Positive Pion Photoproduction near Threshold

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An investigation is made in order to test the compatibility of photoproduction data on positive pions near threshold with some predictions of dispersion theory. The result is that no real disagreement necessarily exists between these theoretical predictions and our still uncertain experimental knowledge.

INTRODUCTION

THE interpretation of threshold pion interactions has been in the past few years the aim of numerous investigations[1] stimulated by some inconsistencies appearing between theoretical predictions and results of measurements. Nevertheless, the situation does not look much improved since it is not yet very clear whether this inconsistency is due to the uncertainty of measured quantities or to the approximations of theoretical calculations or even perhaps to the inadequacy of the theory in describing pion interactions. One point in which theory and experiments seem to disagree considerably is the behavior of pion photoproduction cross sections near threshold. Apart from the rather delicate problem of the ratio between π^- and π^+ production, there is a striking difference between the energy dependence of the 90° differential cross section for π^+ photoproduction as predicted by the approximate solutions of dispersion relations of Chew, Goldberger, Low, and Nambu[2] and the experimental values of this quantity.

In order to investigate how seriously one has to take this discrepancy, we have made a new analysis of photoproduction data on π^+ in the energy range $E_\gamma = 163$ Mev, $E_\gamma = 200$ Mev in the laboratory, combining all available experimental data on angular distributions with a "second approximation" to dispersion relations. In our analysis no attempt is made to resolve completely the photoproduction problem by determining theoretically the photoproduction amplitude; a semiphenomenological treatment is followed. We use only theoretical equations which look more reliable and by connecting the different experimental quantities through these equations, a compatibility test is made. The result of this test is that the equations used are compatible with our present experimental knowledge on photoproduction. We point out that a more accurate measurement of the angular distributions of photopions would yield very detailed information through a semiphenomenological analysis similar to the one given in this work.

As a further result, our analysis seems to indicate that some corrections to the theoretical static limit are larger than estimated by CGLN. The difference between our multipoles and those by Chew, Goldberger, Low, and Nambu go in the right direction to make the fitting of the theory with experiments satisfactory. This might be an interesting point in favor of the theory itself.

"SECOND APPROXIMATION" TO DISPERSION RELATIONS AND DESCRIPTION OF THE METHOD

In the following we shall refer for notation directly to the Chew, Goldberger, Low, and Nambu paper. Photoproduction amplitudes given in CGLN are obtained through a comparison of the dispersion relations expanded in powers of $1/M$ (inverse of the nucleon mass) with the equations of the static theory. However, it is difficult to say whether this expansion is correct under dispersion integrals especially con-

TABLE I. Results from the evaluation of the integrals.[a]

W	$\mathrm{Re}\left(\mathfrak{F}_1+\frac{M+\epsilon_2}{q}\mathfrak{F}_2\right)$	Born term	Rescattering corr.
7.8	−23.78	−16.84	− 6.94
7.87	−24.98	−16.78	− 8.20
7.94	−26.30	−16.72	− 9.58
8.0	−27.52	−16.76	−10.86
W	$\mathrm{Re}3(E_{1+}-M_{1+})$	Born term	Rescattering corr.
7.8	2.52	2.00	0.52
7.87	3.50	2.70	0.81
7.94	4.34	3.23	1.11
8.0	4.94	3.52	1.42
W	$\mathrm{Re}3(M_{2+}-E_{2+}-E_{2-}-M_{2-})$		Rescattering corr.
7.8	−0.56		$\simeq 0$
7.87	−0.97		$\simeq 0$
7.94	−1.33		$\simeq 0$
8.0	−1.57		$\simeq 0$

[a] W is expressed in pion mass units. Multipoles are in units 10^{-15} cm/sterad$^{\frac{1}{2}}$.

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[1] For a general survey of the situation of threshold pion physics see reports of *1958 Annual International Conference on High-Energy Physics at CERN*, edited by B. Ferretti (CERN Scientific Information Service, Geneva, 1958), p. 50 and reports of the 1959 Kiev Conference.

[2] Chew, Goldberger, Low, and Nambu, Phys. Rev. **106**, 1345 (1957) in the following indicated as CGLN. For a comparison of CGLN photoproduction amplitudes with experiments see reference 1. Dispersion relations for photoproduction have also been studied by Logunov, Tavkhelidze, and Solovyov, Nuclear phys. **4**, 427 (1957); and L. D. Solovyov, Nuclear Phys. **5**, 256 (1958).

sidering the convergence of integrals in the high-energy region. The S-wave contribution seems particularly affected by this treatment.

