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Electromagnetic Corrections to the Ratio $\sigma(p+d \rightarrow H^3+\pi^+)/\sigma(p+d \rightarrow He^3+\pi^0)$

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Electromagnetic corrections to the ratio between charged and uncharged pions produced along with either a triton or helium-3 from 600-Mev protons incident on deuterium have been estimated. It was found that the main correction comes from the difference in triton and helium-3 wave functions. It was not found possible to correct unambiguously for the effects of mass difference between charged and uncharged pions. An enhancement of around 10% of positive pions was obtained with an estimated uncertainty of ±3%. The result agrees with experiments at CERN.

INTRODUCTION

In recent years there has been considerable interest in the hypothesis of charge independence. Previous experiments with nucleon- and π-meson scattering, as well as evidence from the ground-state nuclear physics, showed that the hypothesis is at least crudely correct. The more ambitious investigation of Puppi and Stanghellini,[1] using dispersion relations, on the experimental evidence on π^+p and π^-p scattering left some doubt, however, for the exact applicability of a charge-independent theory. As a good experiment for the test of charge independence, several authors have suggested the reactions

$$\begin{aligned} p+d &\rightarrow H^3+\pi^+ \quad (a) \\ p+d &\rightarrow He^3+\pi^0 \quad (b). \end{aligned} \tag{1}$$

If the electromagnetic interactions are neglected in (1), H^3 and He^3 are distinguished only by different z components of total isotopic spin $T=\frac{1}{2}$, with $T_z(H^3) = -\frac{1}{2}$ and $T_z(He^3)=+\frac{1}{2}$. The deuteron has $T=0$ and the pion has $T=1$ with $T_z(\pi^+)=+1$, $T_z(\pi^0)=0$ and one thus obtains, if the electromagnetic field is neglected,

$$d\sigma_a/d\Omega : d\sigma_b/d\Omega = 2:1,$$

for all angles and energies and irrespective of coordinate system. The experiment has the advantage that the initial state is the same for both reactions. It further involves both nucleons and π mesons. The above-mentioned experiment has been performed at Berkeley[2] and at Chicago[3] and has now also been done at CERN.[4]

Charge independence is assumed to apply only for the specific nucleon or meson forces while one has to correct for electromagnetic interactions. If the mass differences between charged and uncharged mesons and nucleons are believed to have an electromagnetic origin, the corresponding correction will be included in a complete electromagnetic correction. From a field theoretical point of view the coupling constants for the π-meson fields are believed to be charge independent but the exact validity of charge independence in an experiment will be destroyed by the electromagnetic interactions.

The electromagnetic field will have essentially two separable effects. First, the wave functions will be perturbed. Secondly, as the masses become different, the kinematics of the two processes will no longer be exactly the same. The final state momenta differ with the result that scattering angles and cross sections transform differently when one moves from the laboratory to the center-of-mass system. The experimental cross sections quoted in this paper have been corrected to the c.m. system.

In the experiment the triton and helium nuclei are counted at the same angle in the laboratory system. We wish to make the test of charge independence in the center-of-mass system, and in this system the reaction products thus come out at different angles and also with different energies or momenta.

TREATMENT

We assume the transition matrix element T to be a function of the variables q, θ, ψ_3, and c where q denotes the pion momentum, θ the angle at which the pion emerges, ψ_3 is the wave function of the triton or helium nucleus, and c denotes the charge (+ or 0) of the pion. We thus assume that the only effect of the different masses for charged and uncharged particles is that the energy conservation law gives different momenta to the pions, while the transition matrix is a function of the pion momenta only and not of the mass. It seems natural to assume a momentum instead of an energy dependence of the matrix. We will, however, come back to this question later on in this paper.

The q and θ dependence could, under our assumptions, be found experimentally just by measuring $\partial\sigma/\partial\theta$ and $\partial\sigma/\partial q$ for, e.g., reaction (1a). In the last experiment one has of course to vary the initial energy.

MODEL FOR THE REACTION

In order to proceed to obtain the corrections due to different ψ_3 and c we have to understand the reactions

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[1] G. Puppi and A. Stanghellini, Nuovo cimento **5**, 1305 (1957); A. Stanghellini, Nuovo cimento **10**, 398 (1958).

