Effect of High Pressure on Some Hot Electron Phenomena in *n*-Type Germanium

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The pressure dependence of the current density vs electric field characteristic for *n*-type germanium at 297°K has been measured to a maximum pressure of 30 000 kg/cm², and to a maximum field of 10 000 v/cm. It is concluded that the electrons are "heated" by the field sufficiently to cause the conduction band valleys along the ⟨100⟩ directions in reduced momentum space to be appreciably populated with electrons at atmospheric pressure. The pressure dependence of the angle ϵ between current and field at 77°K has been measured to a maximum field of 3000 v/cm and a maximum pressure of 5000 kg/cm². ϵ is independent of pressure except for a small increase for applied fields of ~3000 v/cm. Suggested interpretations for the data are given.

THE dependence on electric field E of the electrical conductivity of extrinsic *n*-type germanium was measured first by Ryder and Shockley,[1] and subsequently by many other workers.[2] At room temperature for $E \lesssim 500$ v/cm, Ohm's law holds. For larger fields, there is a gradual decrease in the conductivity until for $E \gtrsim 5000$ v/cm, the current density reaches a saturation value. (For very much larger fields, there is a renewed rise in current but little work has been done in this range.) A simple theoretical treatment, originally by Shockley[3] and subsequently by others,[2] assuming spherical bands and invoking mixed optical and acoustic scattering, is in reasonable agreement with the data. The theoretical situation to date has been reviewed by Conwell.[4]

The experiments reported here were undertaken in order to investigate the influence on the high field conductivity of the "silicon-like" ⟨100⟩ conduction band valleys which are about 0.16 ev above the ⟨111⟩ conduction-band minima.[5] The fraction of the conduction electrons in the upper valleys is $\sim\frac{1}{3}\%$ at 300°K for thermal equilibrium conditions. As the electron temperature (this term is used to denote the mean electron energy without a commitment to a particular form for the distribution function) is increased by an applied field, two effects due to the presence of the ⟨100⟩ minima may be expected to occur: (1) an increase of the carrier density in the upper conduction band, (2) an increase in the frequency of interband scattering of electrons between the sets of minima. Both tend to decrease the drift velocity.[5]

The energy separation of the two sets of minima decreases linearly with increasing hydrostatic pressure[5] until at ~35 000 kg/cm² the energies of the two sets of valleys coincide. To determine the influence of this upper conduction band, we have examined hot-electron effects as a function of hydrostatic pressure.

The pressure apparatus has been described in detail elsewhere.[6] Longitudinal current and the angle between current and field (tranverse effects) were measured as a function of applied field. Electrical circuitry and sample shapes were similar to that used by Sasaki, Shibuya, and Mizuguchi,[7] except that the samples used for longitudinal measurements had no sidearms.

LONGITUDINAL EFFECTS

Figures 1 and 2 show the variation of current density with electric field as a function of pressure at room temperature for two samples, one with the current along a ⟨100⟩ direction, the other along a ⟨110⟩ direction. Because of the mass anisotropy of each valley, the electrons do not gain energy from the field equally for all directions of applied field, so that, in general, the conductivity is not scalar when Ohm's law fails to hold.[8] For the two high symmetry directions chosen, the current and field are colinear, but only for the ⟨100⟩ case is the electron distribution and the contribution to the current the same for each ⟨111⟩ valley. For the other case, the valleys are equivalent in pairs. When one averages over all valleys, however, using the theory of Shibuya[8] and