We will remark, as a first step to obtain more reliable informations from dispersion relations, that by adding Eqs. (9.1) and (9.2) of CGLN we get a subtracted relation in which the only important contribution under integrals is given by the P-wave amplitudes. All integrals are more convergent than in Eqs. (9.1) and (9.2). Having used Eqs. (7.1) and (7.2), we write

$$\begin{aligned}
&\mathrm{Re}[\mathfrak{F}_1+(M+\epsilon_2)/q\,\mathfrak{F}_2]=\mathrm{Re}\{E_0+(2M_{1+}+M_{1-})(M+\epsilon_2)/q+3M_{2-}-3M_{2+}-\tfrac{3}{2}E_{2+}+E_{2-}\\
&\quad+3x[M_{1+}+E_{1+}+(3M_{2+}+2M_{2-})(M+\epsilon_2)/q]+(15/2)x^2(2M_{2+}+E_{2+})\}=-\binom{0}{1}f\mu(M/W)[(W+M)^2-1]^{\frac{1}{2}}\\
&\quad+\frac{(W-M)[(M+\epsilon_1)(M+\epsilon_2)]^{\frac{1}{2}}}{\pi}\int_{M+1}^{\infty}\frac{2W'dW'}{q'(W'-M)[(M+\epsilon_1')(M+\epsilon_2')]^{\frac{1}{2}}}\\
&\quad\times\mathrm{Im}\left\{[E_0q'+M_{1+}(3\omega_q'+2(M+\epsilon_2')-6M\nu_1/k')+M_{1-}(M+\epsilon_2')]\left[\frac{1}{W'^2-W^2}\pm\frac{1-4M\nu_1/(W^2-M^2)}{W'^2+W^2-2M^2-4M\nu_1}\right]\right.\\
&\qquad\left.+\binom{0}{1}\frac{12M\nu_1}{W^2-M^2}\,\frac{E_{1+}-M_{1+}}{W'-M}\right\},\qquad(1)
\end{aligned}$$

where W = total energy (in pion mass units), $x=\cos\theta=(k\omega_q-2M\nu_1)/kq$, $\epsilon_1(\epsilon_2)$ = nucleon energy in the initial (final) state, q = pion momentum, ω_q = pion energy, and k = photon momentum. All quantities refer to the c.m. system. $2M\nu_1=\frac{1}{2}(2M+1)/(M+1)$ (see below). Also we have that f = pion-nucleon renormalized coupling constant, $\mu=\mu_p-\mu_n$ for isotopic superscripts $+$ and $-$, and $\mu=\mu_p+\mu_n$ for isotopic superscript 0, with $\mu_p=1.78e/2M$, $\mu_n=-1.91e/2M$, and e = electronic charge. Only S and P amplitudes have been retained under the dispersive integrals.

The upper sign goes with isotopic superscripts $+$, 0 and the lower with isotopic superscript $-$. On the left-hand side we have indicated also the D-wave terms which are well approximated by the Born terms alone. Higher multipoles have been supposed negligible.

Our procedure consists in using for $\mathrm{Im}M_{1\pm}$ and $\mathrm{Im}E_{1+}$ the approximate expressions of CGLN in order to evaluate integrals, neglecting $\mathrm{Im}E_0$. In this way we expect to attain some advantages: the main contribution to the integrals comes actually from the 33-resonant state. The high-energy region is not so important because of the good convergence of the expressions. No $1/M$ expansion is made in the kinematical factors so that the method represents a second approximation of dispersion relations in an iterative calculation. The momentum transfer at which this expression has to be evaluated has been chosen in order to get $x=\cos\theta=0$ at threshold. In this way no contribution appears from the unphysical region.

