Analysis of (d,t) Pick-Up Reactions*

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A comparison is made between (p,d) [or (d,p)] and (d,t) pick-up reactions involving transitions between the same nuclear levels. Eleven cases are studied, having transfer orbital angular momentum $l=0$, 1 or 2, for nuclei from Li^6 to Mg^{25} (also Sn^{117}), and for incident energies of the order of 15 Mev. It is found that if the differential cross sections of the corresponding (d,p) and (d,t) reactions are plotted as functions of momentum transfer the curves differ by a factor independent of angle. This property holds primarily in the region of the first peak of the angular distribution. Towards larger angles the curves differ in shape. Because of the proportionality between the curves in the forward direction, it is possible to obtain an expression for extracting the stripping reduced width of (d,t) reactions. This reduced width corresponds to the reduced width of the same transition when studied by a (d,p) process. No emphasis is placed on the interpretation of the results in terms of the structure of the triton.

An attempt is made to determine the triton momentum transform directly from an analysis of the $d+d \rightarrow p+t$ experiments, considering these as stripping reactions. It was not possible to apply the curves thus obtained to the (d,t) reactions in the heavier nuclei.

The experiment $F^{19}(d,t)F^{18}$ ground state was performed with 14.8-Mev deuterons, for angles θ_{lab} between 5° and 45°. The results extend the information about transitions with $l=0$.

I. INTRODUCTION

THIS work is a survey and analysis of certain reactions of the type $X(d,t)Y$. An attempt is made to gain insight into the mechanism of the reactions and to assess their value for nuclear spectroscopy.

The mechanism of the transfer of the neutron in (d,t) or (t,d) reactions is usually thought to be the same as that in the corresponding (p,d) and (d,p) reactions. Theories have been given[1] which describe the triton reactions by the same formulas used for deuteron stripping, taking into account the proper kinematic modifications. It remains to be seen how nearly correct these theories are.

The theory of deuteron stripping is normally set up in plane wave Born approximation,[2] and this gives a good fit to experiment, especially for the forward part of the angular distribution, and for the range of energies usually studied, 8–20 Mev, when low-lying states of light nuclei are formed. In Born approximation the cross section can be written as a product of three factors: a factor which describes the separation of a neutron from the projectile, a factor which describes the penetration of the neutron to the surface of the target nucleus, and a factor (the reduced width) which describes the probability that the target nucleus can capture the neutron. When the same Born approximation is used for triton stripping only the first of these factors is different. The probability of separating a neutron from a triton is not the same as that of separating a neutron from a deuteron. If the formulas are expressed in suitable terms the ratio of these two probabilities is a constant factor, independent of angle, and the value of this factor should not depend on the target nucleus which is considered. These predictions of the Born approximation theory are subject to experimental test, namely: (1) that the cross sections of the corresponding (d,t) and (p,d) reactions are, to within a constant factor, the same function of the momentum transfer; (2) that the ratio of amplitudes of corresponding reactions is independent of the target.

Detailed theories of the triton structure, such as that of Irving,[3] or that of Pease and Feshbach,[4] predict the value of the factor in question. However these theories use wave functions which are reasonable approximations only for small distances between the nucleons, and do not have the correct asymptotic form. For that reason a simple asymptotic approximation of the triton wave function is used throughout the present work. Although a comparison with the more fundamental theories is also made (Sec. II), no emphasis is placed upon interpreting the numbers obtained here as giving fundamental information about the structure of the triton. The deficiencies of the Born approximation theory (see below) make such an interpretation seem pointless.

Using the asymptotic approximation, the asymptotic normalization factor B^2 [for the definition of B^2 see Eq. (3)] is determined by the comparison of the (d,t) experiments with the corresponding (d,p) reactions involving the same levels. This procedure has been previously followed in some cases.[5,6]

The greater number of (d,t) reactions recently available allowed a more systematic evaluation of B^2 to be made in the present study. Various cases for transfer angular momentum $l=0$, 1 and 2 for nuclei from Li^6 to Mg^{25} (also a case for Sn^{117}) are considered in Sec. III. It is plausible, from the results, to establish a numerical value for B^2 which is constant for all (d,t) reactions studied. This permits the use of (d,t) reactions to determine the reduced width of a transition. The

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[1] S. T. Butler and E. E. Salpeter, Phys. Rev. **88**, 133 (1952); and M. C. Newns, Proc. Phys. Soc. (London) **A65**, 916 (1952).
[2] P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1952).
[3] J. Irving, Phil. Mag. **42**, 338 (1951).
[4] R. L. Pease and H. Feshbach, Phys. Rev. **88**, 945 (1952).
[5] A. Werner, Nuclear Phys. **1**, 9 (1956). It should be pointed out that this paper contains several mistakes.
[6] M. El Nadi and L. Abou Hadid, Nuclear Phys. **8**, 51 (1958).

knowledge of the stripping reduced widths for (d,p) and (d,n) reactions has been very useful in the determination of nuclear spectroscopic and structural properties.[7,8]

Beyond the first peak the simple Born approximation theory does not give a good fit to (d,p) and (d,t) angular distributions. There is considerable evidence that (d,p) reactions are heavily influenced by distorted waves corrections.[9,10] Distorted waves effects are likely to be altogether different for (d,t) reactions. In addition the triton is a more compact structure than the deuteron, so the asymptotic wave function should not be a good approximation over a large region of this nucleus, where every particle interacts with the others most of the time. For these reasons it is interesting to make the comparison between (d,t) and (p,d) reactions also outside the region of good agreement with the crude theory of stripping. Preliminary information about some of the finer details of the reaction mechanism seems to appear in some cases (Sec. III).