* Present address: Massachusetts Institute of Technology, Department of Electrical Engineering, Cambridge, Massachusetts.
1 E. J. Ryder and W. Shockley, Phys. Rev. **81**, 139 (1951).
2 See J. B. Gunn, *Progress in Semiconductors* (John Wiley & Sons, Inc., New York, 1957), Vol. 2, p. 246, for a review and references to earlier work.
3 W. Shockley, Bell System Tech. J. **30**, 990 (1951).
4 E. M. Conwell, J. Phys. Chem. Solids **8**, 234 (1959).
5 M. I. Nathan, W. Paul, and H. Brooks (to be published). M. I. Nathan, Harvard University, Gordon McKay Technical Report HP-1, 1958 (unpublished).
6 P. W. Bridgman, Proc. Am. Acad. Arts Sci. **72**, 157 (1938). M. G. Holland, Harvard University, Gordon McKay Laboratory Technical Report No. HP-4 (unpublished).
7 W. Sasaki, M. Shibuya, and K. J. Mizuguchi, J. Phys. Soc. Japan **13**, 456 (1958), W. Sasaki, M. Shibuya, K. Mizuguchi, and G. M. Hatoyama, J. Phys. Chem. Solids **8**, 250 (1959).
8 See, for example, M. Shibuya, Phys. Rev. **99**, 1189 (1955).

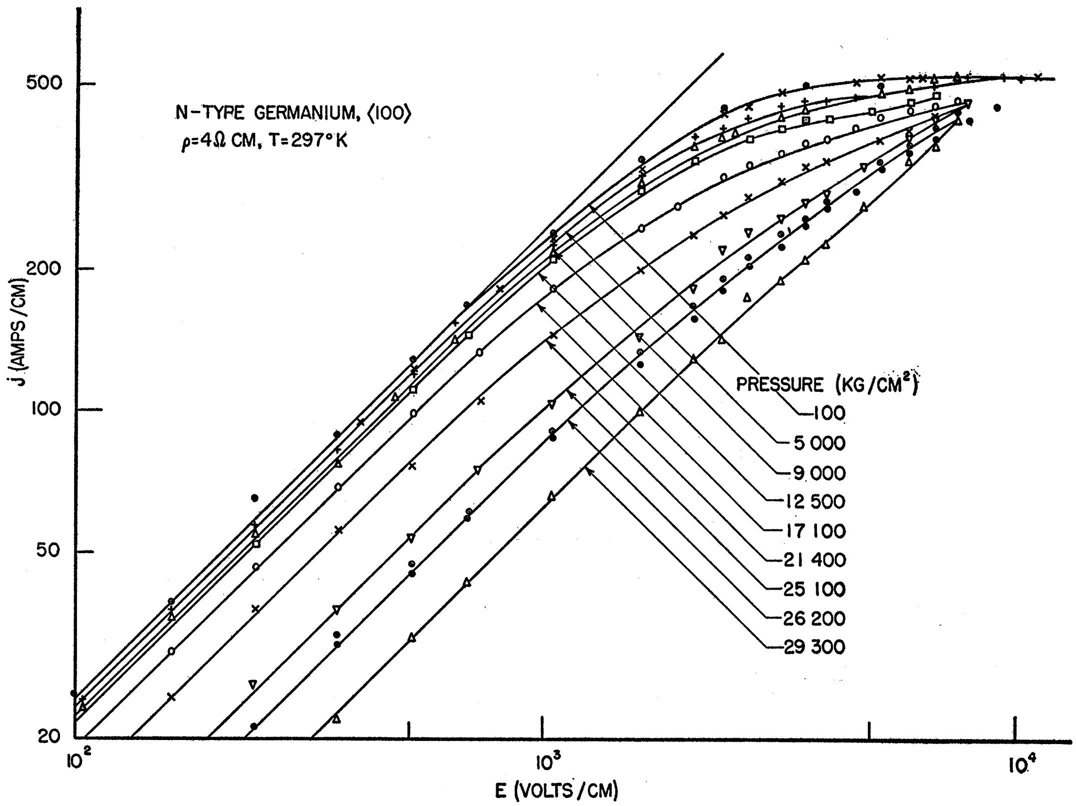

FIG. 1. The variation of current density with electric field as a function of pressure at room temperature for a ⟨100⟩ oriented sample.

the room-temperature anisotropy data of Sasaki *et al.*[7] which implies that the valley populations are essentially equal, very little difference is expected in the characteristics of the two samples.