With the same method described above we have also treated Eqs. (9.3) and (9.4) which we report for reference here.

$$\begin{aligned}
&\mathrm{Re}\mathfrak{F}_3=\mathrm{Re}[3(E_{1+}-M_{1+})+D\text{-wave terms}]=\tfrac{1}{2}q\frac{W-M}{2W}[(M+\epsilon_1)(M+\epsilon_2)]^{\frac{1}{2}}\left\{\frac{ef}{2M\nu_1}\left[\frac{2M}{W+M}\mp\frac{2M(W-M)}{W^2-M^2-4M\nu_1}\right]\right.\\
&\quad\pm f\mu\frac{4M}{4M\nu_1-W^2+M^2}+\frac{1}{\pi}\int_{M+1}^{\infty}\frac{2W'}{W'-M}\,\frac{dW'}{q'[(M+\epsilon_2')(M+\epsilon_2')]^{\frac{1}{2}}}\\
&\quad\left.\times\mathrm{Im}\left(3(E_{1+}-M_{1+})\left[\frac{1}{W'-W}\pm\frac{-W'+W-2M+4M\nu_1/(W'-M)}{W'^2+W^2-2M^2-4M\nu_1}\right]\mp2\frac{\mathfrak{F}_1q'+(M+\epsilon_2')\mathfrak{F}_2}{W'^2+W^2-2M^2-4M\nu_1}\right)\right\},\qquad(2)
\end{aligned}$$

$$\begin{aligned}
&\mathrm{Re}\mathfrak{F}_4=\mathrm{Re}[3(M_{2+}-E_{2+}-M_{2-}-E_{2-})+\text{higher wave terms}]=\tfrac{1}{2}q^2\frac{W-M}{2W}\left(\frac{M+\epsilon_1}{M+\epsilon_2}\right)^{\frac{1}{2}}\\
&\quad\times\left\{-\frac{ef}{2M\nu_1}\left[\frac{2M}{W-M}\mp\frac{2M(W+M)}{W^2-M^2-4M\nu_1}\right]\pm f\mu\frac{4M}{4M\nu_1-W^2+M^2}+\frac{1}{\pi}\int_{M+1}^{\infty}\frac{2W'dW'}{q'(W'-M)[(M+\epsilon_1')(M+\epsilon_2')]^{\frac{1}{2}}}\right.\\
&\quad\left.\times\mathrm{Im}\left(3(E_{1+}-M_{1+})\left[\frac{1}{W'+W}\pm\frac{-2M-W'-W+4M\nu_1/(W'-M)}{W'^2+W^2-2M^2-4M\nu_1}\right]\mp2\frac{\mathfrak{F}_1q'+(M+\epsilon_2')\mathfrak{F}_2}{W'^2+W^2-2M^2-4M\nu_1}\right)\right\}.\qquad(3)
\end{aligned}$$

The convergence of integrals is the same as in Eq. (1).

All integrals have been evaluated analytically with the method indicated in Appendix I. We have considered only contributions from terms containing $\sin^2\alpha_{33}$. Terms containing $\sin 2\alpha_{33}$ and the small phase shifts were estimated and found negligible. CGLN expression for the multipoles are listed in Appendix II. Our results are summarized in Table I. Figures include also the contribution of the isotopic index 0, which is well approximated by the Born term, and concern directly π^+ photoproduction.

As expected, "rescattering corrections" are very large in Eq. (1). In Eq. (3) a cancellation makes the contribution of integrals negligible. Waves higher than $l=1$ in Eq. (2) and $l=2$ in Eq. (3) were eliminated by projection.

Comparison with CGLN multipoles will be made later.

By the procedure described we have obtained predictions of the theory about some particular combinations of the multipoles. All values obtained have probably the same degree of reliability. To go further with our analysis we consider the expression of the angular distribution of photopions which, retaining only S and P waves and SD interference terms for the final state of the pion, reads[3]

$$\frac{d\sigma}{d\Omega}\frac{1}{Q}=a_0+a_1\cos\theta+a_2\cos^2\theta,$$

where $Q=\omega_q q/(1+k/M)^2$ is the statistical factor. a_0, a_1, and a_2 can be easily expressed in terms of the multipoles—supposed to be real—in the following way:

$$\begin{aligned} a_0&=S^2+X^2+Y^2+SD,\\ a_1&=-2SK,\\ a_2&=K^2-X^2-Y^2-3SD,\end{aligned}\tag{4}$$

where

$$\begin{aligned} S&=E_0(q/kQ)^{\frac{1}{2}},\\ D&=(-3M_{2+}+3M_{2-}-E_{2-}-6E_{2+})(q/kQ)^{\frac{1}{2}},\\ X&=\tfrac{3}{2}(E_{1+}-M_{1+})(q/kQ)^{\frac{1}{2}},\\ X+Y&=-(2M_{1+}+M_{1-})(q/kQ)^{\frac{1}{2}},\\ X+Y+K&=-3(M_{1+}+E_{1+})(q/kQ)^{\frac{1}{2}}.\end{aligned}\tag{5}$$

Actually higher multipoles contribute to the angular distribution; however, neglecting measurements at small and large angles ($150°>\theta>30°$), a quadratic parabola gives a reasonable fitting of the data. A more refined analysis would be practivally meaningless considering the type of discussion we have in mind, owing to the present experimental errors.