Section IV contains a discussion of the $d+d\rightarrow p+t$ reaction, treated as proceeding by a stripping mechanism. The triton transform [Eq. (7)] is obtained directly from the experimental results and is compared with the one used in the (d,t) reactions with the heavier nuclei.

In the Appendix the experimental data for the reaction $F^{19}(d,t)F^{18}$ g.s. with 14.8-Mev deuterons is presented.

II. STRIPPING FORMULAS

The differential cross section for a reaction $A(a,a-1)A+1$, i.e., the stripping of the particle of mass a by the nucleus of mass A, as calculated by the simple Born approximation is:

$$\frac{d\sigma_{a,a-1}(\theta)}{d\Omega}=\frac{1}{(2\pi\hbar^2)^2}\frac{k_1}{k_0}\frac{a(a-1)A(A+1)}{(A+a)^2}\times\frac{1}{(2i_0+1)(2J_0+1)}\sum_{m_1M_1}\sum_{m_0M_0}|\langle\mathfrak{M}\rangle|^2. \quad (1a)$$

If antisymmetrized wave functions are used, the sum can be expressed as: (see references 7 and 8)

$$\sum_{m_1M_1}\sum_{m_0M_0}|\langle\mathfrak{M}\rangle|^2=\frac{a}{2}\frac{\hbar^4}{2n+1}\frac{2i_0+1}{2l+1}(2J_1+1)^2[C]^2\mathcal{S}(l)\left(\frac{A+1}{2A}\right)^2\times(q^2+K^2)^2\times[I(k)]^2\times[J(q)]^2, \quad (1b)$$

where a is restricted to the values 2, 3, and 4,

$$I(k)=\int\phi_a^*(\varrho,\mathbf{x})\phi_{a-1}(\varrho)e^{i\mathbf{k}\cdot\mathbf{x}}d\varrho d\mathbf{x}, \quad (1c)$$

and

$$J(q)=\int\phi_{A+1}^*(\xi,\mathbf{r})\phi_A(\xi)e^{i\mathbf{q}\cdot\mathbf{r}}d\xi d\mathbf{r}. \quad (1d)$$

For the inverse reaction

$$\frac{d\sigma_{a-1,a}(\theta)}{d\Omega}=\frac{2J_0+1}{2J_1+1}\frac{2i_0+1}{2i_1+1}\left(\frac{k_0}{k_1}\right)^2\frac{d\sigma_{a,a-1}(\theta)}{d\Omega}. \quad (1e)$$

In the above equations: i_0, i_1 are the spins of the nuclei a, $a-1$, with z components m_0 and m_1, J_0, J_1 are the spins of the nuclei A, $A+1$, with z components M_0 and M_1, $\mathbf{k}_0$, $\mathbf{k}_1$ are the relative momenta of the systems $A+a$, $(A+1)+(a-1)$, n is the spin of the transferred particle, l is the angular momentum transfer, and $[C]^2$ is the isotopic spin coupling factor, i.e., the square of the Clebsch-Gordan coefficient: $C(T_0,\frac{1}{2},T_1;M_{T0},M_{T1}-M_{T0})$. $\mathcal{S}(l)$ is called the "spectroscopic factor," and is essentially the coefficient of fractional parentage connecting the nucleus $A+1$ with the nucleus A.

$$\mathcal{S}(l)=(A+1)\sum_j\langle A+1,J_1,T_1|A,J_0,T_0\rangle_{l,j}^2$$

(see references 7 and 8). T_0, T_1 are the isotopic spins of nuclei A, $A+1$, with z components M_{T0} and M_{T1}. $\mathbf{q}$ is the transfer momentum, $\mathbf{q}=\mathbf{k}_0-[A/(A+1)]\mathbf{k}_1$, K is the wave number of the transferred nucleon in the nucleus $A+1$, $\mathbf{k}$ is the wave vector of the transferred nucleon in the nucleus a; $\mathbf{k}=\mathbf{k}_1-[(a-1)/a]\mathbf{k}_0$; ϱ, $\mathbf{x}$, ξ, and $\mathbf{r}$ are the spatial coordinates in the indicated nuclei.

The integral $J(q)$, Eq. (1d), is treated in exactly the same way in (d,p) and (d,t) reactions, the spatial wave function of the nucleus $A+1$ being written as the product wave function for a neutron of angular momentum l and a core, the nucleus A. The integration is limited to the region outside a cutoff radius r_0, and in this region the neutron is taken as a free particle, its radial wave function $R_l(r)$ being approximated by a Hankel function. The integration can be expressed as follows:

$$\left(\int_{r_0}^{\infty}R_l(r)Y_l^m(\theta,\varphi)\phi_A^*(\xi)\phi_A(\xi)d\xi d\mathbf{r}\right)^2=4\pi(2l+1)\frac{3\theta_0^2}{r_0}\frac{1}{(q^2+K^2)^2}\left(y\frac{j_l(x)}{h_l^{(1)}(iy)}\frac{dh_l^{(1)}(iy)}{dy}-x\frac{dj_l(x)}{dx}\right)^2. \quad (2)$$

In this equation $\theta_0^2=\frac{1}{3}r_0^3R_l^2(r_0)$ is the dimensionless single-particle reduced width. For an actual transition between a state of nucleus A and a state of nucleus

[7] J. B. French, *Nuclear Spectroscopy*, edited by F. Ajzenberg-Selove [Academic Press, Inc., New York (to be published)].
[8] M. H. Macfarlane and J. B. French (to be published).
[9] W. Tobocman, Phys. Rev. **115**, 98 (1959).
[10] J. P. Martin, Ph.D. thesis, University of Pittsburgh, 1959 (unpublished).