Figure 3 shows the pressure dependence of the low field (Ohmic) conductivity for the two samples compared with earlier data.[9] This serves to confirm that the samples and apparatus are behaving properly.

Several main features may be observed in Figs. 1 and 2. Firstly, for the lower-pressure range ($\lesssim$15 000 kg/cm²), the pressure dependence of the conductivity is greater at intermediate fields than at low fields, while the field at which deviations from Ohm's law set in (the "critical field," E_c), remains essentially unchanged. Secondly, at the higher pressures, the samples remain "Ohmic" to much greater fields. Finally, in Fig. 1, where the maximum field was approximately 10 000 v/cm, it is seen that the saturation current is independent of pressure between 0 and 10 000 kg/cm². Because of electrical breakdown of contacts in the high-pressure cylinder, it was not possible to get this data at all pressures.

To give a quantitative explanation of these data is difficult, though much can be said qualitatively by reasoning in terms of a Shockley-type theory.[3] Rather than solve the Boltzmann equation, Shockley assumes a Boltzmann distribution function, and then derives relations between mobility, electric field, and electron temperature by equating the power input to the distribution from the field to the energy-loss rate due to scattering. The power gain from the field is proportional to the square of the field multiplied by the mobility, which in turn, is proportional to the average momentum transfer per collision, irrespective of how inelastic the collisions may be. The power lost to the lattice, on the other hand, may be mainly due to highly inelastic, though relatively infrequent, processes which do not contribute significantly to determining the drift mobility. Thus, in germanium at room temperature, optical-mode scattering probably contributes essentially all the energy loss while contributing at most 20% to the inverse of the mobility.[3,10] If a monoenergetic distribution is assumed, it can be shown straightforwardly by use of the Shockley-type argument that any mechanism which either increases the energy loss or lowers the low-field mobility will cause E_c to increase provided that all the electrons remain in one set of valleys (the ⟨111⟩ in germanium). In order to explain the fact that E_c remains constant between atmospheric pressure and 20 000 kg/cm² while the effective low-field drift mobility decreases by a factor of two, it is necessary to take into account the

[9] P. W. Bridgman, Proc. Am. Acad. Arts Sci. **82**, 71 (1953).

[10] T. Morgan, J. Phys. Chem. Solids **8**, 245 (1959).