[2] K. C. Bandtel, W. J. Frank, and B. J. Moyer, Phys. Rev. **106**, 802 (1957).

[3] A. V. Crewe, E. Garwin, B. Ledley, E. Lillethun, R. March, and S. Marcowitz, Phys. Rev. Letters **2**, 269 (1959).

[4] D. Harting, J. C. Kluyver, A. Kusumegi, R. Rigopoulos, A. M. Sachs, G. Tibell, G. Vanderhaeghe, and G. Weber, Phys. Rev. Letters **3**, 52 (1959).

more fully. We construct an approximate model for the reaction and then we introduce the electromagnetic corrections as well as possible. We have to believe, of course, that the corrections are not too sensitive to the model adopted and that the model is not too far from the truth. We do not require absolute cross sections to good accuracy but require the variation of cross sections with our variables to be accurate enough.

To construct our model we use an impulse approximation type of calculation originally proposed by Chew[5] to describe the scattering of neutrons by deuterium. This or similar approximations have since been successfully applied to other reactions involving larger nuclei. It should work especially well for a loosely bound nucleus like deuterium and at a high energy. In this theory the interaction is reduced to free two-body interactions between the elementary particles (nucleons and pions in this case) while the binding effects appear as form factors as the particles, instead of being described by plane waves as in free collisions, are described by momentum distributions. Thus we bring into our model the matrix elements for pion production from nucleon-nucleon collisions which are better known than our original matrix element called T.

We denote the matrix element for the meson production in two-nucleon collisions by[6]

$$M_{TT'}(PQ), \tag{2}$$

where T and T' are the total isotropic spin for incident and final nucleons, respectively, while P and Q are so defined that the pion and the two nucleons have momenta Q, $P-\frac{1}{2}Q$ and $P+\frac{1}{2}Q$, respectively. In addition to M_{10}, there will also be a $M_{01}{}^{\mathrm{D}}$, the latter being the matrix element for a final deuteron.

The impulse approximation gives π-meson producing amplitudes from collision with one of the deuteron nucleons leaving the other unaffected and these amplitudes will be of the type [compare the papers by Ruderman[7] and Bludman[8]]:

$$f_{TT'}{}^{c}(\theta)=\int\psi_3{}^{c}(rxx)\psi_{\mathrm{D}}(x)\exp[i\mathbf{x}\cdot(\tfrac{1}{2}\mathbf{k}-\tfrac{1}{3}\mathbf{q}^{c})]d\mathbf{x}\times\int M_{TT'}(\mathbf{r}\mathbf{q}^{c})\exp[i\mathbf{r}\cdot(\mathbf{k}-\mathbf{q}^{c}/6)]d\mathbf{r}. \tag{3}$$

Here $\psi_3(rxx)$ is the wave function of the final bound three nucleon system as function of the relative coordinates between the nucleons. $\psi_{\mathrm{D}}(x)$ is the deuteron wave function. As before c is the charge (+ or 0) of the produced pion. Thus $\psi_3{}^{0}=\psi_{\mathrm{He}}$, $\psi_3{}^{+}=\psi_{\mathrm{H}}$. In $\psi_3{}^{c}$ we have put equal the coordinates of the two colliding particles relative to the untouched one. This should be justified as the π-mesons result from close collisions, i.e., $M(\mathbf{r})$ is short ranged. $\mathbf{k}$ is the incident protons momentum.

