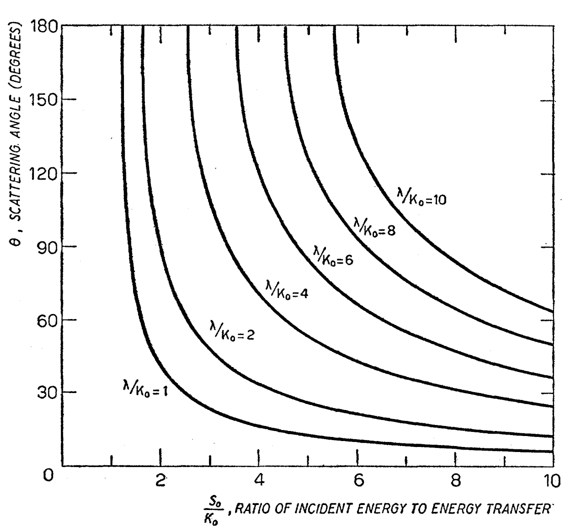

FIG. 2. Curves connecting different scattering situations of inelastic electron scattering, the differential cross sections of which can be compared by formula (5).

[4] R. H. Dalitz and D. R. Yennie, Phys. Rev. **105**, 1598 (1957).

by

$$\sum m_km_l(s_ks_l'+s_ls_k'+\tfrac{1}{2}\lambda^2\delta_{kl}),$$

where

$$m_k=\langle q'r'|j_k+\mathbf{k}\cdot\mathbf{j}k_4^{-2}k_k|r\rangle,$$

and the summation is only over the nucleon spins. Performing in (2) the integrations over d^3r' and dq_0' in the laboratory system, we obtain:

$$d\sigma=\frac{e^2}{(2\pi)^5}\frac{1}{8}\sum\frac{Ms'}{s}\frac{m_km_l}{\lambda^4}(s_ks_l'+s_ls_k'+\tfrac{1}{2}\lambda^2\delta_{kl})$$
$$\times\frac{q'd\Omega_{q'}}{q_0'+r_0'-\mathbf{k}\cdot\mathbf{q}'q_0'/\mathbf{q}'^2}ds_0'd\Omega_{s'},$$

where

$$r_0'=M+k_0-q_0'.$$

From invariance arguments one sees that after performing the integration over the pion directions and the summation over the nucleon spins, one is left with an expression of the form:

$$d^2\sigma/ds_0'd\Omega_{s'}=(s'/s)[f(\mathbf{k}^2,k_0)k_kk_l+g(\mathbf{k}^2,k_0)\delta_{kl}]$$
$$\times(s_ks_l'+s_ls_k'+\tfrac{1}{2}\lambda^2\delta_{kl}), \quad (4)$$

where $f(\mathbf{k}^2,k_0)$ and $g(\mathbf{k}^2,k_0)$ are two unknown functions determined by j_μ, but which have the same value for all processes with momenta satisfying (3). One notices that the form (4) of the cross section for inelastic scattering remains unchanged if more or other particles are produced by the current j_μ as long as these particles are not observed. Thus (4) is valid for general inelastic scattering of electrons or muons by a gauge-invariant current, if only one photon is exchanged and if the electron or muon line is not modified by radiative corrections. The latter could be taken into account by substituting $\bar{u}(\mathbf{s})\gamma_\mu u(\mathbf{s})$ by more complicated expressions. Form factors for the muon could be introduced in this way.

We apply (4) to get a relation between the differential cross sections of inelastic electron scattering and inelastic muon scattering. For s_0, s_0', p_0, p_0' and θ, φ satisfying (3), we have [if $f(\mathbf{k}^2,k_0)\neq 0$]:

$$\frac{d^2\sigma}{dp_0'd\Omega_{p'}}\bigg/\frac{d^2\sigma}{ds_0'd\Omega_{s'}}$$
$$=\frac{p's}{ps'}\frac{2\mathbf{k}\cdot\mathbf{p}\mathbf{k}\cdot\mathbf{p}'+\frac{1}{2}\lambda^2\mathbf{k}^2+h(\mathbf{k}^2,k_0)(2\mathbf{p}\cdot\mathbf{p}'+\frac{3}{2}\lambda^2)}{2\mathbf{k}\cdot\mathbf{s}\mathbf{k}\cdot\mathbf{s}'+\frac{1}{2}\lambda^2\mathbf{k}^2+h(\mathbf{k}^2,k_0)(2\mathbf{s}\cdot\mathbf{s}'+\frac{3}{2}\lambda^2)}, \quad (5)$$

which depends on one parameter $h(\mathbf{k}^2,k_0)$. If we take both p, p' and s, s' to be electron momenta, then (5) gives a relation between the differential cross sections of inelastic electron scattering at different angles and from (3) the corresponding energies [of course for this case in (3) μ has to be replaced by m], which may be used to determine $h(\mathbf{k}^2,k_0)$ and to test the electrodynamic assumptions. For electrons or high-energy muons we may neglect the rest mass in (3), which leads to the simpler condition:

$$s_0(s_0-k_0)(1-\cos\theta)=p_0(p_0-k_0)(1-\cos\varphi). \quad (6)$$

In Fig. 2 we have plotted (6) taking k_0 as energy unit.

