valid, but it is the only direct comparison with theory one can presently make. The errors shown in Fig. 7 have been derived from the error limits quoted in the text by standard statistical methods, with the assumption that all errors are standard deviations.

It is seen that in those cases for which the information is most reliable (high retardation factors and large anomalies) the relation is linear with a slope of unity. It is not possible to justify fully such a simple function in terms of the theory developed by Church and Weneser[55] and by Nilsson and Rasmussen.[8] Barring fortuitous cancellations, this relationship does seem to mean that for the cases examined the anomalous part of the electron-ejection matrix element does not change rapidly when that for gamma-ray emission becomes severely attenuated.

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Nuclear Magnetic Moments from Hyperfine Structure Data

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The nuclear magnetic moments determined from the hyperfine structure of the $^2S_{\frac{1}{2}}$ and $^2P_{\frac{1}{2}}$ states are systematically smaller than those determined by methods of magnetic resonance. The Breit-Crawford-Schawlow correction, which takes into account the finite dimensions of the nucleus, together with the Bohr-Weisskopf correction which takes into account the spatial distribution of nuclear magnetism, succeed in explaining at least the order of magnitude in the preceding difference. However, in the two corrections, certain factors are determined graphically, while others are to a certain extent erroneous, owing to incomplete solving of the Darwin-Gordon differential system. All these difficulties are removed in the present paper and the final result is a completely analytical expression for the total correction. The numerical calculations made for $_{80}Hg^{199}$ starting from the ground state $^2S_{\frac{1}{2}}$ (Hg II) fall in good agreement with the value of the nuclear moment determined by magnetic resonance.

I. INTRODUCTION

DETERMINATION of the nuclear magnetic moment from the hyperfine structure of a certain element is made with the greatest accuracy in the following circumstances:

(1) Considering only the $^2S_{\frac{1}{2}}$ and $^2P_{\frac{1}{2}}$ states, the hyperfine splitting is maximum for the $^2S_{\frac{1}{2}}$ ground state.

(2) If the element studied does not normally contain the necessary electronic configuration, a suitable ionization must be produced, as the Fermi-Segrè formula is rigorously applicable only for atoms with a single valence electron.

Under these circumstances, the magnetic moment is given by the formula

$$\mu_0=\frac{3}{8}\left(\frac{M_p}{m_e}\right)\frac{(\hbar c/e^2)^2}{R_\infty}\frac{n_*^3}{(1-d\sigma/dn)}\frac{Ia_s}{Z_iZ_o^2\chi(\frac{1}{2},Z_i)}\mu_N, \quad (\text{I},1)$$

where a_S is the interval factor of the $^2S_{\frac{1}{2}}$ state, while $\chi(\frac{1}{2},Z_i)$ is the relativistic correction of Racah:

$$\chi(\tfrac{1}{2},Z_i)=\frac{3}{\rho(4\rho^2-1)};\quad \rho=(1-a^2)^{\frac{1}{2}};\quad a=\frac{Z_i}{137}, \quad (\text{I},2)$$

the rest of the notations being the usual ones. The subscript o refers to the fact that the moment is obtained by an optical method.

The magnetic moments determined from the formula (1) are for almost all nuclei smaller than those determined by nuclear magnetic resonance, and the difference is on account of the assimilation of the atomic nucleus with a point magnetic dipole. Setting Y for the factor which takes into account the finite extension of the nucleus and the distribution of nuclear magnetism, D for the diamagnetic factor, and μ_R for the magnetic moment determined by resonance, we must have the following equality:

$$\mu_0/Y=\mu_R/D, \quad (\text{I},3)$$

where D has approximately the expression

$$D=(1-3.19\times10^{-5}Z_i^{\frac{4}{3}}), \quad (\text{I},4)$$

while Y is the product of the Breit-Crawford-Schawlow correction (Q) and that of Bohr and Weisskopf (Λ):

$$Y=Q\Lambda. \quad (\text{I},5)$$

In the following we shall establish a rigorous analytic expression for the Q factor which should meet the requirements of a precision determination as is the case in the magnetic resonance method. First of all we shall solve very precisely the Darwin-Gordon differential system for the wave functions inside the nucleus and we shall perform by a particular method the integrals on the perturbed electronic wave functions outside the nucleus, which leads us to the explicit expression of the magnitude of Y in the Breit-Crawford-Schawlow theory. After obtaining the accurate value of Q, obtaining the Y does not offer outstanding difficulties.

