K^--Meson Elastic Scattering by Emulsion Nuclei*

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Data on elastic nuclear scattering of K^- mesons in emulsion have been obtained. The differential cross section has been calculated by the WKB approximation in the method of partial waves. It is concluded that the real and imaginary potentials necessary to give the correct K^--nucleon cross section also give a good fit to the elastic nuclear scattering data.

I. INTRODUCTION

One of the fundamental problems in strange particle physics is that of determining the parity of K mesons in strong interactions. A procedure suggested in this problem is the use of dispersion theory[1] to relate the sign of the parity to the zero-energy forward scattering amplitudes of $K^{\pm}$ mesons on protons. The magnitude of the amplitudes are known from the low-energy scattering cross sections. Obviously the best way to determine their sign is by observing the interference with Coulomb scattering at low energies for both K^+ and K^- on protons. Instead of this, one may attempt (as has been done for the K^+ case)[2,3] to infer the sign by the interference effects of elastic scattering with nuclei utilizing the optical model. In that case the sign of the real part of the nuclear potential was unfortunately ambiguous at low energies because of the presence of an imaginary part even though it is small. However, the ambiguity in sign disappears at high energies.[3] Reasons of continuity lead to the conclusion of a repulsive potential and negative scattering amplitude at zero energy. This conclusion is also borne out by arguments based on inelastic nuclear scatters.

This work was originally undertaken with the same philosophy underlying that of the K^+ case, i.e., to obtain information on the K^- meson forward scattering amplitude from the elastic nuclear scattering. This unfortunately turned out to also suffer from the same ambiguity but to a greater extent than for the K^+ case because of the largeness of the imaginary part of the nuclear potential. There still exists however, (as in the K^+ case), arguments based on the inelastic scattering which tend to relieve the ambiguity. In essence the elastic scattering is shown to be in agreement with the potential indicated by the known K^--nucleon cross sections.

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[1] P. T. Matthews and A. Salam, Phys. Rev. **110**, 565, 569 (1958); C. Goebel, Phys. Rev. **110**, 572 (1958); D. Amati and B. Vitale, Nuovo cimento **7**, 190 (1958); K. Igi, Progr. Theoret. Phys. (Kyoto) **19**, 238 (1958).

[2] D. Fournet Davis, Phys. Rev. **106**, 816 (1957); Hoang, Kaplon, and Cester, Phys. Rev. **107**, 1698 (1957).

[3] Igo, Ravenhall, Tiemann, Chupp, Goldhaber, Lannutti, and Thaler, Phys. Rev. **109**, 2133 (1958).

II. EXPERIMENTAL DETAILS

The 300 Mev/c "separated" K^- beam of the Berkeley bevatron[4] was incident on the emulsion stack and brought to rest. The stack consisting of 120 stripped pellicles of G-5 emulsion was processed in the usual way. The "on track" method of scanning was employed. The K^- mesons were distinguished from background by a grain count 0.5 cm in from the incident edge and were followed until they interacted or came to rest. A total track length of 50.4 meters was followed in the energy interval $80 \geqslant T \geqslant 17$ Mev. For the elastic scattering data only ending K^--meson tracks were included since it made identification more certain. Tracks ending in stars not containing hyperons, hyperfragments, or π

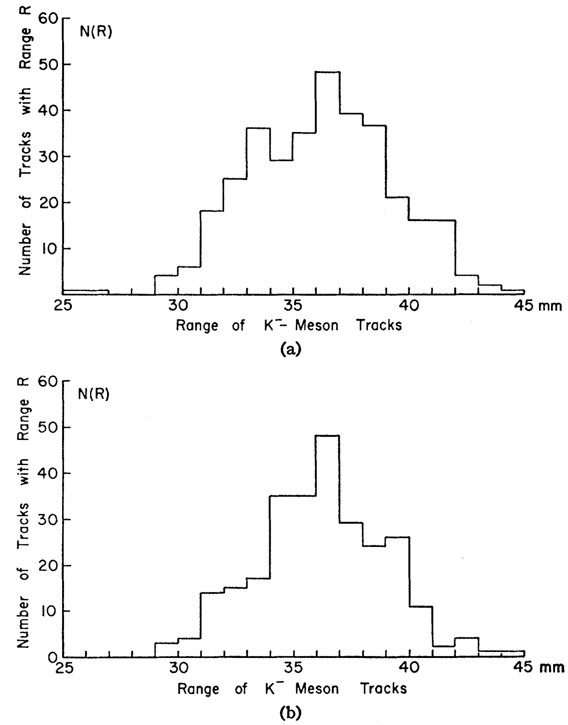

Fig. 1(a). Range histogram of K^--meson tracks without scatters. (b) Range histogram of K^--meson tracks with elastic nuclear scatters.

