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Binding Energy of a Neutron Gas

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The energy of a neutron gas is determined as a function of density using the methods of Brueckner and Gammel. The system is unbound at all densities. The change in energy from a superconducting type of level inversion is estimated and shown to be negligible.

IT has recently been shown[1,2] that very large nuclei made up almost entirely of neutrons may exist metastably if the neutron-neutron attraction is sufficiently strong to bind the system. Such "hyper-nuclei", if bound, would be almost completely stabilized against beta decay by the high Fermi energy of the electrons produced by the decay of roughly 10^{-4} of the neutrons.

To determine the possibility of such bound neutron systems, we have evaluated the energy of a pure neutron gas as a function of density, using the methods of Brueckner and Gammel.[3] (We refer to this in the following as BG). These are particularly accurate for this case since the region of possible interest is at much lower density than is nuclear matter.

In a pure neutron gas, the Fermi momentum is determined by the relation

$$2(\tfrac{4}{3}\pi k_F^3)\Omega/(2\pi)^3=N, \tag{1}$$

$$k_F=1.912/r_0, \tag{2}$$

with r_0 the radius of the mean volume per particle. The energy is determined as in BG by the equation

$$E=\sum_i \frac{p_i^2}{2M}+\tfrac{1}{2}\sum_{ij}(K_{ij,ij}-K_{ij,ji}). \tag{3}$$

Inserting the angular momentum decomposition of the K matrix and using the proper spin weights, we find

$$E=\sum_i \frac{p_i^2}{2M}+\sum_{k_ik_j}(k_ik_j|K|k_ik_j)_{\text{singlet even}} \sum_{k_ik_jm_s}(k_ik_jm_s|K|k_ik_jm_s)_{\text{triplet odd}}. \tag{4}$$

The explicit expression for the triplet K matrix is given in Eq. (63) of BG.

We have evaluated the self-consistent single particle energies and total energy using the Gammel-Thaler potentials listed in Table I. The results for the total energy and potential energy per particle are given in Fig. 1.

The possible singularities of the K matrix very near the Fermi surface due to the attractive forces were not observed since they can affect only states of total momentum very close to zero and also with relative momentum very close to the Fermi momentum. These have zero weight in the computation of energy due to the finite spacing of momentum values chosen. It is, however, possible to estimate the extent to which the attractive singlet-s interaction near the Fermi surface causes level inversion and depression of the ground state

TABLE I. Parameters of Gammel-Thaler potentials. These are of Yukawa form and have a repulsive core at 0.4×10^{-13} cm.

	Strength (Mev)	Inverse range (10^{13} cm^{-1})
Singlet	− 434	1.45
Triplet central odd	− 40	1.50
Triplet tensor odd	− 22	0.80
Triplet L·S odd	−7317.5	3.70

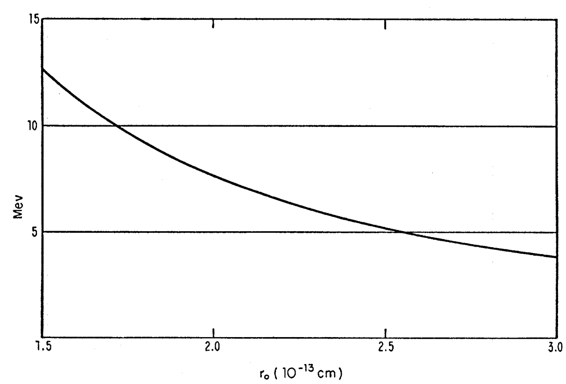

FIG. 1. Total energy per particle.

[1] E. E. Salpeter, Bull. Am. Phys. Soc. 4, 256 (1959).
[2] K. W. Chun, Ph.D. thesis, University of Pennsylvania, 1959 (unpublished).
[3] K. A. Brueckner and J. L. Gammel, Phys. Rev. 109, 1023 (1958).

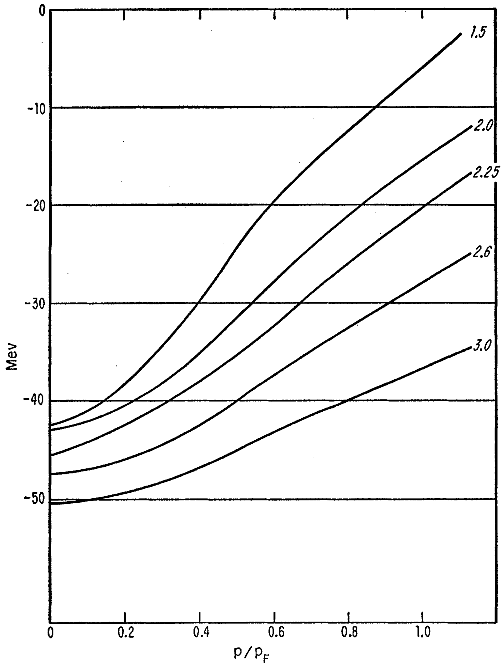

FIG. 2. $\Omega(k_F|K|k_F)/4\pi$ in singlet s state as a function of momentum for several densities.