CORRECTION FOR $\psi_{\mathrm{H}}-\psi_{\mathrm{He}}$ DIFFERENCE

We write the wave function for the triton as

$$\psi_{\mathrm{H}}=N_{\mathrm{H}}e^{-\frac{1}{2}\alpha_{\mathrm{H}}(r_{12}+r_{23}+r_{13})}6^{-\frac{1}{2}}\,\mathrm{Det}|n\alpha,n\beta,p\alpha|, \tag{4}$$

where r_{ij} denotes relative nucleon distances. α and β are ordinary spins; n and p denote neutron and proton, respectively. We put $\alpha_{\mathrm{H}}=0.907\times10^{13}$ cm^{-1}. We expect the main electromagnetic effect in the three-nucleon system to be the p-p Coulomb interaction in He^3 so that we write:

$$\psi_{\mathrm{He}}=[N_{\mathrm{He}}6^{-\frac{1}{2}}\,\mathrm{Det}|p\alpha e^{-\frac{1}{4}\alpha_{\mathrm{He}}(r_{ij}+r_{ik})},p\beta e^{-\frac{1}{4}\alpha_{\mathrm{He}}(r_{ij}+r_{ik})},n\alpha e^{-\frac{1}{4}(2\alpha_{\mathrm{H}}-\alpha_{\mathrm{He}})(r_{ij}+r_{ik})}|+m\phi_{\frac{3}{2}\frac{1}{2}}](1+m^2)^{-\frac{1}{2}}, \tag{5}$$

where the complete wave function is again antisymmetric. $\phi_{\frac{3}{2}\frac{1}{2}}$ is the normalized $T=\frac{3}{2}$ wave function for He^3 now mixed in due to pp Coulomb interaction and m is the admixture. The Eq. (5) gives the wave function for the relative distance between two of the particles with and without Coulomb interaction as $e^{-\frac{1}{2}\alpha_{\mathrm{He}}r_{ij}}$ and $e^{-\frac{1}{2}\alpha_{\mathrm{H}}r_{ij}}$, respectively. To estimate α_{He} we note that the lowest state of the particle in a Coulomb field is described just by the function $e^{-\frac{1}{2}r\alpha}$ with $\alpha=2\mu e^2/\hbar^2$ where μ is the reduced mass. Thus α_{H} corresponds to a charge $\bar{e}^2=\alpha_{\mathrm{H}}\hbar^2/m$; m is the nucleon mass. We get $\bar{e}^2=602\times10^{-20}$ while $e^2=23.04\times10^{-20}$. Thus $\alpha_{\mathrm{He}}=0.872$ and we obtain $N_{\mathrm{H}}^2/N_{\mathrm{He}}^2=1.090$.

To correct for the $\psi_{\mathrm{H}}-\psi_{\mathrm{He}}$ difference we put the wave functions (4) and (5), respectively, into (5). We find that all the $M_{TT'}$ will contribute to the final $T=\frac{1}{2}$ state of the 3-nucleon system. We note, however, that in (3) the correction due to the p-p Coulomb repulsion in He^3 appears only in the integral over $\mathbf{x}$ except when M_{11} is involved. This matrix element implies for π^0 production a $pp\rightarrow pp\pi^0$ reaction and the correction will in this case appear in the coordinate $\mathbf{r}$. The element M_{11} should, however, be the least important and we shall neglect it in this treatment.

We now put for $\psi_{\mathrm{D}}(x)$ in (3) the Hulthén wave function:

$$\psi_{\mathrm{D}}(x)=(e^{-\beta x}-e^{-\gamma x})/x,\quad \beta=0.229\times10^{13}\ \mathrm{cm}^{-1},\quad \gamma=1.371\times10^{13}\ \mathrm{cm}^{-1}. \tag{6}$$

We then obtain for the form factor

$$F=\int\psi_3{}^{c}(rxx)\psi_{\mathrm{D}}(x)\exp(i\mathbf{x}\cdot\mathbf{\Delta}^{c})d\mathbf{x}, \tag{7}$$

where $\mathbf{\Delta}^{c}=\frac{1}{2}\mathbf{k}-\frac{1}{3}\mathbf{q}^{c}$, the expression

$$F=\left(\frac{1}{(\beta+\alpha^{c})^2+\Delta^{c2}}-\frac{1}{(\gamma+\alpha^{c})^2+\Delta^{c2}}\right)N_{c}, \tag{7a}$$

with $\alpha^{+}=\alpha_{\mathrm{H}}$, $\alpha^{0}=(\alpha_{\mathrm{H}}+\alpha_{\mathrm{He}})/2$, $N^{+}=N_{\mathrm{H}}$, $N^{0}=N_{\mathrm{He}}$.

