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Quantized Meson Field in a Classical Gravitational Field*

TSUTOMU IMAMURA†
Department of Physics, University of North Carolina, Chapel Hill, North Carolina

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The behavior of a quantized meson field in a classical gravitational field is examined. Physical quantities such as the expectation value for the number of created mesons are represented in terms of a formal Green's function. They are computed explicitly for the case of a special space-independent gravitational field. The inadequacy of standard iteration procedures is discussed in an Appendix.

1. INTRODUCTION

THE problem of the creation and scattering of quantized particles by an externally impressed gravitational field is a virtually unexplored area of general relativity. On account of the importance and difficulty of the theory of gravitation, the author believes that any aspect of the theory is worth studying if it has characteristics differing from theory of other field theories. In the above-mentioned problem, the form of interaction of the external field is different from that of usual theories in that the characteristic surfaces of the quantized field are affected by the external field. Furthermore there may occur phenomena for which such a semiclassical treatment will be suitable. The author does not assert that the investigation of the present problem is a necessary step to the more important problem of the quantized gravitational field, but it is hoped that some information may nevertheless be obtained as to a semiclassical limit of the latter. In this paper we examine the behavior of a quantized neutral meson field in a classical gravitational field which is simple enough so that exact solutions may be obtained.

In Sec. 2, physical quantities such as the expectation value for the number of created mesons are represented in terms of a formal Green's function. The sole requirement imposed is that the gravitational field be such that well defined state vectors exist in the remote past and future. In Sec. 3, the expectation values are computed explicitly for the case of a special space-independent gravitational field. Some attention is given in the Appendix about the iteration method.

2. PRELIMINARY DISCUSSIONS

In order to have a well-defined state vector in the remote past and future, we shall restrict the gravitational field to be one which has the following properties

$$g^{\mu\nu}(\mathbf{x}, \pm T)=\gamma^{\mu\nu}+O(T^{-\alpha}+L^{-\beta}), \tag{2.1}$$

where $\gamma^{\mu\nu}$ is the Minkowski metric, and which admits the following type of solution for a neutral meson field

$$\begin{aligned}\phi(\mathbf{x}, \pm T)&=\phi^{\pm}(\mathbf{x}, \pm T)+O(T^{-\alpha}+L^{-\beta}),\\ \frac{\partial\phi}{\partial x^\mu}(\mathbf{x}, \pm T)&=\frac{\partial\phi^{\pm}}{\partial x^\mu}(\mathbf{x}, \pm T)+O(T^{-\alpha}+L^{-\beta}).\end{aligned} \tag{2.2}$$

T and L are large constants and α and β are positive real. Greek indices run from 0 to 3, Latin indices from 1 to 3. In the following, equalities which are correct to order $O(T^{-\alpha}+L^{-\beta})$ will be shown by $\approx$. The quantities $\phi^{\pm}(\mathbf{x},t)$ satisfy the free meson equation and can be written as

$$\phi^{\pm}(\mathbf{x},t)=[2(2\pi)^3]^{-\frac{1}{2}}\int d^3k\ (k_0)^{-\frac{1}{2}}\ e^{i\mathbf{k}\cdot\mathbf{x}} \times[a^{\pm}(\mathbf{k})\exp(-i\bar{k}_0t)+a^{\pm\dagger}(-\mathbf{k})\exp(i\bar{k}_0t)] \tag{2.3}$$

where $\bar{k}_0$ represents $(\mathbf{k}^2+m^2)^{\frac{1}{2}}$, $a^\dagger$ means Hermitian conjugate of a, and the a's satisfy:

$$\begin{aligned}[a(\mathbf{k}),a(\mathbf{k}')]&=[a^\dagger(\mathbf{k}),a^\dagger(\mathbf{k}')]=0,\\ [a(\mathbf{k}),a^\dagger(\mathbf{k}')]&=\delta(\mathbf{k}-\mathbf{k}').\end{aligned} \tag{2.4}$$