The method that we follow from this point on consists in taking from experiments two of the coefficients a_i and then determining from the 5 equations we get combining the 3 theoretical predictions (1), (2), and (3) with these coefficients, the 5 parameters S, X, Y, K, and D. We have taken a_0 and a_1 as known since they are the best established experimentally. Then a_2 is calculated and compared with the experimental values. Actually the calculation is not so straightforward since the interference term due to D waves contains a rather complicated combination of $M_{2\pm}$ and $E_{2\pm}$. To supplement the information given by Eq. (3), we may observe that the right-hand side of Eq. (1) is very weakly x dependent. Then we write the approximate relations

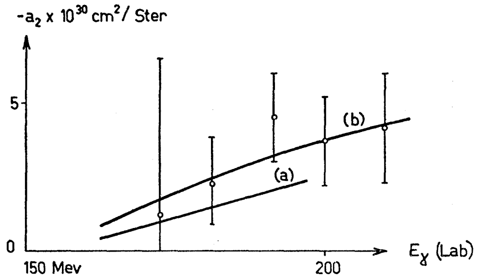

FIG. 1. (a) Present analysis. (b) Interpolation of experimental data.[4]

$$\begin{aligned} M_{1+}+E_{1+}+[(M+\epsilon_2)/q](3M_{2+}+2M_{2-})&\simeq0,\\ 2M_{2+}+E_{2+}&\simeq0.\end{aligned}\tag{6}$$

This is not completely correct since higher multipoles are present; however, we consider the approximation to be in line with the analysis based on 5 parameters of the experimental data, which contain all multipoles. Making use of Eqs. (6), we find easily

$$D=M_{2+}-E_{2+}-E_{2-}-M_{2-}-[2q/(M+\epsilon_2)](M_{1+}+E_{1+}).\tag{7}$$

We are now ready to use all our equations in order to calculate a_2. In Appendix III we give for reference the algebraic system which has to be solved. The result of the calculation is given in Fig. 1.

As input parameters we have taken the interpolation of experimental data given in reference.[4] a_0 is constant and equal to 14.8×10^{-30} cm^2/sterad; a_1 is interpolated by the formula $a_1=-5.7[q/k(k\omega_q)^{\frac{1}{2}}]\times10^{-30}$ cm^2/sterad.

DISCUSSION

The agreement between the calculated a_2 and the experimental values of this quantity can be considered satisfactory. Even if the experimental errors are very large, the energy dependence seems to be quite well

[3] See for instance M. Gell-Mann and K. M. Watson, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Palo Alto, 1954), Vol. 4, p. 241.

[4] Beneventano, Bernardini, Carlson-Lee, Stoppini, and Tau, Nuovo cimento 4, 323 (1956). This paper contains a summary of most of the previous experimental data in the same energy range. Measurements of π^+ angular distribution have been recently made by J. H. Marlmberg and C. Robinson, Phys. Rev. **109**, 158 (1958); and Knapp, Imhof, Kenney, and Mendez, Phys. Rev. **107**, 323 (1958).

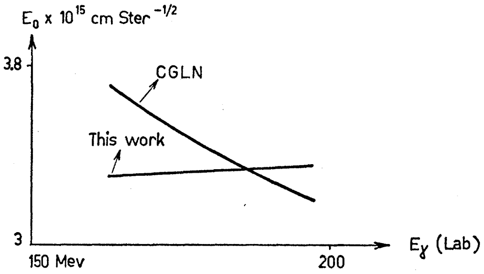

FIG. 2. Energy dependence of E_0.

reproduced. We may perhaps say that this agreement is an argument in favor of the procedure followed and the constancy of a_0 appears not incompatible with the theory. To illustrate this last point, we remark that an a_0 greater than the experimental value used in the lower energy region gives a calculated a_2 smaller in absolute value.

However, appreciable variations of a_0, say 20%, would not bring the calculated values of a_2 outside the experimental errors.