II. ELECTRONIC WAVE FUNCTIONS

For the space region outside the nucleus we shall use the perturbed electronic wave functions of Rosenthal and Breit[1] obtained by solving the Darwin-Gordon differential system in the $(-\epsilon/m_ec^2 \ll 1)$ approximation, where ϵ is the quantized energy of the valence electron.

$$\begin{aligned}\phi_1 &= aC_+J_{2\rho}(2\sqrt{y})+aC_-J_{-2\rho}(2\sqrt{y}),\\ \phi_2 &= C_+\{(k-\rho)J_{2\rho}(2\sqrt{y})+(y)^{\frac{1}{2}}J_{2\rho+1}(2\sqrt{y})\}\\ &\quad +C_-\{(k-\rho)J_{-2\rho}(2\sqrt{y})-(y)^{\frac{1}{2}}J_{-(2\rho+1)}(2\sqrt{y})\}.\end{aligned} \tag{II,1}$$

The notations are those in the above-mentioned paper of Rosenthal and Breit, namely:

$$y=(2Zr)/a_{\rm H},\quad \rho=(k^2-a^2)^{\frac{1}{2}};\quad a=Ze^2/\hbar c;\quad |k|=(j+\tfrac{1}{2}).$$

The passage from the (ϕ_1,ϕ_2) components of the wave function to the (F,G) components in the more current notation, is made by taking account of their different normalization:

$$(F,G)\rightarrow\frac{1}{(4\pi)^{\frac{1}{2}}}\frac{1}{r}(\phi_1,\phi_2);\quad 4\pi\int_0^\infty (F^2+G^2)r^2dr=1. \tag{II,2}$$

Finally, the determination of the C_+, C_- coefficients from the normalization and continuity conditions on the nuclear surface gives us

$$\begin{aligned}C_+ &\approx \psi_n(0)\frac{a_{\rm H}}{2Z_i}(4\pi)^{\frac{1}{2}},\\ \left(\frac{C_-}{C_+}\right) &\approx -\frac{\Gamma(1-2\rho)}{\Gamma(1+2\rho)}\frac{[1+(k-\rho)\zeta_k]}{[1+(k+\rho)\zeta_k]}y_0^{2\rho},\end{aligned} \tag{II,3}$$

where $\zeta_k=-F(r_0)/aG(r_0)$. The functions of the $^2S_{\frac{1}{2}}$ state are obtained for the particular case $k=-1$.

For the space region inside the nucleus, we suppose a homogeneous distribution of the nuclear charge as well as the $-U(r_0)/m_ec^2\gg 1$ $(r_0\approx 1.216\times 10^{-13}A^{\frac{1}{3}}$ cm) condition, which is fulfilled for heavy nuclei. In this case we have to solve the following differential system for the small (F) and large (G) components of the wave function:

$$\begin{aligned}\frac{dF}{dx}+(1-k)\frac{F}{x} &\approx -\tfrac{3}{2}a(1-\tfrac{1}{3}x^2)G,\\ \frac{dG}{dx}+(1+k)\frac{G}{x} &\approx +\tfrac{3}{2}a(1-\tfrac{1}{3}x^2)F;\end{aligned} \tag{II,4}$$

here $x=r/r_0$.