$A+1$, the reduced width θ^2 is $\mathbb{S}(l)\theta_0^2$. This reduced width is assumed to be the same whether the reaction is (d,p) or (t,d). In the Wronskian above $j_l(x)$ and $h_l^{(1)}(iy)$ are spherical Bessel and Hankel functions of the first kind[11]; $x=qr_0$ and $y=Kr_0$.

The other integral, $I(k)$, is in general calculated with specific wave functions for the particles involved. The integral usually is carried throughout all space, and the result of the integration is called the "form factor."

For (d,p) reactions the Hulthén wave function for the deuteron is used in the calculation of $I(k)$, as usually done. Almost the same angular distribution would be obtained if only the first term of the Hulthén wave function were used, the simple asymptotic form $e^{-\alpha r}/r$. (The binding energy of the deuteron is given by $\mathcal{E}=\hbar^2\alpha^2/m$.) The correction in the Hulthén function which is due to the finite range of the interaction potential only adds to the form factor a term slowly varying with angle.

For (d,t) reactions there is no generally accepted wave function available for the triton, comparable with the Hulthén function used above for the deuteron. The calculation of $I(k)$ therefore is performed in the present work using three different reasonable wave functions: (a) the asymptotic approximation:

$$\phi_t(\xi,\mathbf{r})=\frac{B}{(4\pi)^{\frac{1}{2}}}\frac{e^{-\gamma r}}{r}\phi_d(\xi), \tag{3}$$

where B is the normalization factor; $\gamma^2=(4m/3\hbar^2)$ (B.E.), m is the nucleon mass, B.E. the binding energy of the last neutron in the triton, 6.2 Mev; $\phi_d(\xi)$ is the normalized deuteron wave function; $\boldsymbol{\xi}=\mathbf{r}_1-\mathbf{r}_2$; $\mathbf{r}=\mathbf{r}_3-\frac{1}{2}(\mathbf{r}_1+\mathbf{r}_2)$. (b) the Irving[3] wave function:

$$\phi_t(\xi,\mathbf{r})=N\frac{\exp[-\eta(\frac{3}{4}\xi^2+r^2)^{\frac{1}{2}}]}{(\frac{3}{4}\xi^2+r^2)^{\frac{1}{2}}}, \tag{4}$$

where N is the normalization factor, $\eta^{-1}=0.93$ fermi. (c) the Pease and Feshbach[4] wave function (only the S part):

$$\phi_t(\mathbf{x},\mathbf{y},\boldsymbol{\xi})=N'\{A_1\exp[-\tfrac{1}{2}\lambda_1(x+y+\xi)]+A_2\exp[-\tfrac{1}{2}\lambda_2(x+y+\xi)]\}, \tag{5}$$

where N' is the normalization factor; $\mathbf{x}=\mathbf{r}_3-\mathbf{r}_2$; $\mathbf{y}=\mathbf{r}_3-\mathbf{r}_1$; $A_1=1.08r_c^{-\frac{3}{2}}$; $A_2=3.01r_c^{-\frac{3}{2}}$; $\lambda_1=0.9r_c^{-1}$; $\lambda_2=1.8r_c^{-1}$; $r_c=1.184$ fermis.

Using the asymptotic approximation, which is the simplest form, $I(k)$ becomes

$$I(k)=(4\pi)^{\frac{1}{2}}B/(k^2+\gamma^2).$$

Because by the conservation of energy

$$k^2+\gamma^2=\tfrac{2}{3}[(A+1)/A](q^2+K^2),$$

the form factor for (d,t) reactions would be the same as that for (d,p) reactions involving the same transition if the simple asymptotic form also were taken for the deuteron. This fact supports the idea of comparing (d,t) and (d,p) cross sections as function of momentum transfer $\mathbf{q}$.

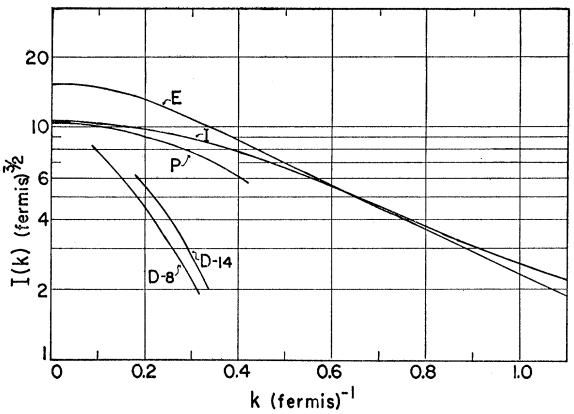

FIG. 1. The triton transform $I(k)$ calculated with different triton wave functions: curve E corresponds to the asymptotic approximation with the normalization factor determined by experiment; curve I to the wave function of Irving; curve P to the wave function of Pease and Feshbach. Curves D are $I(k)$ given directly from the $d+d\rightarrow p+t$ reactions: D-8 with 8.1-Mev incident deuterons, D-14 with 13.8-Mev incident deuterons.

Figure 1 shows a plot of $I(k)$ calculated,[7,12] with forms 3, 4, and 5 for the triton and a Hulthén form for the deuteron wave function. The value of B in form 3 is deduced from experiment (see Sec. III). The range of k involved in the (d,t) reactions studied is from $\sim$0.2 to $\sim$0.75 fermi^{-1} for the forward direction. Within this region, it is interesting to note that form 3, a simple exponential given by the binding energy, and form 4, the wave function of Irving, give approximately the same variation with k. The fit with experiment is, therefore, of the same quality, and it is found to be good. The integral with the Pease and Feshbach wave function requires extensive calculations, and it was not carried out for values of k greater than $\sim$0.4, which, however, is enough to cover the range of interest for the $d+d\rightarrow p+t$ reactions (see Sec. IV).

