of 2− and 3− are predicted for the ground state of Tm^{172} by the method described above. Tm^{172} with either of the two predicted spins and at sufficiently high energy might be expected to decay into the 2+ level of the ground-state rotational band, to the 1172-kev 3+ level, and to the 1− and 3− levels of the $K=0-$ band which would produce the observed radiations mentioned in the Introduction.

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Multiple Scattering of Polarized Electrons

M. K. Sundaresan
Department of Physics, Panjab University, Chandigarh, India
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The theory of multiple scattering derived earlier is used to evaluate the numerical magnitudes of the depolarization produced due to multiple scattering in gold foil of thickness 1 mg/cm² for polarized electrons of velocities: $\vartheta/c=0.6$, 0.7, 0.8, 0.9. The depolarization effect is found to be extremely small. The correction due to multiple scattering to the electrostatic rotation of spins is also computed.

In a recent paper, Mühlschlegel and Koppe[1] have investigated the single and multiple scattering of polarized electrons with a view to obtaining expressions for the amount of depolarization produced due to these processes. They found that, since in the nonrelativistic approximation the polarization due to Mott scattering vanishes, the magnitude of the polarization vector **P** of the initial electrons remains unchanged; however, its direction is altered. In fact they have shown that this change in the direction of the vector **P** is exactly as though the polarized electrons were subjected to a transverse cylindrical electric field of constant magnitude. They have investigated the possibility that, although the single scattering of the electrons leads only to a pure rotation of the polarization vector, the multiple scattering could lead to some depolarization due to the fact that electrons scattered at a given angle, on account of their different paths, would have their polarization vectors pointing in different directions. However, in their final analysis, they have used Molière's theory of multiple scattering which has been recently shown[2] to involve an inconsistent approximation. It is the purpose of this note to use the theory of multiple scattering of reference 2 to evaluate the numerical magnitude of the electrostatic rotation of the spins and the amount of depolarization produced, respectively.

In the theory of multiple scattering, an expression for the "screening angle," χ_α, has been derived on the basis that the experimental single scattering cross section at small angles is correctly represented if terms of the first and second Born approximations alone are retained in the theoretical formula. χ_α is given by[2,3]

$$\chi_\alpha^2=\chi_0^2\left[1+4\alpha\chi_0\left(\frac{1-\beta^2}{\beta}\ln\chi_0+\frac{0.2310}{\beta}+1.4480\beta\right)\right], \quad (1)$$

where $\alpha=zZ/137$ (Z being the atomic number of the scatterer and z the number of units of charge on the particle undergoing scattering), β the velocity of the particle in units of the velocity of light, and

$$\chi_0=\mu(\hbar Z^{\frac{1}{3}}/0.885pa_0). \quad (2)$$

Here p represents the momentum of the particle undergoing scattering, a_0 the Bohr radius, and μ a parameter to be suitably chosen to describe the multiple scattering angular distribution properly. By fitting the angular distributions of electrons multiply scattered in gold and beryllium at high energies (15 Mev), the value of μ is found to be 1.80. A direct calculation of χ_α for beryllium from the variational wave function of the atom yields the value $\mu=2.1$. There are reasons to believe that this value of μ does not change very much with the energy of the incident particle as long as it is relativistic ($\beta\sim1$).

The formula given above for χ_α, due to the very assumptions on which it is based, has greater validity for β values close to unity. We estimate that it should be quite good even for β values for the electron of the order of 0.6. For values of β much smaller than 0.6, errors will be introduced due to the more and more unreliable nature of the Born approximation itself.

[1] B. Mühlschlegel and H. Koppe, Z. Physik **150**, 474 (1958).
[2] B. P. Nigam, M. K. Sundaresan, and Ta-You Wu, Phys. Rev. **115**, 491 (1959).
[3] For comparison we also give Molière's expression for χ_α:
$$\chi_\alpha^2=\chi_{0M}^2[1.13+3.76(\alpha^2/\beta^2)]; \quad \chi_{0M}=\hbar Z^{\frac{1}{3}}/0.885pa_0.$$

TABLE I. Depolarization effect due to multiple scattering from gold foil of thickness 1 mg/cm^2 at $\vartheta=1.0$.

β	0.6	0.7	0.8	0.9
$E_{\rm kin}$ (kev)	127.8	204.6	340.7	661.3
$\chi_c(B)^{\frac{1}{2}}$	0.6124(35.0°)	0.3992(22.81°)	0.2159(12.34°)	0.1149(6.5°)
$[s^2/4B\bar{f}(1.0)]$	2.51×10^{-5}	2.32×10^{-5}	1.02×10^{-5}	4.6×10^{-6}

TABLE II. Electrostatic rotation of spins due to scattering from gold foil of thickness 1 mg/cm^2 at $\vartheta=2.0$.