[4] Barkas, Dudziak, Giles, Heckman, Inman, Mason, Nickols, and Smith, University of California Radiation Laboratory Report UCRL-3627, December, 1956 (unpublished).

mesons as reaction products were distinguished from π^--meson produced stars by their range and a rough estimate of the grain density 3.5 cm from the end of the track. Tracks ending in zero prong stars were identified by a 10% grain count *versus* range curve. In the following of tracks, all angles whose projection in the plane of the emulsion were $\geqslant 5°$ were recorded; this requires a correction to obtain the true angular differential cross section.

Events were classified as elastic scatters from nuclei other than hydrogen if (i) there were no visible prongs or recoils at the scatter,[5] (ii) there were no visible changes in ionization before and after the scatter, and (iii) they fell in the range histogram defined by tracks which did not scatter. Figure 1(a) shows the range histogram of tracks without scatters and 1(b) shows the range histogram of those satisfying the criteria (i to iii) above. The corrected differential elastic scattering cross-section data is shown in Fig. 4.

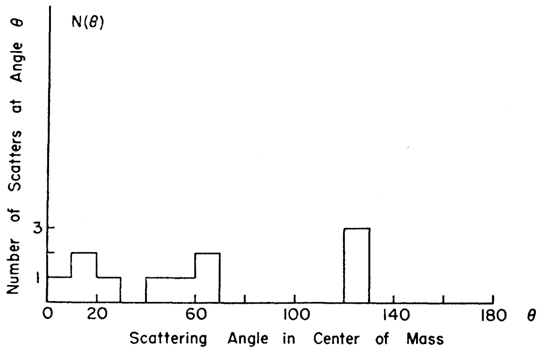

FIG. 2. Angular distribution of inelastic K^--meson scatters.

III. ANALYSIS OF DATA

The differential elastic cross section was calculated using the optical model. The imaginary part of the nuclear potential V was obtained from the relation

$$\mathrm{Im}V = -\bar{\sigma}\rho v\eta\hbar/2. \qquad (1)$$

Here $\bar{\sigma}$ is the average total K^--nuclear cross section, ρ is the nuclear density, assumed constant, v is the velocity of the K^- meson inside the nucleus, and η is the correction factor for the Pauli exclusion principle.[6] The value of $\bar{\sigma}$ is the average of the K^- proton and K^--neutron total cross sections evaluated at the average energy of the K^- meson inside the nucleus. The total cross section includes elastic scattering, charge exchange, and hyperon-π associated production on a nucleon but the Pauli correction factor is applied only to scattering and charge exchange events. The basic

[5] It is possible for an elastic collision of a K^- on carbon to leave a visible nuclear recoil. There was only one such possibility among 245 elastic scatters. Its omission in the elastic nuclear differential cross section was consequently negligible.

[6] R. M. Sternheimer, Phys. Rev. **106**, 1027 (1957).

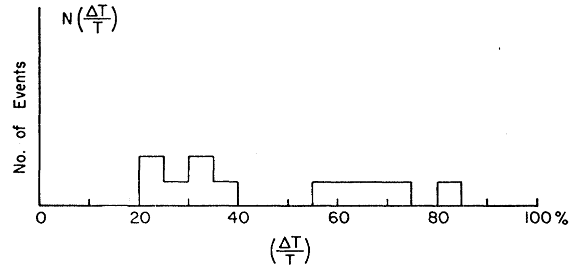

FIG. 3. Number of inelastic events with fractional energy loss $\Delta T/T$.

K^--nucleon cross sections were obtained from the Berkeley hydrogen and deuterium bubble chamber results.[7]

Inside the nucleus the K^- meson has the velocity v associated with the kinetic energy $T_{\mathrm{in}} = T_{\mathrm{out}} - \mathrm{Re}V$; T_{out} is the kinetic energy outside the nucleus taken as 50 Mev for our data. This is certainly an oversimplification since the optical model potential is velocity dependent. However, $\bar{\sigma}$ increases with decreasing v over the energy interval under consideration so that some smoothing results from this. In fact, over the energy range 50–70 Mev for T_{out}, $\eta\bar{\sigma}v$ of Eq. (1) is approximately constant; a variation of $\mathrm{Re}V$ from zero to -20 Mev corresponds to this interval.