Furthermore we shall assume the following properties for simplicity

$$g^{0l}(\mathbf{x},t)=0, \quad \text{for all } \mathbf{x} \text{ and } t.$$

The Hamiltonian for the system has the form,

$$H(\pm t)=\frac{1}{2}\int d^3x\ (-g)^{\frac{1}{2}} \times\left(-g^{00}\frac{\partial\phi}{\partial t}\frac{\partial\phi}{\partial t}+g^{lm}\frac{\partial\phi}{\partial x^l}\frac{\partial\phi}{\partial x^m}+m^2\phi^2\right), \tag{2.5}$$

and satisfies the asymptotic conditions

$$H(\pm T)\approx\frac{1}{2}\int d^3k\ \bar{k}_0a^{\pm\dagger}(\mathbf{k})a^{\pm}(\mathbf{k})+(\text{H.c.}). \tag{2.6}$$

The vacuum state vector for the remote past is defined by $a^-\psi_0=0$, and the incoming single particle states are given by $\psi_{\mathbf{k}}=a^{-\dagger}(\mathbf{k})\psi_0$.

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† On leave of absence from Osaka University, Osaka, Japan.

The expectation value for the number of created mesons in the remote future in the state for which no mesons are present in the remote past is

$$(\psi_0 a^{+\dagger}(\mathbf{k})a^{+}(\mathbf{k})\psi_0). \tag{2.7}$$

The scattering amplitude for the momentum transition $\mathbf{k}$ to $\mathbf{k}'$ is

$$(\psi_0^{+}a^{+}(\mathbf{k})a^{-\dagger}(\mathbf{k})\psi_0), \tag{2.8}$$

where ψ_0^{+} is defined by $a^{+}\psi_0^{+}=0$.

In order to calculate these quantities in terms of the basic vectors ψ_0, $\psi_{\mathbf{k}}\cdots$, one must express $\phi(\mathbf{x}t)$ or a^{+} as a function of a^{-}. By using the meson Green's function, they are represented as

$$\begin{aligned}\phi(\mathbf{x},t)&=\int d^3y\left(\frac{\partial G}{\partial(-T)}(\mathbf{x},t;\mathbf{y},-T)\phi(\mathbf{y},-T)-G(\mathbf{x},t,\mathbf{y},-T)\frac{\partial\phi}{\partial(-T)}(\mathbf{y},-T)\right)\\&\approx i\left(\frac{(2\pi)^3}{2}\right)^{\frac{1}{2}}\int d^4kd^4k'\,G(\mathbf{k},k_0,\mathbf{k}',k_0')e^{i\mathbf{k}\cdot\mathbf{x}-ik_0t-ik_0'T}(k_0')^{-\frac{1}{2}}\\&\qquad\times[(k_0'+k_0')a^{-}(\mathbf{k}')\exp(i\bar{k}_0'T)+(k_0'-k_0')a^{-\dagger}(-\mathbf{k}')\exp(-i\bar{k}_0'T)],\end{aligned}\tag{2.9}$$

and

$$\begin{aligned}a^{+}(\mathbf{k})&\approx(2(2\pi)^3)^{-\frac{1}{2}}\int d^3x\,e^{-i\mathbf{k}\cdot\mathbf{x}}\left[(\bar{k}_0)^{\frac{1}{2}}\phi(\mathbf{x},T)+i(k_0)^{-\frac{1}{2}}\frac{\partial\phi}{\partial T}(\mathbf{x},T)\right]e^{ik_0T}\\&\approx\frac{i}{2}\int dk_0d^4k'\,(k_0\bar{k}_0')^{-\frac{1}{2}}G(\mathbf{k},k_0,\mathbf{k}',k_0')e^{-iT(k_0+k_0')}\\&\qquad\times[(k_0+k_0)(k_0'+\dot{k}_0')a^{-}(\mathbf{k}')-(k_0+\dot{k}_0)(k_0'-k_0')a^{-\dagger}(-\mathbf{k}')\exp(2i\bar{k}_0'T)]\end{aligned}\tag{2.10}$$

where

$$G(\mathbf{x},t,\mathbf{x}',t')=\int d^4kd^4k'\,G(\mathbf{k},k_0,\mathbf{k}',k_0')e^{i\mathbf{k}\cdot\mathbf{x}-i\mathbf{k}'\cdot\mathbf{x}'-ik_0t+ik_0't'}.$$