III. COMPARISON BETWEEN (p,d) AND (d,t) REACTIONS INVOLVING THE SAME TRANSITION

The comparison between (p,d) [or (d,p)] and (d,t) reactions involving the same transition has been made. The similarities of the mechanism of the two processes were observed by examining the variation of the differential cross sections with transfer momentum $\mathbf{q}$ and by comparing the absolute magnitude of the cross sections. The comparison also evaluated the possibility of determining the reduced width of a transition by means of a (d,t) reaction.

[11] L. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1955), 2nd ed., p. 77.

[12] The author is very grateful to P. Iano for performing the calculations with the wave function of Pease and Feshbach.

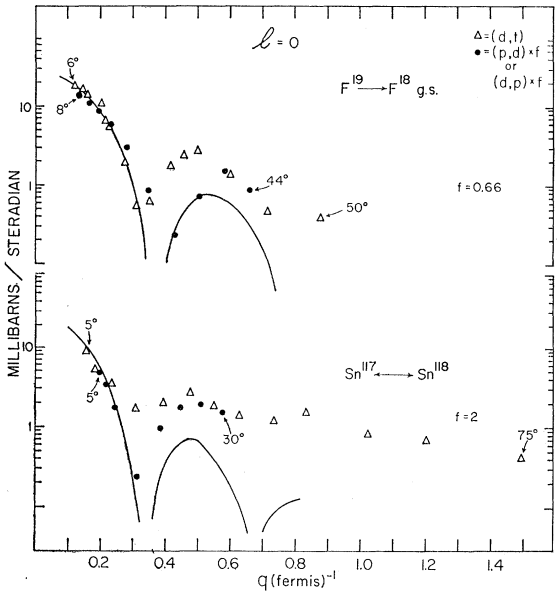

FIG. 2. Comparison between corresponding (p,d) [or (d,p)] and (d,t) reactions, for orbital angular momentum transfer $l=0$. The differential cross sections are shown as function of momentum transfer $\mathbf{q}$. The (d,p) reactions are normalized to the (d,t) cross sections at the region of the first peak of the angular distribution. The actual scattering angles are indicated on each curve for several representative points.

Several cases were studied, for values of $l=0$, 1 and 2. Some of the reactions have been performed at various incident energies; in such cases only the pairs of (d,p) and (d,t) experiments with similar incident energies are presented. In most of the cases studied the incident energies are of the order of 15 Mev; thus, since the Q of the reactions do not differ by much, the differential cross sections extend through the same range of momentum transfer.

Figures 2, 3, and 4 show the experimental differential cross sections plotted against q. The (d,p) reactions are normalized to the (d,t) cross sections at the first peak. The theoretical Butler curves are also shown. In most of the cases it is possible to fit both (d,p) and (d,t) experiments with the same radius, thus the shape of the theoretical curves as functions of q is almost identical. The form factors are slightly different since the Hulthén wave function is used for the deuteron and a simple exponential for the triton. The difference is a factor slowly varying with angle, and whose amplitude may be about 1 or 2 for the cases studied.

Unfortunately the data available do not present angular distributions for angles higher than 90°, and in various (p,d) cases not even for the region of the second peak.

For the range studied it can be said that: (a) in the region of q corresponding to the first peak the curves for (d,p) and (d,t) reactions do differ by a constant factor; (b) the (d,t) reactions, for $l=1$ and 2 have the first valley and the second peak lower than the (d,p) reactions. This is more clearly seen in the carbon cases, Fig. 3, and in the magnesium cases, Fig. 4, where the difference is a factor of the order of 2. The two $l=0$ cases examined do not seem to present the feature described in (b). The different behaviors of (d,p) and (d,t) reactions at large angles may be due to the details of the interaction mechanisms in the pick-up process, or to different contributions from other processes. It would be very interesting to compare the angular distributions at still higher angles.

The comparison of the absolute magnitude of the cross sections was performed with Eqs. (1). For the (d,t) reactions the asymptotic approximation was used, [see Eq. (3)] therefore the fit with experiment gives the

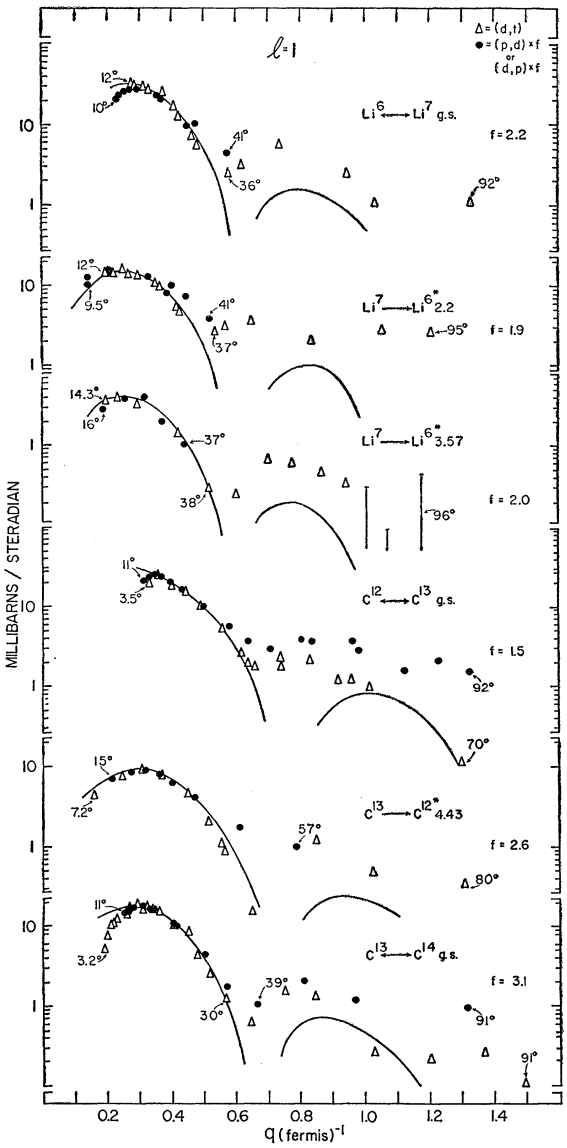

FIG. 3. Comparison between corresponding (p,d) [or (d,p)] and (d,t) reactions for orbital angular momentum transfer $l=1$, as in Fig. 2.