[5] G. F. Chew, Phys. Rev. **80**, 196 (1950).
[6] S. Mandelstam, Proc. Roy. Soc. (London) **A244**, 491 (1958).
[7] M. Ruderman, Phys. Rev. **87**, 383 (1952).
[8] S. A. Bludman, Phys. Rev. **94**, 1722 (1954).

TABLE I. Correction for difference between hydrogen-3 and helium-3 wave functions.

θ_{lab}^0	14		11.3		8.6	
$\theta_{c.m.}^0$	66	144	51	153	38	160
Correction %	8.3	6.4	7.9	6.2	8.1	5.9

We then obtain the correction to the ratio $\sigma_{H^3}/\sigma_{He^3}$, in percent as a function of scattering angle of the H^3 or He^3 system. (See Table I.) The energy of the incident proton is 600 Mev (lab).

We have also to study the effect of the $T=\frac{3}{2}$ state admixture in He^3. If we call $\sigma_{\frac{1}{2}}$ the cross section for $T=\frac{1}{2}$ production of He and $\sigma_{\frac{3}{2}}$ the $T=\frac{3}{2}$ production the ratio due to the admixture alone would be:

$$\frac{2\sigma_{\frac{1}{2}}(1+m^2)}{\sigma_{\frac{1}{2}}+m^2\sigma_{\frac{3}{2}}}, \tag{8}$$

where m is the admixture [Eq. (5)]. Isotopic spin impurities have been studied by MacDonald.[9] A crude (over) estimate of the admixture we get from

$$m^2\leq(E_0-E_1)^2\langle\psi_{\frac{1}{2}}|\mathcal{C}^2|\psi_{\frac{1}{2}}\rangle, \tag{9}$$

where $\mathcal{C}$ is the part of the Hamiltonian that gives $\Delta T=1$ transitions. Effectively $\mathcal{C}=e^2/2r_{ij}$. Putting $\psi_{\frac{1}{2}}=\psi_{He}$ ($\alpha_{He}=\alpha_H$). We obtain

$$m^2\leq 0.4/(E_0-E_1)_{Mev}{}^2=6\times10^{-3}, \tag{9a}$$

with $E_0-E_1=8$ Mev which is the binding energy of helium. We expect $\sigma_{\frac{3}{2}}\ll\sigma_{\frac{1}{2}}$ as in our impulse approximation only the M_{T1} can contribute to a final $T=\frac{3}{2}$ state for the 3-nucleon system. We get an overestimate of the correction by putting $\sigma_{\frac{3}{2}}=0$ and we thus obtain a correction $<0.6\%$ to the ratio. For $\sigma_{\frac{3}{2}}=\sigma_{\frac{1}{2}}$ there would be no correction. We will not include this correction in our final answer.

We have previously found that the correction due to $\psi_{He}-\psi_H$ difference is important. We know, however, that the wave functions (4) or (5) are only approximate and wish to investigate how sensitive our results are to the assumptions about these wave functions.

Bransden, Robertson, and Swan[10] have written

$$\psi_{He}=N_{He}\exp[-(\nu/2)\sum_{ij}r_{ij}{}^2]$$
$$\psi_H=N_H\exp[-(\lambda/2)\sum_{ij}r_{ij}{}^2], \tag{10}$$

and have determined ν and λ from minimizing the energy including the appropriate p-p Coulomb interaction to obtain

$$\nu=0.1404\times10^{26}\text{ cm}^{-2}\quad\lambda=0.1436\times10^{26}\text{ cm}^{-2}.$$

This gives $N_H{}^2/N_{He}{}^2=(\lambda/\nu)^3=1.069$ which is to be compared with our previous value 1.090.[11]

A severe drawback of our functions (4) and (5) is the neglect of the (by now) well justified hard core in the nucleon-nucleon interaction. We have estimated the effect of the hard core on our correction N_H/N_{He} by calculating this ratio with:

$$\psi(r_{ij})=0\quad r_{ij}<r_0=0.55\times10^{-13}\text{ cm},$$
$$\psi(r_{ij})=Ne^{-\frac{1}{2}\alpha(r_{12}+r_{23}+r_{13})}, \tag{11}$$

with $\alpha=0.907\times10^{13}$ cm^{-1} and $\alpha=0.890\times10^{13}$ cm^{-1} to obtain

$$N(\alpha=0.907)/N(\alpha=0.890)=1.066,$$

while without the hard core

$$N(\alpha=0.907)/N(\alpha=0.890)=1.057.$$

We thus conclude that the hard core tends to increase the correction due to normalization by around 1%.