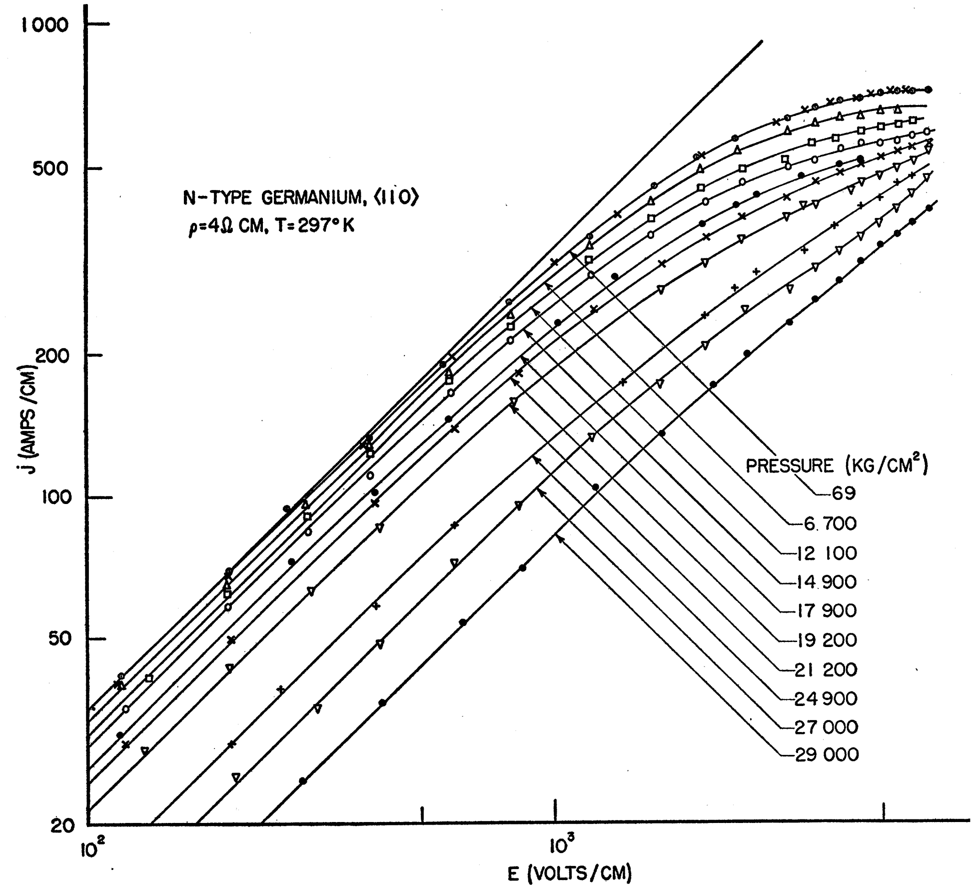

FIG. 2. The variation of current density with electric field as a function of pressure at room temperature for a ⟨110⟩ oriented sample.

effect of the ⟨100⟩ valleys. The electrons with energy greater than the band-edge energy of the ⟨100⟩ valleys are distributed between the ⟨100⟩ and ⟨111⟩ valleys and are subject to interband scattering; their mobility is greatly reduced and to a first approximation their contribution to the conductivity may be neglected. As the field is increased above the Ohmic range and as the electron temperature increases, the number of electrons with energy above the band edge of the ⟨100⟩ valleys increases. Hence the conductivity is reduced more at high fields than at low fields. This tends to lower E_c. As the pressure increases, this effect becomes more important because the energy separation between the ⟨100⟩ and the ⟨111⟩ valleys decreases. Therefore, E_c will tend to be decreased more at higher pressure. This is probably the mechanism which offsets the effect of the decreased low-field mobility with increasing pressure on E_c and keeps it constant. It also accounts for the increased pressure dependence of the effective mobility in the intermediate field region (~3000 v/cm).

From the interband scattering theory, the pressure coefficient of the conductivity can be calculated[5] as a function of temperature. If a Boltzmann distribution is assumed for hot electrons and optical-mode scattering is neglected, the calculation applies, provided the temperature is interpreted as the electron temperature, T_e. On this basis, the pressure coefficient of the conductivity or the current density should increase with increasing T_e for kT_e small compared with the energy separation of the conduction band edges and should decrease for kT_e large compared with the energy separation. This is

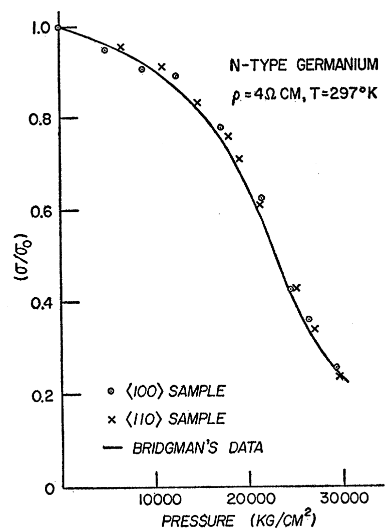

FIG. 3. The pressure dependence of the low field (Ohmic) conductivity for the samples of Figs. 1 and 2 compared with earlier data (see reference 9).

qualitatively consistent with the electric-field dependence of the pressure coefficient of the current density up to 9000 kg/cm^2 shown in Fig. 1. The current density is independent of pressure for fields of ~10 000 v/cm. This suggests that kT_e is large compared to the energy separation.