In the case of a space-independent gravitational field, several conditions on G are given from general requirements, namely,

$$G(\mathbf{k},k_0,\mathbf{k}',k_0')=-G^{*}(-\mathbf{k},-k_0,-\mathbf{k}',-k_0') \tag{2.11}$$

from the Hermiticity of ϕ,

$$G(\mathbf{k},k_0,\mathbf{k}',k_0')=G(-\mathbf{k},k_0,-\mathbf{k}',k_0') \tag{2.12}$$

from reflection invariants, and

$$\begin{aligned}&\int G(\mathbf{k},k_0,k_0')G(\mathbf{k},p_0,p_0')(k_0'-p_0')e^{-it(k_0+p_0)-iT(k_0'+p_0')}d^3kdk_0dk_0'dp_0dp_0'\approx0\\&-g^{00}(t)\int G(\mathbf{k},k_0,k_0')G(\mathbf{k},p_0,p_0')(k_0k_0'-k_0p_0')e^{-it(k_0+p_0)-iT(k_0'+p_0')}d^3kdk_0dp_0'dp_0dp_0'\approx(2\pi)^6[-g(t)]^{\frac{1}{2}}\\&\int G(\mathbf{k},k_0,k_0')G(\mathbf{k},p_0,p_0')(k_0k_0'p_0-k_0p_0p_0')e^{-it(k_0+p_0)-iT(k_0'+p_0')}d^3kdk_0dk_0'dp_0dp_0'\approx0\end{aligned}\tag{2.13}$$

from the consistency for a canonical quantization; $[\phi(\mathbf{x},t),\phi(\mathbf{x}',t)]=0$ etc. Here $G(\mathbf{k},k_0,k_0')$ is defined by

$$G(\mathbf{k},k_0,k_0')\delta^3(\mathbf{k}-\mathbf{k}')=G(\mathbf{k},k_0,\mathbf{k}',k_0'). \tag{2.14}$$

3. MESON FIELD IN A SPACE-INDEPENDENT GRAVITATIONAL FIELD

Using the form, (2.14) for the Green's function, we may write the Hamiltonian in terms of a^{-} as

$$\begin{aligned}H(t)&\approx\tfrac{1}{4}(2\pi)^6[-g(t)]^{\frac{1}{2}}\int d^4kdk_0'dp_0dp_0'\,e^{-i(k_0+p_0)t-i(k_0'+p_0')T}\\&\qquad\times G(\mathbf{k},k_0,k_0')G(\mathbf{k},p_0,p_0')(-\bar{k}_0)^{-1}[g^{00}(t)k_0p_0+g^{lm}k_lk_m+m^2]\\&\qquad\times[(k_0'+\bar{k}_0)(p_0'+\bar{p}_0)e^{2ik_0T}a^{-}(\mathbf{k})a^{-}(-\mathbf{k})+(k_0'+\bar{k}_0)(p_0'-\bar{p}_0)a^{-}(\mathbf{k})a^{-\dagger}(\mathbf{k})]+(\mathrm{H.c.})\\&\approx\int d^3k[A_{\mathbf{k}}a^{-}(\mathbf{k})a^{-}(-\mathbf{k})+B_{\mathbf{k}}a^{-}(\mathbf{k})a^{-\dagger}(\mathbf{k})]+(\mathrm{H.c.}).\end{aligned}\tag{3.1}$$

It is easily seen that $B_{\mathbf{k}}$ is real, from (2.11) and (2.12). The Hamiltonian is seen to be quadratic in a^- and $a^{-\dagger}$. Moreover, momentum is conserved. The Hamiltonian (3.1) has exactly the same form as that used by Bogolijubov[1] in his treatment of the theory of superconductivity and can be diagonalized by the technique which he has introduced. The result is

$$H(t)\approx\int d^3k(B_{\mathbf{k}}{}^2-|A_{\mathbf{k}}|^2)^{\frac{1}{2}}\alpha^\dagger(\mathbf{k})\alpha(\mathbf{k})+(\mathrm{H.c.}), \tag{3.2}$$

where

$$\alpha(\mathbf{k})=e^{i\theta_{\mathbf{k}}/2}u_{\mathbf{k}}a^-(\mathbf{k})-e^{-i\theta_{\mathbf{k}}/2}v_{-\mathbf{k}}a^{-\dagger}(-\mathbf{k}),\quad A_{\mathbf{k}}=|A_{\mathbf{k}}|e^{i\theta_{\mathbf{k}}},$$
$$u_{\mathbf{k}}=\cosh x_{\mathbf{k}},\quad v_{\mathbf{k}}=\sinh x_{\mathbf{k}},\quad \tanh 2x_{\mathbf{k}}=-|A_{\mathbf{k}}|/B_{\mathbf{k}}.$$

The condition

$$B_{\mathbf{k}}>|A_{\mathbf{k}}| \tag{3.3}$$

is a necessary one for Bogolijubov's transformation to be permissible. It will be shown later that it is satisfied in our case as long as the space metric remains positive definite.