We have also estimated the effect of the hard core on the ratio of the form factors (7) but found that the only effect came from the normalization constants N_c.

CORRECTION DUE TO $q_{\pi^+}-q_{\pi^0}$ DIFFERENCE

We have previously stated as a natural assumption that the matrix element is a function of q and that this dependence can be measured experimentally. The cross section for the reaction (1a) was measured at 590 and 600 Mev of the incident proton in laboratory system and the result was $\sigma(590):\sigma(600)=1.063\pm0.036$.[12] Correcting for the difference in phase space and incident proton velocity at these two energies we obtain

$$\Delta T^2/T^2=0.0358^{+0.0130}_{-0.0147}, \tag{12}$$

and there will thus be a correction of 3.58% to the ratio due to difference in meson momenta.

We could correct in a similar way for the fact that the mesons come out at slightly different c.m. angles. It was, however, not found practical to measure the angular dependence of the cross sections, so we have to rely upon our model to make this correction. We calculate the difference in c.m. $\Delta\theta=\theta_{H^3}-\theta_{He^3}$ and then obtain the correction in the form factor. These corrections are given in Table II. There is also an angular dependence in the matrix elements $M_{TT'}$. From Puppi,[13] we obtain

[9] W. M. MacDonald, Phys. Rev. **101**, 271 (1956).

[10] B. H. Bransden, H. H. Robertson, and P. Swan, Proc. Phys. Soc. (London) **A69**, 877 (1956).

[11] Considering the difference in methods we think that the agreement is satisfactory. At the same time, however, we point out that the value $N_H{}^2/N_{He}{}^2=1.090$, which is used in our calculations, is an estimate only. A more convincing way of calculation would have been to use a variation method on the functions (4) and (5) as was used for the functions (10). Unfortunately, the functions (10) do not lead to explicit expressions for the form factor (F).

[12] The author wishes to thank Dr. D Harting, Dr. J. C. Kluyver, Dr. A. Kusumegi, Dr. R. Rigopoulos, Dr. A. M. Sachs, Dr. G. Tibell, Dr. G. Vanderhaeghe, and Dr. G. Weber for obtaining these unpublished results.

[13] G. Puppi, *1958 Annual International Conference on High-Energy Physics at* CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

TABLE II. Correction for difference between center-of-mass angles for charged and uncharged pions as obtained from the form factor (7).

$\theta_{\text{lab}}{}^0$	14		11.3		8.6	
$\theta_{\text{c.m.}}{}^0$	66	144	51	153	38	160
$\Delta\theta$(rad) 10^4	84	−84	54	−54	37	−37
Correction %	+0.86	−0.88	+0.44	−0.46	+0.22	−0.24

$|M_{10}{}^{\text{D}}|^2 \sim 0.5+\cos^2\theta$, while the other matrix elements are more isotropic. Table III gives the correction to be expected from $|M_{10}{}^{\text{D}}|^2$ and the total angular correction, i.e., the correction from Table II added.

Experimentally,[14] however, it is known that the angular distribution is flat for the small c.m. angles.[15] (This may be due to a hard core[8] and other corrections such as the other $M_{TT'}$ neglected in this estimate of angular dependence.) We thus neglect the angular correction for the small c.m. angles.

To obtain the corrections in this section we relied upon the assumption that the matrix element is a function of pion momentum. Another sensible guess would be that it is a function of energy (or of both energy and momentum). The question cannot be directly answered by experiments but one has to rely upon a theory of pion-production which is, however, as yet incomplete. If, however, we believe that the variation of the matrix element is due to the final state pion-nucleon interaction which is not too strong, the the reaction should be momentum dependent. If, on the other hand, the interaction is very strong, then the theory of resonances should be applicable and an energy dependent matrix element would seem more natural.[16] We can only conclude that the question cannot as yet be satisfactorily answered. We note that the same difficulty appears in any test of charge independence involving charged and uncharged pions.