TABLE I. Values of the normalization constant B^2 calculated for various (d,t) reactions.

Reaction	Level (Mev)	E_i (Mev)	Q (Mev)	r_0 (fermis)	$B^2\theta^2$ (fermis)$^{-1}$	θ^2	B^2 (fermis)$^{-1}$	Ref.
				$l=0$				
$F^{19}(d,t)F^{18}$	g.s.	14.8	−4.144	7	0.0062	...	0.40	a
							0.70	
$F^{19}(p,d)F^{18}$	g.s.	18.5	−8.175	6		0.017		b
						0.009		c
$Sn^{118}(d,t)Sn^{117}$	g.s.	14.8	−3.1	8	0.0038	...	0.25	d
$Sn^{117}(d,p)Sn^{118}$	g.s.	14.8	7.2	8		0.015		d
				$l=2$				
$Na^{23}(d,t)Na^{22}$	g.s.	14.8	−6.17	6.5	0.014	...	0.72	e
$Na^{23}(p,d)Na^{22}$	g.s.	18	−10.2	5.3		0.021		b
$Mg^{25}(d,t)Mg^{24}$	g.s.	14.8	−1.07	5.4	0.0062	...	0.73	f
$Mg^{25}(p,d)Mg^{24}$	g.s.	17	5.107	5.2		0.0079		b
$Mg^{24}(d,p)Mg^{25}$	g.s.	14.8		5.0		0.0085		f
$Mg^{25}(d,t)Mg^{24}$	1.37	14.8		6		...	0.8	f
$Mg^{25}(p,d)Mg^{24}$	1.37	17		5.2		0.022		b
				$l=1$				
$Li^7(d,t)Li^6$	g.s.	14.8	−0.988	5.6	0.0628	...	1.20	g
$Li^6(d,p)Li^7$	g.s.	14.8	5.027	5		0.053		g
$Li^7(p,d)Li^6$	g.s.	17.5				0.053		c
$Li^7(d,t)Li^6$	2.2	14.8		6	0.037		1.02	g
							1.37	
$Li^7(p,d)Li^6$	2.2	17.5	−7.227	6		0.036		c
						0.027		h
$Li^7(d,t)Li^6$	3.57	14.8		6.5	0.038	...	1.36	i
$Li^7(p,d)Li^6$	3.57	17		5		0.028		h
$C^{13}(d,t)C^{12}$	g.s.	14.8	1.302	4.6	0.031	...	0.94	j
	g.s.	3.3		5.6	0.032		1.28	k
$C^{12}(d,p)C^{13}$	g.s.	14.8	2.271	4.6		0.033		l
	g.s.	3.3		6.1		0.025		k
$C^{13}(d,t)C^{12}$	4.43	14.8		5	0.023	...	0.76	j
$C^{13}(p,d)C^{12}$	4.43	17	−7.15	5		(0.030)[m]		b
$C^{14}(d,t)C^{13}$	g.s.	14.8	−1.92	5.6	0.048	...	0.73	n
$C^{13}(d,p)C^{14}$	g.s.	14.8	5.94	5.4		0.069		l
$N^{14}(d,t)N^{13}$	g.s.	14.8	−4.286	5.5	(0.023)[o]	...	(0.5)	p
$N^{14}(p,d)N^{13}$	g.s.	18.5	−8.317	5.4		0.046		q,b

[a] Present article—Appendix.
[b] E. F. Bennett, Ph.D. thesis, Princeton University, 1958 (unpublished).
[c] J. B. Reynolds and K. G. Standing, Phys. Rev. **101**, 158 (1956).
[d] B. L. Cohen, J. B. Mead, R. E. Price, K. S. Quisenberry, and C. Martz, Phys. Rev. **118**, 499 (1960).
[e] W. F. Vogelsang and J. N. McGruer, Phys. Rev. **109**, 1663 (1958).
[f] E. W. Hamburger and A. G. Blair, Phys. Rev. (to be published).
[g] S. H. Levine, R. S. Bender, and J. N. McGruer, Phys. Rev. **97**, 1249 (1955).
[h] D. R. Maxon and E. F. Bennett, Phys. Rev. **116**, 131 (1959).
[i] See reference 13.
[j] S. Mayo and A. I. Hamburger, Phys. Rev. **117**, 832 (1960).
[k] H. D. Holmgren, J. M. Blair, B. E. Simmons, T. F. Stratton, and R. V. Stuart, Phys. Rev. **95**, 1544 (1954).
[l] J. N. McGruer, E. K. Warburton, and R. S. Bender, Phys. Rev. **100**, 235 (1955).
[m] Value obtained taking $\theta^2=0.033$ for $C^{13}(p,d)C^{12}$ g.s. at 17 Mev. See reference j.
[n] W. E. Moore, J. N. McGruer, and A. I. Hamburger, Phys. Rev. Letters **1**, 29 (1958).
[o] Value based on a rough fit of two points measured.
[p] E. K. Warburton, Ph.D. thesis, University of Pittsburgh, 1956 (unpublished).
[q] K. G. Standing, Phys. Rev. **101** 152 (1956).

quantity $B^2\theta^2$. The values of θ^2 were taken from the corresponding (d,p) experiments. The values of B^2 thus determined from the cases studied are listed in Table I.