However, the observed pressure coefficient at intermediate fields is almost an order of magnitude smaller than is expected from the calculation for kT_e comparable with the energy separation. Therefore, the conclusion about T_e is not warranted, and the decreased pressure dependence of the current density for applied fields of ~10 000 v/cm is not consistent with this simple picture.

For pressures higher than ~20 000 kg/cm^2, the current-field characteristics change more rapidly with pressure. The main feature of the data is that at the highest pressure the curve is essentially linear. This is readily understood. By ~25 000 kg/cm^2, the populations of the two bands are approximately equal at low fields. The mobility of both bands is quite low, owing to the large amount of interband scattering. In addition, the average energy loss per collision is considerably larger than at zero pressure. The argument for the latter statement is as follows: scattering between conduction bands is a transition which may involve a transverse acoustic phonon, since this is not forbidden by symmetry.[11] These transverse phonons have energies of the order of 100°K[12] and their number is quite large at room temperature. When such transitions are sufficiently frequent to determine the carrier mobility or momentum balance, as is the case for $P \cong 30\,000$ kg/cm^2, they must be an order of magnitude more frequent than the optical-mode scattering. Since optical-mode phonons, which provide the dominant loss mechanism at atmospheric pressure, have energies only about three times those of the interband phonons, the onset of a large amount of interband scattering at the higher pressures both decreases the rate at which the electrons gain energy from the field and increases the rate of energy loss. The distribution then becomes very hard to "heat", and Ohm's law is obeyed to relatively high fields.

The pressure dependence of the J vs E curve was also measured at 77°K to a maximum pressure of 5000 kg/cm^2. There is no pressure dependence of the curve exceeding the experimental error (~3%), but the pressure range obtainable is not large enough for a comparison with theory.

TRANSVERSE EFFECTS

The angle, ϵ, between the current and the electric field was measured at 77°K by measuring the voltage transverse to the current direction in samples cut from a (110) plane of such orientation as to give maximum anisotropy in the high-field conductivity. The current was fixed 30° from a ⟨100⟩ direction and the field was in the (110) plane. Sasaki *et al.*[7] have found that the variation of ϵ with E goes through a maximum for an E of 10^3 v/cm for their samples. However, it has since been suggested by Gunn (private communication) that this maximum in E may be related to a geometric distortion of the field lines in the neighborhood of the transverse probes. When the current density reaches saturation over the length of the crystal not including the transverse probes, the longitudinal field in the region where the transverse probes are located reaches some specific value E_0. E_0 is lower than the field in the rest of the sample, since the current density is lower in the region where the transverse arms are. As the applied field is increased, E_0 tends to remain fixed since the current density is almost constant. The transverse voltage would then become independent of E and $\tan\epsilon$ would decrease as E^{-1}.

Nathan[13] has since studied these geometric effects and found that in fact, Gunn's suggestion applies to the maximum observed in ϵ for samples of the dimensions used by Sasaki *et al.*[7] and to a lesser extent, those of Koenig,[14] and those first used in the present work. The data of Fig. 4, however, was taken on samples with arms of the order of 10^{-2} cm square. It is felt that the

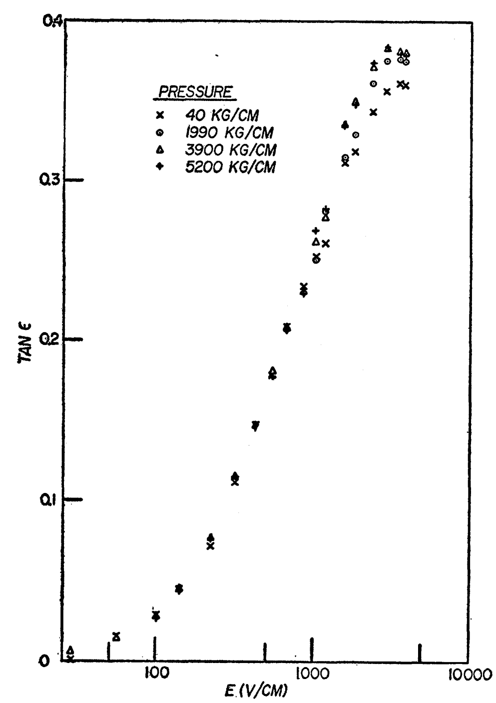

FIG. 4. The variation with pressure of $\tan\epsilon$, the tangent of the angle between the electric field and the current, measured at 77°K for a sample the orientation of which was chosen to give maximum effect.