The expectation value of the number of created mesons with momentum k, is given by

$$\langle N^+(k)\rangle\approx\frac{1}{4}\int dk_0dk_0'dp_0dp_0'\,G(\mathbf{k}k_0k_0')G(\mathbf{p}p_0p_0')e^{-iT(k_0+k_0'+p_0+p_0')}$$
$$\times[k_0k_0'-\bar{k}_0(k_0-k_0')-\bar{k}_0{}^2][p_0p_0'-\bar{k}_0(p_0-p_0')-\bar{k}_0{}^2](\bar{k}_0)^{-2}(\psi_0a^-(-\mathbf{k})a^{-\dagger}(-\mathbf{k})\psi_0). \tag{3.4}$$

The vacuum state in the remote future is defined by

$$0=a^+(\mathbf{k})\psi_0{}^+\approx\frac{i}{2}\int dk_0dk_0'\,G(\mathbf{k}k_0k_0')e^{-iT(k_0+k_0')}(\bar{k}_0\bar{k}_0')^{\frac{1}{2}}[(k_0+k_0)(k_0'+k_0)a^-(\mathbf{k})-(k_0+k_0)(k_0'-k_0)a^{-\dagger}(-\mathbf{k})e^{2ik_0T}]\psi_0{}^+,$$

and the scattering amplitude for the process, in which the initial and the final momenta of meson are different is equal to zero on account of definitions of ψ_0 and $\psi_0{}^+$.

In order to get more explicit expressions for these physical quantities, we shall consider the following gravitational field:

$$\begin{aligned} g^{\mu\nu}&=\gamma^{\mu\nu} && \text{for}\quad \infty>t>\tau+L^{-1},\ -\tau-L^{-1}>t>-\infty,\\ g^{\mu\nu}&=\eta^{\mu\nu}(\text{const}) && \text{for}\quad \tau>t>-\tau. \end{aligned} \tag{3.14}$$

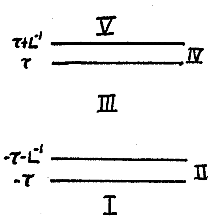

FIG. 1. Diagram of the regions considered.

In the regions I, III, and V of the Fig. 1, the solution of the field equation is easily found to be

$$\phi^{\mathrm{I}}(\mathbf{x},t)=[2(2\pi)^3]^{-\frac{1}{2}}\int d^3k(\bar{k}_0)^{-\frac{1}{2}}e^{i\mathbf{k}\cdot\mathbf{x}}[a^-(\mathbf{k})\exp(-i\bar{k}_0t)+a^{-\dagger}(-\mathbf{k})\exp(i\bar{k}_0t)],$$

$$\phi^{\mathrm{III}}(\mathbf{x},t)=[2(2\pi)^3]^{-\frac{1}{2}}\int d^3k(k_\eta)^{-\frac{1}{2}}e^{i\mathbf{k}\cdot\mathbf{x}}[b(\mathbf{k})e^{-ik_\eta t}+b^\dagger(-\mathbf{k})e^{ik_\eta t}], \tag{3.15}$$

$$\phi^{\mathrm{V}}(\mathbf{x},t)=[2(2\pi)^3]^{-\frac{1}{2}}\int d^3k(\bar{k}_0)^{-\frac{1}{2}}e^{i\mathbf{k}\cdot\mathbf{x}}[a^+(\mathbf{k})\exp(-i\bar{k}_0t)+a^{+\dagger}(-\mathbf{k})\exp(i\bar{k}_0t)],$$

where $k_\eta=(\eta^{lm}k_lk_m+m^2/-\eta^{00})^{\frac{1}{2}}$. In the regions II and IV, field equation is

$$g^{00}\frac{\partial^2\phi}{\partial t^2}+\left\{(-g)^{-\frac{1}{2}}\frac{\partial}{\partial t}[(-g)^{\frac{1}{2}}g^{00}]\right\}\frac{\partial\phi}{\partial t}+g^{lm}\frac{\partial^2\phi}{\partial x^l\partial x^m}-m^2\phi=0. \tag{3.16}$$

[1] N. N. Bogolijubov, J. Phys. U.S.S.R. 11, 23 (1947).

