Deuteron Magnetic Moment and Momentum Dependence of Two-Nucleon Potential*

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The correction to the deuteron magnetic moment [μ_d] is calculated, in the manner pointed out by Feshbach, for the potential derived recently by Sugawara and Okubo from pion field theory. This potential includes, besides an $\mathbf{L}\cdot\mathbf{S}$ potential, a quadratic term, $-V_2(\mathbf{r})(p^2/2\kappa^2)+\text{H.c.}$ [$V_2(\mathbf{r})$ being the second-order static potential, p and κ the nucleon (relative) momentum and rest mass, respectively]. It is shown in particular that this new term gives a positive correction to μ_d. Numerical magnitudes are estimated using phenomenological deuteron wave functions fitted to all known deuteron data, the hard-core radius [r_C] and the D-state probability [P_D] being adjustable parameters. Results are shown graphically as functions of P_D for two values of r_C. It is seen that the corrections depend sensitively on these two parameters. If there were no other appreciable corrections to μ_d than those discussed here, *ps-ps* theory would lead to 6% for P_D, while μ_d would not be fitted in the *ps-pv* case as well as for the Gammel-Thaler potential, since the correction due to the quadratic term is not large enough to cancel the correction due to the $\mathbf{L}\cdot\mathbf{S}$ potential.

FESHBACH[1] has shown that the momentum-dependent term in the two-nucleon potential can appreciably affect the deuteron magnetic moment [μ_d]. The same consideration is, therefore, applied here to the pion-theoretical potential derived recently by the present author and Okubo.[2] The potential consists of three terms,

$$V=V(\text{static})+V_{LS}(r)\mathbf{L}\cdot\mathbf{S}-\{V_2(\mathbf{r})(p^2/2\kappa^2)+\text{H.c.}\},\quad (1)$$

where $V_2(\mathbf{r})$ is the second-order static potential, and p and κ are the nucleon (relative) momentum and rest mass, respectively.

The correction to μ_d in units of the nuclear magneton can be expressed in terms of the deuteron S- and D-wave functions, u and w, respectively, as

$$(\Delta\mu_d)_{p^2}=\frac{3\sqrt{2}}{2\kappa}\left[\int V_T(r)uwdr+\frac{\sqrt{2}}{4}\int V_C(r)w^2dr-\frac{\sqrt{2}}{2}\int V_T(r)w^2dr\right],\quad (2)$$

and

$$(\Delta\mu_d)_{LS}=\frac{\kappa}{6}\left[\int r^2V_{LS}(r)u^2dr-\frac{\sqrt{2}}{2}\int r^2V_{LS}(r)uwdr-\int r^2V_{LS}(r)w^2dr\right],\quad (3)$$

where[3]

$$V_2(\mathbf{r})\equiv-[V_C(r)+V_T(r)S_{12}],$$

and

$$\int[u^2+w^2]dr=1.\quad (4)$$

It is now obvious that $(\Delta\mu_d)_{p^2}$ is positive and $(\Delta\mu_d)_{LS}$ has the same sign as $V_{LS}(r)$ itself.

The above integrals have been numerically evaluated using phenomenological deuteron wave functions[4] fitted to all known deuteron data except the hard-core radius [r_C] and the D-state probability [P_D]. These two parameters are chosen as

$$r_C=0.4316 \text{ and } 0.5611\times10^{-13}\text{ cm},$$
$$P_D=3,\ 4 \text{ and } 5\%,\quad (5)$$

and the shape-dependent parameter is assumed to be zero, since it gives rise to only a minor change in the deuteron wave function.[4] The last term in (3) is neglected for simplicity. The results are plotted in Fig. 1 as functions of P_D for two values of r_C. The dashed lines indicate $(\Delta\mu_d)_{p^2}$ only. The solid curves are the sum of (2) and (3). The *ps-ps* curves refer to the *ps-ps* potential (the first paper of reference 2), while those designated as