For $l=0$ there are two cases studied, fluorine and tin. The two (p,d) experiments on fluorine, at 18.5 Mev, disagree on the value of the absolute cross section, and thus yield different values of B^2, 0.4 and 0.7 fermi^{-1}. The tin case, a much heavier element than the others studied, gives $B^2=0.25$ (fermi^{-1}). These results appear inconclusive.

For $l=1$ and $l=2$ the variations of the values of B^2 do not seem to depend in any systematic way upon the nucleus considered or upon the state considered. Therefore, an average value $B^2=0.95$ fermi^{-1} is taken for $l=1$, and $B^2=0.73$ fermi^{-1} is taken for $l=2$. These two values are chosen from the cases where the (d,t) and (d,p) reactions were done in the same experimental conditions. The uncertainty in the determination of B^2 in these cases is estimated to be of the order of $\pm 20\%$, coming both from experimental errors and from uncertainty in the fit with theory. For the other cases the uncertainty depends on the errors on the determination of the absolute cross sections. It is in general of the order of $\pm 40\%$.

A few cases of comparison between (d,He^3) and (d,n) reactions are presented in Table II. The value of B^2 for He^3 in the lithium case agrees very well with the one found for the triton.[13] In the boron cases only ratios are

[13] E. W. Hamburger, Ph.D. thesis, University of Pittsburgh, 1959 (unpublished).

TABLE II. Values of the normalization constant B^2 for some (d,He^3) reactions. E_i = energy of the incident particle in the laboratory system; $Q=Q$ of the reaction; r_0 is the cutoff radius; B^2 is the normalization factor of the triton (He^3) wave function; θ^2 is the dimensionless reduced width of the transition.

				$l=1$				
Reaction	Level (Mev)	E_i (Mev)	Q (Mev)	r_0 (fermis)	$B^2\theta^2$ (fermis)$^{-1}$	θ^2	B^2 (fermis)$^{-1}$	Ref.
$Li^6(d,He^3)He^5$	g.s.	14.8	0.840	5.5	0.081	...	1.0	a
$Li^6(n,d)He^5$	g.s.	14	−2.427	4.5		0.080		b
$B^{11}(He^3,d)C^{12}$	g.s.	5	10.463	5	Ratio	Ratio		c
$B^{11}(d,n)C^{12}$	g.s.	9	13.731	4.5	$\frac{B^2\theta^2(4.43)}{B^2\theta^2(\text{g.s.})}$	$\frac{\theta^2(4.43)}{\theta^2(\text{g.s.})}$		d
$B^{11}(He^3,d)C^{12}$	4.43	5		5	=0.232	=0.228		c
$B^{11}(d,n)C^{12}$	4.43	9		4.5				d

[a] See reference 13.
[b] G. M. Frye, Jr., Phys. Rev. **93**, 1086 (1954).
[c] H. D. Holmgren, E. A. Wolicki, and R. L. Johnston, Phys. Rev. **114**, 1281 (1959).
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compared and they indicate the same B^2 for the excited and ground states.

IV. THE $d+d \rightarrow p+t$ REACTIONS

The description of the $(d+d)$ reactions as a direct interaction was suggested by Butler and Symonds,[14] and has been applied with success to several experiments.[15–17]

The main purpose of the present analysis is to obtain information about the triton transform $I(k)$ which was used in the preceding section. Only a few of the experiments available are studied, those which presented the more complete data, at incident energies of the order of 10 Mev.

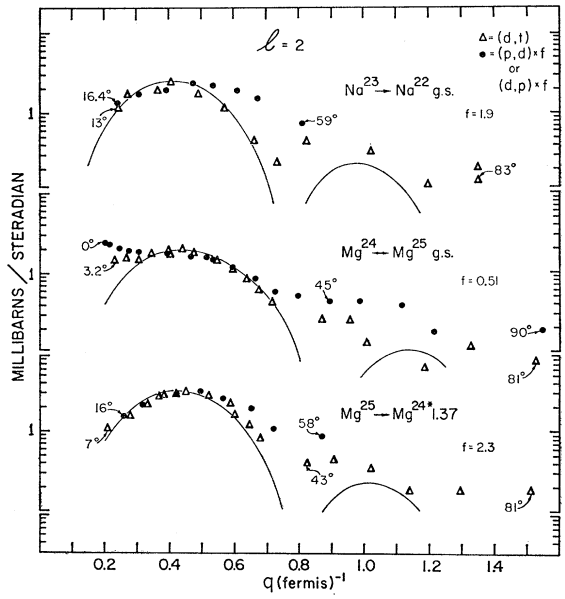

FIG. 4. Comparison between corresponding (p,d) [or (d,p)] and (d,t) reactions for orbital angular momentum transfer $l=2$, as in Fig. 2.

The data of the $D(d,p)T$ reactions measured by Brolley, Putnam, and Rosen,[18] with five incident energies from 6 to 14 Mev was analysed as a stripping process. Also the reaction $D(d,n)He^3$ at 8.4 Mev, studied by Daehnick and Fowler[16] was reanalysed, without subtracting an isotropic background, as such a subtraction is not clearly justified.

Because the target and incident particles are identical the angular distributions are symmetrical about 90° and the theoretical cross section is given by the sum of three terms[19]

$$d\sigma(\theta)/d\Omega \propto f^2(\theta)+f^2(\pi-\theta)+\tfrac{2}{3}f(\theta)f(\pi-\theta), \quad (6)$$

where $f^2(\theta)$ is the cross section when the observed particle comes from the incident beam, $f^2(\pi-\theta)$ when it comes from the target deuterons, and the cross term describes the interference amplitude.

