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Seven-Dimensional Charge Space

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Some implications of a charge space of seven (rotational) dimensions are considered for (I) particle classification, (II) strong interactions, and (III) weak interactions. The principal conclusions from this point of view are that (I) all particles except the photon and graviton can be incorporated in 7 dimensional charge space; lepton conservation must be abandoned, which automatically introduces parity nonconservation into β decay; (II) the one boson, two fermion interaction is predominantly pseudovector in form and induces no mass differences; the $\Sigma-\Lambda$ mass difference arises from interference with a fundamental two boson, two fermion interaction of lower charge symmetry; the $\Xi-N$ mass difference has an "intrinsic" basis and is not due entirely to strong interactions; (III) the weak universal Fermi interaction has signature -1 under time reversal; the strangeness change $\Delta S=\pm 1$ is associated with 256 independent (in principle) terms; the rule $\Delta I=\pm\frac{1}{2}$ needs further qualification because $\mathbf{I}=\mathbf{T}+\mathbf{U}$, where $\mathbf{T}$ and $\mathbf{U}$ are independent operators.

CHARGE space should be ascribed the dimensionality that affords the most compact description of elementary particles and their interactions. The present paper examines the case of a rotational space with seven dimensions.[1] Application to observed effects is made in a way to maximize the 7 dimensional symmetry and thus obtain a system that is simplest with respect to charge space; this at least provides an ideal scheme against which to measure any deviations of real particles. The discussion comprises three parts: particle classification, strong interactions, and weak interactions.

I. PARTICLE CLASSIFICATION

In this part we attempt to organize particles according to their "bare" properties, insofar as these can be inferred from observable particles. The classification includes all present particles except the photon and graviton, which are distinguished as the only ones having classical rather than quantum wave equations. These excluded fields supply external calibrations for the 7 dimensional scheme; specifically, all the bare-particle expressions below are supposed to be diagonal in charge and mass.

1. The Baryons

The known baryons can be arranged in a column,

$$\Psi=\begin{bmatrix}\Sigma^+\\ \Sigma^{0+}\\ \Sigma^{0-}\\ \Sigma^-\\ p\\ n\\ \Xi^0\\ \Xi^-\end{bmatrix}\quad\begin{array}{ccc}\rho_3 & \sigma_3 & \tau_3\\ +1 & +1 & +1\\ & & -1\\ & -1 & +1\\ & & -1\\ -1 & +1 & +1\\ & & -1\\ & -1 & +1\\ & & -1\end{array}\qquad \Sigma^{0\pm}=(\Sigma^0\pm\Lambda)/\sqrt{2},\quad(1)$$

and associated with the definitions of three independent Pauli spin operators in charge space, $\boldsymbol{\rho}$, $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$ as shown. The diagonal charge and mass operators are

$$q=S_3+T_3+U_3,\quad m=M-S_3\Delta,$$
$$2\mathbf{S}=\tfrac{1}{2}(1-\rho_3)\boldsymbol{\sigma},\quad 2\mathbf{T}=\boldsymbol{\tau},\quad 2\mathbf{U}=\tfrac{1}{2}(1+\rho_3)\boldsymbol{\sigma},\qquad(2)$$
$$M\approx 1.1\text{ Bev},\quad \Delta\approx 0.4\text{ Bev}.$$

The Σ-Λ and smaller mass differences are neglected as arising from the interactions, but the Ξ-N mass difference is regarded as "intrinsic" for two reasons discussed in later sections: there is a parallel μ-e mass difference, for which no explanation in terms of interactions seems possible; and baryon-boson interactions of the high charge symmetry considered here cannot cause a Ξ-N mass difference.[2]

The total wave function of a baryon is supposed to be of the form ΨX, where X is the usual real space wave function. This means that X is the same (except for intrinsic mass variations) for each of the eight particles in Eq. (1); namely, it has spin $\frac{1}{2}$ and parity $+$ (by definition). The eight varieties of baryon in Eq. (1) represent the simplest spinor in a 7 dimensional charge space, just as X is the simplest spinor in a 5 dimensional (rotational) real space.

Equation (1) suggests that there are no more hyperons to be found, for the simplest spinor in 7 dimen-

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[1] J. Tiomno, Nuovo cimento 6, 69 (1957); here the full 7 dimensional symmetry is not used but decomposed into disjoint 4- and 3-spaces.

[2] Another argument for intrinsic Ξ—(Λ,Σ)—N mass differences is that the strangeness selection rule on electromagnetic interaction then becomes a simple consequence of gauge invariance. Write the free Lagrangian as $L=\bar{\Psi}(\gamma_\mu\partial_\mu+m)\Psi$ and introduce the electromagnetic interaction by $\partial_\mu\rightarrow\partial_\mu-ie\,qA_\mu$, where m and q are charge space operators. Then the gauge transformation $A_\mu\rightarrow A_\mu+\partial_\mu\Omega$ is compensated by $\Psi\rightarrow\exp\{ie\,q\Omega\}\Psi$ only if

$$[m,q]=0.$$

With the intrinsic m assignment of Eq. (2) this is just the condition necessary to eliminate γ transitions between Σ^+ and p, Ξ^- and Σ^-, μ and e (below). From the present point of view intrinsic Δm are the main feature of "strangeness": they eliminate γ transitions, and the pion metastability of hyperons occurs because accidentally $m_\pi<\Delta/2<m_K$. A suggestion of intrinsic mass differences and a certain baryon-lepton analogy has also been made by Gamba, Marshak, and Okubo, Proc. Natl. Acad. Sci. (U.S.) 45, 881 (1959).

sional space has just eight entries, and these are all filled. Of course, if charge space were really 9 dimensional, the simplest spinor would have 16 components; or the next higher spinor in 7 dimensional space might have 24 components. In any case, further hyperons must appear in multiples of 8, and the most likely multiple indicated at present is zero.

2. The Bosons

A column of bosons diagonal in charge and mass is

$$\phi=\begin{array}{c}\\ \begin{bmatrix} K^+ \\ K^- \\ K^{0-} \\ K^{0+} \\ \pi^+ \\ \pi^- \\ \pi^0 \end{bmatrix}\end{array} \quad \begin{array}{ccc} S_3 & T_3 & U_3 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & 0 & +\frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{array} \quad K^{0+}=K^0, \quad K^{0-}=\bar{K}^0. \tag{3}$$

There are no boson operators exactly analogous to $\boldsymbol{\rho}$, $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$ for the fermions; but the bosons can be classified in terms of independent three-vectors $\mathbf{S}$, $\mathbf{T}$, and $\mathbf{U}$ that do possess exact baryon analogs,[3] given in Eq. 2 and discussed further in part II. The charge and mass operators are

$$q=S_3+T_3+U_3, \quad m=M'+(S_3)^2\Delta', \\ M'\approx 0.1 \text{ Bev}, \quad \Delta'\approx 1.6 \text{ Bev}. \tag{4}$$

Perturbation treatment of the boson self-energy loop, averaged over suitable fermion pairs with intrinsic mass differences according to Eq. (2), always yields an expression like Eq. (4). One is thus inclined to regard the K–π difference qualitatively as a reflection of the intrinsic baryon mass difference already assumed, despite inability to say anything about relative magnitudes because of divergences.

The 7-element column (3) represents a vector in charge space. As before the total wave function is ϕy, where y is a real space wave function identical for all charge states (except for intrinsic mass effects); in particular, K as well as π must have spin and parity 0^-. Also, there is no room in Eq. (3) for the discovery of additional bosons of the same class. It is somewhat easier to find additional bosons of other classes than in the baryon case, however: the one-component charge scalar would be the π^{00} that has been several times postulated[4,5] though never observed. If charge space were 9 dimensional, there would be two more bosons in the vector ϕ. These two possibilities are the only simple ones, for the tensor of next lowest order in 7 dimensional charge space has 21 components. Mere complexity argues against the usefulness of any such construct. The charged vector meson hypothesized[6,7] as a β decay intermediary has no place in this scheme, where several charged and neutral components must be assigned to each boson state in real space.

The absence of the π^{00}, at first sight perhaps surprising, is in accord with a simple compound model.[8] If the charge vector (K,π) is composed of a baryon-antibaryon pair, they are presumably in a $^1S_0$ state to form a pseudoscalar. The exclusion principle then requires for the charge scalar π^{00} a $^1P_1$ state, which may have considerably less "binding" than the $^1S_0$ state.[9] which may have considerably less "binding" than the If the bosons were elementary particles in the same sense as the baryons, one would rather expect the simplest element π^{00} to have the lowest mass.

A feature of both baryons and bosons is the correlation of what might be called their real space and charge space "statistics." That is, the spinor Ψ is associated with a spinor X, the tensor ϕ with the tensor y. This association appears first for 7 dimensional charge space and is one of the most appealing arguments against lower dimensionalities.

There are some hints here that bosons in a 7 dimensional scheme are most economically regarded as secondary rather than primary particles: the absence of the π^{00} is explainable on a compound model, and the K–π mass difference may simply reflect the intrinsic baryon mass difference. Accordingly, we shall exclude any fundamental boson-boson or boson-lepton interactions.

3. The Leptons

This section attempts to fit the leptons into 7 dimensional charge space, proceeding by analogy with the above. The association of real and charge space statistics indicates an eight-element column; one diagonal in mass and charge is

$$\psi'=\begin{bmatrix} e^+ \\ \nu^0 \\ \gamma_5\nu^{0c} \\ \gamma_5 e^- \\ \mu^+ \\ \lambda^0 \\ \gamma_5\lambda^{0c} \\ \gamma_5\mu^- \end{bmatrix}, \tag{5'}$$

[3] The quantities I_3, J_3, J_3' of reference 1 are here T_3, $(S+U)_3$ and $(S-U)_3$, respectively. T. Okabayashi, Progr. Theoret. Phys. (Kyoto) **21**, 867 (1959), uses S_3, T_3, and U_3 directly, denoting them by $\frac{1}{2}\eta$, ϑ_3 and $\frac{1}{2}\xi$.

[4] Y. Nambu, Phys. Rev. **106**, 136 (1957).

[5] A. M. Baldin, Nuovo cimento **8**, 569 (1958); Y. Yamaguchi, Progr. Theoret. Phys. (Kyoto) **19**, 622 (1958); S. N. Gupta, Phys. Rev. **111**, 1436, 1698 (1958); W. Krolikowski, Nuclear Phys. **10**, 213 (1959).

[6] J. Schwinger, Ann. phys. **2**, 407 (1957).

[7] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[8] E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949); M. A. Markov, Doklady Akad. Nauk SSSR **101**, 54, 449 (1955); S. Sakata, Progr. Theoret. Phys. (Kyoto) **16**, 686 (1956); R. W. King and D. C. Peaslee, Phys. Rev. **106**, 360 (1957); H. P. Stapp, Phys. Rev. Letters **1**, 296 (1958); L. B. Okun, J. Exptl. Theoret. Phys. U.S.S.R. **34**, 469 (1958) [translation: Soviet Phys. JETP **7**, 322 (1958)].