For the $S_{\frac{1}{2}}$ $(k=-1)$ electrons, the solutions of the system (II,4) obtained by the development in series are

$$\begin{aligned}F_{(i)}(x) &= -G(0)[0.5ax-(0.1a+0.112\,498a^3)x^3\\ &\quad +(0.048\,213a^3+0.009\,040a^5)x^5\\ &\quad -(0.006\,942a^3+0.005\,915a^5\\ &\quad +0.000\,378a^7)x^7+(0.000\,379a^3\\ &\quad +0.001\,561a^5+0.000\,327a^7\\ &\qquad +0.000\,007a^9)x^9+\cdots],\\ G_{(i)}(x) &= +G(0)[1-0.375a^2x^2+(0.1a^2\\ &\quad +0.042\,187a^4)x^4-(0.008\,333a^2\\ &\quad +0.021\,428a^4+0.002\,260a^6)x^6\\ &\quad +(0.004\,315a^4+0.001\,674a^6\\ &\quad +0.000\,071a^8)x^8-(0.000\,404a^4\\ &\quad +0.000\,530a^6+0.000\,068a^8\\ &\qquad +0.000\,001a^{10})x^{10}+\cdots].\end{aligned} \tag{II,5}$$

The $G(0)$ constant, as determined from the condition of sticking to the nuclear surface of the interior and exterior wave-function components has the following expression in the approximation $y_0\ll 1$:

$$G^2(0)\approx\psi_n{}^2(0)\frac{y_0^{-2(1-\rho)}}{\Gamma^2(1+2\rho)}\left[\frac{2\rho}{1-(1-\rho)\zeta}\right]^2\times(1+a^2\zeta^2)\frac{1}{q(a)}, \tag{II,6}$$

where

$$\begin{aligned}q(a) &= 1-0.406\,666a^2+0.072\,975a^4\\ &\qquad -0.0007\,441a^6+0.000\,497a^8-\cdots,\\ \zeta(a) &= \tfrac{2}{5}\{1+0.106\,233a^2+0.016\,649a^4\\ &\qquad +0.002\,788a^6+0.000\,479a^8+\cdots\}.\end{aligned} \tag{II,7}$$

III. THE BREIT-CRAWFORD-SCHAWLOW CORRECTION

The hyperfine splitting is—in the relativistic theory—proportional to $\int_0^\infty FGdx$. By setting the subscript zero for the wave functions corresponding to a point nucleus atom, the Q correction factor is defined by the ratio[2]

$$Q=\int_0^\infty FGdx\Big/\int_0^\infty F_0G_0dx. \tag{III,1}$$

As $F_0G_0<FG$, the following transformation may be made:

$$\begin{aligned}Q &= \int_x^\infty F_0G_0dx\Big/\int_0^\infty F_0G_0dx\\ &= 1-\left[\int_0^x F_0G_0dx\Big/\int_0^\infty F_0G_0dx\right].\end{aligned} \tag{III,2}$$

[1] J. Rosenthal and G. Breit, Phys. Rev. **41**, 459 (1932).

[2] M. Crawford and A. Schawlow, Phys. Rev. **76**, 1310 (1949).

By using the following mathematical formula for calculating the integrals from Bessel function products,

$$\int_0^\infty \frac{J_\mu(\xi)J_\nu(\xi)}{\xi^\lambda}d\xi=2^{-\lambda}$$
$$\times\frac{\Gamma(\lambda)\Gamma\frac{1}{2}(\mu+\nu-\lambda+1)}{\Gamma\frac{1}{2}(\mu-\nu+\lambda+1)\Gamma\frac{1}{2}(\mu+\nu+\lambda+1)\Gamma\frac{1}{2}(-\mu+\nu+\lambda+1)}. \quad \text{(III,3)}$$

the expression of Q becomes

$$Q=\left\{1-\frac{2\rho(4\rho^2-1)}{3\Gamma^2(1+2\rho)}(1+\rho)\frac{\chi^{2\rho-1}}{2\rho-1}y_0^{2\rho-1}\right\}. \quad \text{(III,4)}$$

The undetermined parameter χ from (III,4) is graphically estimated and is found to be about one unit. We shall now show that indeed $\chi\sim 1$ and that moreover it does not depend on the nuclear radius but only on Z. Let us now introduce the following notations:

$$\begin{aligned} &F=F^++F^- \quad \text{and} \quad G=G^++G^- \quad \text{for} \quad x\geqslant 1;\\ &F=F_{(i)} \qquad\quad \text{and} \quad G=G_{(i)} \qquad\quad \text{for} \quad x\leqslant 1; \end{aligned} \quad \text{(III,5)}$$

where F^+ is proportional to C_+, F^- to C_- [according to (II,1) and (II,2), etc.]. The integral between infinite limits from $F\,G$ (III,1) is now decomposed into a number of integrals which may be performed separately:

$$\int_0^\infty FGdx=\int_0^\infty F^+G^+dx-\int_0^1 F^+G^+dx$$
$$+\int_0^1 F_{(i)}G_{(i)}dx+\int_1^\infty [F^+G^-+F^-G^+]dx$$
$$+\int_1^\infty F^-G^-dx. \quad \text{(III,6)}$$

In the following we refer only to $s_{\frac{1}{2}}$ electrons.