energy. The relevant matrix elements of the K matrix are shown in Fig. 2 as a function of momentum and in Fig. 3 at the Fermi surface as a function of density. We take the formula for the ground-state energy shift per particle as given by Bardeen, Cooper, and Schrieffer.[4]

$$\frac{\Delta E}{N}=-2\frac{N(0)(\hbar\omega)^2}{N}\exp\left(\frac{-2}{N(0)V_0}\right). \tag{5}$$

$N(0)$ is the level density at the Fermi surface

$$N(0)=\frac{k_F M^*\Omega}{2\pi^2\hbar^2}, \tag{6}$$

with M^* the effective mass defined by the relation

$$\frac{1}{M^*}=\frac{1}{M}+\frac{1}{p_F}\left[\frac{dV(p)}{dp}\right]_{p_F}, \tag{7}$$

Ω the normalization volume, and $-V_0$ the average matrix element of the interaction at the Fermi surface, which we take from Fig. 3. The characteristic energy we take to be the kinetic energy at the Fermi surface, i.e.,

$$\hbar\omega=\hbar^2k_F^2/2M^*. \tag{8}$$

The resulting shift per particle is given in Fig. 4.[5] This is as a typical value 1.2×10^{-2} Mev at $r_0=2.60\times10^{-13}$ cm; comparison with Fig. 1 shows that this is much too small to affect the total energy appreciably.

We have also looked for a possible minimum in the energy at densities approciably lower than that corresponding to $r_0=3.0\times10^{-13}$ cm. As the density goes to zero, the energy is given exactly by Eq. (4), with the odd states giving no contribution as the momentum of the Fermi gas goes to zero, and only the singlet-S state remaining in the even state sum. To lowest order in the

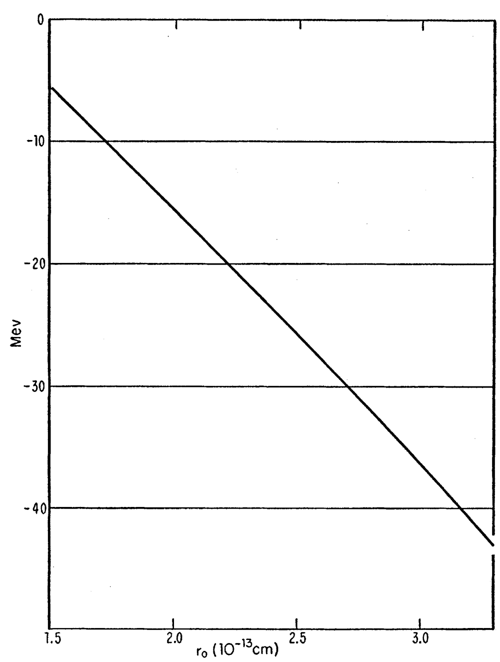

FIG. 3. $\Omega(k_F|K|k_F)/4\pi$ in singlet s state at the Fermi surface as a function of density.

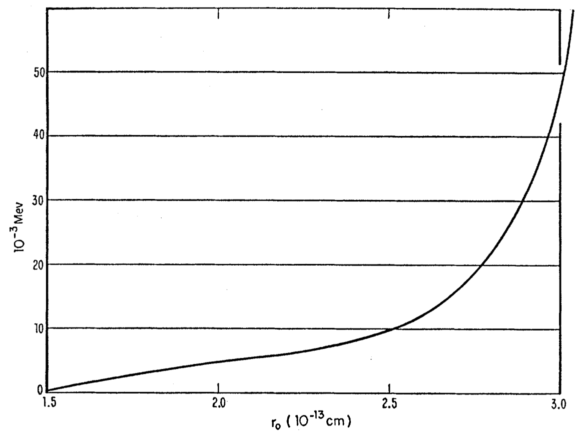

FIG. 4. Ground-state energy shift per particle due to level inversion, as a function of density.

[4] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. **108**, 1175 (1957).