[9] The π^{00} will then be a vector meson as in reference 4 rather than a pseudoscalar as in reference 5.

where ν^{0c}, λ^{0c} are the charge conjugates of ν^0, λ^0. Here γ_5 is the usual Dirac matrix in real space; it is appended so that the total wave function of a bare particle can again be written as $\psi\xi$ with ξ a constant function in real space (except for intrinsic mass) for all elements of ψ, while at the same time assigning opposite intrinsic parities to particle and antiparticle. This peculiar feature of Eq. (5′) is dwelt on below; with ρ_3, σ_3, τ_3 assignments as in Eq. (1), the charge and mass formulas are

$$q=S_3+T_3+U_3, \quad m=M_0-S_3\Delta_0, \quad M_0=0, \quad \Delta_0\approx 0.2 \text{ Bev}. \tag{6′}$$

The change in sign of mass indicated by Eq. (6′) is in accord with the presence of the γ_5. The μ mass appears here as intrinsic, and the e mass as resulting from interactions, although this is a theoretical mystery at present.[10] If one assumes Δ_0 the same for bare baryons as for leptons, then $\Delta'/\Delta_0\approx 2$ measures the baryon mass renormalization, suggesting $M_0\approx 0.6$ Bev, for the unrenormalized baryon mass.

The correspondence between Eqs. (6′) and (2) determines uniquely the μ, e positions in Eq. (5′). From this alone a few conclusions can be drawn:

(i) any type of electromagnetic transition between μ and e is forbidden[11] to all orders by the same strangeness rule[2] that prevents $\Sigma^+\rightarrow p+\gamma$, $\Lambda\rightarrow n+\gamma$, etc.; this holds whether the neutrinos in $\mu\rightarrow e+\nu+\nu$ are identical or not.

(ii) The electrodynamic corrections to the μ and e magnetic moments must both be $\alpha/2\pi$ to lowest order[12]; to higher orders the corrections should differ only in intrinsic mass effects.

(iii) An excuse arises for the existence of the μ meson, otherwise irritatingly superfluous. There are two positions for charged particles of each sign in Eq. (5′), which must be filled by leptons similar in space-like character but linearly independent.

This last feature is absent from charge spaces of lower dimensionality than seven; with higher dimensionality, it is necessary to add more independent charged leptons, for which empirical evidence is lacking.

The most striking aspect of Eq. (5′) is the necessity of using both particle and antiparticle in order to fill up the eight entries in the column. This implies that somehow or other the distinction between leptons and antileptons is of less import than that between baryons and antibaryons.[13] If the superscript a denotes antiparticle, then Eqs. (1) and (5′) imply

$$(\Psi X)\underset{\text{(nonlinear)}}{\longleftrightarrow}(\Psi X)^a, \qquad (\psi'\xi)\underset{\text{(linear)}}{\longleftrightarrow}(\psi'\xi)^a. \tag{7}$$

This suggests as a working hypothesis [called hypothesis (Z) hereafter] that leptons and antileptons are experimentally indistinguishable, which is their essential difference from baryons; with corollaries

(i) no measurement can determine "relative leptonness" of two leptons (*e.g.*, e^- and μ^+ or μ^-; e^- and ν^0 or ν^{0c}).

(ii) massless leptons are effectively Majorana neutrinos.

Corollary (i) is automatically satisfied by the electromagnetic interaction, which is diagonal in all particles of Eq. (5′). The appendix shows that for β decay this condition imposes factors $(1\pm\gamma_5)$ between all lepton pairs and prohibits mixture of (A, V) with (S, T, P) terms in the interaction form without derivatives. For lepton and antilepton to be indistinguishable in boson-lepton interactions requires the mass of ν^0 (and of λ^0, if it is to play any role in β decay) to vanish identically and two charged (i.e., massive) leptons not to interact with a boson. This last precludes boson-lepton interactions with any rotational invariance in charge space except about the Q_3 axis; from the absence of strong boson-lepton interactions one then infers that all strong boson-fermion interactions must have a higher degree of invariance in charge space (part II).

If the masses of ν^0 and λ^0 both vanish, it seems unnecessary to maintain a distinction between the two neutrinos, so we assume $\lambda^0\equiv\nu^0$ and multiply each by a normalizing factor $1/\sqrt{2}$. Whatever the intrinsic character of ν^0, only the Majorana-type combinations $(\nu^0+\nu^{0c})/\sqrt{2}=\nu$ and $\gamma_5(\nu^{0c}-\nu^0)/\sqrt{2}=\lambda$ occur in β decay, according to the argument in part III, Eq. (29). This is the basis for corollary (ii). The only effect of ascribing Dirac character to ν^0—which we now do—is to prevent λ from vanishing identically.

A necessary *ad hoc* step, justified only because it leads to Eq. (29), a satisfactory starting point for a universal Fermi interaction, is to exchange the 3rd and 6th entries in Eq. (5′). This is effected by the transformation

$$R=\tfrac{1}{2}(1+\sigma_3\tau_3)+\tfrac{1}{4}[(1-\sigma_3\tau_3)+\rho_3(\sigma_3-\tau_3)] +\frac{1}{2\sqrt{2}}(\rho_+\tau_+\sigma_-+\rho_-\tau_-\sigma_+),$$

which yields

$$\psi=R\psi'=\begin{bmatrix} e^+ \\ \nu^0/\sqrt{2} \\ \nu^0/\sqrt{2} \\ \gamma_5 e^- \\ \mu^+ \\ \gamma_5\nu^{0c}/\sqrt{2} \\ \gamma_5\nu^{0c}/\sqrt{2} \\ \gamma_5\mu^- \end{bmatrix}, \tag{5}$$

[10] Katayama, Taketani, and Ferreira, Progr. Theoret. Phys. (Kyoto) **21**, 818 (1959).

[11] Davis, Roberts, and Zipf, Phys. Rev. Letters **2**, 211 (1959); Berley, Lee, and Bardon, Phys. Rev. Letters **2**, 357 (1959).

[12] Lundy, Sens, Swanson, Telegdi, and Yovanovitch, Phys. Rev. Letters **1**, 38 (1958); Garwin, Hutchinson, Penman, and Shapiro, Phys. Rev. Letters **2**, 516 (1959).

[13] A distinction between baryons and leptons on the basis of their particle-antiparticle relations has also been suggested by G. Schremp, Phys. Rev. **113**, 936 (1959).

and replaces the operator S_3 in Eq. (6′) by

$$\left(\frac{1-\rho_3}{2}\right)\left(\frac{\sigma_3+\tau_3}{2}\right)+\rho_3\left(\frac{1+\tau_3}{2}\right)\left(\frac{1-\sigma_3}{2}\right).$$

The second of these terms is self-contradictory in assigning different masses to identical particles in Eq. (5) and is therefore discarded, leaving the satisfactory form

$$m=M_0-\left(\frac{1-\rho_3}{2}\right)\left(\frac{\sigma_3+\tau_3}{2}\right)\Delta_0. \tag{6}$$

The procedures of the last paragraph are arbitrary and tentative: it is not clear that Eq. (5) is unique or even the most suitable lepton function. In lieu of anything better, however, we shall use it in the universal Fermi interaction of part III.

Whatever the inadequacies of hypothesis (Z), it is at least satisfying that charge space considerations should yield a baryon-lepton distinction. Otherwise one would have two equal families of fermions in charge space and no reason to suppose that the roster was complete. As it is, however, there exists one family each of full-bodied and eviscerated fermions; and the principle of distinction between them cannot generate any new families of fermions or bosons.

If hypothesis (Z) is correct, one should explain why lepton conservation seems to work so well for β-decay processes. As illustrated in the appendix, this is a result of the $(1\pm\gamma_5)$ factors in the interaction: they produce a 100% ν polarization that makes all simple β-processes go *as if* they satisfied lepton conservation with a 2-component neutrino. Actually, we here deny lepton conservation, and later discussion will also deny any physical significance to the 2-component neutrino, which appears as an algebraic accident of the four-fermion interaction.

II. STRONG INTERACTIONS

The absence of strong boson-lepton interactions, which would necessarily be charge asymmetric, suggests that strong boson-baryon interactions be given maximum symmetry in 7 dimensional charge space. As in the lepton case, charge space considerations imply some restriction on the real space form of the interactions. The sections below discuss the charge rotation operators for fermions and bosons and the interaction forms between two baryons and one or two bosons. The real space wave functions are henceforth absorbed into Ψ and ϕ, which are now subject to both real and charge space operators, distinguished by respective Greek and Roman subscripts.

1. Fermion Charge Rotations

To exhibit 7 dimensional rotational properties, a Dirac-type formalism is convenient. Define in terms of $\boldsymbol{\rho}$, $\boldsymbol{\sigma}$, $\boldsymbol{\tau}$ associated with spinor (1) the quantities Γ_A,

$$\begin{aligned}
&\Gamma_1=\rho_1\sigma_1, \quad \Gamma_5=\rho_3\sigma_1, \quad \Gamma_{AB}+\Gamma_{BA}=2\delta_{AB},\\
&\Gamma_2=\rho_1\sigma_2, \quad \Gamma_6=\rho_3\sigma_2, \quad \Gamma_{1234567}=(i)^3,\\
&\Gamma_3=\rho_1\sigma_3, \quad \Gamma_7=\rho_3\sigma_3,\\
&\Gamma_4=\rho_2,
\end{aligned} \tag{8}$$

This set of Γ_A is by no means unique: an 8-fold variation is obtainable by transformations

$$\begin{aligned}
&\Gamma_A'=R'\Gamma_A(R')^{-1},\\
&R^x=(R^x)^{-1}=\tfrac{1}{2}(1+\rho_3+\sigma_3-\rho_3\sigma_3),\\
&R^y=(R^y)^{-1}=\tfrac{1}{2}(1+\rho_3+\tau_3-\rho_3\tau_3),\\
&R^z=(R^z)^{-1}=\tfrac{1}{2}(1+\sigma_3+\tau_3-\sigma_3\tau_3).
\end{aligned} \tag{9}$$

The Γ_A obtainable in this way are shown in Table I. Changing the relative signs of the different terms comprising R in Eq. (9) leads only to variations in sign among the Γ_A of Table I, and not to any new forms; it is sufficient to consider only the transformations (9). In fact, all the forms of Table I correspond simply to relative sign changes of some components of Eq. (1), as seen by considering $R\Psi$; and since there are no interference measurements for detecting such changes, all columns in Table I are physically equivalent.[14] We shall therefore use representation (8) throughout for the Γ_A.

There are 21 rotation operators,

$$\Sigma_{AB}=(\Gamma_{AB}-\Gamma_{BA})/2i, \tag{10}$$

corresponding to the full set of 7 dimensional rotations. It is convenient to express nine of these operators as three commuting 3-vectors:

$$\begin{aligned}
&S_3=\tfrac{1}{4}(\Sigma_{12}-\Sigma_{34}) \quad \text{and cyclic},\\
&U_3=\tfrac{1}{4}(\Sigma_{12}+\Sigma_{34}) \quad \text{and cyclic},\\
&T_3=\tfrac{1}{2}\Sigma_{56}, \quad T_2=\tfrac{1}{2}\Sigma_{75}, \quad T_1=\tfrac{1}{2}\Sigma_{67},
\end{aligned} \tag{11}$$

with eigenvalues $(0, \frac{1}{2})$ for S and U, $\frac{1}{2}$ for T. The operators $\mathbf{S}$, $\mathbf{T}$, $\mathbf{U}$ are all that remain valid upon a breakdown of the 7 dimensional symmetry into disjoint 4 dimensional and 3 dimensional spaces associated, respectively, with ($\mathbf{S}$, $\mathbf{U}$) and $\mathbf{T}$. Even if this breakdown is not assumed, several physical quantities can be

TABLE I. Independent forms of Γ_A. Note the uniformity of Γ_7.