The first integral is identical with that in the denominator of the expression of Q in (III,1)—its evaluation being made without any difficulty by using (III,3)

$$\int_0^\infty F^+G^+dx=-a\psi_n^2(0)\frac{3}{2\rho(4\rho^2-1)}y_0^{-1}. \quad \text{(III,7)}$$

The second integral is obtained very simply by the development in series of Bessel's functions and by stopping at the first term

$$\int_0^1 F^+G^+dx\approx -a\psi_n^2(0)\frac{(1+\rho)}{\Gamma^2(1+2\rho)}\frac{y_0^{-2(1-\rho)}}{(2\rho-1)}. \quad \text{(III,8)}$$

The third integral may be evaluated term by term by using the analytical expressions in (II,5) for the wave functions. The calculations are very tedious but not intricate. One obtains

$$\int_0^1 F_{(i)}G_{(i)}dx=-\frac{9}{10}a\psi_n^2(0)\frac{\rho^2}{\Gamma^2(1+2\rho)}$$
$$\times\frac{(1+a^2\zeta^2)}{[1-(1-\rho)\zeta]^2}y_0^{-2(1-\rho)}\xi(a), \quad \text{(III,9)}$$

where $\xi(a)$ is given by the development

$$\xi(a)=1+0.162\,675a^2+0.025\,192a^4 +0.004\,446a^6+0.001\,047a^8+\cdots. \quad \text{(III,10)}$$

We want to emphasize that the various developments involving the electronic wave functions inside the nucleus are determined within a relative error of about 10^{-3}, even for the extreme case $Z=137$.

The two remaining integrals can no longer be integrated in the relatively easy way used for the others. For them we shall have to consider the following auxiliary mathematical theorem, whose demonstration, being so simple, will not be reproduced here.

Let us take two functions $f(x)$ and $f_0(x)$ which meet the following conditions:

(1) $\lim_{x\to 0} f(x)=\pm\infty$; $\lim_{x\to 0} f_0(x)=\pm\infty$;

(2) $\lim_{x\to 0}\{f(x)/f_0(x)\}=1$;

(3) $\lim_{x_0\to 0}\int_{x_0}^\infty f(x)dx=\pm\infty$; $\lim_{x_0\to 0}\int_{x_0}^\infty f_0(x)dx=\pm\infty$;

(4) $\int_{x_0}^\infty f(x)dx\neq 0,\ \infty$; $\int_{x_0}^\infty f_0(x)dx\neq 0,\ \infty$

for $x_0\neq 0,\ \infty$.

From the above properties it results that

$$\int_{x_0}^\infty f(x)dx\approx\int_{x_0}^\infty f_0(x)dx \quad \text{if} \quad x_0\ll 1.$$

In our case, the theorem is satisfied and it allows us to estimate the integrals we are interested in by taking only the first term in the various Bessel functions which appear.

Thus the fourth integral will be

$$\int_1^\infty (F^+G^-+F^-G^+)dx$$
$$\approx -a\frac{y_0}{4\pi r_0^2}C_+^2\left(\frac{C_-}{C_+}\right)\frac{2}{\Gamma(1+2\rho)\Gamma(1-2\rho)}\int_{y_0}^\infty\frac{dy}{y^2}$$
$$\approx +a\psi_n^2(0)\frac{2}{\Gamma^2(1+2\rho)}\frac{1-(1+\rho)\zeta}{1-(1-\rho)\zeta}y_0^{-2(1-\rho)}. \quad \text{(III,11)}$$

