Drift Velocity of Electrons in Helium

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The drift velocity of electrons in helium at 300°K has been measured for E/p values between 4×10^{-4} and 40 volt/cm-mm Hg. The data for $E/p<1$ volt/cm-mm Hg were obtained from measurements of electron transit time in a modernized version of the double shutter tube developed by Bradbury and Nielsen. The data at high E/p were obtained from microwave measurements of the electron density in a positive column of a low-pressure discharge. The measured drift velocities are in good agreement with previous results in the E/p range from 10^{-1} to 3 volt/cm-mm Hg. At E/p less than 3×10^{-3} volt/cm-mm Hg the electrons are essentially in thermal equilibrium with the gas. Margenau's theoretical expression for the drift velocity of electrons in a gas for which the cross section for momentum transfer is independent of electron energy is found to fit the data for $E/p<1$ volt/cm-mm Hg to the accuracy of measurements. The cross section which gives the best fit of the theory to the data is 6×10^{-16} cm^2.

I. INTRODUCTION

IN the course of studies of the behavior of slow electrons under various experimental conditions[1,2] we have measured the drift velocity of electrons in helium at 300°K over a much wider range of E/p, the electric field to pressure ratio, than had been covered in the experiments of Townsend and Bailey,[3] Nielsen,[4] or Hornbeck.[5] The measurements at low E/p were made using a modern version of the double shutter drift velocity tube developed by Bradbury and Nielsen.[6] The values at high E/p were obtained from studies of the positive column of a low pressure, low current discharge by making simultaneous measurements of the electron density and the discharge current. We shall discuss each of these experiments and then compare the results with previous experiments and with theory.

II. LOW-FIELD MEASUREMENTS

The measurements of electron drift velocity at low fields were made using the electrode structure shown schematically in Fig. 1. The cathode, S, is illuminated with ultraviolet radiation from a high-pressure mercury lamp. The photoelectrons liberated from the cathode are acted upon by the uniform electric field maintained by the guard rings. Alternate wires of the planar grids G_1 and G_2 are connected together.[7] Direct current voltages can be applied between the sets of grid wires so as to reduce the transmitted electron current to some small value, say 0.5%. The first grid is made transmitting by periodically applying short rectangular voltage pulses such as to reduce the fields between alternate grid wires to zero. The electrons arriving at the second grid will be collected by the grid wires except when the voltage between the wires is reduced by the application of a second voltage pulse. By varying the time delay between the pulses applied to the first grid and those applied to the second grid and measuring the current which arrives at the collector, C, one obtains a plot of the rate of arrival of electrons at the second grid as a function of time. Such a plot is shown in Fig. 2 for a fixed value of E/p and two different gas pressures.

If the voltages applied between the wires of the grids are small enough so as not to affect the motion of the electrons in the uniform field region between the grids, and if the pulses applied to the second grid are sufficiently narrow then the curves shown in Fig. 2 represent the rate of arrival of electrons at the second grid as a function of the time. Under these conditions the time at which the current is a maximum is the transit time for the electrons. If the pulses applied to the grids are sufficiently narrow, the ratio of the width of the peak to the transit time is proportional to the square root of the ratio of the quantity D/μ to the voltage applied between the grid wires.[8] D/μ is the ratio of the electron diffusion coefficient, D, to the electron mobility, μ, and is a measure of the average energy of the electrons. Since the value of E/p is the same for the two curves of Fig. 2, the values of D/μ and the transit time are unchanged while the width of the pulse decreases with increasing voltage and pressure.

Figure 3 shows a plot of the measured drift velocities, i.e., the distance between grids divided by the measured transit times, as a function of E/p over the range of E/p which could be conveniently covered with this