A	Γ_A	Γ_{A^x}	Γ_{A^y}	Γ_{A^z}	$\Gamma_{A^{xy}}$	$\Gamma_{A^{xz}}$	$\Gamma_{A^{yz}}$	$\Gamma_{A^{xyz}}$
1	$\rho_1\sigma_1$	$\rho_2\sigma_2$	$\rho_1\sigma_1\tau_3$	$\rho_1\sigma_1\tau_3$	$\rho_2\sigma_2\tau_3$	$\rho_2\sigma_2\tau_3$	$\rho_1\sigma_1$	$\rho_2\sigma_2$
2	$\rho_1\sigma_2$	$-\rho_2\sigma_1$	$\rho_1\sigma_2\tau_3$	$\rho_1\sigma_2\tau_3$	$-\rho_2\sigma_1\tau_3$	$-\rho_2\sigma_1\tau_3$	$\rho_1\sigma_2$	$-\rho_2\sigma_1$
3	$\rho_1\sigma_3$	ρ_1	$\rho_1\sigma_3\tau_3$	$\rho_1\sigma_3$	$\rho_1\tau_3$	ρ_1	$\rho_1\sigma_3\tau_3$	$\rho_1\tau_3$
4	ρ_2	$\rho_2\sigma_3$	$\rho_2\tau_3$	ρ_2	$\rho_2\sigma_3\tau_3$	$\rho_2\sigma_3$	$\rho_2\tau_3$	$\rho_2\sigma_3\tau_3$
5	$\rho_3\tau_1$	$\rho_3\tau_1$	τ_1	$\rho_3\tau_1\sigma_3$	τ_1	$\rho_3\tau_1\sigma_3$	$\tau_1\sigma_3$	$\tau_1\sigma_3$
6	$\rho_3\tau_2$	$\rho_3\tau_2$	τ_2	$\rho_3\tau_2\sigma_3$	τ_2	$\rho_3\tau_2\sigma_3$	$\tau_2\sigma_3$	$\tau_2\sigma_3$
7	$\rho_3\tau_3$	$\rho_3\tau_3$	$\rho_3\tau_3$	$\rho_3\tau_3$	$\rho_3\tau_3$	$\rho_3\tau_3$	$\rho_3\tau_3$	$\rho_3\tau_3$

[14] The author is indebted to M. T. Vaughn for clarifying this point.

expressed in terms of **S**, **T**, **U**. The strangeness is

$$S=2S_3-\mathfrak{N}, \tag{12}$$

where $\mathfrak{N}$ = baryon number; and the charge vector

$$\mathbf{Q}=\mathbf{S}+\mathbf{T}+\mathbf{U}, \tag{13}$$

has the charge number as its third component, $q=Q_3$.

The component Q_2 of the charge vector figures in "antiparticulation,"

$$\Psi^a=A\bar{\Psi}=(-1)^{2J}e^{i\pi Q_2}C\bar{\Psi}, \tag{14}$$

where J is the particle spin and C is the real space charge conjugation operator.[15] A well-known feature of A is that

$$A(\Gamma_D O)A^{-1}=\pm(\tilde{\Gamma}_D\tilde{O}), \tag{15}$$

where O is a real space operator and the signature is $-$ for $O=SAP$, $+$ for $O=TV$.

2. Boson Charge Rotations

An elementary procedure for the bosons is to form Kemmer-type matrices,[16] $B_A=\frac{1}{2}(\Gamma_A+\Gamma_A')$, where Γ_A and Γ_A' are independent. This 64-rowed representation is reducible, containing representations of 1, 7, 21, and 35 rows. The 7-rowed B_A ($A=1$ to 6) have a simple representation exactly analogous to that of the usual[16] 5-rowed β_μ ($\mu=1$ to 4); the corresponding column ϕ consists simply of the real elements ϕ_A ($A=1$ to 7) where

$$\begin{aligned}&(2)^{-\frac{1}{2}}(\phi_1\mp i\phi_2)=K^{\pm}, \quad (2)^{-\frac{1}{2}}(\phi_5\mp i\phi_6)=\pi^{\pm},\\ &(2)^{-\frac{1}{2}}(\phi_3\pm i\phi_4)=K^{0\pm}, \quad \phi_7=\pi^0,\end{aligned} \tag{16}$$

The charge rotation operators now follow the usual definition[16] for Kemmer matrices,

$$\begin{aligned}&\Sigma_{AB}=(B_{AB}-B_{BA})/i, \quad A,B=1 \text{ to } 6,\\ &\Sigma_{A7}=B_A\eta_7/i, \quad \eta_7=\eta_{123456}, \quad \eta_A=2B_A{}^2-1.\end{aligned} \tag{17}$$

One can again define quantities **S**, **T**, **U** by Eq. (11) with the inclusion of an extra factor 2 throughout, as is usual in going from the spin $\frac{1}{2}$ to spin 1 case; the assignments resulting from Eqs. (16) and (17) are those given in Eq. (3). It is not possible to work backwards from **S**, **T**, **U** to a quantity like the fermion $\boldsymbol{\rho}$, although $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ may be obtained.

An important feature of assignment (3) is that the isotopic spin

$$\mathbf{I}=\mathbf{T}+\mathbf{U} \tag{18}$$

is composed of two parts **T** and **U** that serve, respectively, for pions and kaons. Since these parts are independent, no simple rule like $\Delta I=\frac{1}{2}$ is sufficient to describe weak, strangeness-violating decays; one must specify in detail what happens to T and U. This remark applies not only to boson decays like $K\rightarrow 2\pi$ but also to hyperon decays. The example of a weak interaction assembled in part III has in fact no particular connection with $\Delta I=\frac{1}{2}$, although it gives many of the same results.

For the bosons, antiparticulation is particularly simple,

$$A\phi=-\phi. \tag{19}$$

3. Two Fermion, One Boson Interaction

The simplest interaction form is a complete scalar in charge space,

$$G_1\bar{\Psi}\Gamma_A(O\phi_A)\Psi, \tag{20}$$

where O is some real space operator. Equation (20) implies complete symmetry among all π- and K-baryon interactions.[17] If O is the usual pseudoscalar operator, one must attribute the observed order-of-magnitude difference in $G_\pi{}^2$ and $G_K{}^2$ to renormalization effects. There are two objections to this: in a highly symmetric scheme there is no obvious source of such a large difference in vertex renormalizations; and as discussed in part III, the weak decay interactions suggest that the vertex renormalization is small in absolute magnitude, substantially less than a factor 2. The dominant part of Eq. (20) must therefore be

$$O\phi_A=i\gamma_5\gamma_\eta\phi_{A\eta}, \quad \phi_{A\eta}=(\partial_\eta\phi_A)/\kappa_A, \tag{21}$$

where κ_A is the observed rest mass of the meson. This normalizing factor appears quite naturally if one uses linearized (i.e., Kemmer) equations in real space for the boson wave functions: then the five quantities, ϕ, φ_η all appear on an equal footing, so that (21) is the exact PV analog of the usual PS interaction

$$O'\phi_A=\gamma_5\phi_A. \tag{21'}$$

Of course it is possible to have some admixture of (21′) with (21); the present exploratory discussion will employ only Eq. (21), however.

A diagram[18] of Eq. (20) is shown in Fig. 1. Such a diagram may be useful in describing higher spinors in 7 dimensional space.[19] For example, the $J=T=\frac{3}{2}$ resonance observed in pion-nucleon scattering must have some generalization in going from 3 to 7 dimensional charge space. The appropriate diagram is constructed by placing an additional cube on each face of the cube in Fig. 1, and then removing the central cube. This structure contains 48 corners or elements and is the next simplest spinor to Eq. (1). Of these 48 elements, 16 represent $J=\frac{3}{2}$ resonances in $\pi-N$, $\pi-\Xi$, $\pi-\Lambda$ and $\pi-\Sigma$ scattering; the others represent possible

[15] A is a straightforward extension of a similar operator long used in pion-nucleon considerations: A. Pais and R. Jost, Phys. Rev. **87**, 871 (1952); L. Michel, Nuovo cimento **10**, 319 (1953).

[16] N. Kemmer, Proc. Roy. Soc. (London) **173**, 91 (1939).

[17] This form with Eq. (21′) is suggested by R. E. Behrends, Nuovo cimento **11**, 424 (1959), along with an alternate form for baryons of different parity in order to explain the mass differences along the lines of reference 6; see H. Katsumori and K. Shimoura, Progr. Theoret. Phys. (Kyoto) **20**, 578 (1959).

[18] N. Dallaporta, Nuovo cimento **7**, 200 (1958).

[19] Thanks are expressed to Professor H. A. Jahn for helpful remarks on this subject.

$J=\frac{3}{2}^+$ resonances in K-baryon scattering. Whether these latter resonances actually appear is a moot question and would require much more detailed investigation than attempted here.

4. Two Fermion, Two Boson Interactions

Although interaction (20) amplifies any intrinsic mass differences that may exist, it does not originate any, in particular the Σ–Λ mass difference. Moreover, Eq. (20) entirely forbids[20] the reactions $K^-+p\rightarrow K^{0-}+n$, $\pi^++p\rightarrow K^++\Sigma^+$ and imposes experimentally violated restrictions on other production processes.[21] These difficulties can be removed by introducing a second basic interaction,

$$(G_2/M)\{\bar{\Psi}I_A\gamma_\eta\Psi\}\{\phi E_A\phi_\eta\}, \tag{22}$$

where I_A, E_A are charge space operators and $M\approx 1.1$ Bev is a normalizing mass. To yield a Σ–Λ mass difference, Eq. (22) must not be invariant under all 21 operators of 7 dimensional rotations; but to show charge independence and strangeness conservation, it must be invariant under at least $\mathbf{Q}$ and S_3. A systematic way of obtaining such a form is the following: arrange the 21 Σ_{AB} in seven linear combinations of three each and call these E_A, while setting

$$I_A=\Gamma_A. \tag{23}$$

Invariance under $\mathbf{Q}$ and S_3 is sufficient to determine the E_A except for two coefficients:

$$\begin{aligned}
&E_1=\Sigma_{27}+\Sigma_{63}-\Sigma_{54}, && E_5=a(\Sigma_{23}+\Sigma_{14})-b\Sigma_{67},\\
&E_2=\Sigma_{35}+\Sigma_{71}-\Sigma_{64}, && E_6=a(\Sigma_{31}+\Sigma_{24})-b\Sigma_{75},\\
&E_3=\Sigma_{16}+\Sigma_{52}-\Sigma_{74}, && E_7=a(\Sigma_{12}+\Sigma_{34})-b\Sigma_{56}.\\
&E_4=\Sigma_{51}+\Sigma_{62}+\Sigma_{73}, &&
\end{aligned} \tag{24}$$

One way of defining the coefficients a, b is to introduce an operator that behaves under $\mathbf{Q}$ like a 3 dimensional tensor of second rank,

$$\begin{aligned}
&V_{11}=\tfrac{4}{3}[\Sigma_{15}-\tfrac{1}{2}(\Sigma_{26}+\Sigma_{37})], && V_{12}=V_{21}=\Sigma_{16}+\Sigma_{25},\\
&V_{22}=\tfrac{4}{3}[\Sigma_{26}-\tfrac{1}{2}(\Sigma_{15}+\Sigma_{37})], && V_{23}=V_{32}=\Sigma_{27}=\Sigma_{36},\\
&V_{33}=\tfrac{4}{3}[\Sigma_{37}-\tfrac{1}{2}(\Sigma_{15}+\Sigma_{26})], && V_{31}=V_{13}=\Sigma_{35}+\Sigma_{17}.
\end{aligned} \tag{25}$$

Equation (22) is invariant under V_{ij} (for bosons, $\frac{1}{2}V_{ij}$ for fermions) if

$$a=b=1. \tag{24a}$$

Whatever the physical meaning of V_{ij}, it is satisfactory that Eq. (22) should be invariant under 9 charge rotation operators, which is the number associated with the breakdown of 7 dimensional symmetry into disjoint 4 dimensional plus 3 dimensional.