In a quite similar way we perform the last integral

$$\int_1^\infty F^- G^- dx$$

$$\approx -a\frac{y_0}{4\pi r_0^2}C_+^2\left[\frac{C_-}{C_+}\right]^2\frac{(1-\rho)}{\Gamma^2(1-2\rho)}\int_{y_0}^\infty\frac{dy}{y^{2(1+\rho)}}$$

$$\approx -a\psi_n^2(0)\frac{(1-\rho)}{\Gamma^2(1-2\rho)}\left[\frac{1-(1+\rho)\zeta}{1-(1-\rho)\zeta}\right]^2\frac{y_0^{-2(1-\rho)}}{(1+2\rho)}. \quad \text{(III,12)}$$

Now we add up all integrals and, considering (III,1) and (III,6), we obtain the following expression for the Q correction factor (Breit-Crawford-Schawlow):

$$Q=1-\frac{2\rho(4\rho^2-1)}{3\Gamma^2(1+2\rho)}(1+\rho)\frac{\Xi}{(2\rho-1)}y_0^{2\rho-1}, \quad \text{(III,13)}$$

where

$$\Xi=\left\{1-\frac{9}{10}\rho^2\frac{(2\rho-1)}{(1+\rho)}\frac{(1+a^2\zeta^2)}{[1-(1-\rho)\zeta]^2}\xi(a)+2\frac{(2\rho-1)}{(1+\rho)}\left(\frac{1-(1+\rho)\zeta}{1-(1-\rho)\zeta}\right)-\frac{1-\rho}{1+\rho}\frac{2\rho-1}{2\rho+1}\times\left(\frac{1-(1+\rho)\zeta}{1-(1-\rho)\zeta}\right)^2\right\}. \quad \text{(III,14)}$$

Now, comparing (III,4) with (III,13) we obtain an analytic formula for the undetermined parameter χ

$$\chi^{2\rho-1}\equiv\Xi. \quad \text{(III,15)}$$

Obviously, it does not depend on the nuclear radius but only on Z. For $\rho\rightarrow 1$ the limiting value $\frac{3}{4}$ is obtained.

IV. THE BOHR AND WEISSKOPF CORRECTION

Taking into account the spatial distribution of nuclear magnetism as well as its double origin (intrinsic spin magnetic moment and orbital magnetic moment) leads to the necessity of another correction. In brief, their theory is the following:

We write the separate interation of the optical electron with the two parts of the nuclear moment[3]

$$W=W_S+W_L=\int_{(\tau)}\psi^* e\alpha[\mathbf{A}_S(\mathbf{r})+\mathbf{A}_L(\mathbf{r})]\psi d\tau$$

$$=\pm\frac{16}{3}\pi e g_I I\int_{(\tau)}\rho(\mathbf{R})\{\alpha_S q_S+\alpha_L q_L\}d\tau_R \quad \begin{cases}+({}^2S_{\frac{1}{2}})\text{ state}\\ -({}^2P_{\frac{1}{2}})\text{ state}\end{cases}, \quad \text{(IV,1)}$$

where $\rho(\mathbf{R})=\varphi^*(\mathbf{R})\varphi(\mathbf{R})\approx$const for a homogeneous distribution of nuclear magnetism, and for $R\leqslant R_0$; $\rho(\mathbf{R})\approx 0$ outside the nucleus. By inserting the explicit values for the q_S and q_L, we obtain the expression of the Λ correction factor.

$$\Lambda=\int_{(\tau)}\rho(\mathbf{R})\{\alpha_S\eta_S+\alpha_L\eta_L\}d\tau_R, \quad \text{(IV,2)}$$

where

$$\eta_S=1-\left[\int_0^R FGdr\bigg/\int_0^\infty FGdr\right];$$
$$\eta_L=1-\left[\int_0^R\left(1-\frac{r^3}{R^3}\right)FGdr\bigg/\int_0^\infty FGdr\right]; \quad \text{(IV,3)}$$

or, in its better known form,[4]

$$\Lambda=1-(\alpha_S\epsilon_S+\alpha_L\epsilon_L), \quad \text{(IV,4)}$$

where

$$\epsilon_S=\left\{\int_0^\infty FGdr\right\}^{-1}\left\langle\int_0^R FGdr\right\rangle;$$
$$\epsilon_L=\left\{\int_0^\infty FGdr\right\}^{-1}\left\langle\int_0^R\left(1-\frac{r^3}{R^3}\right)FGdr\right\rangle. \quad \text{(IV,5)}$$