CORRECTION DUE TO $\pi^+-\pi^0$ CHARGE DIFFERENCE

We also have to estimate the effect of having in one final state a charged pion (leaving a triton) but in the other an uncharged pion (leaving helium). In this discussion we still keep to the pure impulse approximation, i.e., neglect multiple scatterings which will be discussed in the next section. We just wish to investigate the case when the π^0 leaves the two colliding nucleons in free space but the π^+ leaves them in a Coulomb field of these two nucleons. To study this effect we consider the pion-nucleon interaction. As is well known, this is mainly a p-state interaction. From a knowledge of the p-wave phase shift the wave function is known up to the region of interaction. If we believe that the Coulomb interaction is negligible within the region of nuclear interaction,[17] we then know the wave function in this region in the charged case also except for its amplitude, which is obtained from fitting to the Coulomb wave function outside this region.

From Brueckner,[18] we obtain for a 220-Mev lab energy $\delta_{33}=105°$. The wave function in the outer region is given by

$$\varphi(kr)=G(kr)+\cot\delta F(kr),$$

where G and F are the irregular and regular wave functions for the charged or uncharged case. The logarithmic derivative is

$$f=\frac{G'(kr)+\cot\delta F'(kr)}{G(kr)+\cot\delta F(kr)}. \tag{13}$$

With $r=1.4\times10^{-13}$ cm, $k=1.7\times10^{13}$ cm^{-1}, we get $f=-11.18$. In terms of f we get

$$\varphi(kr)=1/[F'(kr)-fF(kr)], \tag{14}$$

from which we calculate the ratio

$$R=\frac{\varphi_{\text{coul}}}{\varphi_{\text{neut}}}, \tag{15}$$

with f the same in the charged and uncharged case as stated above. With $e^2/\hbar v=0.007$ we obtain $R=1.005$. Thus the amplitude for the case when the positive pion is produced and interacts with the proton in the triton nucleus is 1.005 times the amplitude when produced at a neutron. The charge effect will be largest when the two colliding nucleons are left in a singlet state, whence the amplitude will be $a-\frac{1}{3}a'$ if the pion-nucleon has a $T=\frac{3}{2}$ interaction,[6] where a and a' are the amplitudes from production of each of the nucleons. As $a\approx a'$, there would be at most a 1.6% correction in the squared amplitude if $a=1.005a'$. This correction will, however, be partly cancelled by the $a'-\frac{1}{3}a$ amplitude as well as the triplet amplitudes $a+\frac{1}{3}a'$. There may, of course, be some important interference effects but these are practically impossible to handle. We will neglect this Coulomb effect in our final estimate. We only note that

TABLE III. Total correction for difference between center-of-mass angles for charged and uncharged pions.

$\theta_{\text{lab}}{}^0$	14		11.3		8.6	
$\theta_{\text{c.m.}}{}^0$	66	144	51	153	38	160
Correction %	−0.94	−0.69	−0.59	−0.33	−0.32	−0.17
Total angular correction %	−0.08	−1.57	−0.15	−0.79	−0.10	−0.41

[14] W. J. Frank, K. C. Bandtel, R. Madey, and B. J. Moyer, Phys. Rev. 94, 1716 (1954).
[15] The quoted experiment involves 340 Mev while we discuss 600 Mev incident protons. Roughly the same angular pattern is expected.
[16] The difficulties indicated in this section were brought out in a discussion with Professor R. G. Sachs and Professor Y. Yamaguchi.
[17] L. Van Hove, Phys. Rev. 88, 1358 (1952).
[18] K. Brueckner, Phys. Rev. 86, 106 (1952).

the effect will be to increase π^+ relative to π^0 production.[19]

CORRECTION FOR DIFFERENT PHASE SPACES

Due to the $\pi^+-\pi^0$ and He^3-H^3 mass differences the phase space factors will be different for the two reactions. The relativistic phase space factors will be:

$$P_{\pi^+}=\frac{q_{\pi^+}E_{H^3}E_{\pi^+}}{E_{\pi^+}+E_{H^3}}=540.0,$$
$$P_{\pi^0}=\frac{q_{\pi^0}E_{He}E_{\pi^0}}{E_{\pi^0}+E_{He}}=543.9. \tag{16}$$

Thus the correction is 0.72%.