It may be useful to write out the second factor $\phi E_A\phi_\eta$ of Eq. (22) according to Eqs. (24) and (24a):

$$\begin{aligned}
(E_1\pm iE_2)/\sqrt{2}&\rightarrow\pm[\pi^0K^\mp-\pi^\mp K_3\pm i\pi^\mp K_4]_\eta,\\
E_3&\rightarrow[\pi^-K^+-\pi^+K^-+i\pi^0K_4]_\eta,\\
E_4&\rightarrow -i[\pi^-K^++\pi^+K^-+\pi^0K_3]_\eta,\\
(E_5\pm iE_6)/\sqrt{2}&\rightarrow\pm[K_3K^\mp-\pi^0\pi^\mp\pm iK_4K^\mp]_\eta,\\
E_7&\rightarrow[K^-K^+-\pi^-\pi^++iK_4K_3]_\eta,\\
[AB]&\equiv\phi_A\phi_{B\eta}-\phi_B\phi_{A\eta}.
\end{aligned} \tag{26}$$

Here the charged mesons are all destruction operators.

An approximate magnitude for G_2 may be inferred from analysis of s-wave π-nucleon scattering,[22]

$$G_2/4\pi\approx 0.3\gtrsim G_1^2/4\pi\approx 0.08. \tag{27}$$

Of course the magnitude of G_2 depends on the value assumed for the normalizing mass M; the choice $M\approx 1.1$ Bev corresponds to the cutoff energy in the treatment of π-nucleon scattering with PV coupling. In any case, it seems that G_2 and G_1^2 have the same order of magnitude, so that (22) and (20) could contribute comparably to strange particle production. Evaluation shows that (22) contributes directly to $\pi^++p\rightarrow K^++\Sigma^+$ and that (22) and (20) together can give general agreement with other production processes.[21] A Σ–Λ mass difference arises from interference between (20) and (22), though not for either interaction separately.

Of course Eq. (22) opens a Pandora's box: one can equally well assume independent, basic interactions of two fermions with 3, 4, $\cdots n\cdots$ bosons; and Eq. (27) gives no assurance of decreasing G_n with increasing n. Terms higher than G_2 will not be considered here; the higher terms presumably show at least as much rota-

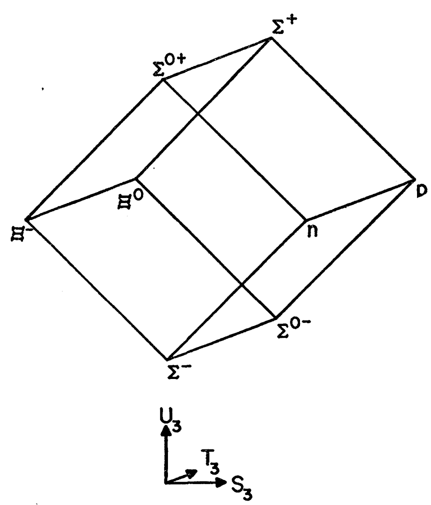

FIG. 1. The symmetric strong interaction (20). The corners of the cube are baryons, the connecting links are K and π mesons appropriate to the charge and strangeness change.

[20] It also forbids $K^-+p\rightarrow K^++\Xi^-$ but allows $K^++p\rightarrow K^{0-}+\Xi^0$.

[21] A. Pais, Phys. Rev. **110**, 574 (1958). See also Feinberg, Kabir, and Weinberg, Phys. Rev. Letters **3**, 527 (1959).

[22] Drell, Friedman, and Zachariasen, Phys. Rev. **104**, 236 (1956); A. Klein and B. H. McCormick, Progr. Theoret. Phys. (Kyoto) **20**, 876 (1958).

tional invariance as Eq. (22) and are also invariant under C and A.

III. THE WEAK INTERACTION

This section attempts to follow the general ideas of 7 dimensional charge space in constructing a universal Fermi interaction. We get only as far as showing how this can generally be done and exhibiting a couple of examples; it seems premature to propose a detailed interaction form on the basis of presently available data. First order perturbation theory is used throughout.

1. Introduction

The four-fermion interaction is taken as

$$\sum_A c_A\{\bar{\Psi}P_A\gamma_\eta(1+\epsilon_A\gamma_5)\Psi\}\{\bar{\Psi}P_A'\gamma_\eta(1+\epsilon_A'\gamma_5)\Psi\}, \quad (28)$$

where the Ψ are baryon functions of both charge and real space, the P_A are charge space operators, and c_A are coefficients. According to the appendix this form by itself does not assure baryon conservation, so we must additionally assume both Ψ functions in a single bracket to have the same baryon number; with a complete set of P_A there is no loss of generality in taking all four Ψ in Eq. (28) to have the same baryon number. It is also possible to arrange things so that lepton interactions are described by pairwise substitution of ψ for Ψ in either bracket but not by substitution for one Ψ in each bracket, which might violate baryon conservation. This substitution must employ a form of ψ invariant under antiparticulation; for Eq. (28) can be formally rewritten in terms of Ψ^a with appropriately transformed P_A, and according to hypothesis (Z) there can be no distinction between putting $\Psi\rightarrow\psi$ in the original form of Eq. (28) and $\Psi^a\rightarrow\psi$ in the transformed (28). This requirement is satisfied by the rule

$$\Psi\rightarrow\psi^+=\tfrac{1}{2}(\psi+\psi^a)=\begin{bmatrix}\tfrac{1}{2}(1+\gamma_5)e^+\\ \tfrac{1}{2}\nu\\ \tfrac{1}{2}\nu\\ -\tfrac{1}{2}(1-\gamma_5)e^-\\ \tfrac{1}{2}(1+\gamma_5)\mu^+\\ \tfrac{1}{2}\lambda\\ \tfrac{1}{2}\lambda\\ -\tfrac{1}{2}(1-\gamma_5)\mu^-\end{bmatrix}, \quad (29)$$

using Eq. (5) for ψ and the A appropriate to Eq. (8). The quantities

$$\nu=(\nu^{0c}+\nu^0)/\sqrt{2},\quad \lambda=\gamma_5(\nu^{0c}-\nu^0)/\sqrt{2}, \quad (30)$$

are linearly independent; in what follows they will be treated as orthogonal Majorana neutrinos.[23]

Equation (29) guarantees maximum polarization in the observed sense for all lepton processes; this polarization results not from two-component neutrinos, however, but from effectively "two-component" electrons and μ mesons! This simply illustrates that the two-component neutrino—or two-component lepton in general—is merely a mathematical convenience and has no special physical significance.

The choice of ρ factors for P and P' in Eq. (28) can be made in two nonequivalent ways: either $(P,P')\sim(\rho_1,\rho_2)$ or $(1,\rho_3)$, or $P\sim(1,\rho_3)$ and $P'\sim(\rho_1,\rho_2)$. The second form is necessary if Eq. (28) is to account for the pion decay of hyperons; but there is no evidence against including the first as well; we provisionally omit it for simplicity. Then in terms of strangeness, the P' factor induces $\Delta S=\pm1$, the P factor $\Delta S=0$ (if we specifically choose forms to exclude the $\Delta S=\pm2$ that is also allowed in general for P). Then all weak decay processes involve exactly $\Delta S=\pm1$: in ordinary β decay the leptons carry $\Delta S=\pm1$, in $K-\mu$ decay the leptons carry $\Delta S=0$. Which process one describes depends on which bracket in Eq. (28) undergoes $\Psi\rightarrow\psi^+$; substitution of both brackets leads to $\mu-e$ decay.

The factors $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ in P and P' are associated with electric charge conservation, which implies that Eq. (28) is a scalar, 3-component of a vector, or 33-component of a tensor under the rotation operator $\mathbf{Q}$. These three possibilities have alternating behavior under time reversal, which in the present scheme is equivalent to transposition of the charge space operators: the scalar is of course invariant, the 3-component of a vector cross product has -1 signature, and the tensor 33-component has signature $+1$. Since a uniform signature is required for the observed distinction between θ^1 and θ^2, the possibilities separate into two nonmixing groups—vector, or scalar plus tensor. The charge vector has the attractive feature that each bracket in Eq. (28) contains one charged and one neutral particle, as required for leptons (appendix). Adopting the vector form, we write

$$\begin{aligned}H(\beta)=ig\sum_A c_A[&\{\bar{\Psi}P_A{}^+\gamma_\eta(1+\epsilon_A\gamma_5)\Psi\}\\ &\times\{\bar{\Psi}P_A{}'^-\gamma_\eta(1+\epsilon_A')_5)\Psi\}\\ &-\{\bar{\Psi}P_A{}^-\gamma_\eta(1+\epsilon_A\gamma_5)\Psi\}\\ &\times\{\bar{\Psi}P_A{}'^+\gamma_\eta(1+\epsilon_A'\gamma_5)\Psi\}], \quad (31)\end{aligned}$$

where the second term is the Hermitian conjugate of the first. Here the superscripts $\pm$ indicate that the charge space operators contain factors $\sigma^\pm$ or $\tau^\pm$ (and in the case of P' a factor $\rho^\pm$ or $\rho^\mp$ as well), to which respective factors τ_3, σ_3 may be adjoined. The relation $H(\beta)^T=-H(\beta)$ does not contradict the usual tests of time-reversal invariance in β decay, which would only reveal phase differences between different coefficients in Eq. (31) but can never show the presence of a constant multiplying phase factor i. This choice implies that in the present scheme $K_3=\theta^1\rightarrow2\pi$, while $K_4=\theta^2$; thus (K_1, K_2, K_3) is the 3-vector analogy to (π_1, π_2, π_3). The form (31) precludes a number of unobserved processes, like $\mu\rightarrow3e$, $\mu+N\rightarrow N+e$, $K^0\rightarrow\pi^0+\mu+e$, $K^\pm\rightarrow\pi^\pm+\nu+\nu$; all but the last of these would reveal "relative leptonness" of μ and e.

[23] Dual neutrinos also appear in A. Salam and J. C. Ward, Nuovo cimento 11, 568 (1959).

Any weak interactions involving bosons are assumed to occur entirely through interaction (31) with the mediation of strong interactions[24,25] like (20) and (22). The β-decay boson[26,27] is specifically excluded: the general flavor of part I is that bosons are secondary particles, so that an elementary four-particle interaction should involve only fermions; and the absence[11] of $\mu \to e+\gamma$ provides some experimental evidence against the β-decay boson.[27] The principle of conserved currents in β-decay[28,7] is not in accord with the absence of primary boson-lepton interactions, nor is it especially indicated for Eq. (31), where it would seem to imply $P^{\pm}=Q^{\pm}$. In the detailed discussion below it appears unlikely that present data can be fit with so simple a choice for P; for example, the decay $\Xi^- \to e^- + \nu + n$ must then proceed with the same strength as ordinary β decay.