We must conclude that a constant a_0 is not incompatible with the other experimental data: however, at the same time a behavior similar to the one predicted by CGLN is not excluded.[5] In order to resolve this question definitely, a more accurate knowledge of the angular distribution is necessary.

Another interesting feature to discuss is the difference between our combinations of multipoles and those by CGLN. The energy dependence of E_0 (see Fig. 2) rests essentially on the assumed constancy of a_0. The difference between CGLN values and ours yields an indirect estimate of their unknown function $N^{(-)}$. We want now to compare the right-hand side of Eqs. (1) and (2) with the values of the same combinations of multipoles in the CGLN approximation. It is not quite clear which set of phase-shifts has to be used in evaluating CGLN expressions since this evaluation depends critically on the assumed low-energy dependence of all phase-shifts. In Table II the values of the CGLN expression for the combination of multipoles included in Eqs. (1) and (2) are calculated using (a) the whole set of phase shifts by Anderson,[6] (b) setting equal to 0 the small P waves. Chiu and Lomon[7] values for the small phase shifts have also been used and the result is practically the same as for (a). The unknown CGLN function $N^{(-)}$ is set equal to 0. $\mathrm{Re}3(E_{1+}-M_{1+})$ agrees very well with our values while for $\mathrm{Re}(\mathfrak{F}_1+[(M+\epsilon_2)/q]\mathfrak{F}_2)$ an indication that our values are higher than those by CGLN seems to exist, independently of the set of phase shifts used.

TABLE II. Values of the expressions in Eqs. (1) and (2) in CGLN approximation.

W		$\mathrm{Re}\left(\mathfrak{F}_1+\frac{M+\epsilon_2}{q}\mathfrak{F}_2\right)$	$\mathrm{Re}3(E_{1+}-M_{1+})$
7.8 8.0	(a) -18.3 -20.8	(b) -19.8 -24.7	(a) 2.5 4.7

$M_{1\pm}$ and E_{1+} can also be obtained separately from our combined analysis. However, their values are differences of large numbers and thus are not very accurate. We wish to point out that M_{1+} and E_{1+} agree substantially with the CGLN values while lower values are found for the positive quantity M_{1-}. This might explain the values found for the right-hand side of Eqs. (1) and (2).

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APPENDIX I

We want to give a brief summary of the method followed in evaluating dispersive integrals. The expressions that have been calculated were of the form

$$\int_{M+1}^{\infty} f(W',W)\frac{\sin^2\alpha_{33}}{q'^3}dW', \qquad \mathrm{P}\int_{M+1}^{\infty} g(W',W)\frac{\sin^2\alpha_{33}}{q'^3}dW',$$

where $f(W',W)$, $g(W',W)$ represent rational functions of their arguments. Functions g have a simple pole for $W'=W$ and P indicates the principal value. We have taken the following effective-range formula for $\tan\alpha_{33}$:

$$\tan\alpha_{33}=\frac{aq^3}{\omega^*(1-r\omega^*)},$$

where $\omega^*=W-M$ and $r=1/\omega_r$; ω_r=resonance energy. We have put the values $a=0.11$ and $r=1/2.16$ obtained by Anderson and Davidon[8] through a best fit of all data on scattering experiments.

We have used this formula because of its simplicity: the values of the integrals depend essentially on the behavior of α_{33} in the resonance region and the low-

[5] Barbaro, Goldwasser, and Carlson-Lee have recently measured a_0 at 161 Mev and find $a_0=18.5\pm1.3\times10^{-30}$ cm²/sterad in agreement with CGLN predictions. A. Barbaro (private communication).

[6] H. L. Anderson, *Proccedings of the Sixth Annual Rochester Conference on High-Energy Physics* (Interscience Publishers, New York, 1956), p. 20.

[7] H. Y. Chiu and E. L. Lomon, Ann. Phys. **6**, 50 (1959).