Each term in Eq. (6) was expressed in terms of Eq. (1) for an $A(d,p)A+1$ reaction. However in this analysis, only the first term of Eq. (6) actually was considered for the description of the forward peak, since for angles smaller than 30° the contributions from the other two terms were found to be very small.

The deuteron-proton integral in Eq. (1) is performed in the usual way, with the Hulthén wave function for the deuteron. The other integral, i.e.,

$$\int \phi_t(\xi,\mathbf{r})\phi_d(\xi)e^{i\mathbf{q}\cdot\mathbf{r}}d\xi d\mathbf{r}, \quad (7)$$

is the same integral which was called $I(k)$ in Sec. II, where $\mathbf{k}$ was defined as $\mathbf{k}=\mathbf{k}_d-\frac{2}{3}\mathbf{k}_t$, a definition which corresponds to $\mathbf{q}$ in the present reactions. It was thought that it might be possible to bypass theoretical considerations of the integral $I(k)$ by determining it

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empirically, by fitting the $d+d\rightarrow p+t$ angular distributions. The integral thus determined could be then used in the (d,t) experiments with heavier nuclei.

In Fig. 1, the curve denoted by D-8 is $I(k)$ obtained from the D(d,n)He^3 experiment with 8.4-Mev deuterons and the curve D-14 is $I(k)$ from the D(d,p)T experiment with 13.8-Mev deuterons. These curves cover the range of k corresponding to the forward angles. It is to be noted that they are parallel. Curves obtained at other energies also are parallel to the ones plotted, although their magnitudes vary. This variation may be seen from the table of reduced widths (Table III).

The disagreement between the curves for $I(k)$ given directly by the $d+d\rightarrow p+t$ reactions and the curves which fit the (d,t) experiments with heavier nuclei seems to show that the simple theoretical treatment used does not permit a direct correlation between these two kinds of reactions.

On the other hand, it is interesting that the D(d,p)T experiments do give good agreement with an expression analogous to that used for (d,p) reactions in heavier nuclei. In other words, the curves D (Fig. 1) are described by the integral of Eq. (7) if a cutoff radius is taken. Therefore, form 7 written as a product of a reduced width, a form factor, and a Wronskian (calculated at $r_0=6$ fermis) was found to give good fits to all the $d+d$ reactions analysed here. The significance of a cutoff radius for deuterons is not apparent. However reduced widths for the triton have been extracted for each case and are listed in Table III. They are found to increase with increasing incident energy. The (d,t) and $(d,$He$^3)$ experiments at similar incident energies give the same value for the reduced width of triton and He^3.

V. CONCLUSIONS

The comparison between (p,d) and (d,t) experiments involving the same transitions, and studied in the same range of momentum transfer q and incident energy, shows that in the region corresponding to the first peak of the angular distribution the differential cross sections of these two kinds of reactions have the same variation with $\mathbf{q}$. This result is in agreement with the simple Born approximation treatment of the reactions. The comparison also shows that there seems to exist certain regularities in the differences between the curves in the region not well described by the crude theory. It should be expected that these differences would be explained by a more elaborate treatment.

TABLE III. Reduced widths for triton and He^3 at various energies.

Reaction	Q (Mev)	E_i (Mev)	r_0 (fermis)	θ^2
D(d,p)T	4.01	6.1	5.9	0.024
		8.1	5.9	0.035
		10.3	5.9	0.044
		12.15	5.9	0.058
		13.8	5.9	0.071
D(d,n)He^3	3.27	8.4	6	0.037

The simple relation found between (p,d) and (d,t) experiments suggests that there does exist a coefficient B^2 which is the same for all experiments. This seems to be true within the range of experimental uncertainty, at least for light nuclei in the range of energy studied. A suitable average value would seem to be $B^2\simeq 0.8$ fermi^{-1}. This value for B^2 may be used to extract reduced widths in (d,t) or $(d,$He$^3)$ reactions, a practical formula for the reduced width then being

$$\theta^2=0.625\times 10^{-2}\left(\frac{A+3}{A}\right)^2\frac{1}{t}\frac{1}{[C]^2}\frac{1}{r_0^3(\text{fermis})^3}\times\frac{d\sigma/d\Omega(\text{mb/sr})}{\sigma_{\text{tab}}\times F^2}.$$

Here σ_{tab} is the factor tabulated for (d,p) and (d,n) reactions by Lubitz[20]; $t=(A+1)k_t/Ak_d$; $F^2=[1+0.008\times(x^2+y^2)]^2$, where $x=qr_0$ and $y=Kr_0$. (This factor F^2 is introduced because the table is computed with the Hulthén wave function for the deuteron rather than a simple asymptotic form.)'The other symbols are defined in Sec. II. Reduced widths obtained with this formula should have the same values as in the corresponding (p,d) or (n,d) reactions.

The attempt to use in the formula for (d,t) reactions the triton transform $I(k)$ as determined by the $d+d\rightarrow p+t$ reactions was not successful. This may indicate that there is a difference between the detailed processes by which a deuteron picks up a neutron in a $d+d$ reaction and in a (d,t) reaction in a heavier nucleus.

APPENDIX

The Reaction F$^{19}(d,t)$F^{18} g. s. with 14.8-Mev Deuterons

The reaction F$^{19}(d,t)$F^{18} g.s. was studied previously by El Bedewi and Hussein[21] with 9-Mev deuterons. However the angular distribution obtained in that experiment was incomplete. For an $l=0$ curve the slope of the first peak is very steep, and the cross section is only large at very small angles ($<12°$). Therefore, several points in that region are required in order to have a fit which would give a reliable value for the reduced width. For this reason the experiment was repeated using 14.8-Mev deuterons.