Decay processes like $\Xi \to N+\pi$ are completely forbidden to first order by Eq. (31). The pion is associated with the P-factor of (31) through a baryon-antibaryon loop, so that the real baryons are connected by an operator P' which must have strangeness change $\Delta S=\pm 1$. This is then the selection rule for weak pion emission.

2. Formulation

Present fragmentary evidence suggests that matrix elements for e and μ are of equal magnitude in lepton decay of heavy particles. An arrangement suitable to express this condition is

$$\begin{aligned}
A_\eta(jk;\epsilon) &= [(\Xi^0\Xi^-)_\eta{}^{\epsilon}+j(\Sigma^+\Sigma^{0+})_\eta{}^{-\epsilon}] \\
&\quad +k[(pn)_\eta{}^{-\epsilon}+j(\Sigma^{0-}\Sigma^-)_\eta{}^{\epsilon}], \\
B_\eta(jk;\epsilon) &= [(n\Xi^-)_\eta{}^{\epsilon}+j(\Sigma^+\Sigma^{0-})_\eta{}^{-\epsilon}] \\
&\quad +k[(p\Xi^0)_\eta{}^{-\epsilon}+j(\Sigma^{0+}\Sigma^-)_\eta{}^{\epsilon}], \\
A_\eta'(jk;\epsilon) &= [(\Xi^-\Sigma^{0+})_\eta{}^{\epsilon}+j(\Xi^0\Sigma^+)_\eta{}^{-\epsilon}] \\
&\quad +k[(\Sigma^{0-}p)_\eta{}^{-\epsilon}+j(\Sigma^-n)_\eta{}^{\epsilon}], \\
B_\eta'(jk;\epsilon) &= [(\Xi^-\Sigma^{0-})_\eta{}^{\epsilon}+j(n\Sigma^+)_\eta{}^{-\epsilon}] \\
&\quad +k[(\Sigma^{0+}p)_\eta{}^{-\epsilon}+j(\Sigma^-\Xi^0)_\eta{}^{\epsilon}], \\
& j^2=k^2=\epsilon^2=1, \quad (pn)^{\epsilon}=\{\bar{\psi}_p\gamma_\eta(1+\epsilon\gamma_5)\psi_n\}, \quad \text{etc.}
\end{aligned} \tag{32}$$

Here A, B are operators of P type, A', B' are P' type, so that Eq. (31) can be written symbolically as

$$H(\beta)=ig\Sigma\{P_\eta(jk;\epsilon)P_\eta'(j'k';\epsilon')-\text{H.c.}\}, \tag{33}$$

with P, P' taken from Eq. (32). This form shows that $H(\beta)$ contains 256 independent terms. Substitution of Eq. (29) yields

$$\begin{aligned}
P_\eta(j+;-) &\to [(\lambda\mu^-)_\eta{}^{-}+j(\nu e^-)_\eta{}^{-}]=L_\eta(j), \\
P_\eta'(j'+;-) &\to -[(\mu^-\nu)_\eta{}^{-}+j'(e^-\lambda)_\eta{}^{-}]=L_\eta'(j'), \\
& P,\, P' \to 0 \text{ otherwise.}
\end{aligned} \tag{34}$$

Terms with μ^+, e^+ follow from Eq. (34) by charge conjugation. The requirement for μ, e equivalence is now that all $P(j+;-)$ terms in $H(\beta)$ have only one sign of j, and that all $P'(j'+;-)$ terms have only one sign of j'. This eliminates 60 of the original 256 terms in Eq. (33), a not very serious restriction.

According to Eqs. (33), (34), four-lepton (i.e., two-neutrino) interactions must be proportional to

$$\begin{aligned}
H(\lambda\nu) &= ig[L_\eta(j)L_\eta'(j')-\text{H.c.}] \\
&= ig[(\lambda\mu^-)_\eta{}^{-}(\mu^-\nu)_\eta{}^{-}+jj'(e^-\lambda)_\eta{}^{-}(\nu e^-)_\eta{}^{-}-\text{H.c.} \\
&\quad +j(\nu e^-)_\eta{}^{-}(\mu^-\nu)_\eta{}^{-}+j'(\lambda\mu^-)_\eta{}^{-}(e^-\lambda)_\eta{}^{-}-\text{H.c.}] \\
&= ig[\{(\lambda\nu)_\eta{}^{-}-(\nu\lambda)_\eta{}^{-}\}\{(\mu^-\mu^-)_\eta{}^{-}-jj'(e^-e^-)_\eta{}^{-}\} \\
&\quad +\{j'(\lambda\lambda)_\eta{}^{-}-j(\nu\nu)_\eta{}^{-}\} \\
&\qquad \times\{(e^-\mu^-)_\eta{}^{-}-(\mu^-e^-)_\eta{}^{-}\}] \\
&= ig[2(\lambda\nu)_\eta\{(\mu^-\mu^-)_\eta{}^{-}-jj'(e^-e^-)_\eta{}^{-}\} \\
&\quad +\{j(\nu\nu)_{\eta 5}-j'(\lambda\lambda)_{\eta 5}\}\{(e^-\mu^-)_\eta{}^{-}-(\mu^-e^-)_\eta{}^{-}\}],
\end{aligned} \tag{35}$$

with $(\lambda\nu)_\eta=\bar{\psi}_\gamma\gamma_\eta\psi_\nu$, $(\nu\nu)_{\eta 5}=\bar{\psi}_\nu\gamma_\eta\gamma_5\psi_\nu$. Here the second form follows from the first by Fierz exchange, and the third results from using the Majorana properties of the neutrinos. The last line of $H(\lambda\nu)$ gives exactly the same measurable features for $\mu-e$ decay (lifetime, angular distribution, electron polarization) as obtained from a conventional form with a single Dirac neutrino and coupling constant g. The previous line accounts for neutrino-electron (or μ meson) scattering; on the present scheme it is actually an exchange process whereby one type of neutrino is converted into the other, thus preserving the rule $\Delta S=\pm 1$ for all weak interactions.

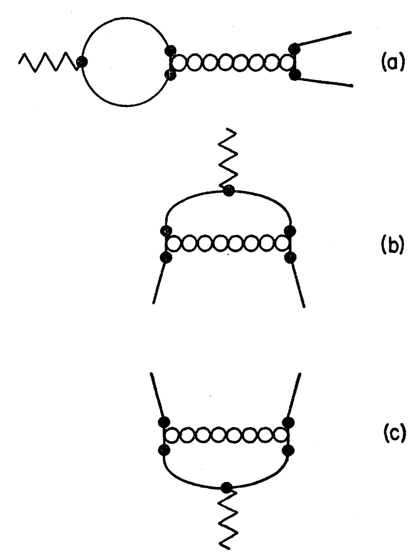

FIG. 2. Diagrams for weak pion decay of hyperons.

[24] N. Dallaporta, Nuovo cimento 1, 962 (1953); G. Costa and N. Dallaporta, Nuovo cimento 2, 519 (1955).
[25] M. Gell-Mann and A. H. Rosenfeld, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1957), Vol. 1, p. 407.
[26] E. R. Caianiello, Phys. Rev. 81, 625 (1950); Y. Tanikawa, Phys. Rev. 108, 1615 (1957).
[27] G. Feinberg, Phys. Rev. 110, 1482 (1958).
[28] S. S. Gershtein and J. B. Zeldovich, J. Exptl. Theoret. Phys. U.S.S.R. 29, 698 (1955) [translation: Societ Phys. JETP 2, 576 (1957)].

TABLE II. Charge space factors for weak pion emission.

Term in $H(\beta)$	Corresponding $\Sigma_A N_A P_A'$ in $H(\pi)$, suppressing ϕ and undex η
$\epsilon[A(jk;\epsilon)A'(j'k';\epsilon')-\mathrm{H.c.}]=$	$-\sqrt{2}[(1+j)(1-k)+j\delta(\epsilon,-\epsilon')][A'(jj',-kk';\epsilon')-\mathrm{H.c.}]-\delta(\epsilon\epsilon')\zeta(jj';\epsilon')[X(jj',-kk')-\mathrm{H.c.}]$
$\epsilon[A(jk;\epsilon)B'(j',k';\epsilon')-\mathrm{H.c.}]=$	$-\sqrt{2}(1+j)(1-k)[B(jj',-kk';\epsilon')-\mathrm{H.c.}]-\delta(\epsilon\epsilon')\{(jj'+kk')[N(jj',-\epsilon')-\mathrm{H.c.}]$ $+(jk'+j'k')[M(jj',-\epsilon')-\mathrm{H.c.}]\}$
$\epsilon[B(jk;\epsilon)A'(j'k';\epsilon')-\mathrm{H.c.}]=$	$\sqrt{2}\delta(\epsilon,-\epsilon')\{-j[B'(jj',-kk';\epsilon')-\mathrm{H.c.}]+k'[B'(jj',-kk';-\epsilon')-\mathrm{H.c.}]\}$ $+\delta(\epsilon\epsilon')\{(1+jj'kk')[N(jj';\epsilon')-\mathrm{H.c.}]+(jk'+j'k)[M(jj';-\epsilon')-\mathrm{H.c.}]\}$
$\epsilon[B(jk;\epsilon)B'(j'k';\epsilon')-\mathrm{H.c.}]=$	$\sqrt{2}\delta(\epsilon,-\epsilon')k'[A'(jj',-kk';-\epsilon')-\mathrm{H.c.}]-\delta(\epsilon\epsilon')kk'\zeta(jj',\epsilon')[X(jj',kk')-\mathrm{H.c.}]$

Here $X(jk)=[(\Sigma^{0+}\Xi^{0})^{+}-j(\Sigma^{0+}\Xi\ )^{-}]+k[j(n\Sigma^{0-})^{+}-(n\Sigma^{0-})^{-}]$,
$M(j\epsilon)=(p\Sigma^{+})^{\epsilon}+j(\Sigma^{-}\Xi^{-})^{-\epsilon}$,
$N(j\epsilon)=(n\Sigma^{0+})^{\epsilon}+j(\Sigma^{0-}\Xi^{0})^{-\epsilon}$,
$\delta(\epsilon\epsilon')=[(\epsilon-\epsilon')/2]^2$,
$\zeta(j\epsilon)=\frac{1}{2}[(1+j)+\epsilon(1-j)]$.