[1] J. L. Pack and A. V. Phelps, Phys. Rev. **100**, 1229(A) (1955).
[2] L. S. Frost and A. V. Phelps, Phys. Rev. **98**, 559(A) (1955); and **100**, 1234(A) (1955).
[3] J. S. Townsend and V. A. Bailey, Phil. Mag. **46**, 657 (1923).
[4] R. A. Nielsen, Phys. Rev. **50**, 950 (1936).
[5] J. A. Hornbeck, Phys. Rev. **83**, 374 (1951).
[6] N. E. Bradbury and R. A. Nielsen, Phys. Rev. **49**, 388 (1936).
[7] L. B. Loeb, *Basic Processes in Gaseous Electronics* (University of California Press, Berkeley, 1955), Chap. I.
[8] See reference 7, Chap. II, and R. A. Duncan, Australian J. Phys. **10**, 54 (1957). The symmetry of the wave form in our experiments at a given E/p and pressure is better than that shown by Duncan since our measurements were made with a fixed frequency of pulses applied to the grids. Using Duncan's equations the fractional half-width of our pulse is expected to be given by $(\Delta t/t)^2=(4D\ln2)/(\mu V)$ where V is the voltage between the grids. Therefore, the D/μ values from Fig. 2 are 0.024 electron volt compared to a thermal value of 0.026 for 300°K. In spite of considerable effort we were unable to obtain a pressure independent set of values of D/μ at higher and lower E/p using this technique.

apparatus. Distortion of the electric field in the drift region is minimized by maintaining the average of the potentials of the grids at the potential of the corresponding guard ring at all times and by the use of small rectangular pulses to operate the grids rather than the large amplitude sine waves used by Bradbury and Nielsen.[6] In spite of these precautions, the apparent drift velocity at very low E/p is found to depend slightly on the drift distance. The experimental points for two drift distances are indicated by the circles and triangles of Fig. 3. Since the measured drift velocities for a given drift distance appear to depend only on E/p, contact differences of potential are not the source of the discrepancy.[9] The discrepancy is assumed to be accounted for by allowing the effective length of the drift distance to vary with E/p. This end effect is eliminated by calculating the drift velocity from the ratio of the difference of the two drift distances to the difference of the transit times. The final drift velocity

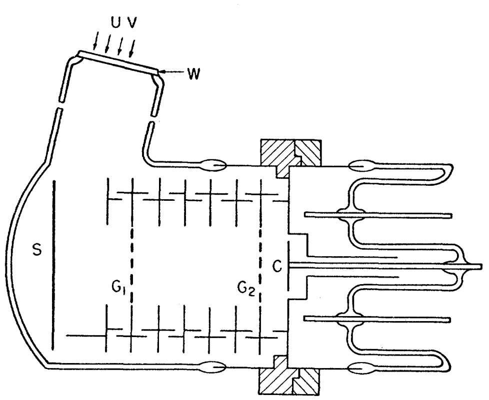

FIG. 1. Schematic of drift velocity tube showing arrangement of electrodes, guard rings, and light shields.

data are indicated by the solid curve of Fig. 3. These results will be compared with theory in Sec. III.

At this point we shall discuss a few of the experimental details pertinent to this measurement. The drift tube is of metal, glass, and fired lava construction with gold wire gasket vacuum seals. The cathode end of the tube envelope is constructed of Pyrex glass with a transition to the ultra pure silica window and a glass to Kovar seal with a nonmagnetic stainless steel flange. All of the electrodes are gold-plated Advance metal except the grids, which are 3-mil gold-plated molybdenum wires 30 mils apart and mounted on fired lava supports. The lava supports are shielded with gold-plated Advance metal to reduce charging effects to a minimum. The ultraviolet source is a high-pressure mercury lamp removed from its outer envelope and

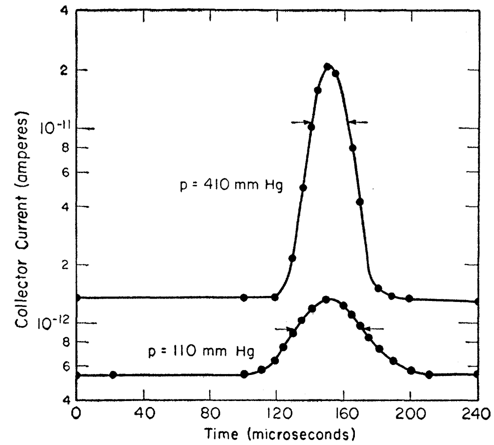

FIG. 2. Collector current *vs* time delay between pulse applied to grids for $E/p=6\times10^{-3}$ volt/cm-mm Hg, $d=6.35$ cm and helium pressures of 110 and 410 mm Hg. The transit time is indicated by the time of maximum collector current and is 151 microseconds. The arrows indicate the widths of these curves at half maximum. Note the 16 fold increase in maximum collector current as the gas pressure is increased from 110 to 410 mm Hg.

operated in a nitrogen filled, water cooled jacket. The lamp is operated at about 100 volts and 1 ampere, and is stabilized by monitoring the emitted light with a photocell and using the output to control the discharge current. As indicated in Fig. 1, the guard ring electrodes are constructed so as to shield the drift area from the glass walls and to reduce stray light reaching the grid wires and collector.