An interesting application of (5) would be the measurement of high-energy inelastic muon scattering at different angles with fixed λ^2, k_0. Any sufficiently strong nonelectromagnetic interaction of the muon with nucleons or pions would result in a disagreement of the cross sections with one value of $h(\mathbf{k}^2, k_0)$ in (5), but for the time being such an experiment seems difficult because of the low intensities of the available muon beams.

3. POLARIZATION EFFECTS

The muons obtained from pion decay are produced with a longitudinal polarization and one may expect an asymmetry in the pion production by polarized muons. In the static approximation, where the recoil momentum is neglected, as well as in the case that the recoil nucleon is not observed, the only scalar one can form with the polarization pseudovector $\boldsymbol{\zeta}$ parallel $\mathbf{s}$ is $(\boldsymbol{\zeta}\times\mathbf{s}')\cdot\mathbf{q}'$ and this shows that a coincident detection of the final muon and pion is necessary to get a polarization effect. If we write the photoproduction current $\mathbf{j}$ in the form:

$$\mathbf{j}=A\boldsymbol{\sigma}+B\mathbf{q}'\boldsymbol{\sigma}\cdot\mathbf{k}+C\mathbf{k}\boldsymbol{\sigma}\cdot\mathbf{k}+D\mathbf{q}'\boldsymbol{\sigma}\cdot\mathbf{q}'+E\mathbf{k}\boldsymbol{\sigma}\cdot\mathbf{q}'+F\mathbf{q}'\times\mathbf{k},$$

then the use of the appropriate projection operators in the evaluation of the muon spin sums in (2), after summation over the nucleon spins, gives rise to an additional contribution, proportional to

$$\mathrm{Im}[(A+C\mathbf{k}^2)B^*-(A+D\mathbf{q}^2)E^* +(EB^*+CD^*)\mathbf{q}\cdot\mathbf{k}](\mathbf{q}'\times\mathbf{k})\cdot\mathbf{s}, \quad (7)$$

the differential cross section. Here the incident muon has been assumed to be completely longitudinally polarized in the system which is used for $\mathbf{j}$.

We have evaluated (7) for the case in which $\mathbf{s}$, $\mathbf{s}'$, and $\mathbf{q}'$ form an orthogonal triad in the center-of-mass system of the nucleon and meson, using for $\mathbf{j}$ the matrix elements of Fubini, Nambu, and Wataghin, reference 3, Eq. (14) and keeping all the terms. The nucleon form factors were taken in the form $(1+\lambda^2r^2/12)^{-2}$ with $r_m^V=r_m^S=r_e^V=0.8$ fermi, $r_e^S=0$. Our results are given in Table I. $W=1200$ Mev lies near the maximum of the π^+ photoproduction cross section while $W=1236$ Mev is the δ_{33} resonance. Under "asymmetry" we listed the percentage of the contribution of the expression (7) which gives rise to an up-down asymmetry, to the differential cross section. The magnitude of the differential cross section $d^3\sigma/ds_0'd\Omega_{s'}d\Omega_{q'}$ is of the order $10^{-37}-10^{-36}$ cm^2 Mev^{-1} steradian^{-2}, thus very high incident muon intensities are needed for an observation of the effect. For the lowest values of s_0, $s_0-\mu=326.9$ Mev and $s_0-\mu=378.0$ Mev the polarization vanishes because after the collision the muon is at rest in the center-of-mass system of nucleon and pion. The values for $s_0-\mu=684.8$ Mev and $s_0-\mu=746.7$ Mev are given in brackets because they correspond to $\lambda=587$ Mev and $\lambda=603$ Mev, respectively, where the Fubini-Nambu-Wataghin matrix elements should not be trusted too much.

TABLE I. Asymmetry of the pion production in collisions of longitudinal polarized muons with protons. Pion, initial muon, and final muon are taken to be orthogonal in the c.m. system of nucleon and pion. W is the total energy of nucleon and pion in their c.m. system; $s_0-\mu$ is the incident kinetic energy of the muon in the laboratory. Negative values indicate an inverted asymmetry.

W (Mev)	$s_0-\mu$ (Mev)	Asymmetry in the production of π^+	π^0	W (Mev)	$s_0-\mu$ (Mev)	Asymmetry in the production of π^+	π^0
1200	326.9	0	0	1236	378.0	0	0
	398.5	7.0	34.8		451.7	4.4	13.3
	452.2	8.1	37.5		507.0	5.7	15.6
	505.9	6.5	34.7		562.3	6.3	15.6
	684.8	(−13)	(+15.2)		746.7	(−3.4)	(+9.1)

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