For baryon decays it is necessary to combine $H(\beta)$ with the pion part of strong interaction (20) via a baryon loop as in Fig. 2. In this diagram the two dumbells with umbilical connection (for free momentum transfer in a local four-field interaction) represent $H(\beta)$; diagram (*a*) involves $\mathrm{Tr}[\Gamma_D P(jk;\epsilon)]$ with $D=5$, 6, 7, which vanishes unless $P=A(+-;\epsilon)$; diagrams (*b*) and (*c*) comprise terms in $(\Gamma_D PP'+PP'\Gamma_D)$; and the diagram conjugate to (*a*) would involve $\mathrm{Tr}\,[\Gamma_D P']\equiv 0$. The kinematic factors associated with these diagrams are such that diagram (*a*) yields a form

$$H(\pi)=igG_1J_0(M/2\pi)^2\sum_A \epsilon_A N_A(\pi)\{P_{A\eta}'\phi_\eta-\mathrm{H.c.}\},\tag{36}$$

where $J_0(M/2\pi)^2$ is a divergent integral with characteristic mass $M\approx 1.1$ Bev, and $N_A(\pi)$ are coefficients associated with the charge space factors just mentioned. The charge subscripts on $\phi_\eta=\partial_\eta\phi/m_\pi$ is dropped for simplicity, as it will be obvious from P'. Diagrams (*b*) and (*c*) yield kinematical factors identical to first order in perturbation theory with (36), provided that $\epsilon=\epsilon'$. If $\epsilon=-\epsilon'$, these diagrams yield instead a leading term (using perturbation theory with *PV* coupling for the pion) proportional to

$$igG_1(m_\pi/M)J_0(M/2\pi)^2\sum[\{\bar{\Psi}(1\pm\gamma_5)\Psi\}\phi-\mathrm{H.c.}].\tag{37}$$

We take this term to be of order $(m_\pi/M\approx 15\%$ relative to (36) and omit it entirely, since it is of the same order as neglected vertex corrections from strong interactions; these vertex corrections appear to be of order 20% in the comparison of $\mu-e$ with nuclear β decay. On this lowest-order basis the charge space factors in Eq. (36) corresponding to Fig. 2 have been computed in Table II.

Diagram 2(*a*) also serves for $\pi-\mu(\pi-e)$ decay. On substitution (34) one has

$$\begin{aligned}H(\pi 2)&=ig_0G_1J_0(M/2\pi)^2N(\pi 2)\{L_\eta'(j')\phi_\eta-\mathrm{H.c.}\}\\&=N(\pi 2)h(\pi 2).\end{aligned}\tag{38}$$

The loop integral J_0 is supposed the same as in Eq. (36). $N(\pi 2)$ is a numerical factor depending on $\Sigma c_A\,\mathrm{Tr}\,[\Gamma_D P_A]$. Values of $N(\pi 2)$ and similar factors are presented for individual $H(\beta)$ terms in Table III.

The conjugate diagram to 2(*a*) would describe $K-\mu(K-e)$ decay:

$$\begin{aligned}H(K2)&=igG_1J_0(M/2\pi)^2N(K2)\{L_\eta(j)\phi_\eta-\mathrm{H.c.}\}\\&=N(K2)h(K2),\end{aligned}\tag{39}$$

where $\phi_\eta=\partial_\eta\phi_A/\kappa_A$ with $A=1$, 2.

For three-body leptonic decay the situation is more complicated: even with neglect of vertex corrections, $K(3)$ decay can be effected by interaction (20) acting twice [Fig. 3(a)] or interaction (22) acting once [Fig. 3(b)]. No arguments appear for selecting one of these over the other; although (3b) leads to a quadratic and (3a) only to a logarithmic divergence in perturbation theory, the gauge condition for photon propagation assigns to both the same order of magnitude in units of M^2. These divergences make uncertain the kinematic factors in the "effective Hamiltonian" of $K(3)$ decay. For simplicity we assume a form $(\phi E'\phi_\eta)$ for the boson functions, as in Eq. (22). This is unlikely to be correct in detail but will permit a survey of the charge space factors and perhaps some order-of-magnitude comparisons in decay rates. Accordingly,

$$\begin{aligned}H(K3)&=ig\frac{M}{(2\pi)^2}[G_1^2J_1N(K31)+G_2J_2N(K32)]\\&\qquad\times[(\pi^0K^+)_\eta L_\eta(j)-\mathrm{H.c.}]\\&=N(K31)h(k31)+N(K32)h(32),\end{aligned}\tag{40}$$

TABLE III. Charge space coefficients for leptons.

Term in $H(\beta)$	Corresponding lepton term
$A(jk;\epsilon)A'(j'k';\epsilon')$	$tt'H(\lambda\nu)+t[n_1+k'u(j'\epsilon';j)]$ $+t'[n_2+kp(\epsilon;j)]$
$A(jk;\epsilon)B'(j'k';\epsilon')$	$tt'H(\lambda\nu)+t[n_3+j'w(j\,k';\epsilon';j)]$ $+t'[n_2+kp(\epsilon;j)]$
$B(jk;\epsilon)A'(j'k';\epsilon')$	$tt'H(\lambda\nu)+t[n_1+k'u(j'\epsilon';j)]+t'v(k\epsilon;j')$
$B(jk;\epsilon)B'(j'k';\epsilon')$	$tt'H(\lambda\nu)+t[n_3+j'w(j'k',\epsilon';j)]+t'v(k\epsilon;j')$

where $t=\frac{1}{4}(1+k)(1-\epsilon)$ and likewise for t'; and
$n_1=\sqrt{2}(1-j')(j-k')\epsilon'h(K2)-\sqrt{2}(1-j')(1+k')h(K31)$
$-\sqrt{2}(1+j')(1+k')[h(K32)-h(\theta^1 32)-h(\theta^2 32)]$,
$n_2=\sqrt{2}(1+j)(1-k)\epsilon h(\pi 2)-\sqrt{2}(1+j)(1+k)h(\pi 31)$
$+\sqrt{2}(1-j)(1+k)h(\pi 32)$,
$n_3=-\sqrt{2}(1+k')[(1-j')h(\theta^1 31)+(1+j')h(\theta^2 31)]$.

TABLE IV. Parameters of $H(K\pi)$.

$\Sigma N_A P_A'$ in $H(\pi)$	E_1	E_2 [a]
$\sqrt{2}[A'(++;\epsilon')-\text{H.c.}]$	0	$8[(\pi^+ + \pi^-)K_3 + (K^+ + K^-)\pi^0]_\eta$
$[M(-\epsilon')+N(-\epsilon')-\text{H.c.}]$	0	$8[K^+\pi^- + K^-\pi^+ - \pi^0 K_3]_\eta$
$\sqrt{2}[A'(-+;\epsilon')-\text{H.c.}]$	$8[(K^+ + K^-)\pi^0]_\eta$	0
$\sqrt{2}[B'(-+;\epsilon')-\text{H.c.}]$	$-8[(\pi^+ + \pi^-)K_3]_\eta$	0
$[M(-\epsilon')-N(-\epsilon')-\text{H.c.}]$	$8[\pi^0 K_3]_\eta$	0
$[X(+-)-\text{H.c.}]$	$8[K^+\pi^- + K^-\pi^+]_\eta$	0

[a] The first entry in this column contains an additional term $8i[(\pi^+ - \pi^-)K_4]_\eta$ which vanishes identically on the necessary symmetrization between the pions.

where the notation $(\pi^0 K^+)_\eta$ follows[29] Eq. (26), and the factors N are given in Table III. J_1 and J_2 are dimensionless divergences presumably of the same order as J_0 and assumed to have negligible dependence on kinematic factors. The corresponding form for leptonic decay of θ^1, θ^2 is

$$H(\theta 3)=ig\frac{M}{(2\pi)^2}[G_1{}^2 J_1 N(\theta 31)+G_2 J_2 N(\theta 32)] \times\left[\left\{\begin{matrix}(\pi^+K_3)_\eta\\ i(\pi^+K_4)_\eta\end{matrix}\right\}L_\eta(j)-\text{H.c.}\right] \quad (41)$$
$$=N(\theta 31)h(\theta 31)+N(\theta 32)h(\theta 32).$$

By shifting the loops in Fig. 3 to the left side as in Fig. 2(a), one has a diagram suitable for π_3 decay, $\pi^\pm \to \pi^0 + \mu^\pm(e^\pm)+\nu$, where μ emission or absorption can occur only with virtual pions. The corresponding interaction is

$$H(\pi 3)=ig[M/(2\pi)^2][G_1{}^2 J_1 N(\pi 31)+G_2 J_2 N(\pi 32)] \times[(\pi^0\pi^-)_\eta L_\eta'(j')-\text{H.c.}] \quad (42)$$
$$=N(\pi 31)h(\pi 31)+N(\pi 32)h(\pi 32),$$

in complete analogy with Eq. (40). A similar form can be written for $K^\pm \to K_3(K_4)+\mu^\pm(e^\pm)+\nu$.

Of course interaction (33) gives rise to direct lepton decay of baryons on substitution of Eq. (34). It is convenient to define

$$\begin{aligned}
p(\epsilon;j)&=ig[(pn)_\eta{}^{-\epsilon}L_\eta'(j)-\text{H.c.}],\\
u(l\epsilon;j)&=ig\{[l(\Sigma^-n)_\eta{}^{\epsilon}+(\Sigma^0{}^-p)_\eta{}^{-\epsilon}]L_\eta(j)-\text{H.c.}\},\\
v(l\epsilon;j)&=ig\{[(n\Xi^-)_\eta{}^{\epsilon}+l(p\Xi^0)_\eta{}^{-\epsilon}]L_\eta'(j)-\text{H.c.}\},\\
w(l\epsilon;j)&=ig\{[(n\Sigma^+)_\eta{}^{-\epsilon}+l(\Sigma^{0+}p)_\eta{}^{-\epsilon}]L_\eta(j)-\text{H.c.}\}.
\end{aligned} \quad (43)$$

The terms (43) represent the hyperon β decays most feasible to observe at present. Their coefficients are also listed in Table III.

To the present order of accuracy the process $K \to 2\pi$ is described by using the two-boson loops of Fig. 3 to close the open lines in Fig. 2. Note that the corresponding diagram with both π mesons in a single loop cannot contribute, as the pions must be in a $T=1$ state for both Figs. 3(a) and 3(b). The effective interaction is most simply based on the $H(\pi)$ of Eq. (36) and Table II:

$$H(K\pi)=igG_1 J_0[M^3/(2\pi)^4][\phi(G_1{}^2J_1E_1+G_2J_2E_2)\phi_\eta]\phi_\eta, \quad (44)$$

where the charge operators E_1 and E_2 are given in Table IV; the charge index on the ϕ outside the brackets is suppressed, as it follows by charge conservation from the other factors. Table IV lists all nonvanishing contributions to E_1 and E_2.

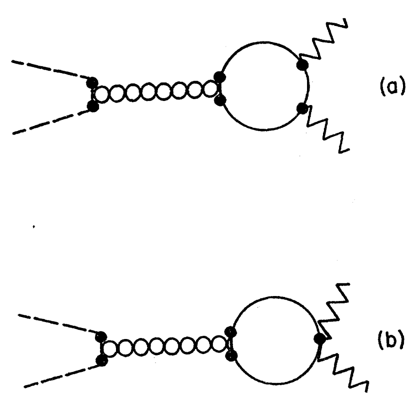

FIG. 3. Diagrams for K_3 decay.

[29] Note that the H.c. of $(\pi^0K^+)_\eta$ is $(K^-\pi^0)_\eta = -(\pi^0K^-)_\eta$.

3. Application

This section outlines some applications of the preceding expressions. No attempt is made to find a definite, complete form for $H(\beta)$, but examples are given of how the present scheme can account for available data. Efforts to transcribe the formulas of the last section in terms of charge space operators $\boldsymbol{\rho}$, $\boldsymbol{\sigma}$, and $\boldsymbol{\tau}$ have not so far yielded any algebraically attractive symmetry in agreement with observations.