CORRECTIONS TO THE IMPULSE APPROXIMATION

In order to obtain the previous corrections we used as a model the impulse approximation. There are thus two essential approximations involved. First, there is the use of matrix elements $M_{TT'}(2N\rightarrow 2N\pi)$ off the energy shells of the involved reactions although experimentally they are only known *on* the energy shell. However, it is fairly well established that the reactions $2N\rightarrow 2N\pi$ depend mainly on the final state, so we do not expect any severe errors to be brought in here. It is actually only in the angular correction that we considered the $M_{TT'}$ and we did not find this correction important.

Secondly, there is the effect of multiple scattering. In general, we expect this effect to decrease with increasing energy and be small for a 600-Mev proton incident on deuterium or a 200-Mev pion emerging from H^3 or He^3. Also, the pion scattering on the two colliding particles is already included by use of the appropriate $M_{TT'}$. We could correct for the distortion of the incident proton by putting, instead of initial plane wave with sharp momentum k, a wave with some spread Δk and similarly in the final state. Then we would have to integrate over these Δk and Δq. The correction would thus be an averaging effect and should be small.

These corrections to the impulse approximations will, however, only give corrections to our previous corrections, as with the electromagnetic effects neglected, but the more exact treatment we do get the ratio $\sigma_{\pi^+}/\sigma_{\pi^0}=2/1$.

Now there is however also the possibility of electromagnetic scatterings where isospin is not conserved. Thus the initial state which is mainly a $T=\frac{1}{2}$ state can produce a $T=\frac{3}{2}$ state through an electromagnetic scattering. As a total $T=\frac{3}{2}$ state gives a ratio $\sigma_{\pi^+}/\sigma_{\pi^0}=\frac{1}{2}$ there will be a negative correction to the uncorrected ratio 2.[20] We can imagine that the incident proton wave function around one nucleon in the deuteron is admixed with some $T=\frac{3}{2}$ state in addition to $T=\frac{1}{2}$, and that the $T=\frac{3}{2}$ state results from inelastic, electromagnetic scatterings from the other nucleon in deuteron. As there is thus involved a double scattering, an inelastic scattering, and an electromagnetic scattering, all of which tend to be small, we argue that the involved correction is negligible. The same correction would also appear in the final state.

We also point out another correction which we have neglected, and which is also of electromagnetic and nonisospin conserving type. The incident proton emits virtual photons which can produce pions by interaction with deuteron.[21] However, this correction is apt to be small.

TABLE IV. Ratio of cross sections for charged and uncharged pions (measured at the same laboratory angle). The electromagnetic corrections included are referred to in the text.

Pion c.m. angle	114°	36°	129°	27°	142°	20°
Ratio R	2.22	2.15	2.21	2.16	2.22	2.17

SUMMARY AND DISCUSSION OF CORRECTIONS

We have thus found that the Coulomb corrections are of the order of several percent and not negligible. We found the largest correction to be due to the difference between H^3 and He^3 wave functions and especially due to the normalization of these wave functions. We did not find it possible to correct unambiguously for the fact that the mesons, due to different masses, come out with different momenta. It seems that the pion production has to be better understood theoretically in order to do such a correction. We have not tried to correct for the $T=\frac{3}{2}$ admixture in an intermediate state, or for the virtual photoproduction of mesons.

Adding together the corrections from Tables I and III (the corrections for the small c.m. angles put equal to zero as discussed) and from (12) and (16), we obtain for the ratio of $\sigma_{\pi^+}/\sigma_{\pi^0}$ as a function of pion c.m. angle (shown in Table IV).

