Two-Nucleon Potential from Pion Field Theory with Pseudovector Coupling*

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The two-nucleon potential is derived from *ps-pv* pion field theory up to orders $g^2(p/\kappa)^2$ and $g^4(p/\kappa)$, using the method outlined in the preceding paper, where it was applied to *ps-ps* theory. It is shown that the only quadratic term is $-V_2(\mathbf{r})(p^2/2\kappa^2)+$H.c. [$V_2(\mathbf{r})$ is the second-order static potential], just as in the *ps-ps* case. The static part is almost the same as the *ps-ps* potential. However, a big difference appears in the $\mathbf{L}\cdot\mathbf{S}$ potential; because of the difference in kinematical corrections from *ps-ps* and *ps-pv* vertices, large $\mathbf{L}\cdot\mathbf{S}$ potentials result from both one-pion and two-pion exchange diagrams (with no nucleon pairs), though no $\mathbf{L}\cdot\mathbf{S}$ potential follows from such diagrams in case of *ps-ps* theory. The entire $\mathbf{L}\cdot\mathbf{S}$ potential has the right sign in the odd state and is of the same sign and of larger magnitude [by a factor of two or three] in the even state. We show that this isospin dependence of the $\mathbf{L}\cdot\mathbf{S}$ potential is not appreciably modified even if we add, besides the *ps-pv* coupling term, two pion-pair terms which are fitted to low-energy *S*-wave pion-nucleon scattering. This big difference in the $\mathbf{L}\cdot\mathbf{S}$ potential could eventually be used to discriminate between *ps-ps* and *ps-pv* theories. Various $\mathbf{L}\cdot\mathbf{S}$ potentials, theoretical and phenomenological, are shown on graphs for comparison.

1. INTRODUCTION

SINCE the method outlined in the preceding paper[1] seems to be a satisfactory way of deriving the two-nucleon potential up to $g^4(p/\kappa)$ and $g^2(p/\kappa)^2$, we now apply the same to *ps-pv* theory, primarily to see if there is any significant difference between the *ps-ps* and *ps-pv* potentials which could eventually be used to discriminate between these theories.

It is found that there is a big difference in the $\mathbf{L}\cdot\mathbf{S}$ potential, though the static potential stays almost the same. The details are presented in this paper.

Of course, the *ps-pv* coupling alone can hardly be the correct coupling, since it apparently cannot explain low-energy *S*-wave pion-nucleon scattering. We, therefore, supplement it by adding

$$H'=(\lambda_1/\mu)\bar{\psi}\psi\phi^2+i(\lambda_2/\mu^2)\bar{\psi}\gamma_\mu\boldsymbol{\tau}\psi\boldsymbol{\phi}\times(\partial\boldsymbol{\phi}/\partial x_\mu),\quad(1)$$

where $\partial\boldsymbol{\phi}/\partial t$ stands for the canonical conjugate to $\boldsymbol{\phi}$ and λ_1 and λ_2 are chosen so that they reproduce low-energy pion-nucleon scattering [$\lambda_1\approx\lambda_2\approx0.4$].[1] We show that these pion-pair terms give only minor effects upon both static and $\mathbf{L}\cdot\mathbf{S}$ potentials resulting from the *ps-pv* coupling term alone.

It is shown in particular that the one-pion exchange diagram is the source of the largest $\mathbf{L}\cdot\mathbf{S}$ potential up to the orders in question. A canonical transformation converts most of the $g^2(p/\kappa)^2$-term into a large $\mathbf{L}\cdot\mathbf{S}$ potential of order $g^4(p/\kappa)$, leaving $-V_2(\mathbf{r})(p^2/2\kappa^2)+$H.c. [$V_2(\mathbf{r})$ being the second-order static potential] as the only essentially quadratic potential, just as in the *ps-ps* case. The static potential stays almost the same as the *ps-ps* potential; the entire static potential, therefore, looks like almost the same as $V_2(\mathbf{r})$ down to distances of the order of the pion Compton wavelength.

The source of the large $\mathbf{L}\cdot\mathbf{S}$ potential in case of *ps-pv* theory can be traced back to the purely kinematical corrections from the *ps-pv* vertices. Therefore, the $\mathbf{L}\cdot\mathbf{S}$ potential reported in this paper has a well-established origin.