The average collector current ranged from 10^{-15} to 10^{-9} ampere and was measured using a vibrating reed electrometer. As is suggested by the data of Fig. 2 we are able to obtain sufficient currents at low E/p by making measurement at high gas pressures. This feature results from the very rapid decrease in net cathode emission and grid transmission with gap voltage at low E/p. No appreciable deterioration in

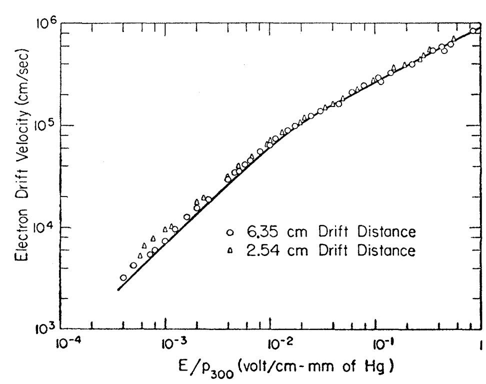

FIG. 3. Electron drift velocity as a function of E/p in helium at 300°K. The drift velocities computed from measured transit times are indicated by the points. The smooth curve shows the drift velocity after correction for end effects. The linear dependence of the corrected drift velocity curve on E/p for $E/p<3\times10^{-3}$ volt/cm-mm Hg indicates that the electrons are in thermal equilibrium with the gas.

[9] An additional source of error in the effective length of the drift distance is the distance required for the electrons to reach an equilibrium energy distribution in the presence of the field. However, such an effect should be pressure dependent and at the pressure of these measurements should be negligible.

cathode emission was observed over several weeks. The drift tube is supplied with gas from an "ultrahigh vacuum" system of standard design.[10] The system is baked-out at 250°C for 14 hours and reaches an ultimate pressure of 10^{-9} mm Hg with a rate of rise of 10^{-9} mm Hg/min when closed off from the helium flask and 10^{-7} mm Hg/min when exposed to the helium flask. The helium used is Airco reagent grade with no known impurities. The helium pressure is measured with a mercury manometer coupled to the vacuum system with a null-reading metal diaphragm manometer[11] so as not to contaminate the vacuum system with mercury. Pressure measurements can be made to an accuracy of ± 0.3 mm Hg and voltage measurements are made with a voltmeter calibrated to $\frac{1}{4}\%$ of full-scale.

III. HIGH-FIELD MEASUREMENTS

The drift velocity of electrons in helium at high values of E/p were determined from simultaneous measurements of the current, the electric field gradient, and the electron density in the positive column of a low-pressure helium discharge.

Experiment

The discharge was contained in a 13-mm Vycor tube about 30 cm long and was operated at pressures between 0.25 and 3 mm Hg of helium and a current of 1 to 10 milliamperes. The discharge current was maintained constant by means of a current regulated power supply and was observed to be free of oscillations, i.e., to have less than a 5% ac signal when observed with an oscilloscope with an amplifier and cables passing from 10 cps to 10 000 cps. No standing striations were observed in the positive column region used. The electric field gradient was determined by measuring the difference in floating potentials of two 3-mil tungsten wire probes inserted into the positive column discharge at points separated by 15 cm. The probes were outgassed by drawing a large electron current to them and heating them to a red heat. The discharge currents were less than 10 ma at pressures between 0.3 and 3 mm Hg so that the maximum measured temperature rise at the wall of the tube was 2%.