[8] H. L. Anderson, Lectures on the Physics of Mesons, Roma, 1957 (unpublished).

energy dependence is not critical. We have

$$\sin^2\alpha_{33}=\frac{a^2q^6}{\omega^{*2}(1-r\omega^*)^2+a^2q^6}. \tag{1A}$$

To make the calculation easier we have used the following approximation for q:

$$q\simeq[(W+M)/2M](\omega^{*2}-1)^{\frac{1}{2}},$$

and for the denominator of Eq. (1) we have used

$$\omega^{*2}(1-r\omega^*)^2+a^2q^6\simeq\eta^2\omega^{*2}P_1P_2,$$

where $\eta^2=1/232.1$. P_1 and P_2 are quadratic expressions in ω^*. $P_1=(\omega^*-\alpha)(\omega^*-\alpha^+)$, $P_2=(\omega^*-\beta)(\omega^*-\beta^+)$, with $\alpha=-1.951+i7.431$; $\beta=1.951+i0.378$.

Integrals can now be evaluated by factoring the integrand. They are reduced to the form

$$\sum_1^n A_i\int_1^\infty\frac{(\omega^{*2}-1)^{\frac{1}{2}}}{\omega^*-a_i}d\omega^*,$$

where a_i are the roots of the denominator.

The A_i satisfy

$$\sum_1^n A_i=0,$$
$$\sum_1^n A_ia_i=0,$$
$$\cdots$$
$$\sum_1^n A_ia_i^{n-1}=1.$$

APPENDIX II

The expressions for the multipoles extracted from CGLN amplitudes are

$$M_{1+}=ef\sqrt{2}qk[-\tfrac{1}{3}F_M/(1+\omega^*/M)+(g_p+g_n)/6M\omega^*$$
$$-\lambda h^{--}+\tfrac{1}{3}\lambda(h_{11}-h_{31})-1/gie^{i\alpha_{33}}\sin\alpha_{33}F_M]$$
$$M_{1-}=ef\sqrt{2}qk[\tfrac{2}{3}F_M/(1+\omega^*/M)-\tfrac{1}{3}(g_p+g_n)/M\omega^*$$
$$+\tfrac{1}{3}\lambda(h_{11}-h_{31})]$$
$$E_{1+}=ef(\sqrt{2}/3)kq[F_Q/(1+\omega^*/M)+\tfrac{1}{3}ie^{i\alpha_{33}}\sin\alpha_{33}F_Q].$$

Notations are the same as in CGLN.

APPENDIX III

The algebraic system to be solved to obtain the five parameters S, X, Y, K, D, is reported here.

The three theoretical predictions (1)–(3) have been combined with the first two of Eqs. (4) using the expressions for S, X, Y, K, D, given by Eqs. (5). All multipoles can be supposed real in the energy region considered. Using Δ, Θ, and Γ for the values of the right-hand side of Eqs. (1)–(3) reported in Table I we find

$$S-[(M+\epsilon_2)/q](X+Y)+\tfrac{1}{2}D=(\Delta+\tfrac{1}{2}\Gamma)(q/kQ)^{\frac{1}{2}}$$
$$X=(\Theta/2)(q/kQ)^{\frac{1}{2}}$$
$$[2q/(M+\epsilon_2)](X+Y+K)-3D=-\Gamma(q/kQ)^{\frac{1}{2}}$$
$$S^2+X^2+Y^2+SD=a_0$$
$$-2SK=a_1.$$

Then

$$a_2=S^2-2SD+K^2-a_0.$$

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Mass Splittings within Baryon Charge Multiplets*

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We have calculated the effect upon the mass splittings within each isobaric multiplet of a phenomenological boson mass difference. It has been possible to sum the diagrams to all orders in the strong couplings but the results are only valid to first order in the mass difference. These results can be compactly expressed as derivatives with respect to the intermediate masses of a function related to the proper self energy. The second-order perturbation results are also calculated.

INTRODUCTION

OVER the past few years, there have been several attempts at explaining the mass differences within charge multiplets by means of the electromagnetic self-energy arising from electric charge and charge-magnetic moment interactions.[1] These attempts have been beset by many difficulties, not the least of which is the presence of divergent integrals which must be cut off in a customary but unsatisfactory manner. Moreover, in addition to this purely electromagnetic difficulty, there exist the extremely complex contributions from the combined effects of the electromagnetic and strong interactions—contributions which

* Work performed under the auspices of the U. S. Atomic Energy Commission.

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[1] See for example, R. Feynman and G. Speisman, Phys. Rev. **94**, 500 (1954); Marshak, Okubo, and Sudarshan, Phys. Rev. **106**, 599, (1957); J. Sakurai, Phys. Rev. **115**, 1304 (1959); H. Katsumori, Progr. Theoret. Phys. (Kyoto) **17**, 803 (1957).