The following assumptions will be made about lepton decay: (i) the μ- and e-matrix elements are identical in magnitude for all processes; and (ii) no decay process corresponding to u, v, or w in Eq. (43) occurs to 1st order. The second is a rather extreme assumption but seems in harmony with present observations;[30] although

[30] Crawford, Cresti, Good, Kalbfleisch, Stevenson, and Ticho, Phys. Rev. Letters 1, 377 (1958); Nordin, Orear, Reed, Rosenfeld, Solmitz, Taft, and Tripp, Phys. Rev. Letters 1, 380 (1958); Leitner, Nordin, Rosenfeld, Solmitz, and Tripp, Phys. Rev. Letters 3, 186 (1959).

lepton modes of Λ and $\Sigma^{\pm}$ decay have been seen, their apparent rate is an order of magnitude slower than expected on a naive universal Fermi interaction. In the present calculation this represents first order forbiddenness, the observed decays presumably arising from vertex corrections by K mesons.[31] A good deal more experimental and theoretical work on this point is clearly required; in the meanwhile assumption (ii), if not actually correct, will at least serve to test the flexibility of the present scheme. The experimental foundation of (i) is also quite weak; although the known cases of lepton decay (pn, π_2, K_3) seem to agree with it, they represent a very small fraction of the total possible lepton processes—and agreement with (ii) for bosons does not necessarily imply agreement for all baryon pairs contributing to the loops.

By manipulating Table III to satisfy assumptions (i) and (ii), one can obtain as independent terms

$$\begin{array}{l}\sqrt{2}[h(K2)-h(K31)],\quad \sqrt{2}[h(K32)-h(\theta^1 32)-h(\theta^2 32)],\\ \sqrt{2}h(\theta^1 31),\quad \sqrt{2}h(\theta^2 32),\quad \sqrt{2}h(\pi 2),\quad \sqrt{2}h(\pi 31),\\ \sqrt{2}h(\pi 32),\quad H(\lambda\nu),\quad \{(pn)_\eta^- L_\eta'\}.\end{array} \tag{45}$$

Note that the G_2 terms automatically give equality of the lepton decay rates[25] for θ^1 and θ^2 at a rate twice that for $K^{\pm}$ lepton decay[32]; these conclusions are not necessarily true for the G_1 terms but may be satisfied for a sufficiently symmetric $H(\beta)$.

In the absence of any better guess, suppose all boson terms to contribute equally, with

$$N^2=2, \tag{46}$$

for each h in Eq. (45). If assumption (i) is relaxed to demand μ and e equivalence only for presently observed processes, it is possible to obtain $H(\lambda\nu)$ terms with different values of j and j' and thus extract only the part $H(\mu e)$, while $H(ee)$ and $H(\mu\mu)$ vanish. It is perhaps of interest that the terms in Table III giving rise to $H(\lambda\nu)$ can at most yield $(pn)^+$ for nuclear β decay, which gives a relative sign for the A and V coupling terms opposite to that observed. It is fundamental to the present scheme, based on charge analogies between baryons and leptons, that the (pn) and (μe) decay processes cannot arise from the same term in $H(\beta)$.

Assumptions (46) permit the assignment of values to the loop integrals J_0 in Eqs. (38) and (39). The decay rates are

$$r(\pi 2)=r_0(G_1^2/4\pi)J_0^2N^2(\pi 2)(m_\mu/m\pi)\times[1-(m_\mu/m\pi)^2]^2, \tag{47}$$
$$r_0=g^2M^4m_\mu/(2\pi)^4=10^{10}\ \text{sec}^{-1}.$$

From the observed $r(\pi 2)=3.9\times 10^7$ sec^{-1},

$$J_0^2=0.19\approx 0.2. \tag{48}$$

Similarly, an observed rate of $r(K2)=4.8\times 10^7$ sec^{-1} yields

$$J_0^2=0.16\approx 0.2. \tag{49}$$

For K_{e3} and $K_{\mu 3}$ the decay rates are

$$\begin{aligned} r(K3)&=[|L|^2/(2\pi)^3]Wm_K^5,\\ L&=ig[M\sqrt{2}/(2\pi)^2m\pi]G_{12}J_{12},\\ \sqrt{2}G_{12}J_{12}&=G_1^2J_1N(K31)+G_2J_2N(K32).\end{aligned} \tag{50}$$

Here W is a statistical factor dependent on the kinematic form assumed in Eq. (40) but generally of order 10^{-2}. For the kinematic form (40)

$$K_{\mu 3}/K_{e3}\approx 0.6, \tag{51}$$

which is just within present uncertainties. Taking an average $r(K3)$ of 3.3×10^6 sec^{-1}, one obtains

$$(G_{12}J_{12})^2\approx 0.8, \tag{52}$$

so that J_1 and/or J_2 seems of much the same order as J_0.

For the $\pi-\pi$-lepton interaction of Eq. (42)

$$\begin{aligned} H(\pi 3)&=ig[M\sqrt{2}/(2\pi)^2m_\pi](G_{12}J_{12}')\times[\phi^*\partial_\eta\phi-\partial_\eta\phi^*\phi]L_\eta'(j')\\ &=ig'[\phi^*\partial_\eta\phi-\partial_\eta\phi^*\phi]L_\eta'(j'),\end{aligned} \tag{53}$$

where $g'\approx\pm 0.3g$ if we assume

$$(J_{12}')^2\approx J_{12}^2. \tag{54}$$

One thus infers an $(H\pi 3)$ similar to that obtained from the principle of conserved lepton currents,[4] but with a coefficient about $\frac{1}{3}$ as large and of indeterminate sign. This estimate of g' is of course subject to much uncertainty[33]; there is no necessity on the present scheme to have $g'\equiv g$, although the two appear comparable in magnitude.

For the pion decay of baryons

$$\begin{aligned} r(\pi)&=(G_1^2/4\pi)J_0^2[g(M/2\pi)^2]^2m_\pi[N_s^2W_s+N_p^2W_p],\\ W_s&=[(M_i-M_f)/m_\pi]^2[(E_f+M_f)/M_i](p/m_\pi),\\ W_p&=[(M_i+M_f)/m_\pi]^2[(E_f-M_f)/M_i](p/m_\pi),\end{aligned} \tag{55}$$

where i and f refer to initial and final baryon states, and p is the momentum of the decay. The quantities of N_s and N_p are the charge space factors for the respective γ_η and $\gamma_\eta\gamma_5$ terms in Eq. (36), as taken from Table II. The kinematic factors W_s and W_p are listed in Table V.

TABLE V. Parameters of hyperon-π decay.

Decay	W_s	W_p	A^2
$\Lambda\to\pi^-$	1.96	0.74	5.2±0.5
$\to\pi^0$	1.98	0.82	5.5±0.8
$\Sigma^+\to\pi^+$	7.04	5.03	2.7±1.0
$\to\pi^0$	6.80	4.75	2.7±1.0
$\Sigma^-\to\pi^-$	7.46	5.29	2.5±0.8
	$\langle A^2\rangle_{\text{Av}}=3.7\pm 1$		

[31] S. Oneda, Nuclear Phys. **9**, 476 (1959).
[32] Okubo, Marshak, Sudarshan, Teutsch, and Weinberg, Phys. Rev. **112**, 665 (1958).
[33] If the κ_A^{-1} factors are omitted from $\phi_{A\mu}$, then $|g'|\approx 0.1|g|:M$. Sugawara, Phys. Rev. **112**, 2128 (1958).

All elements of Table II involving $\pi^\pm$ emission contain a factor $\sqrt{2}$, while those for π^0 emission do not; these factors arise from the strong interaction. Thus the $\pi^+/\pi^0=2$ ratio observed in Λ decay simply indicates that the corresponding terms in Table II appear with the same coefficients.[34] It is tempting to assume that the same is true for all terms in H_π and set

$$\begin{aligned} \Lambda &\to \pi^+ && |N_s|=|N_p|=A, \\ &\to \pi^0 && |N_s|=|N_p|=A/\sqrt{2}, \\ \Sigma^+ &\to \pi^+ && (|N_s|,|N_p|)=(\sqrt{2}A,0), \\ &\to \pi^0 && |N_s|=|N_p|=A, \\ \Sigma^- &\to \pi^- && (|N_s|,|N_p|)=(\sqrt{2}A,0). \end{aligned} \tag{56}$$

Here the coefficients of Λ decay have an extra factor $1/\sqrt{2}$ relative to the others, since the interactions are all formulated in terms of Σ^{0+} and $\Lambda=(\Sigma^{0+}-\Sigma^{0-})/\sqrt{2}$. Polarization experiments[35] indicate that for $\Sigma^\pm\to\pi^\pm$ one of the terms N_s or N_p vanishes but do not decide the choice.

The values of A obtained by comparing Eqs. (55), (56) with the observed decay rates are listed in Table V. Generous allowances are included for experimental error, to which is added the uncertainty from the (s,p) choice for $\Sigma^\pm\to\pi^\pm$; an average value of J_0^2 is taken from Eqs. (48) and (49). The result in Table V is

$$|A|\approx 2(1\pm 15\%), \tag{57}$$

the fluctuation being of the order expected from neglect of vertex corrections and Eq. (37). This simple numerical value of A hints at a certain degree of symmetry in $H(\beta)$; on the other hand, the present crude comparison does not establish assumption (56) as actually correct.

In the process $K\to 2\pi$ the charge space properties of the G_1^2 and G_2 terms are particularly distinct, the G_2 interaction being simpler. The first-order absence of $K^\pm\to 2\pi$ implies equal coefficients for the two nonvanishing terms under E_2 in Table IV; then in θ^1 decay $\pi^\pm/\pi^0=2$, but the relative signs of the amplitudes are opposite to those of a two-pion $T=0$ state. For the G_1 interaction the absence of $K^\pm\to 2\pi$ implies vanishing of two separate terms under E_1, while the $\pi^\pm/\pi^0$ ratio in θ^1 decay is arbitrary.

All such statements apply only to the s-wave parts of the interaction $H(\pi)$; that is, to the combinations $\{A.(++;+)+A'(++;-)\}$, etc., in $\Sigma N_A P_A'$. Since these same combinations are responsible for pion decay by hyperons, the decays $\Sigma^+\to\pi^+$ and $\Sigma^-\to\pi^-$ must be s wave if the corresponding terms (G_1 and G_2, respectively) are to contribute to $K\to 2\pi$. Measurement of the $\Sigma^\pm\to\pi^\pm$ decay processes to distinguish between s wave and p wave emission would be of immediate interest in this regard. The $\Delta I=\frac{1}{2}$ rule requires that one be s wave, the other p wave,[25] while the present scheme would also allow both to be s wave.

Equation (57) suggests a coefficient 2 to multiply the $\Sigma N_A P_A'$ terms in Table IV. The $\theta^1\to 2\pi$ decay rate is then

$$r(K\pi)=(3p/4\pi^2 m_K^2)\{(G_1^2/4\pi)J_0^2(G_{12}J_{12}'')^2\} \times(16K)^2 g^2(M/2\pi)^6, \tag{58}$$

where $p^2=m_K^2/4-m_\pi^2$ and $2K=(m_K/m_\pi)^2+(m_K/m_\pi)-2$. This last factor depends on the assumed kinematic form of Eq. (44) and is not very reliable; it yields $r(K\pi)\approx 3\times 10^{10}$ sec^{-1}, provided that $(J_{12}'')^2=(J_{12})^2$.