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[11] C. Herring and E. Vogt, Phys. Rev. **101**, 944 (1956).
[12] B. N. Brockhouse and P. K. Iyengar, Phys. Rev. **108**, 894 (1957).

[13] M. I. Nathan (to be published).
[14] S. H. Koenig, Proc. Phys. Soc. (London) **73**, 959 (1959).
[15] G. Weinreich, T. M. Sanders, and H. G. White, Phys. Rev. **114**, 33 (1959).

results are truly representative of bulk properties for the range of data presented, since further reduction (by etching) of the size of the side arms left the shape of $\tan\epsilon$ vs E unaltered.

The observed variation of ϵ with E can be understood in fairly simple terms. It is simplest to think in terms of only two $\langle 111\rangle$ valleys, each oriented differently with respect to the electric field so that beyond the Ohmic region the electron temperatures will be different in each. Under these conditions, the conductivity becomes anisotropic (since the mobility is field dependent), as may be seen by a simple geometric argument.[7] Carriers from the tail of the hotter distribution, those above the energy of the phonon involved in intervalley scattering[15] will scatter by spontaneous emission of the $\sim 300°K$ intervalley phonons to the cooler valley. Similarly, those carriers in the cooler valley, for which it is energetically possible, will scatter to the hotter valley. As long as the electron temperature is less than $\sim 300°K$, a steady state will be reached for which there are more and more carriers in the cooler valley as the field is increased, so the total net transfer rate will be the same in both directions. The net effect is to increase the measured anisotropy. The data of Sasaki suggests that the population ratio of the valleys is $\sim 3:1$ at the highest values of E.

When the electron temperature increases in all $\langle 111\rangle$ valleys to a magnitude such that much of the distribution is at an energy large compared to the intervalley phonon energy, the steady-state population tends towards more equal population in all valleys, since all carriers can then spontaneously radiate intervalley phonons and scatter from one valley to the other. This would reduce the anisotropy at high fields. The maximum in the $\tan\epsilon$ vs E curve of Fig. 4, it is felt, is due to the onset of this condition.

With an increase in hydrostatic pressure, the maximum value of ϵ increases as is shown in Fig. 4. A possible mechanism for this is similar to that responsible for the initial increase of ϵ at lower values of E. For large E, the tail of the distribution for the hottest $\langle 111\rangle$ valley will overlap the $\langle 100\rangle$ valleys. By symmetry (remembering that the $\langle 111\rangle$ minima are at the zone boundary), these electrons will scatter equally to all $\langle 100\rangle$ valleys, and from there, again equally, back to the $\langle 111\rangle$ valleys. This two-step process when energetically possible, can be stimulated by phonons that are partially excited at 77°K, since the selection rules[11] that forbid inter-$\langle 111\rangle$-valley scatterings by transverse acoustic phonons do not apply. Therefore, it may well compete with the one-step inter-$\langle 111\rangle$-valley process which at these fields is acting to equalize the valley population. With an increase in pressure, the $\langle 100\rangle$ valleys are lowered relative to the $\langle 111\rangle$ valleys, making this two-step anisotropy-enhancing mechanism more probable.

At still higher fields, it may well be that the population in the upper minima will be sufficient to make a significant contribution to the conductivity directly. The effect would be to reduce the anisotropy with increasing E, and for fixed E to reduce the anisotropy with increasing hydrostatic pressure. This effect hasn't been observed for the fields attained.

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