2. SECOND-ORDER POTENTIAL

Following exactly the same procedure outlined in the preceding paper,[1] we first show that the wave-function renormalization [(13) of A] does not have to be modified up to order $g^2(p/\kappa)$. The first remarkable difference occurs in the second-order potential in (18) of A: We now have to add the following extra term of order $g^2(p/\kappa)^2$ to the third term of (18):

$$\frac{g^2}{4\pi}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\frac{i}{4\kappa^2\mu^2}\left[p^2,\left[(\boldsymbol{\sigma}_1\cdot\mathbf{p})(\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla})\frac{e^{-\mu r}}{r}+(\boldsymbol{\sigma}_2\cdot\mathbf{p})(\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla})\frac{e^{-\mu r}}{r}+\text{H.c.}\right]\right]_-.\quad(1)$$

This is, however, of the commutator form between p^2 and some function. Thus it is totally transformed, according to the argument in Sec. 5 of A, into a term of order $g^4(p/\kappa)$. The result is

$$\frac{g^4\mu^2}{(4\pi)^2\kappa}[3-2(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)]\left[\left(\frac{1}{x}+\frac{1}{x^2}\right)^2-2\left(\frac{1}{x^2}+\frac{1}{x^3}\right)\times\left(\frac{1}{x}+\frac{3}{x^2}+\frac{3}{x^3}\right)\mathbf{L}\cdot\mathbf{S}\right]e^{-2x}.\quad(2)$$

Thus the only quadratic term is, as before, $-V_2(\mathbf{r})\times(p^2/2\kappa^2)+$H.c., as it should be because of the equivalence of these two couplings.

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[1] M. Sugawara and S. Okubo, preceding paper [Phys. Rev. **117**, 605 (1959)], to be cited as A in this paper.

3. FOURTH-ORDER POTENTIAL

The pion-uncrossing diagrams are shown to give exactly the same contribution as before; they give rise to a velocity-dependent term which cancels out exactly a term [the fifth term in (18) of A] due to wave-function renormalization [(13) of A].

On the other hand, the pion-crossing diagrams give an $\mathbf{L}\cdot\mathbf{S}$ potential. The additional term is

$$\frac{g^4\mu^2}{(4\pi)^2\kappa}\{3+2(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)\}\left[\left(\frac{1}{x}+\frac{1}{x^2}\right)^2-2\left(\frac{1}{x^2}+\frac{1}{x^3}\right)\times\left(\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}\right)\mathbf{L}\cdot\mathbf{S}\right]e^{-2x}. \quad (3)$$

It is now seen that the one-pion exchange contribution (2) is even larger than the two-pion exchange contribution (3). The latter agrees with the previous calculation,[2] while (2) is an entirely new term.

In case of *ps-pv* coupling, diagrams including nucleon pairs are certainly negligible. Thus the entire potential resulting from *ps-pv* coupling is just the sum of $V_{ps\text{-}ps}$(no-pair) [(22) of A] and (2) and (3):

$$V_{ps\text{-}pv}=V_{ps\text{-}ps}(\text{no-pair})+\frac{g^4\mu^2}{(4\pi)^2\kappa}6\left(\frac{1}{x}+\frac{1}{x^2}\right)^2e^{-2x}-\frac{g^4\mu^2}{(4\pi)^2\kappa}12\left[\left(\frac{1}{x^2}+\frac{1}{x^3}\right)\left(\frac{1}{x}+\frac{2}{x^2}+\frac{2}{x^3}\right)-\frac{(\boldsymbol{\tau}_1\boldsymbol{\tau}_2)}{3}\left(\frac{1}{x^2}+\frac{1}{x^3}\right)\left(\frac{2}{x^2}+\frac{2}{x^3}\right)\right]e^{-2x}\mathbf{L}\cdot\mathbf{S}, \quad (4)$$

where $V_{ps\text{-}ps}$(no-pair) does not include any $\mathbf{L}\cdot\mathbf{S}$ potential.

It is added that both (2) and (3) [thus the $\mathbf{L}\cdot\mathbf{S}$ potential in $V_{ps\text{-}pv}$] are entirely due to the difference between the *ps-ps* and *ps-pv* vertex corrections. Thus the origin of this $\mathbf{L}\cdot\mathbf{S}$ potential is purely kinematical and has no ambiguity.

As is seen from (4), the correction to the static potential is very small, while the $\mathbf{L}\cdot\mathbf{S}$ potential is quite appreciable. This $\mathbf{L}\cdot\mathbf{S}$ potential is plotted in the final section. It is characterized as having the same sign in both even and odd states [the right sign in the odd state], while the magnitude in the even state is nearly 2 to 3 times as large as in the odd state.

4. PION-PAIR TERMS CONTRIBUTIONS

As was stated in the introduction, *ps-pv* coupling alone can hardly be the correct coupling. Therefore, we introduce (1) in addition to the *ps-pv* term and estimate the contributions from these pion-pair terms. These are nearly the same as the nucleon-pair contributions in case of *ps-ps* theory, simply because of the formal similarity. They are explicitly

$$V_{ps\text{-}pv}(\text{pairs})=V_{ps\text{-}ps}(\text{one-pair})+V_{ps\text{-}ps}(\text{two-pair})-\frac{\lambda_1 g^2\mu^2}{(4\pi)^2\kappa}\frac{6}{\pi}\left[\frac{K_0(2x)}{x^3}+\left(\frac{2}{x^2}+\frac{1}{x^4}\right)K_1(2x)\right]+\frac{\lambda_1 g^2\mu^2}{(4\pi)^2\kappa}\frac{12}{\pi}\left[\frac{2K_0(2x)}{x^3}+\frac{3K_1(2x)}{x^4}\right]\mathbf{L}\cdot\mathbf{S}, \quad (5)$$

where $V_{ps\text{-}ps}$(one-pair) and $V_{ps\text{-}ps}$(two-pair) are given, respectively, by (26) of A and (25) of A. These are, however, shown to be minor compared with $V_{ps\text{-}pv}$ given by (4) as regards both the static and the $\mathbf{L}\cdot\mathbf{S}$ potentials.