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APPENDIX

This section outlines conditions necessary to prevent the four-fermion interaction from revealing lepton-antilepton distinctions. Consider an ordinary β decay, determined by $|H(\beta)|^2=|\Sigma_\alpha f_\alpha \bar{\psi}_y O_\alpha \psi_x|^2$; here the lepton functions ψ and operators O_α all refer to real coordinate space, and the coefficients f_α are nuclear matrix elements.[36] The covariant Casimir projection operator is

$$P_+=(\gamma_\mu p_\mu+im)/2p_4, \tag{A.1}$$

and for the antiparticle $\psi^a=C\bar{\psi}$

$$P_+{}^a=(\gamma_\mu p_\mu-im)/2p_4, \tag{A.2}$$

so that the only lepton-antilepton distinction observable here is the sign of the m term. A measurement on lepton y alone is described by

$$\begin{aligned} |H(\beta)|^2=\sum_{\alpha,\delta}\left\{ f_\alpha f_\delta^* \bar{\psi}_y O_\alpha\left(\frac{\gamma_\mu p_\mu{}^x\pm im^x}{2p_4{}^x}\right)O_\delta'\psi_y+\text{c.c.}\right\}, \\ O_\delta'=\gamma_4 O_\delta{}^\dagger\gamma_4. \end{aligned} \tag{A.3}$$

The most general condition to eliminate the m^x term from Eq. (A.3) is

$$O_{\alpha,\delta}=Q_{\alpha,\delta}(1+\epsilon\gamma_5),\quad \epsilon^2=1, \tag{A.4}$$

with arbitrary $Q_{\alpha,\delta}$; this is necessarily parity-nonconserving.[37,7]

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[34] And may have nothing to do with $\Delta I=\frac{1}{2}$ rule: R. H. Dalitz, Revs. Modern Phys. **31**, 823 (1959).

[35] Cool, Cork, Cronin, and Wenzel, Phys. Rev. **114**, 912 (1959).

[36] Derivative couplings in H_β introduced by mesonic corrections can all be cast into the coefficients f_α, so that the O_α comprise only Dirac matrices.

[37] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956); E. C. G. Sudarshan and R. E. Marshak, Phys. Rev. **109**, 1860 (1958).

Next consider the two-component reduction of Eq. (A.3) according to

$$\psi_y=\begin{bmatrix}\xi_y\\ \dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{(E+m)}\xi_y\end{bmatrix},\quad \psi_y{}^a=\begin{bmatrix}\dfrac{\boldsymbol{\sigma}\cdot\mathbf{p}}{(E+m)}\xi_y{}^a\\ \xi_y{}^a\end{bmatrix},$$

$$\xi_y{}^a=\sigma_2\xi_y{}^*. \quad (A.5)$$

The net difference between the two forms (A.5) is an operation γ_5, so that the two-component reduction of $\bar{\psi}_y O_\alpha\gamma_\mu O_\delta'\psi_y$ will (not) change sign between particle and antiparticle if the number of factors γ_μ in O_jO_k is odd (even). The terms in Eq. (A.3) with $\alpha=\delta$ must be even in γ_μ; to eliminate particle-antiparticle distinction, odd terms must be absent also when $\alpha\neq\delta$. This separates the Q_α into two noninterfering groups, (A, V) and (S, T, P); that is, there are no ordinary Fierz interference terms.

It seems unlikely that selection between these two groups can be based on considerations of $|H(\beta)|^2$; one must resort to algebraic symmetry conditions on H_β itself, which is not directly observable. Of course (A, V) are equivalent when multiplied by $(1\pm\gamma_5)$, so that only one coupling constant need be specified; the other group requires two independent constants, one for T and one for (S, P). The (A, V) choice is thus inherently simpler.

We now suppose that there is a universal four-fermion interaction

$$H(\beta)\sim\{\bar{\Psi}P\Psi\}\{\bar{\Psi}P'\Psi\}, \quad (A.6)$$

written in some "normal order" where leptons may substitute for baryons in either bracketed pair. Then by the above, both P and P' must contain $(1\pm\gamma_5)$ factors, although the two $\pm$ need not be correlated.

The above discussion is superficially similar but not identical with the mass reversal argument,[38] which involves $H(\beta)$ itself (not $|H(\beta)|^2$) and depends on a strict assumption of particle conservation throughout, while the present argument stems from a denial of this principle. The net result is to render the present argument less precise and detailed in its conclusions: in the two factors of a four-fermion interaction, it allows arbitrary sign choices in $(1\pm\gamma_5)$, and it cannot decide between (A, V) and (S, T, P).

Lepton decay also occurs for bosons, at least through the mediation of baryon loops, when it must have the phenomenological form

$$\{\bar{\psi}_y\gamma_\mu(1+\epsilon\gamma_5)\psi_x\}\partial_\mu\varphi$$
$$=\{\bar{\psi}_y[(m^x-m^y)-(m^x+m^y)\epsilon\gamma_5]\psi_x\}. \quad (A.7)$$

This form allows particle-antiparticle comparison of various x and y unless m^x or m^y always vanishes. Since the charged leptons have $m\neq 0$, this provides an argument that (i) the mass of the neutrino and of the $\lambda^0=\mu^0$ lepton must be identically zero; (ii) the form (A.6) must contain at least one neutral fermion in each bracket.

Equations (A.1)–(A.3) neglect the Coulomb field for the leptons, but the conclusion should be valid in any case. Suppose ψ and $|H(\beta)|^2$ in (A.3) to be expanded as power series in Z; then the argument above suffices for even powers of Z in $|H(\beta)|^2$, and the odd powers vanish by the reality of $|H(\beta)|^2$ since the expansion parameter of ψ is proportional to iZ. If a β type interaction like $p+e^-\rightarrow p+e^-$ could occur, it would in principle be possible to determine leptonness by observing interference of the Coulomb and β type scattering; but all such four-charged-fermion interactions are excluded by (ii) above.

Of course double β decay without neutrino emission is no longer forbidden by lepton conservation, but the matrix element vanishes identically because of the $(1\pm\gamma_5)$ factors:

$$\int\{\bar{\psi}_e\gamma_\mu(1+\epsilon\gamma_5)\psi_\nu\bar{\psi}_e\gamma_\eta(1+\epsilon\gamma_5)\psi_\nu\}$$
$$\sim\int\{\bar{\psi}_e\gamma_\mu(1+\epsilon\gamma_5)\psi_\nu\bar{\psi}_\nu\gamma_\eta(1-\epsilon\gamma_5)\psi_{\bar{e}}\}$$
$$\sim\int\frac{d^4k}{k^2}\{\bar{\psi}_e\gamma_\eta(1+\epsilon\gamma_5)\gamma_\lambda k_\lambda\gamma_\eta(1-\epsilon\gamma_5)\psi_{\bar{e}}\}\equiv 0, \quad (A.8)$$

where k_λ is the virtual neutrino momentum. This conclusion is contained in the simple statement that the $(1\pm\gamma_5)$ factors make β decay behave *as if* there were lepton conservation with two-component neutrinos, although neither assumption is necessary: this extends to other cases, such as the absence of inverse β decay to yield a negatron when the incident neutrinos are produced during negatron β decay, as in a reactor.

On the present view the interactions internal to charge space (i.e., not involving the electromagnetic and gravitational fields, which are external) are "strong" or "weak" according as they do or do not distinguish between fermion and antifermion. This is the primary distinction, from which follow corollary[39] differences in parity conservation and degree of charge symmetry. If heavy bosons are not elementary but simply fermion-antifermion pairs with much greater "binding" than fermion-fermion pairs,[8] then the mere existence of heavy bosons implies a strong interaction that distinguishes between fermion and antifermion. Only baryons can participate in this interaction, since leptons and antileptons are intrinsically indistinguishable. This

[38] J. J. Sakurai, Nuovo cimento **7**, 649 (1958).

[39] The present view is not in accord with the idea of distinguishing strong and weak interactions by requiring CP invariance for both and extracting P invariance for strong interactions by virtue of symmetries in charge space: S. N. Gupta, Can. J. Phys. **35**, 1309 (1957); J. J. Sakurai, Phys. Rev. **113**, 1679 (1959); G. Feinberg and F. Gürsey, Phys. Rev. **114**, 1153 (1959).

observed fact suggests that the strong interaction has invariance under C and hence under P if only CP invariance is assumed. Also, the necessity for exchange antisymmetry in the baryon-antibaryon model of the boson implies rather simple charge space properties for the interaction. On this compound model the statement that strong interactions conserve strangeness is a tautology.

The interaction that does not distinguish fermions from antifermions can be—and presumably is—universal to both baryons and leptons. It must be weak enough not to form bound boson states, or free leptons would not be observed; according to the argument above it must also contain $(1\pm\gamma_5)$ factors and be parity-violating. The choice of a charge space vector form for the interaction, required to avoid revealing relative leptonness through $(\mu e)\pi$ interactions, implies $T=CP=-1$ rather than invariance and also a lower degree of charge symmetry than in the strong interaction.

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Inconsistency of Cubic Boson-Boson Interactions

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It is shown that there does not exist a ground state for a system of spin zero bose fields coupled only by local interactions involving three powers of the fields. Thus these interactions alone are not suitable for a model of interacting fields.

I. INTRODUCTION

It is necessary, if a field theory is to give a consistent description of physical reality, that there be a state of lowest energy. If the energy spectrum has no lower bound, then the system can undergo a "radiative" collapse. We shall be concerned in this paper with a group of local interactions of spin zero bose fields that do not give rise to a state of lowest energy. These are interactions that involve three powers of the fields, i.e., for one field, ϕ, interacting with itself,[1]

$$H_{\text{int}}=\lambda\int\phi^3(x)d^3x, \tag{1}$$

for two fields, ϕ and χ,

$$H_{\text{int}}=\lambda\int\chi(x)\phi^2(x)d^3x, \tag{2}$$

and for three fields, ϕ, χ, and θ,

$$H_{\text{int}}=\lambda\int\chi(x)\theta(x)\phi(x)d^3x. \tag{3}$$

The field operators are taken to be Hermitean and thus λ must be a real constant, with dimensions of an inverse length. These three interactions are the so-called "super-renormalizable" ones.[2]

For each of the interactions given above we shall show that the assumption of the existence of a ground state leads to a contradiction, and hence that these interactions alone between the fields are not physically realizable.

II. PROOF OF THE NONEXISTENCE OF THE GROUND STATE

First consider the case of one scalar field with the cubic self-interaction (1). The dynamics are described by a Lagrangian

$$L=\tfrac{1}{2}\int d^3x[\phi(x)(\partial_\mu\partial^\mu-m^2)\phi(x)-\lambda\phi^3(x)], \tag{4}$$

where

$$\partial_\mu\partial^\mu=\nabla^2-(\partial/\partial t)^2,$$

and m^2 is taken to be a finite real number which may be zero. The requirement of invariance of L under spatial reflection implies that ϕ must be a scalar field. From L is derived the form of the energy operator, P^0, for the field

$$P^0=\tfrac{1}{2}\int d^3x[(\phi^0)^2+(\phi^k)^2+m^2\phi^2+\lambda\phi^3], \tag{5}$$

where

$$\phi^\mu(x)=\partial^\mu\phi(x).$$

Integrating by parts and introducing the real symmetric

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[1] M. Fierz has already pointed out that in the classical limit this interaction (1), does not lead to a positive definite energy. *Proceedings of the Fifth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, 1955), p. 67.

[2] W. Thirring, Helv. Phys. Acta **26**, 33 (1953).