Thus, to determine $\mathrm{Im}V$ from Eq. (1), $\mathrm{Re}V$ should be known. The available evidence to date indicates that this is negative. The original argument was advanced by Ceccarelli[8] and is supported by our data on the inelastic scattering. The angular distribution of the inelastic scatterings is shown in Fig. 2; there are more forward than backward scatters. In Fig. 3 is plotted the distribution of fractional energy loss; the average is about 47%. These together constitute evidence for an attractive potential since the observed fractional energy loss is

$$\langle f_{\mathrm{out}}\rangle = \frac{T_{\mathrm{out}} - \mathrm{Re}V}{T_{\mathrm{out}}}\langle f_{\mathrm{inside}}\rangle,$$

where

$$\langle f_{\mathrm{inside}}\rangle = \frac{(T_{\mathrm{in}} - T_{\mathrm{in}}')}{T_{\mathrm{in}}} \quad \text{and} \quad \langle f_{\mathrm{out}}\rangle = \frac{(T_{\mathrm{out}} - T_{\mathrm{out}}')}{T_{\mathrm{out}}},$$

where T is the value of the K^- kinetic energy before the scatter and T' is its value after the scatter. If one considers the average value of $\langle f_{\mathrm{inside}}\rangle$ over an isotropic angular distribution in the center-of-mass system (which is consistent with the available data[9] for the energy range under consideration) one calculates that $\langle f_{\mathrm{inside}}\rangle = 0.45$. If the potential is attractive, the back scattered events which are favored by the Pauli prin-

[7] Ascoli, Hill, and Yoon (to be published).

[8] M. Ceccarelli, *Proceedings of the Seventh Annual Rochester Conference on High-Energy Nuclear Physics, 1957* (Interscience Publishers, New York, 1957).

[9] M. F. Kaplon, *1958 Annual International Conference on High-Energy Physics at* CERN, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958).

ciple result in K^- mesons of sufficiently low energy that they are dropped within the potential and do not emerge from the nucleus. Those that do have sufficient energy to get out then correspond to relatively small energy transfers (preferentially forward scatters) so that the observed distribution favors forward angles and $(\langle f_{\text{in}}\rangle_{T_{\text{in}}'>|\text{Re}V|})<0.45$. This requires $\text{Re}V<0$ to obtain $\langle f_{\text{out}}\rangle\sim0.47$.

The magnitude of ReV is estimated from the dispersion relation

$$\text{Re}V=(-2\pi\rho\hbar^2/m)\,\text{Re}f(0),$$

and the optical theorem

$$\bar{\sigma}=(4\pi/k)\,\text{Im}f(0),$$

and the relation

$$\bar{\sigma}_s(0)=[\text{Re}f(0)]^2+[\text{Im}f(0)]^2,$$

where $\bar{\sigma}$ is the total cross section defined previously and $\bar{\sigma}_s(0)$ and $f(0)$ are the average nucleon cross section and amplitude for elastic scattering in the forward direction. This gives

$$|\text{Re}V|\sim10\text{ Mev}.$$

As stated above, it is reasonable to assume that $\eta\bar{\sigma}v$ is constant in the energy region around 60 Mev ($E=50$ Mev; $\text{Re}V=-10$ Mev). Using the value of $\bar{\sigma}$ and v in this region gives

$$\text{Im}V\sim-20\text{ Mev}.$$

The first attempt in calculating the differential elastic nuclear scattering amplitude was by using the modified Born approximation[10]

$$f(0)=(-2m/\hbar^2)[(zZe^2/q_0^2)+\tfrac{1}{3}R_0(\text{Re}V+i\,\text{Im}V)]F(q_0).$$

Here $q_0=2k_0\sin(\theta/2)$, $k_0=p/\hbar$, p is the momentum outside of the potential well, m = mass of the K^- meson, and $z=-1$ for the charge of the K^- meson. The cross section $\bar{\sigma}(\theta)=|f(\theta)|^2$ was calculated for both the heavy elements where $\bar{Z}=41$, $R_0=1.23A^{\frac{1}{3}}\times10^{-13}$ cm, and for the light elements where $\bar{Z}=7$, $R_0=1.36A^{\frac{1}{3}}\times10^{-13}$ cm and the results weighted proportionately by the number of atoms/cm^3 in emulsion to give the average cross section. The form factor $F(q_0)$ for a uniform, Gaussian, and exponential distribution were tried. With the values $\text{Re}V=-10$ Mev (constructive interference with the Coulomb potential) and $\text{Im}V=-20$ Mev, the calculated cross section was too large to fit the data.

However, a good fit was obtained using the partial wave method to calculate the nuclear elastic scattering amplitude. The WKB approximation[11] was used to determine the nuclear phase shifts $\delta_l=\alpha_l+i\beta_l$ in

$$f(\theta)=f_c(\theta)+\sum_{l=0}^{\infty}\frac{(2l+1)}{k}e^{i(2\eta_l+\delta_l)}\sin\delta_l P_l(\cos\theta),$$

where

$$f_c(\theta)=\frac{n}{2k\sin^2(\theta/2)}\exp\{in\ln[\sin^2(\theta/2)]+i\pi+2i\eta_0\},$$