The electron density was calculated from measurements of the frequency shift of a rectangular resonant cavity surrounding a section of the positive column of the discharge tube. Since this technique has been described in several publications[12] we will give only the details pertinent to the experiment. The rectangular cavity was resonant at about 10 cm with the electric field along the axis of the discharge tube and the magnetic field in a plane perpendicular to the axis of the tube. The electric field distribution inside the cavity was determined using small metallic probes according to the technique described by Maier and Slater.[13] The relative frequency shift produced by the probes and, therefore, the square of the microwave electric field strength, with and without the Vycor tube is shown in Fig. 4.

Theory

The drift velocities are calculated from the ratio of the measured current, I, to the average electron density $\bar{n}$. Since the axial dc electric field is constant throughout the positive column the electron drift velocity, w, in the axial direction is constant[14] and given by

$$w=\frac{\int J da}{e\int n da}=\frac{I}{\pi R^2 e\bar{n}}. \tag{1}$$

Here J is the current density, da is an element of area perpendicular to the axis of the discharge tube of radius R, and e is the electronic charge. The microwave measurements yield an average of the electron density, $\langle n\rangle$, in terms of the measured shift in the resonant frequency, $\Delta\omega$, produced by the electrons.[12] Thus,

$$\langle n\rangle=\frac{\int_T nEE_a dv}{\int_T EE_a dv}=\frac{2m\epsilon_0\omega^2(1+\nu_m^2/\omega^2)}{e^2}\frac{\Delta\omega}{\omega}\frac{\int_{CT} EE_a dv}{\int_T EE_a dv}, \tag{2}$$

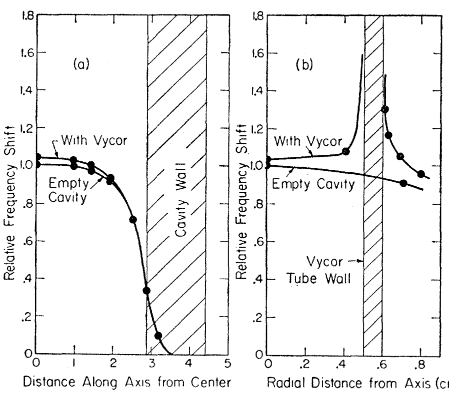

FIG. 4. Relative frequency shift and square of electric field strength as a function of position in the microwave cavity with and without the Vycor discharge tube. Figure 4(a) shows average values of the relative frequency shift along the axis of the cylindrical cavity as produced by a $\frac{1}{8}$-in. bronze sphere mounted on a nylon thread. Figure 4(b) shows average values of the relative frequency shift along a line perpendicular to the cavity axis at its center as produced by 0.013-inch diameter$\times\frac{1}{4}$-inch wire. The maximum frequency shift was $\sim$0.05%.

[10] D. Alpert, *Handbuch der Physik* (Springer-Verlag, Berlin, 1958), Vol. 12.

[11] The manometer used is an improved version of one described by Alpert, Matland, and McCoubrey, Rev. Sci. Instr. **22**, 370 (1951).

[12] D. J. Rose and S. C. Brown, J. Appl. Phys. **23**, 1028 (1952).

[13] L. C. Maier and J. C. Slater, J. Appl. Phys. **23**, 68 (1952).

[14] Here we have assumed that electron energy distribution is unaffected by the ambipolar space charge fields. The error resulting from this assumption is very difficult to estimate because of a lack of a complete theory. Limiting cases have been solved by I. B. Bernstein and T. Holstein, Phys. Rev. **94**, 1475 (1954) and by W. P. Allis and D. J. Rose, Phys. Rev. **93**, 84 (1954). The fact that our results are independent of pressure indicates that the error is small.

where ω is the resonant angular frequency of the cavity with no discharge, m is the electron mass, ϵ_0 the permitivity of free space, ν_m is the frequency of momentum transfer collisions between electrons and the helium gas atoms, E is the actual electric field present in the cavity and E_a is the electric field associated with the mode of the cavity being used for the measurements. The integrals to be evaluated over the discharge tube are indicated by T and those to be evaluated over the discharge tube and microwave cavity by CT.[15] Equation (2) is valid only when the ratio of $\nu_m^2/\omega^2 \ll 1$ unless ν_m is independent of electron velocity.[15] In these experiments, $\nu_m^2/\omega^2 < 0.025$ and ν_m is nearly independent of electron energy. Also, Eq. (2) is valid only when the electron density is sufficiently small.[16] Our electron densities varied from 2×10^9 to 4×10^{10} electron/cc.