A Teflon (CF_2) target, 2.8 mg/cm^2 thick, was bombarded with the deuteron beam of the cyclotron of the University of Pittsburgh. An angular distribution of the reaction F$^{19}(d,t)$F^{18} g.s. was taken at the forward angles, from $\theta_{\text{lab}}=5°$ to 45°. The tritons, magnetically

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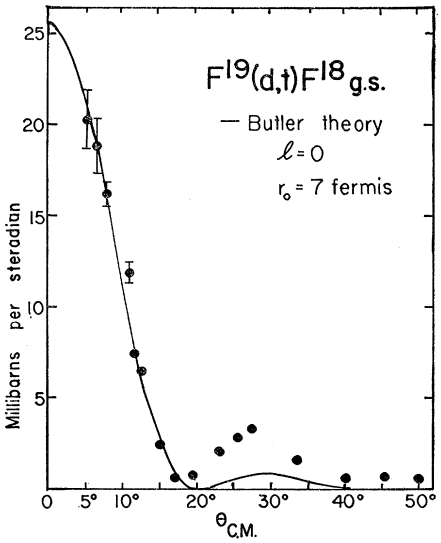

FIG. 5. Angular distribution of the reaction $F^{19}(d,t)F^{18}$ g.s. with 14.8-Mev deuterons.

analysed, were detected in Kodak NTB, 50μ thick, nuclear emulsions. The angular acceptance was limited to 1°.

The experimental angular distribution is shown in Fig. 5 together with an $l=0$ Butler curve using $r_0=7$ fermis. The absolute cross section was calculated by comparison with measurements of the $C^{12}(d,p)C^{13}$ g.s. reaction using the same Teflon target. The cross section of that reaction has been previously measured to $\pm 20\%$ at this laboratory.[13] The result of the experiment in terms of the reduced width is presented in Table I, and its comparison with the corresponding (p,d) reaction is discussed in Sec. III.

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Conversion, *K*-Auger, and *L*-Auger Spectra of Hg^{199}†

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The high resolution of the spectrometer made possible the detailed study of K, L, M, $N+O$ conversion lines and the K- and L-Auger spectra of Au^{199} with the following results (here ω and a are the fluorescence and Auger yields and f_{ij} is the Coster-Kronig transfer probability): K-Auger lines, $\omega_K=0.952\pm0.003$, $KLL:KLX:KXY=1.00:0.496\pm0.015:0.094\pm0.003$, and $KL_1L_1:KL_1L_2:KL_1L_3:KL_2L_2:KL_2L_3:KL_3L_3=1.00:1.32\pm0.1:0.85\pm0.06:0.40\pm0.03:1.28\pm0.08:0.76\pm0.05$; L-Auger lines, $LMM:LMX:LXY=1.00:0.30\pm0.03:0.015\pm0.004$, and $a_L=0.590\pm0.04$, $\omega_L=0.410\pm0.04$, $a(L_1)=0.16\pm0.02$, $a(L_2)=0.46\pm0.04$, $\omega(L_2)=0.32\pm0.03$, and Coster-Kronig yields, $f(L_2L_3X)=0.22\pm0.04$, $f(L_1L_2X)+f(L_1L_3X)=0.74\pm0.04$. In addition considerable detail was obtained on the KLX and L-Auger fine structure. The results of all of the known L-Auger yield work since 1952 have been tabulated in this paper.

The conversion line results are compared and combined with those of two other groups to give an optimum set of relative intensities.

From these are obtained for the 51-kev transition, $\alpha(L_1):\alpha(L_2):\alpha(L_3):\alpha(M):\alpha(N):\alpha(O)=1.00:0.087\pm0.010:0.012\pm0.007:0.212\pm0.04:0.068\pm0.005:0.016\pm0.001$; 156-kev transition, $\alpha(K):\alpha(L_1):\alpha(L_2):\alpha(L_3):\alpha(M):\alpha(N+O)=1.00:0.144\pm0.015:0.830\pm0.028:0.586\pm0.018:0.418\pm0.017:0.107\pm0.005$; 209-kev transition, $\alpha(K):\alpha(L_1):\alpha(L_2):\alpha(L_3):\alpha(M):\alpha(N+O)=1.00:0.155\pm0.005:0.029\pm0.003:0.0085\pm0.0003:0.050\pm0.006:0.0130\pm0.004$, where α is the internal conversion coefficient.

In addition, by use of Rose's Tables the 51-kev transition was determined to be $3.3\pm1\times10^{-4}$ $E2$, and the 209-kev transition 0.113 ± 0.01 $E2$, and the $E2$ assignment of the 158-kev transition was confirmed to better than 1%. The relative gamma-ray intensities are 209 kev:51 kev:158 kev$=1.000:0.045\pm0.002:4.59\pm0.23$.

I. INTRODUCTION

ALTHOUGH the electron spectrum of the radionuclide Au^{199} has been studied with various types of spectrometers, discrepancies exist in the conversion line results and the K- and L-Auger lines have never been studied with high resolution. Therefore, when the Vanderbilt University, iron-free $\pi\sqrt{2}$ spectrometer became operational, a thorough study of the electron line spectrum of Au^{199} from 5 kev to 210 kev was undertaken. The continuous beta-ray spectrum was studied only sufficiently to establish a baseline for the various line spectra.

The well established decay scheme of Au^{199} is shown in Fig. 1.[1-3] The energies of the upper two gamma rays have recently been measured to be $209.17+0.12$ kev and 158.27 ± 0.35 kev.[4] The character of the 158-kev

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