Due to the lack of a detailed theory of pion production, which leads to an uncertainty in corrections such as the momentum correction, and due to the neglected corrections we expect the error in the result to be ±3%.

Experimentally[4] R was measured at 129° to be $R=2.26\pm0.11$ which is thus in agreement with our estimated value $R=2.21$.

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[19] We expect a still smaller correction in collisions for which the proton in the triton is the spectator nucleon and the two colliding nucleons are neutrons in the final state. Thus, this correction is neglected.

[20] The author wishes to thank Professor R. G. Sachs for illuminating discussions of this point.

[21] The author wishes to thank Professor E. M. Henley for illuminating discussions of this point.

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Inelastic Scattering of High-Energy Protons Exciting a Collective Level of Nucleus

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An analysis is made of the angular distribution and polarization of 185-Mev protons inelastically scattered by carbon. A rough calculation using the distorted wave Born approximation shows, even though the quantitative agreement is poor, that the 4.4-Mev level of C^{12} may be interpreted as a collective state.

I. INTRODUCTION

In analyzing the polarization of high-energy nucleons elastically scattered by nuclei, it is customary to use an optical model potential which has a spin-orbit interaction term proportional to the gradient of the spin independent part. Such a potential may be written as

$$V=V_C\rho(\mathbf{r})+V_S(\hbar/\mu c)^2\frac{1}{r}\frac{d\rho}{dr}\boldsymbol{\sigma}\cdot\mathbf{l}, \tag{1}$$

where $\hbar\mathbf{l}$ is the nuclear angular momentum operator and ρ contains the spatial dependence of the potential. The purpose of this paper is to extend the use of this potential to the treatment of inelastic scattering in terms of the nuclear collective model.

Ruderman,[1] and independently Maris,[2] proved that the treatment of the inelastic scattering in terms of the collective model in the Born approximation gives the same explicit expressions for both elastic and inelastic polarization. But the agreement of the Born approximation calculation with the experimental results is known to be only qualitative. The disagreement becomes significantly large in the neighborhood of the diffraction minimum. An improvement over the Born approximation is achieved by taking into account the distortion of the wave functions. Since the low excited states, as well as the ground states, of many nuclei are well described by the independent particle model, many authors have studied the inelastic scattering in terms of this model.[3] On the other hand, the collective model has been equally successful in bringing out certain characteristics of nuclei and there have also been attempts to interpret the inelastic scattering along this line.[4,5] In reference 5 (further referred to as I) the 96-Mev proton data of Strauch and Titus[6] were analysed. Since no polarization effect was measured in this experiment, the analysis was made, for the sake of simplicity, without including the spin-orbit interaction term of (1). In order to bring out the collective nature of the nucleus, the nuclear boundary was taken as a spheroid

$$r=R[1+\beta Y_2^0(\Omega')], \tag{2}$$

where Ω' is measured from a principal axis of the spheroid which, in turn, makes an angle Ω_0 in the space-fixed coordinate system. In this paper we follow the method of I but include the spin-orbit term of (1).

The nuclear wave functions are simply $Y_0^0(\Omega_0)$ and $Y_2^m(\Omega_0)$ for a 0+ state and a 2+ state, respectively. The nucleon wave functions are constructed by taking the distortion of the waves into account. For high-energy incident nucleons the fractional change of the wave number is negligible and hence the incident beam

[1] M. Ruderman, Phys. Rev. **98**, 267 (1955).
[2] Th. A. J. Maris, Nuclear Phys. **3**, 213 (1957).
[3] P. Benoist, C. Marty, and Ph. Meyer, Physica **22**, 1173 (1956); C. A. Levinson and M. K. Banerjee, Ann. Phys. **2**, 471 (1957); H. A. Bethe, Ann. Phys. **3**, 190 (1958); E. J. Squires, Nuclear Phys. **6**, 504 (1958).
[4] H. S. Köhler, Nuclear Phys. **9**, 49 (1958/59).
[5] K. Nishimura, Nuclear Phys. **7**, 425 (1958).
[6] K. Strauch and F. Titus, Phys. Rev. **103**, 200 (1956).