In the limit $L \longrightarrow \infty$, discontinuities occur in the derivative of ϕ. If we suppose ϕ, $\partial\phi/\partial x^\mu$ and $\partial^2\phi/\partial x^l \partial x^m$ to be of order 1, then

$$\left(\frac{\partial^2\phi}{\partial t^2}\bigg/\frac{\partial\phi}{\partial t}\right)+\frac{\partial}{\partial t}[(-g)^{\frac{1}{2}}g^{00}]/(-g)^{\frac{1}{2}}g^{00}+O(1)=0.$$

By integration from $\mp\tau\mp L^{-1}$ to $\mp\tau$ one gets

$$\left[\ln\frac{\partial\phi}{\partial t}+\ln(-(-g)^{\frac{1}{2}}g^{00})\right]_{\mp\tau\mp L^{-1}}^{\mp\tau}=O(L^{-1}).$$

This gives the following relations

$$\frac{\partial\phi}{\partial t}(\mathbf{x},\pm\tau\pm L^{-1})=Y\frac{\partial\phi}{\partial t}(\mathbf{x},\pm\tau)+O(L^{-1}),\tag{3.17}$$

$$\phi(\mathbf{x},\pm\tau\pm L^{-1})=\phi(\mathbf{x},\pm\tau)+O(L^{-1}),\tag{3.18}$$

where $Y=(-(-\eta)^{\frac{1}{2}}\eta^{00})$. This solution is compatible with the above assumption that ϕ, $\partial\phi/\partial x^\mu$ and $\partial^2\phi/\partial x^l\partial x^k$ are of order 1, when it is connected with well-behaved solutions in the regions I, III, and V.

Using (3.15), (3.17), and (3.18), we can express a^+ and b in terms of a^-:

$$b(\mathbf{k})=\tfrac{1}{2}e^{-ik_\eta\tau}\left\{\left[\left(\frac{k_\eta}{\bar{k}_0}\right)^{\frac{1}{2}}+\left(\frac{\bar{k}_0}{k_\eta}\right)^{\frac{1}{2}}Y^{-1}\right]a^-(\mathbf{k})\exp(i\bar{k}_0\tau)+\left[\left(\frac{k_\eta}{\bar{k}_0}\right)^{\frac{1}{2}}-\left(\frac{\bar{k}_0}{k_\eta}\right)^{\frac{1}{2}}Y^{-1}\right]a^{-\dagger}(-\mathbf{k})\exp(-i\bar{k}_0\tau)\right\},\tag{3.19}$$

$$a^+(\mathbf{k})=\left[\cos 2k_\eta\tau-\frac{i}{2}(\sin 2k_\eta\tau)\left(\frac{k_\eta Y}{\bar{k}_0}+\frac{\bar{k}_0}{k_\eta Y}\right)\right]a^-(\mathbf{k})\exp(2i\bar{k}_0\tau)-\frac{i}{2}(\sin 2k_\eta\tau)\left(\frac{k_\eta Y}{\bar{k}_0}-\frac{\bar{k}_0}{k_\eta Y}\right)a^{-\dagger}(-\mathbf{k})\exp(-2i\bar{k}_0\tau).\tag{3.20}$$

The other way to get the expressions of physical quantities is to use Green's functions in regions I, V, and III, that is,

$$(2\pi)^{-3}\int d^3k\, e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}(\bar{k}_0)^{-1}\sin k_0(x_0-y_0),$$

$$(2\pi)^{-3}\int d^3k\, e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}(k_\eta)^{-1}\sin k_\eta(x_0-y_0),$$

and relations (3.17) and (3.18) with the formula

$$G(\mathbf{x},t,\mathbf{x}',t')=\int d^3x''[G(\mathbf{x},t,\mathbf{x}'',t'')(\partial/\partial t'')G(\mathbf{x}'',t'',\mathbf{x}',t')-(\partial/\partial t'')G(\mathbf{x},t,\mathbf{x}'',t'')G(\mathbf{x}'',t'',\mathbf{x}',t')][-g(t'')]^{\frac{1}{2}}g^{00}(t'').\tag{3.21}$$