[5] We have set $M^*=M$ in Eq. (6) and Eq. (8) since evaluation of Eq. (7) shows that M^* differs from M by at most 6% over the density range of interest.

density, the K matrix is given by the scattering limit:

$$(k_ik_j|K|k_ik_j) \underset{\rho\to 0}{=} -(4\pi/M\Omega)k_{ij}^{-1}\delta_0(k_{ij}), \qquad (9)$$

with k_{ij} the relative momentum. The sum over k_i, k_j in Eq. (6) now can be replaced by an integral over the relative momentum, with the result

$$\frac{E}{N}=\frac{3p_F^2}{10M}-\frac{8}{\pi M}\int^{k_F} dk\, k\delta_0(k)\left(1-\frac{3}{2}\frac{k}{k_F}+\frac{1}{2}\frac{k^3}{k_F^3}\right). \qquad (10)$$

The s-state phase shift is given by the effective range formula

$$k\cot\delta_0=-(1/a)+\tfrac{1}{2}r_0k^2, \qquad (11)$$

with

$$a=-23.6\times 10^{-13}\text{ cm.}$$
$$r_0=2.65\times 10^{-13}\text{ cm.} \qquad (12)$$

The energy determined by Eq. (10) is given in Fig. 5 as a function of r_0. Again we see that there is no minimum and, as expected, that the low-density formula gives considerably too much interaction energy even for densities as low as $r_0=3\times 10^{-13}$ cm. This is readily understood since the large singlet scattering phase shifts are markedly reduced by the perturbation of the interaction arising through the effects of the exclusion principle.

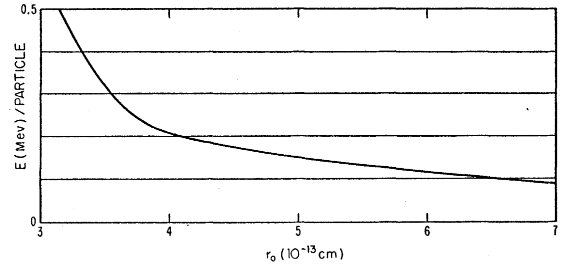

FIG. 5. Energy at very low density as given by scattering phase shifts.

We conclude that a neutron gas is not bound at any density and also that there is no relative minimum in the energy as a function of density. A constrained neutron gas would, however, show superfluidity.

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K^- Absorption and $\pi-\Sigma$ Phase Shifts*

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The relations between $\bar{K}+N\to\pi+Y$ absorption amplitudes and pion-hyperon scattering amplitudes that are implied by the unitarity of the scattering matrix are considered. It has been shown by Kawarabayashi that if Λ production and the K^0-K^- mass difference are neglected, the zero kinetic energy K^--p absorption data of the Berkeley hydrogen bubble chamber group imply that at least one of the angular momentum $\frac{1}{2}$ pion-hyperon scattering amplitudes is much larger than are any of the $j=\frac{1}{2}$ pion-nucleon amplitudes at a corresponding energy. It is demonstrated that the conclusion of Kawarabayashi remains valid if one includes the effects of Λ production and the K^0-K^- mass difference.

It is generally assumed that the interactions of π mesons with nucleons, Σ and Λ particles are primarily responsible for the binding of Λ's in nuclear matter. If this assumption is correct the $\pi\Sigma\Lambda$ interaction is among the strongest of all particle interactions, so that an understanding of πY interactions is essential to understanding the strange particles. Unfortunately, direct πY scattering experiments cannot be done. However, as has been noted by many people, some information concerning the πY scattering amplitudes may be obtainable from analyzing the results of $\bar{K}+N\to\pi+Y$ absorption experiments.

In this paper we are concerned with the phase difference between isotopic spin one and zero $\pi-\Sigma$ scattering amplitudes that may be indicated by the absorption data. We make the usual isotopic spin assignments and follow Day, Snow, and Sucher[1] in assuming that the K^-'s stopped in liquid hydrogen are nearly all absorbed from S orbitals. For definiteness we assume that the $KN\Lambda$ and $KN\Sigma$ interactions have odd intrinsic parity, so that the $\pi\Lambda$ and $\pi\Sigma$ states produced by stopped $\bar{K}$ particles are also S states. (The consequences of the opposite parity assumption are discussed later.) We assume that the branching ratios for the different final states produced by stopped K^-'s in hydrogen are those given by the Berkeley bubble chamber group,[2] i.e., $\Sigma^-+\pi^+$ (45%), $\Sigma^++\pi^-$ (21%), $\Sigma^0+\pi^0$ (27%), and $\Lambda^0+\pi^0$ (7%). These data

* Supported by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] T. B. Day, G. A. Snow and J. Sucher, Phys. Rev. Letters 3, 61 (1959).

[2] L. W. Alvarez, Proceedings of the 1959 International Conference on physics of High-Energy Particles at Kiev, July, 1959 (to be published).