$$\eta_l-\eta_{l-1}=\tan(n/l)\quad n=(-Ze^2/\hbar v).$$

The value of α_l is determined by ReV and β_l is deter-

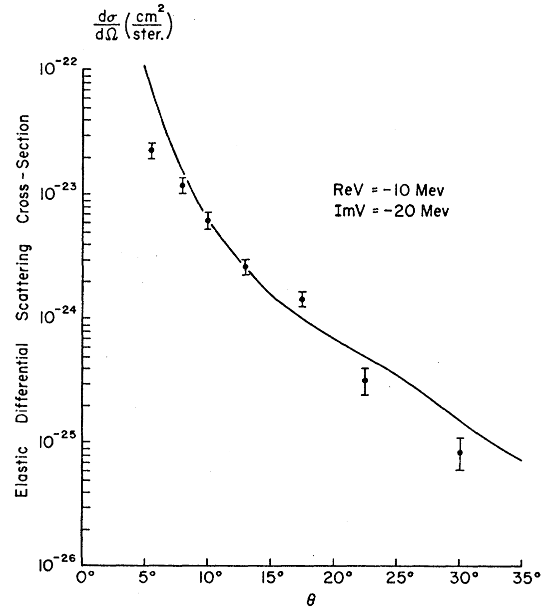

FIG. 4. K^--meson-nuclear elastic scattering cross section for $\text{Re}V=-10$ Mev, $\text{Im}V=-20$ Mev.

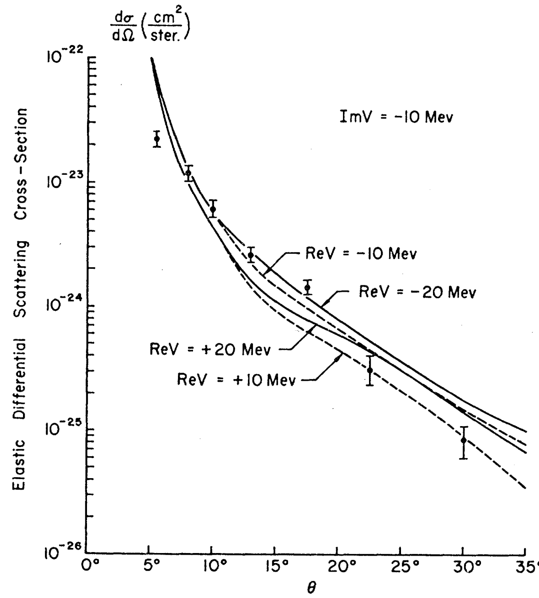

FIG. 5. K^--meson-nuclear elastic scattering cross section $\text{Im}V=-10$ Mev.

[10] A. Pevsner and J. Rainwater, Phys. Rev. **100**, 1431 (1955).
[11] L. I. Schiff, *Quantum Mechanics* (McGraw-Hill Book Company, Inc., New York, 1949).

mined by ImV. If β_l is large, the nuclear part becomes

$$\sum_{l=0}^{\infty} \frac{2l+1}{k} e^{i(2\eta_l)} \left(\frac{-1}{2i}\right) P_l(\cos\theta),$$

so that the cross section becomes independent of α_l and β_l. This is the case when $|\mathrm{Im}V| \geqslant 20$ Mev.

The calculated elastic nuclear cross section for $|\mathrm{Im}V| \geqslant 20$ Mev is shown in Fig. 4. The calculation is independent of the value of ReV. As a special case it gives the cross section for Re$V=-10$ Mev, Im$V=-20$ Mev. The curve shown is the weighted average with the summation extending to $l=5$ for the heavy elements and to $l=3$ for the light elements. The experimental values are seen to agree well with the calculated curve. The low experimental value at 6° is probably due to inefficiency in detecting small angles and the rapid variation of the correction in the vicinity of the cutoff. However, in this case the K^--nuclear elastic scattering data cannot in itself determine either the sign or the magnitude of the real potential.

If a smaller value is used for ImV, say -10 Mev, a good fit to the data can also be obtained. The curves for Im$V=-10$ Mev are plotted for the four values of Re$V=-20$ Mev, $+20$ Mev, -10 Mev, and $+10$ Mev in Fig. 5. In this case the elastic scattering tends to fit the curves where ReV is attractive (negative potential) but is insensitive to the magnitude of ReV.

IV. CONCLUSIONS

Arguments are presented from a qualitative analysis of the K^--nuclear inelastic scattering that the real part of the nuclear potential is attractive. The imaginary part of the potential is determined from the known K^--nucleon data to be Im$V=-20$ Mev. The elastic nuclear scattering is calculated by the method of partial waves. Agreement with the experimental data is obtained for $|\mathrm{Im}V| \geqslant 20$ Mev but the results are insensitive to the magnitude or sign of ReV. Calculations with smaller absolute values of ImV are sensitive to the sign but not the absolute magnitude of ReV, indicating here that Re$V<0$. It is concluded that the real and imaginary potential necessary to give the correct K^--nucleon cross section also gives a good fit to the elastic nuclear scattering data.

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