The results are, for instance,

$$(2\pi)^6\int G(\mathbf{k},k_0,k_0')e^{-ik_0t-ik_0T}\approx(\bar{k}_0)^{-1}\cos k_\eta(t+\tau)\sin\bar{k}_0(-\tau+T)+(k_\eta Y)^{-1}\sin k_\eta(t+\tau)\cos\bar{k}_0(-\tau+T)\tag{3.22}$$

for $\tau>t>-\tau$ and

$$(2\pi)^9\int G(\mathbf{k},k_0,k_0')e^{-ik_0t-ik_0'T}\approx[(k_\eta Y)^{-1}\sin 2k_\eta\tau\cos\bar{k}_0(-\tau+t)+(\bar{k}_0)^{-1}\cos 2k_\eta\tau\sin\bar{k}_0(t-\tau)]\cos\bar{k}_0(T-\tau)$$
$$+[\cos 2k_\eta\tau\cos\bar{k}_0(t-\tau)-k_\eta Y(\bar{k}_0)^{-1}\sin 2k_\eta\tau\sin\bar{k}_0(t-\tau)](\bar{k}_0)^{-1}\sin\bar{k}_0(T-\tau)\tag{3.23}$$

for $t>\tau$.

From (3.1), (3.2) and (3.23), one can get the Hamiltonian in region III as

$$H\approx\frac{1}{2}\int d^3k\, k_\eta\alpha^\dagger(\mathbf{k})\alpha(\mathbf{k})+(\text{H.c.}).\tag{3.24}$$

It is not hard to show that $\alpha(\mathbf{k})$, which is calculated from (3.2) is equal to $e^{ik_\eta\tau}b(\mathbf{k})$, which is given in (3.19). Expression (3.24), rewritten in terms of b and $b^\dagger$ can also be obtained by using the usual Hamiltonian formalism. In this example, comparing (3.2) and (3.24) we can see that the above-mentioned condition (3.3) is just that η^{lm} be positive definite. This is physically agreeable.

The expectation value of the number of created mesons is obtained by using (3.4) and (3.23) or (3.20). We find

$$\langle n^+(\mathbf{k})\rangle \simeq \tfrac{1}{4}[k_\eta Y/\bar{k}_0-\bar{k}_0/k_\eta Y]^2 \sin^2(2k_\eta\tau). \quad (3.25)$$

It is noteworthy that the expectation value of the total number of created mesons, $\int\langle n^+(\mathbf{k})\rangle d^3k$, is infinite. Another curious fact is that $\langle n^+(\mathbf{k})\rangle$ tends to a finite value in the limit of the breaking of the condition (3.3).

The only nonvanishing component of the Riemann tensor is (in the limit $L\rightarrow\infty$)

$$\begin{aligned}R_{0l0m}(t)=&\tfrac{1}{2}(\eta_{lm}-\gamma_{lm})[\delta'(t-\tau)-\delta'(t+\tau)]\\&+\tfrac{1}{8}[(\gamma^{00}+\eta^{00})(\eta_{00}-\gamma_{00})(\eta_{lm}-\gamma_{lm})\\&+(\gamma^{ks}+\eta^{ks})(\eta_{km}-\gamma_{km})(\eta_{sl}-\gamma_{sl})]\\&\times[\delta(t-\tau)\delta(t-\tau)+\delta(t+\tau)\delta(t+\tau)]. \quad (3.26)\end{aligned}$$

In spite of the singular expression of the second term of the right-hand side of (3.26) the following relation may be admitted on account of the symmetric character in the neighborhood of singular points:

$$\int R_{0l0m}(t)(t-\tau)(t+\tau)dt=(\eta_{lm}-\gamma_{lm})\tau. \quad (3.27)$$

Expression (3.25) can be rewritten as

$$\begin{aligned}\langle n^+(\mathbf{k})\rangle=&\tfrac{1}{4}[(\det\eta_{lk})^{\frac{1}{2}}(\eta^{lm}k_lk_m+m^2)^{\frac{1}{2}}(\bar{k}_0)^{-1}\\&-\bar{k}_0(\det\eta_{lk})^{-\frac{1}{2}}(\eta^{lm}k_lk_m+m^2)^{\frac{1}{2}}]^2\\&\times\sin^2(\eta^{lm}k_lk_m+m^2)[2\tau/(-\eta^{00})^{\frac{1}{2}}], \quad (3.28)\end{aligned}$$

and we can therefore express this quantity in terms of the Riemann tensor using relation (3.27). The quantity η^{00} which appears in the last term can be considered as a simple correction of the time unit.