Here the averages are taken with the real distribution of the magnetism in the nucleus which is in fact unknown. These averages can be performed in a relatively simple way for a homogeneous distribution. For a distribution which does not differ very much from the homogeneous one, we shall be able to use the following approximation, somewhat equivalent with the result of Bohr and Weisskopf

$$\left\langle\int_0^\chi f(x,\chi)FGdx\right\rangle\approx\frac{5}{3}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle\left\langle\int_0^\chi f(x,\chi)FGdx\right\rangle_{\text{Av hom.distr.}}, \quad \text{(IV,6)}$$

if $f(0,\chi)=1$.

By using (IV,5), (IV,6) as well as the accurate value of the integral in the denominator of the (IV,5) expression, as results from (III,7) and (III,13) we can express the Λ factor in a quite analytical way

$$\Lambda=1-\frac{(1+a^2\zeta^2)}{[1-(1-\rho)\zeta]^2}\frac{(2\rho)^2}{\Gamma^2(1+2\rho)}\frac{2\rho(4\rho^2-1)}{3Q}\times\frac{39}{280}\lambda_1(a)\frac{5}{3}y_0^{2\rho-1}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{\text{Av}}\times\left\{\alpha_S+\left(1-0.387\,016\frac{\lambda_2(a)}{\lambda_1(a)}\right)\alpha_L\right\}, \quad \text{(IV,7)}$$

[3] A. Bohr and V. Weisskopf, Phys. Rev. 77, 94 (1950).

[4] H. Kopferman *Kernmomente* (Akademische Verlagsgesellschaft, Frankfurt a Main, 1956), second edition, Sec. 73.

here

$$\lambda_1(a)=1+0.225\,063a^2+0.037\,471a^4+0.005\,381a^6+0.000\,767a^8+\cdots,$$

$$\lambda_2(a)=1+0.149\,762a^2+0.019\,232a^4+0.002\,852a^6+0.000\,641a^8+\cdots.$$

We want to emphasize that by obtaining the factor Q in Sec. II, accurately, it was no longer necessary to use the inaccurate value in the Bohr-Weisskopf paper for the integral in the denominator of (IV,5).

The appearance of Q in the Λ factor as well as the proportionality of $(1-\Lambda)$ with $y_0^{2\rho-1}$ enables us to join the two corrections Q and Λ in a single one, $Y=Q\Lambda$, which will have the following expression:

$$Y=1-\frac{2\rho(4\rho^2-1)}{3\Gamma^2(1+2\rho)}(1+\rho)\frac{\Xi}{(2\rho-1)}y_0^{2\rho-1}\left\{1+\frac{(2\rho-1)}{\Xi}\times\frac{(1+a^2\zeta^2)}{(1+\rho)}\left[\frac{2\rho}{1-(1-\rho)\zeta}\right]^2\frac{5}{3}\left\langle\left(\frac{R}{R_0}\right)^2\right\rangle_{\mathrm{Av}}\left[\frac{39}{280}\lambda_1(a)\alpha_S+\left(\frac{39}{280}\lambda_1(a)-\frac{66}{1225}\lambda_2(a)\right)\alpha_L\right]\right\}. \quad \text{(IV,8)}$$

We also make a point of emphasizing that according to the (IV,6) formula, the ratio between the coefficient of α_L and that of α_S is not a constant value but depends on Z; indeed

$$\left\langle\int_0^R\left(1-\frac{r^3}{R^3}\right)FGdr\right\rangle\Big/\left\langle\int_0^R FGdr\right\rangle \approx\int_0^1(1-x^3[1-3\ln x])FGdx\Big/\int_0^1(1-x^3)FGdx =\left(1-0.387\,016\frac{\lambda_2(a)}{\lambda_1(a)}\right), \quad \text{(IV,9)}$$

while in the above-mentioned work of Bohr and Weisskopf this ratio is 0.62.