β	0.6	0.7	0.8	0.9
$s^{\frac{1}{2}}\vartheta$	0.2442	0.2284	0.1733	0.1307
$a(\nu)$	0.4147	0.3384	0.3391	0.3370
η	0.2428(8.5°)	0.2274(5.2°)	0.1729(2.1°)	0.1305(0.9°)

A detailed discussion of our expression (1) for χ_α has already been presented in our earlier paper.[2]

In the theory of multiple scattering an angle χ_c is introduced which is defined by

$$\chi_c^2=4\pi Nte^4Z(Z+1)z^2/(pc\beta)^2,$$

where N is the number of particles per cc in the scattering foil and t is the thickness of the scattering foil. A quantity b is introduced in the theory defined by

$$b=\xi\ln(\chi_c^2/4)-\ln(\chi_\alpha'^2/4),$$

where

$$\ln\frac{1}{\chi_\alpha'}=\ln\frac{1}{\chi_\alpha}+\tfrac{1}{2}-C+\frac{2\alpha\chi_0}{\beta}(1-\beta^2)(1-C),$$

$$\text{and}\quad \xi=1+\frac{2\alpha\chi_0}{\beta}(1-\beta^2).$$

(Here C is Euler's constant $0.577\cdots$). A transformation is made to another parameter B by means of

$$B-\xi\ln B=b,$$

and the multiple scattering angular distribution $f(\theta,t)$ is written in terms of B as an expansion parameter. It is clear that a change in the definition of χ_α from that due to Molière will have a corresponding effect on B.

Mühlschlegel and Koppe have given an expression for the depolarization due to multiple scattering in the form

$$P=P_0[1-(s^2/4B\bar{f})],$$

where P_0 is the initial polarization, P the final polarization after multiple scattering, $s=[1-(1-\beta^2)^{\frac{1}{2}}]^2\chi_c^2B$, and $\bar{f}$ is given by

$$\bar{f}(\vartheta)=f^0(\vartheta)+(1/B)f^{(1)}(\vartheta)+(1/2!B^2)f^{(2)}(\vartheta)$$

with $\vartheta=(\theta/\chi_cB^{\frac{1}{2}})$. In the expression for $\bar{f}$, the terms $f^{(1)\prime}$ and $f^{(2)\prime}$ of our previous paper have not been included, because their effect on the distribution function is small at the small angles involved in multiple scattering. A proper treatment of the large angle scattering must include these terms and the calculations of Mühlschlegel and Koppe have to be carried out without resorting to the small angle approximation. This we have not done here. However, formally the expression they have given for the depolarization due to multiple scattering is valid in our treatment also; only, the quantity B is to be calculated using the correct expression for χ_α given above. The change from Molière's B to our B is not negligible especially at the lower β values where Molière's B are somewhat smaller than the correct B. However, due to the occurrence of s^2 in the numerator and B in the denominator in the expression for the depolarization, the amount of depolarization is anyway extremely small and the error due to using Molière's theory does not make itself felt. In Table I we have presented the numerical value of $(s^2/4B\bar{f})$ at $\vartheta=1.0$ for scattering from gold foil of thickness 1 mg/cm^2 and β values ranging from 0.6 to 0.9. It is seen that the amount of depolarization is extremely small.

Mühlschlegel and Koppe have also derived an expression for the electrostatic rotation of the spins and it is given by

$$\eta=\sqrt{s}\vartheta[1-sa(\vartheta)].$$

Experimentally, $P_\perp$ which is the component of the polarization vector $\mathbf{P}$ perpendicular to the momentum is of significance. If η is the angle between $\mathbf{P}$ and the chosen z axis, then we have $P_\perp=-P\sin(\theta-\eta)$. Again the expression given above for η is formally correct. The correction to the electrostatic rotation due to multiple scattering is represented by the function $a(\vartheta)$ in their theory, and curves are plotted for $a(\vartheta)$ as a function of ϑ for different values of Molière's B should be replaced by our B in calculating the electrostatic rotation. In Table II we have given the value of η at $\vartheta=2.0$ for scattering from gold foil of thickness 1 mg/cm^2 and β values ranging from 0.6 to 0.9.

It is observed from the above that the calculated depolarization effects in multiple scattering are extremely small. Hence, it does not seem possible to attribute the deviations in the measured value of polarization from the theoretical value of $-\vartheta/c$ in the experiment of Frauenfelder et al.[4] to effects of multiple scattering in their even thinner analyzing foils.

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[4] H. Frauenfelder, R. Bobone, E. von Goeler, N. Levine, H. R. Lewis, R. N. Peacock, A. Rossi, and G. De Pasquali, Phys. Rev. **106**, 386 (1957); **107**, 909 (1957).