Since the electron density is independent of distance along the discharge tube, we are only concerned with the difference in $\bar{n}$ and $\langle n\rangle$ due to the fact that the microwave electric field is not constant as a function of radius. The microwave electric field is greatest near the discharge tube walls so that any deviations from an assumed electron density distribution near the wall are emphasized. The ratio $\bar{n}/\langle n\rangle$ is calculated on the assumption that the electron density was distributed according to a Bessel function. Therefore, a decreased electron density near the wall due to departures from the ambipolar limit[17] causes $\bar{n}/\langle n\rangle$ to increase from the calculated value of 0.92 toward unity and makes the computed w too small.

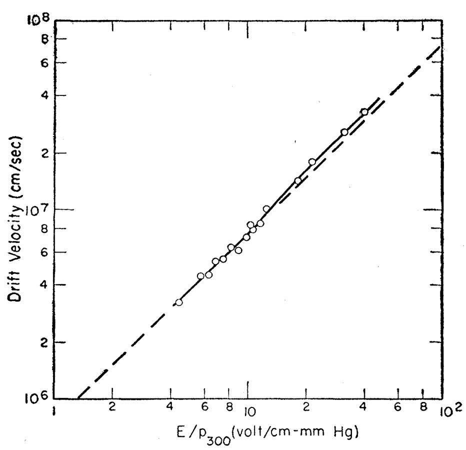

FIG. 5. Electron drift velocity as a function of E/p from discharge tube measurements.

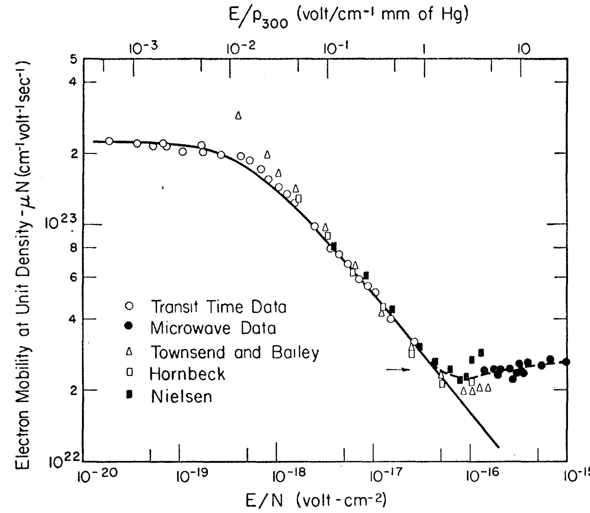

FIG. 6. Electron mobility at unit density in helium at 300°K. The open and solid circles show the results of the present experiments while the other points show the results obtained by Townsend and Bailey, by Nielsen, and by Hornbeck.

The drift velocities calculated from these measurements are shown in Fig. 5 as a function of E/p. The dashed line shows the variation of the drift velocity with E/p which is expected if the electron collision frequency at unit density were constant and equal to 7.2×10^{-8} cm^3/sec. The solid curve is an average of the experimental points.

IV. DISCUSSION

The results of the low-field and high-field drift velocity measurements are combined in Fig. 6 where the mobility at unit gas density, i.e., the product of the drift velocity and the gas density divided by the electric field is plotted as a function of the electric field divided by the gas density. Although not the customary units, the units chosen eliminate questions of normalization of pressure to some arbitrary value. At low E/N the circles show typical data when corrected for end effects. In the low-field region we have fitted the zero frequency limit of the theoretical expression for electron mobility obtained by Margenau[18] to our experimental data. Thus, Margenau shows that for electrons moving in a gas in which the cross section, Q, for momentum transfer is independent of electron energy

$$\mu N=\frac{8e\alpha^{\frac{1}{2}}}{3(2\pi mkT)^{\frac{1}{2}}mQ}\frac{W((\alpha-3)/2,\,(\alpha+2)/2,\,\alpha)}{W((2\alpha-1)/4,\,(2\alpha+3)/4,\,\alpha)} \quad (3)$$

where $\alpha=(M/6m)(eE/kTQN)^2$. Here μ is the mobility,

[15] The ratio of the integral of the product of the electric fields over the cavity plus discharge tube to the integral of the product of the fields in the empty cavity in Eq. (2) is taken to be equal to the ratio of the same integrals evaluated with and without a Vycor rod at the center of the cavity of such a diameter as to give the same frequency shift as for the Vycor tube. The electric field distribution for the dielectric rod was calculated according to the relations due to E. Feenberg and given by Hsieh, Goldey, and Brown, J. Appl. Phys. **25**, 302 (1954).