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APPENDIX

One might try to treat the problem considered here by means of the Yang-Feldman integral equation[2]

$$\begin{aligned}\phi(\mathbf{x}t)=\phi_0(\mathbf{x}t)+\int\Delta^{\text{ret}}(\mathbf{x}t,\mathbf{x}'t')&\\\times\Big\{g^{00}\frac{\partial^2}{\partial t^2}+(-g)^{-\frac{1}{2}}\frac{\partial}{\partial t}[(-g)^{\frac{1}{2}}g^{00}]\frac{\partial}{\partial t}&\\+g^{lk}\frac{\partial^2}{\partial x^l\partial x^k}-\gamma^{\mu\nu}\frac{\partial^2}{\partial x^\mu\partial x^\nu}\Big\}\phi(\mathbf{x}'t').& \quad (A.1)\end{aligned}$$

Iterating this equation, we get, for the first approximation,

$$\begin{aligned}\phi(\mathbf{x}t)=\phi_0(\mathbf{x}t)+&\int_{-\tau}^{t}\Delta(\mathbf{x}t,\mathbf{x}'t')\\&\times\left(\eta^{00}\frac{\partial^2}{\partial t^2}+\eta^{lk}\frac{\partial^2}{\partial x^l\partial x^k}-m^2\right)\phi_0(\mathbf{x}'t')\\&+\int_{-\tau-L^{-1}}^{-\tau}\Delta(\mathbf{x}t,\mathbf{x}'t')\\&\times\left(g^{00}\frac{\partial^2}{\partial t^2}+(-g)^{-\frac{1}{2}}\frac{\partial}{\partial t}[(-g)^{\frac{1}{2}}g^{00}]\frac{\partial}{\partial t}\right.\\&\left.+g^{lm}\frac{\partial^2}{\partial x^l\partial x^m}-m^2\right)\phi_0(\mathbf{x}'t'), \quad (A.2)\end{aligned}$$

where

$$\begin{aligned}\phi_0(\mathbf{x}t)=\int d^3k\, e^{i\mathbf{k}\cdot\mathbf{x}}\Big[&\cos\bar{k}_0(t+T)\phi(\mathbf{k},-T)\\&+(\bar{k}_0)^{-1}\sin\bar{k}_0(t+T)\frac{\partial\phi}{\partial t}(\mathbf{k},-T)\Big].\end{aligned}$$

Under normal initial conditions, ϕ_0, $\partial\phi_0/\partial x^\mu$ and $\partial^2\phi_0/\partial x^\mu\partial x^\nu$ are of order 1. Therefore the second integral on the right-hand side of (A.2) becomes

$$\begin{aligned}\int d^3x'\,\Delta(\mathbf{x}t,\mathbf{x}'-\tau)\frac{\partial}{\partial t}\phi(\mathbf{x}',-\tau)&\\\cdot[(-\eta)^{\frac{1}{2}}\eta^{00}+1][-g(-\tau-\theta L^{-1})]^{-\frac{1}{2}}& \quad (A.3)\end{aligned}$$

where θ is a constant, $1\geq\theta\geq 0$. On the other hand, the corresponding first order term of the expansion of the exact solution is

$$\int d^3x'\,\Delta(\mathbf{x}t,\mathbf{x}'-\tau)\frac{\partial}{\partial t}\phi_0(\mathbf{x}',-t)[(-\eta)^{\frac{1}{2}}\eta^{00}+1]. \quad (A.4)$$

The other first order terms are just what we get from the first integral of the right-hand side of (A.2). Obviously these two expressions, (A.3) and (A.4), are different from each other except in special cases. The difference between them arises from the fact that in the iteration method, $\partial^2\phi/\partial t^2$, which is really of order L, is replaced by the quantity $\partial^2\phi_0/\partial t^2$, which is of order 1. This situation represents an important character of the present problem, arising from the fact that the characteristic surfaces themselves are perturbed, and shows that attempts to use standard perturbation methods in problems of this type are likely to lead only to difficulties.

[2] C. N. Yang and D. Feldman, Phys. Rev. 79, 972 (1950).