V. THE Y FACTOR FOR $p_{\frac{1}{2}}$ ELECTRONS

Apart from the $s_{\frac{1}{2}}$ electrons, only the $p_{\frac{1}{2}}$ electrons are those for which the Y structure correction is important, on account of the proportionality between $(1-Y)$ and the $(2\rho-1)$ power of y_0.

In order to obtain the $Y(p_{\frac{1}{2}})$ structural factor, we shall first establish general relations between the integrals which are included in the definition of $Y(s_{\frac{1}{2}})$ and the corresponding ones for $Y(p_{\frac{1}{2}})$. As a final result, $Y(p_{\frac{1}{2}})$ is expressed in terms of $Y(s_{\frac{1}{2}})$.

From an examination of the Darwin-Gordon differential system for the electronic wave functions inside the nucleus, we can write the following relation:

$$\int_0^1 f(x)[F_{(i)}G_{(i)}]_{k=+|k|}dx =\frac{[F(1)G(1)]_{k=+|k|}}{[F(1)G(1)]_{k=-|k|}}\int_0^1 f(x)[F_{(i)}G_{(i)}]_{k=-|k|}dx =-\frac{(|k|-\rho)}{(|k|+\rho)}\int_0^1 f(x)[F_{(i)}G_{(i)}]_{k=-|k|}dx, \quad \text{(V,1)}$$

where $f(x)$ is an arbitrary analytic function.

By direct integration we also obtain the corresponding relations between the other kinds of integrals, for the states $k=+|k|$ and $k=-|k|$:

$$\int_0^\infty [F^+G^+]_{k=+|k|}dx =-\frac{(2|k|-1)}{(2|k|+1)}\int_0^\infty [F^+G^+]_{k=-|k|}dx, \quad \text{(V,2)}$$

$$\int_0^1 [F^+G^+]_{k=+|k|}dx =-\frac{(|k|-\rho)}{(|k|+\rho)}\int_0^1 [F^+G^+]_{k=-|k|}dx, \quad \text{(V,3)}$$

$$\int_1^\infty [F^+G^-+F^-G^+]_{k=+|k|}dx =-\frac{(|k|-\rho)}{(|k|+\rho)}\int_1^\infty [F^+G^-+F^-G^+]_{k=-|k|}dx, \quad \text{(V,4)}$$

$$\int_1^\infty [F^-G^-]_{k=+|k|}dx =-\frac{(|k|-\rho)}{(|k|+\rho)}\int_1^\infty [F^-G^-]_{k=-|k|}dx. \quad \text{(V,5)}$$

Considering all these relations as well as the definitions of the Q factor, we obtain the following binding relation:

$$\frac{Q(+|k|)-1}{Q(-|k|)-1}=\frac{2|k|+1}{2|k|-1}\left(\frac{|k|-\rho}{|k|+\rho}\right). \quad \text{(V,6)}$$

Further on, according to the definition of the Λ correction factor, we derive the relation for its transformation by means of Q:

$$\frac{Q(+|k|)[1-\Lambda(+|k|)]}{Q(-|k|)[1-\Lambda(-|k|)]}=\frac{2|k|+1}{2|k|-1}\frac{(|k|-\rho)}{(|k|+\rho)}. \quad \text{(V,7)}$$

From (6) and (7) one obtains without any difficulty the relation of transformation for the product $Y=Q\Lambda$, which is the one we had sought:

$$\frac{Y(+|k|)-1}{Y(-|k|)-1}=\frac{(2|k|+1)}{(2|k|-1)}\frac{(|k|-\rho)}{(|k|+\rho)}. \quad \text{(V,8)}$$

In particular for $|k|=1$ (the only case for which the corrections are significant), we have

$$Y(p_{\frac{1}{2}})=1-3\left(\frac{1-\rho}{1+\rho}\right)[1-Y(s_{\frac{1}{2}})]. \qquad \text{(V,9)}$$