[16] The limits of the microwave measurements are discussed by K. B. Persson, Phys. Rev. **106**, 191 (1957). Our use of a cavity mode with the electric field parallel to the axis of the discharge reduces the errors due to field perturbation to a minimum. See S. C. Brown, IRE, Trans. on Microwave Theory Tech. **MTT-7**, 69 (1959).

[17] W. P. Allis and D. J. Rose, Phys. Rev. **93**, 84 (1954).

[18] H. Margenau, Phys. Rev. **69**, 508 (1946).

N is the gas density, e and m are the charge and mass of the electron, M is the mass of the gas atoms, k is Boltzmann's constant, T is the gas temperature, E is the dc electric field, and $W(a,b,c)$ is the confluent hypergeometric function. Q is the only adjustable parameter in Eq. (3).

Equation (3) is plotted as the solid curve of Fig. 6 using a cross section for momentum transfer of $Q=6.2\times10^{-16}$ cm^2. The theory and experiment fit very well up to $E/N=5\times10^{-17}$ volt-cm^2 ($E/p=2$ volt/cm-mm Hg). At this value of E/N the average electron energy is about 2 electron volts.[19] Therefore, we can say that our experiments are consistent with the assumption that the cross section for momentum transfer is constant for electron energies below about 2 ev. This result is in agreement with the data of Normand[20] and of Gould and Brown[21] which show that the cross section is essentially constant for electron energies up to about 2 ev. The magnitude of the elastic cross section required to fit the theory to the experiment is about 10% larger than that obtained by Phelps, Fundingsland, and Brown[22] and by Gould and Brown and 10% less than the value obtained by Anderson and Goldstein.[23] Note that for $E/N<10^{-19}$ volt/cm^2 the electron mobility is independent of E/N as expected for electrons in thermal equilibrium with the gas. Thus, we have been able to study the properties of electrons which are in thermal equilibrium with the gas without having to apply the large amounts of power required to break down the gas using the microwave techniques.

Above an E/N of 5×10^{-17} volt/cm^2 theoretical calculations of Barbiere[24] show that the electron energy distribution is modified by the effect of inelastic collisions such as to reduce the average electron energy and increase the electron mobility. However, at this value of E/p the average electron energy[19,24] is about 2.5 electron volts and the cross section for momentum transfer collisions for the higher energy (>4 ev) electrons varies approximately as the reciprocal of the electron velocity. According to theory[25] when the collision cross section varies inversely as the electron velocity, the electron mobility is constant and independent of the form of the electron energy distribution. Thus, it is not obvious whether the leveling off of the electron mobility at high E/N is due to the effects of inelastic collisions or simply due to the fact that the mobility is nearly independent of the distribution function. The arrow shows the electron mobility expected if the electron collision frequency per atom, i.e., the product of the electron velocity and momentum transfer collision cross section, were 7.2×10^{-8} cm^3/sec as given by Reder and Brown[26] as an average value for their electron energy distribution function calculations. We see that the experimental mobility values are gradually increasing in this region. This suggests that the cross section for momentum transfer collisions is decreasing more rapidly than the reciprocal of the electron velocity. Such a variation with electron velocity is observed experimentally in beam experiments.[20]

V. ACKNOWLEDGMENTS

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[20] C. E. Normand, Phys. Rev. **35**, 1217 (1930). This data must be converted into momentum transfer cross sections using the angular scattering data obtained by C. Ramsaur and R. Kollath Ann. Physik **12**, 529 (1932). See, for example, D. Barbiere, Phys. Rev. **84**, 653 (1951).
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[25] W. P. Allis, *Handbuch der Physik* (Springer-Verlag, Berlin, 1956), Vol. 21.
[26] F. H. Reder and S. C. Brown, Phys. Rev. **95**, 885 (1954).