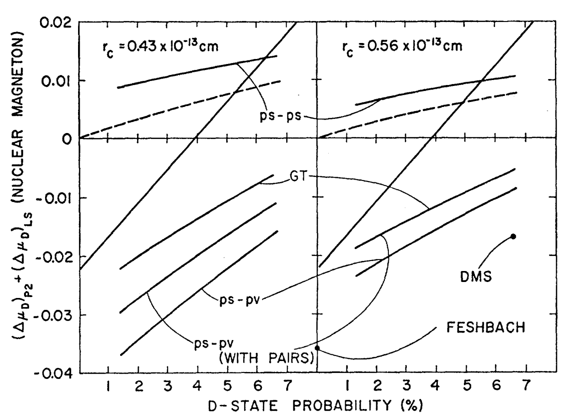

FIG. 1. The plot of the sum of (2) and (3) as functions of D-state probability for two values of the hard-core radius, r_C. The dashed curves represent our new term (2) only. Various curves correspond to several theoretical and phenomenological potentials.

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[1] H. Feshbach, Phys. Rev. **107**, 1626 (1957).

[2] M. Sugawara and S. Okubo, this issue [Phys. Rev. **117**, 605 (1960)]; preceding paper [Phys. Rev. **117**, 611 (1960)].

[3] The last term in (3) differs from Feshbach's result [reference 1] by a numerical factor.

[4] These are tabulated by L. Hulthén and M. Sugawara, in *Encyclopedia of Physics*, edited by S. Flügge (Springer-Verlag, Berlin, 1957), Vol. 39, p. 92.

ps-pv and *ps-pv* (with pairs) refer to two versions of the *ps-pv* potential (the second paper of reference 2). The G-T curves correspond to the Gammel-Thaler $\mathbf{L}\cdot\mathbf{S}$ potential,[5] to which, however, our new correction (2) has been added. The two points on the figure show the previous calculations of $(\Delta\mu_d)_{LS}$ for the Gammel-Thaler potential[5] by Feshbach[1] and others.[6] In these calculations,[1,6] $r_C=0.4\times10^{-13}$ cm and P_D is fixed as placed on the figure.

Two conclusions are drawn from the figure. First, the correction depends sensitively on both r_C and P_D, or the internal detail of the deuteron wave function. Secondly, our new correction $(\Delta\mu_d)_{p^2}$ is not a large correction compared with $(\Delta\mu_d)_{LS}$ in case of the Gammel-Thaler[5] and the *ps-pv*[2] potentials.

The corrections (2) and (3) are supposed to be almost the sum of the relativistic and exchange current corrections to μ_d, which were previously estimated by the present author.[7] The nonadditivity correction is not included in (2) and (3), since all the self-energy diagrams are dropped in deriving the potential.[2] The conclusion reached in the previous work[7] that P_D seems to be slightly smaller than 4% according to pion field theory, does not agree with the present calculation since it suggests that P_D has to deviate from 4% appreciably. This difference is not due to the difference in approaches (previously[7] μ_d was evaluated directly from pion field theory, while it is now estimated from the momentum-dependent term in the potential), but rather to the difference in the approximations made; we did not previously[7] go far enough to match those momentum-dependent terms which are investigated in the present paper.

The straight line which completely traverses both parts of Fig. 1 is the plot of $\mu_d-\mu_p-\mu_n+\frac{3}{2}(\mu_p+\mu_n-0.5)P_D$ (μ_p and μ_n are proton and neutron moments in units of nuclear magneton); this is the line on which the correction $\Delta\mu_d$ should lie, where $\Delta\mu_d$ is the total correction to be added to the *S*- and *D*-state contributions. If there were no other significant corrections besides those given by (2) and (3), the figure indicates that *ps-ps* theory would lead to $\approx$6% for P_D, while there is no way of fitting the deuteron magnetic moment in the other cases.

[5] J. Gammel and R. Thaler, Phys. Rev. **107**, 291 (1957); **107**, 1337 (1957).

[6] DeSwart, Marshak, and Signell, Nuovo cimento **6**, 1189 (1957).

[7] M. Sugawara, Phys. Rev. **99**, 1601 (1955); and Progr. Theoret. Phys. (Kyoto) **14**, 535 (1955).

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