VI. COMPUTATION OF THE NUCLEAR MAGNETIC MOMENT OF $_{80}Hg^{199}$

As an illustrative example, we shall compute the nuclear magnetic moment of $_{80}Hg^{199}$ starting from the ground state $6s\,^2S_{\frac{1}{2}}$ of the $_{80}Hg^{199}$ II ion. For this state, the quantities involved in the (I,1) formula are: $n_*=1.703\,396$; $Z_i=80$; $Z_0=2$; $\chi(\frac{1}{2},80)=2.257\,306$; $(1-d\sigma/dn)=1.236\,539$ (determined by the method of finite differences and by the extrapolation of the resulting series on the basis of the D'Alembert convergence criterion). The interval factor a_{6s} determined by the hyperfine structure[5] is 1.358 cm^{-1}. With these data, we find the following value for μ_0:

$$\mu_0=0.442\,149\mu_N. \qquad \text{(VI,1)}$$

In order to compute Y we take into account the fact that the nuclear state of $_{80}Hg^{199}$ is given by the odd neutron $p_{\frac{1}{2}}$ and that $_{80}Hg^{199}$ fits in Schmidt's diagram. The characteristic values which are included in the expression of Y are: $\alpha_L=0$; $\alpha_S=1$, $r_0=1.216\times10^{-13}A^{\frac{1}{3}}$ cm, $(5/3)\langle(R/R_0)^2\rangle_{\text{Av}}\approx1$. The value of Y is

$$Y=0.879\,825. \qquad \text{(VI,2)}$$

On the other hand, the magnetic resonance test gives

$$\mu_R=0.4993\cdots, \qquad \text{(VI,3)}$$

while the diamagnetic correction D is

$$D=0.990\,348. \qquad \text{(VI,4)}$$

By using the ratios (I,3), we obtain the following values for the moment:

(1) From hyperfine structure,

$$\mu=\mu_0/Y=0.5031. \qquad \text{(VI,5)}$$

(2) From magnetic resonance,

$$\mu=\mu_R/D=0.5041. \qquad \text{(VI,6)}$$

[5] S. Mrozowski, Phys. Rev. 57, 207 (1940).

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Excited Levels in Mn^{56}†

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The half-lives of the first three excited levels of Mn^{56} have been measured by looking at the cascade gamma rays from neutron capture in Mn^{55}. They are found to be 10.7_{-3}^{+2} mμsec, 4.9±0.6 mμsec, and ⩽0.5 mμsec. A fairly plausible assignment of spins and parities to the first, second, and third excited levels of Mn^{56} would seem to be 2^+, 1^+, and 2^+, respectively. The technique described in the paper should be useful in those cases in which excited levels of unstable nuclei cannot be reached through beta decay.

I. INTRODUCTION

Excited states of Mn^{56} were investigated by Green *et al.*[1] by means of the $Mn^{55}(d,p)Mn^{56}$ reaction. More recently, gamma-ray spectra from neutron capture in resonances of Mn^{55} have been investigated by Kennett *et al.*[2] In Fig. 1, from the paper of Kennett *et al.*, are shown the first three excited states of Mn^{56} and the transitions observed following neutron capture in Mn^{55}. The 210-kev level and the 109-kev level are fed directly from the capture state. The 210-kev level is fed predominantly at the 1080-ev ($J=3^-$) resonance; the 109-kev level at the 337-ev ($J=2^-$) resonance. The ground level of Mn^{56} is known to be a 3^+ state.[3,4]

This paper describes the measurement of the half-lives of the first three excited levels of Mn^{56}. A beam

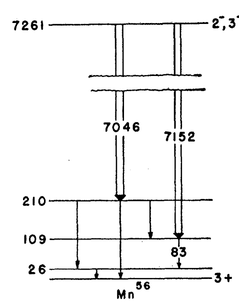

FIG. 1. Excited states of Mn^{56} and the transitions observed following neutron capture in Mn^{55}.

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[2] Kennett, Bollinger, and Carpenter, Phys. Rev. Letters 1, 76 (1958).

[3] Childs, Goodman, and Kieffer, Phys. Rev. Letters 1, 296 (1958).

[4] Strominger, Hollander, and Seaborg, Revs. Modern Phys. 30, 628 (1958).