5. QUANTITATIVE DISCUSSIONS AND CONCLUSIONS

The two-nucleon potential consists, in general, of a static potential, an $\mathbf{L}\cdot\mathbf{S}$ potential, and $-V_2(\mathbf{r})(p^2/2\kappa^2)$ +H.c. [$V_2(\mathbf{r})$ being the second-order static potential], up to orders $g^4(p/\kappa)$ and $g^2(p/\kappa)^2$. The quadratic term is, therefore, the same as in the *ps-ps* case, as it should be because of the equivalence theorem.

The static part is almost the same as the *ps-ps* potential, as is seen from (4) and (5). Thus the entire static potential resembles the second-order static potential down to distances of the order of the pion Compton wavelength, except for the central force in the triplet even state.

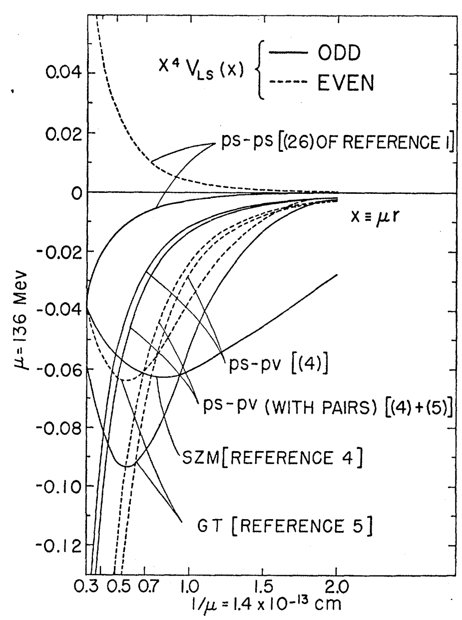

FIG. 1. Plot of x^4 times the $\mathbf{L}\cdot\mathbf{S}$ potential, in units of μ, against $x\equiv\mu r$. Solid curves refer to the odd state, dashed curves to the even state. Various pion theoretical potentials are compared with the phenomenological ones.

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[2] S. Okubo and R. E. Marshak, Ann. phy. 4, 166 (1958).

The $\mathbf{L}\cdot\mathbf{S}$ potential in (4) and (5) are shown in Fig. 1, where x^4 times the coefficient of $\mathbf{L}\cdot\mathbf{S}$ is plotted in units of μ against $x\equiv\mu r$. Solid curves refer to the odd state, dotted ones to the even state. The *ps-ps* curves indicate the $\mathbf{L}\cdot\mathbf{S}$ potential from *ps-ps* theory [(26) of A], the *ps-pv* curves show those of (4), and the *ps-pv* (with pairs) curves show those of the sum of (4) and (5). The curves S-Z-M[3] and G-T[4] are the phenomenological ones.

It is seen from the figure that the $\mathbf{L}\cdot\mathbf{S}$ potential from *ps-ps* theory might be too small in magnitude, while the one from *ps-pv* theory is quite appreciable, though it might be smaller in the odd state. However, the present evidence on the $\mathbf{L}\cdot\mathbf{S}$ potential is very vague and we can hardly conclude anything definite. It is simply pointed out that these two theories predict very different $\mathbf{L}\cdot\mathbf{S}$ potentials, and these differences could eventually be used to discriminate between these theories. We recall, however, that the only source of the $\mathbf{L}\cdot\mathbf{S}$ potential in case of *ps-ps* theory is the nucleon-pair diagrams the estimate of which is still preliminary, while there seems no ambiguity for the $\mathbf{L}\cdot\mathbf{S}$ potential in *ps-pv* theory.

It was pointed out by Feshbach[5] that the negative $\mathbf{L}\cdot\mathbf{S}$ potential in the even state may cause great trouble in explaining the deuteron magnetic moment. This is important especially in *ps-pv* theory, where the $\mathbf{L}\cdot\mathbf{S}$ potential seems even stronger than the Gammel-Thaler potential in the even state. We can show, however, that the quadratic term $[-V_2(\mathbf{r})(p^2/2\kappa^2)+\text{H.c.}]$ gives a new correction which cancels partially the Feshbach effect.[5] This point is discussed in detail in the following paper.

We finally recall the comment given at the end of our previous paper[1] on the higher order terms neglected in the present paper.

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[3] Signell, Zinn, and Marshak, Phys. Rev. Letters **1**, 416 (1958).

[4] J. Gammel and R. Thaler, Phys. Rev. **107**, 291 (1957); **107**, 1337 (1957).

[5] H. Feshbach, Phys. Rev. **107**, 1626 (1957).